Towards an accurate and efficient description of cosmic Large-Scale Structure

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more to come

Understanding cosmological observations, Benasque 2017

Observer's dream:

SDSS

DARK ENERGY SURVEY

2MASS
2 MICRON ALL SKY SURVEY

PAU

LSST

Euclid consortium

SKA
SQUARE KILOMETRE ARRAY

DARK ENERGY SPECTROSCOPIC INSTRUMENT

Theorist's dream:

- primordial non-gaussianity

  ➡ interactions in the inflationary sector

- baryon acoustic oscillations

  ➡ dark energy equation of state

- evolution of perturbations

  ➡ neutrino mass

  properties of dark matter (e.g. fifth force, WDM) and dark energy (e.g. clustering)

Reality:

We have to understand dynamics of (Λ)CDM = dust

The fundamental description is known (?): collisionless particles interacting through gravity

Vlasov -- Poisson system for the distribution function $f(\mathbf{x},\mathbf{v},t)$

$$\frac{\partial f}{\partial t}+\mathbf{v}\cdot\frac{\partial f}{\partial \mathbf{x}}-\nabla\phi\cdot\frac{\partial f}{\partial \mathbf{v}}=0\ , \qquad \nabla^2\phi=4\pi G\int f\, d^3\mathbf{v}$$

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- numerical solution: N-body simulations

  \+ valid up to arbitrary k

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- analytical perturbative methods at $k \lesssim 0.3 \ \mathrm{h}^{-1}\mathrm{Mpc}$

  – are approximate

  + theoretical control of physical processes, flexibility

Simplifying the problem

Newtonian approximation at $l \ll H^{-1} \sim 10^4\ \mathrm{Mpc}$

DM particles move by $uH^{-1} \sim 10\ \mathrm{Mpc}$

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vorticity decays at linear level ➡ work with $\theta \propto \nabla \cdot \mathbf{u}$

Two perturbative schemes

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$$\delta_{\rho,\ \mathrm{NL}}$$

$$\uparrow$$

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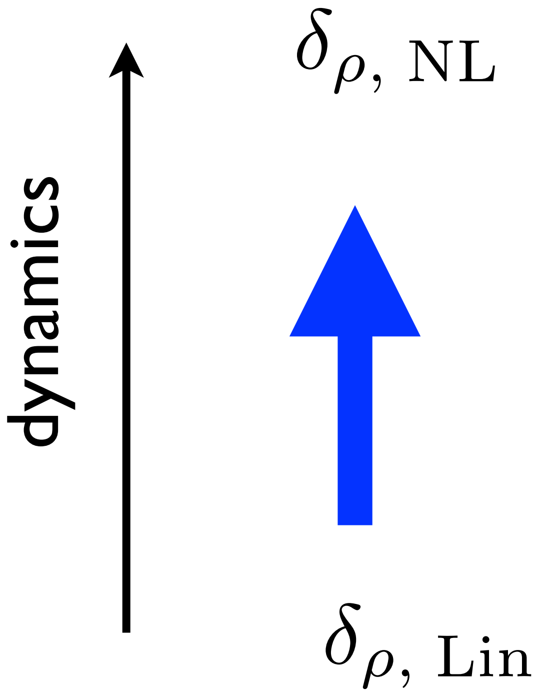

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Standard Perturbation Theory

$$
\begin{tikzcd}
\delta_{\rho,\ \mathrm{NL}} \arrow[r] & \mathcal{P}_{NL}[\delta_\rho] \\
\delta_{\rho,\ \mathrm{Lin}} \arrow[u, "\text{dynamics}"] &
\end{tikzcd}
$$

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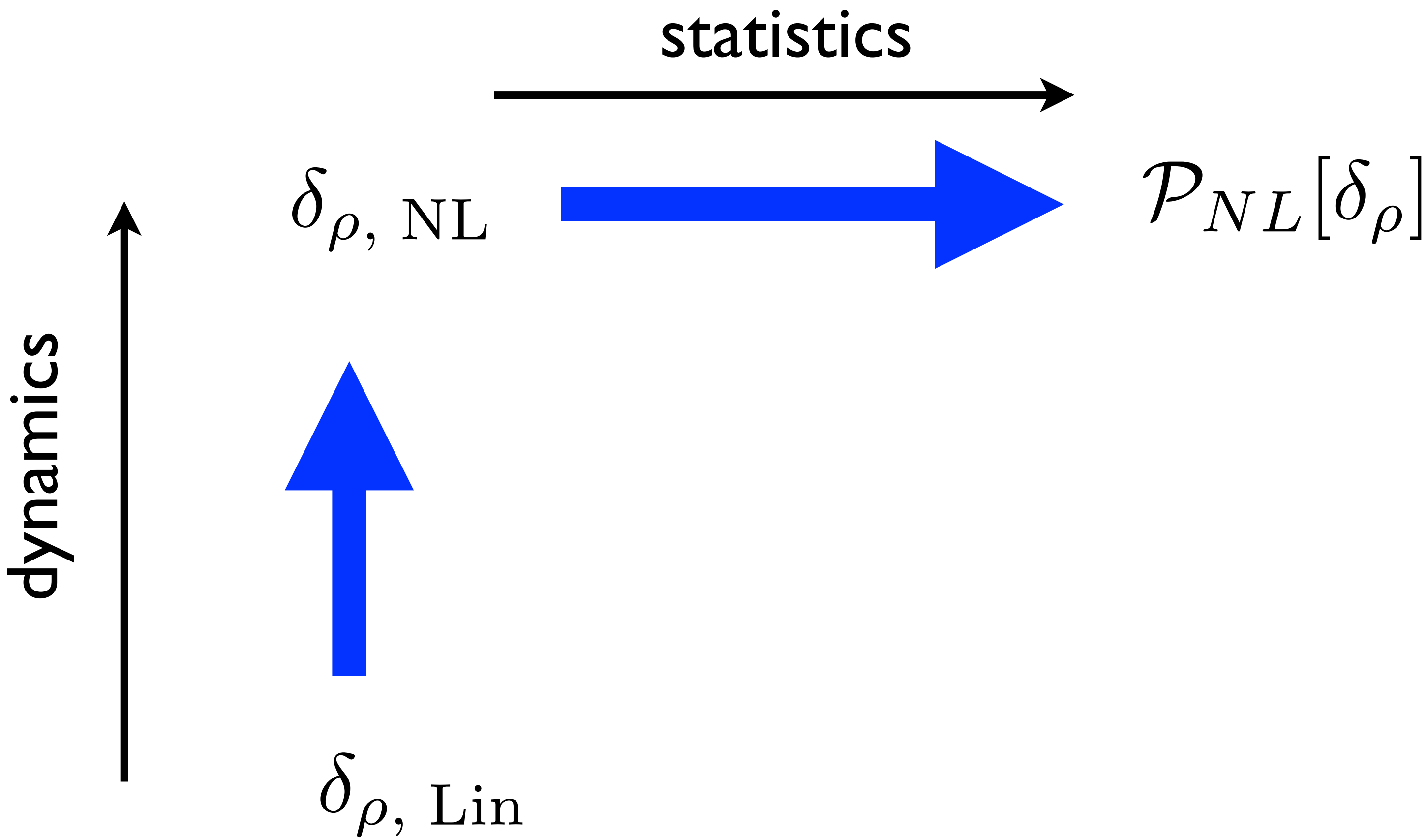

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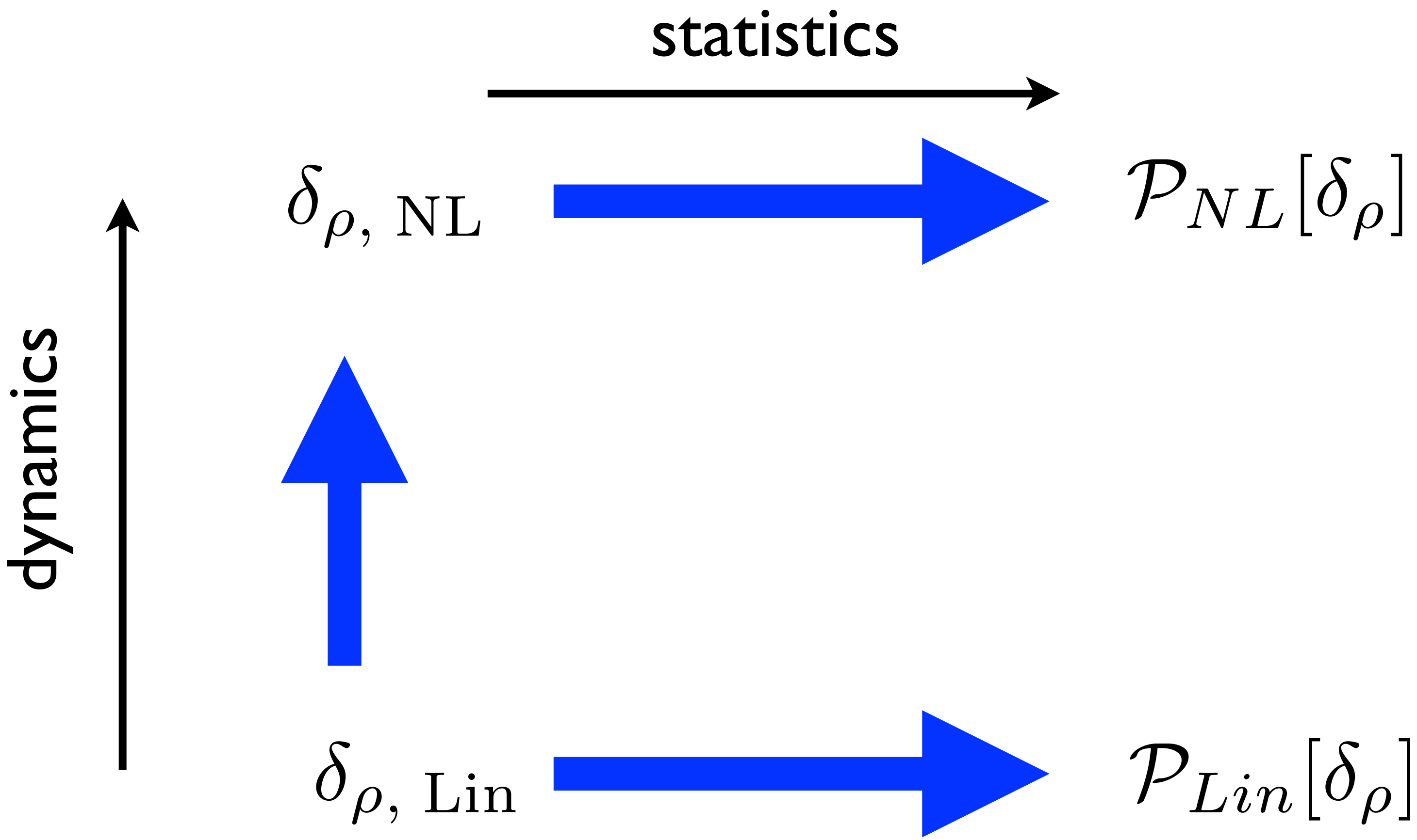

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statistics →

dynamics ↑

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Time-Sliced Perturbation Theory

Valageas (2004)

Blas, Garny, Ivanov, S.S. (2015,2016)

TSPT: time-sliced perturbation theory

Main ideas: Focus on equal-time correlators

Instead of evolving fields, evolve the probability distribution function

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SPT: $\displaystyle\int d\psi_0 \, e^{-\Gamma_0[\psi_0]}\psi(\tau;\psi_0)^2$

$$\Gamma_0[\psi_0] = \frac{\psi_0^2}{2P}$$

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TSPT: $$\int d\psi \, e^{-\Gamma[\psi;\tau]}\psi^2 \qquad \Gamma[\psi;\tau] = \sum_n \frac{\Gamma_n(\tau)}{n!}\,\psi^n$$

Two integrals must coincide

equation for the "vertices"

$$\frac{d}{d\tau}\left(d\psi e^{-\Gamma[\psi;\tau]}\right) = 0$$

$$\dot{\Gamma}_n = -n\Omega\Gamma_n \underbrace{- \sum_{m=2}^{n} C_n^m A_m \Gamma_{n-m+1} + A_{n+1}}_{\text{contains only } \Gamma_{n'} \text{ with } n' < n}$$

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The same logic for fields in space with the substitution:
integral $\Longrightarrow$ path integral

Generating functional for cosmological correlators

$$Z[J,\tau] = \int [\mathcal{D}\delta_\rho] \exp\left\{ -\Gamma[\delta_\rho;\tau] + \int J\delta_\rho \right\}$$

$$\Gamma = \frac{1}{2}\int \frac{|\delta_\rho(k)|^2}{\bar{P}_L(k)} + \sum_{n=3}^{\infty} \frac{1}{n!}\int \Gamma_n(\tau)\delta_\rho^n$$

Γ is an action of a (nonlocal) 3d Euclidean QFT;

τ --- an external parameter

Analogy with QFT cntd.

- For gaussian initial conditions the time dependence factorize

$$\Gamma = \frac{1}{D^2(\tau)}\bar{\Gamma}$$

effective coupling constant $g^2(\tau)$

NB. For primordial NG

$$\Gamma = \frac{1}{g^2}\bar{\Gamma} + \frac{1}{g^3}\hat{\Gamma}$$

$\sim f_{NL} g_0$

- Neat diagrammatic technique

$$\overset{k}{\rule{3cm}{2pt}} \quad = g^2 \bar{P}_L(k)$$

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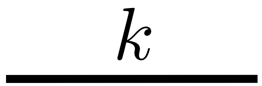

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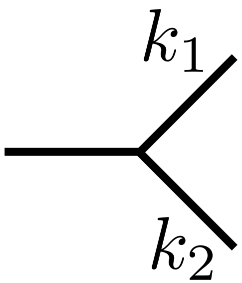
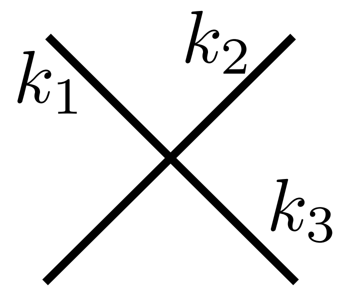


$$= \frac{1}{g^2} \bar{\Gamma}_3(k_1, k_2)$$



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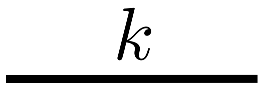
- Neat diagrammatic technique



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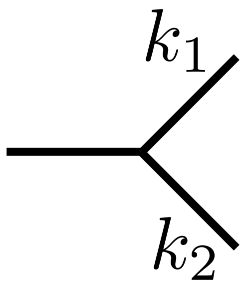

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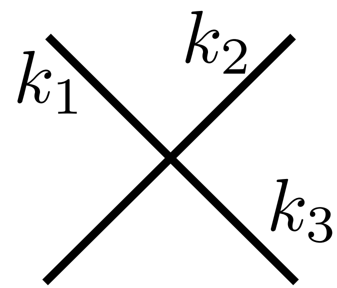

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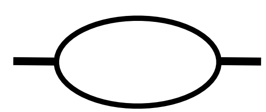
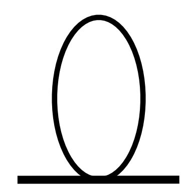
$\langle \delta_\rho \delta_\rho \rangle =$ +  +  + ...

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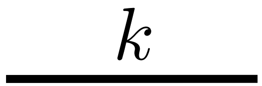

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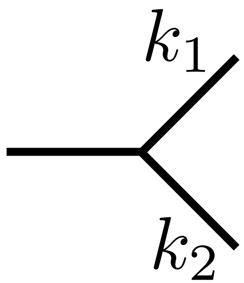

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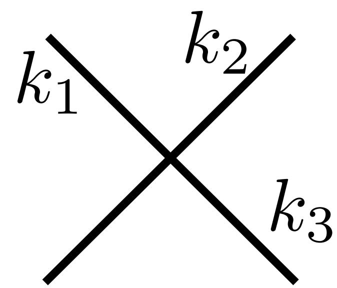

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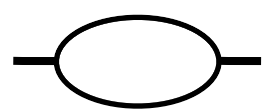
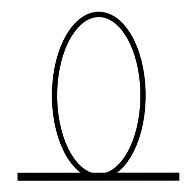
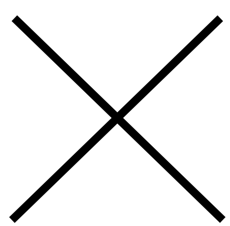
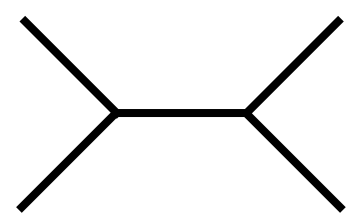
$\langle \delta_\rho \delta_\rho \rangle =$ +  +  + ...

$\langle \delta_\rho \delta_\rho \delta_\rho \delta_\rho \rangle =$  +  + ...

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Individual contributions diverge at small momenta if

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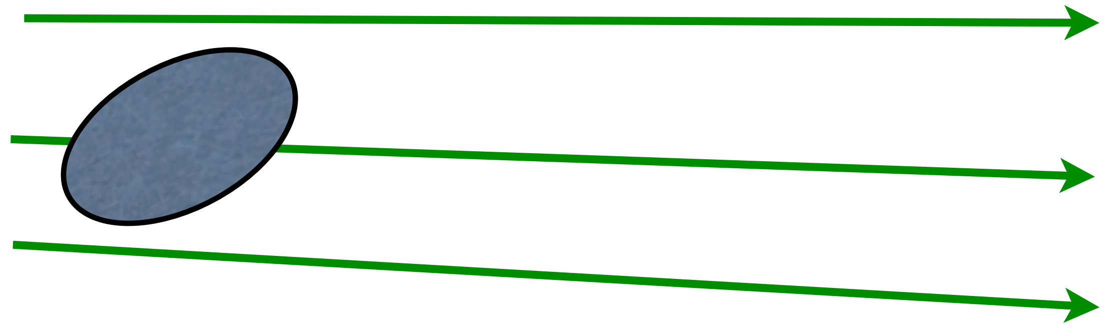

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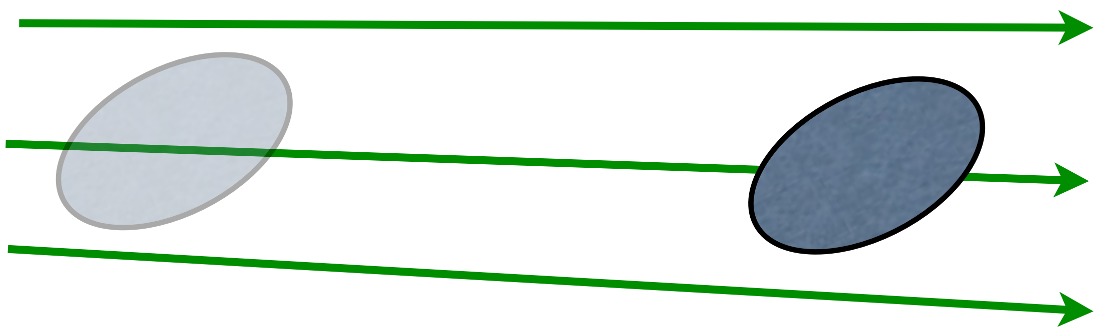

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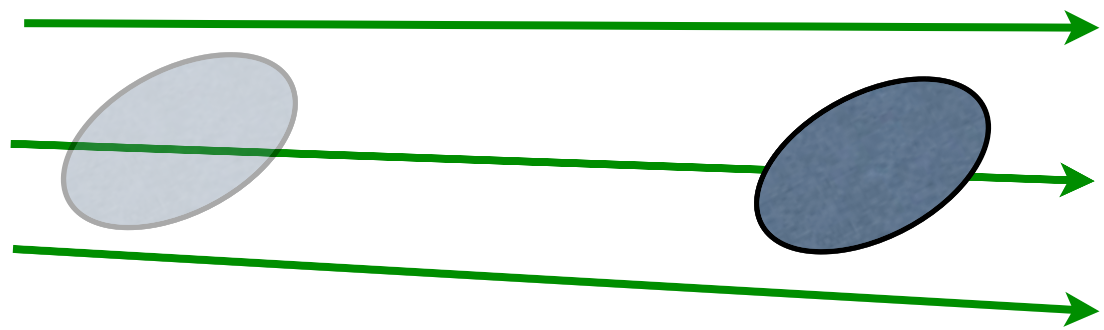

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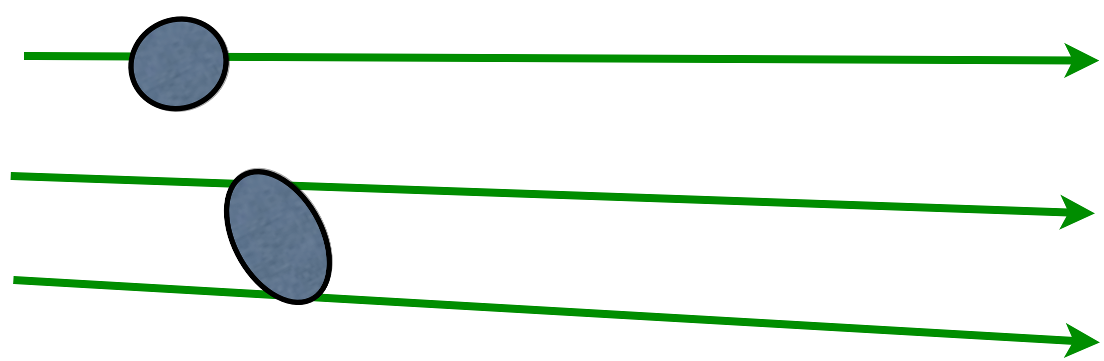

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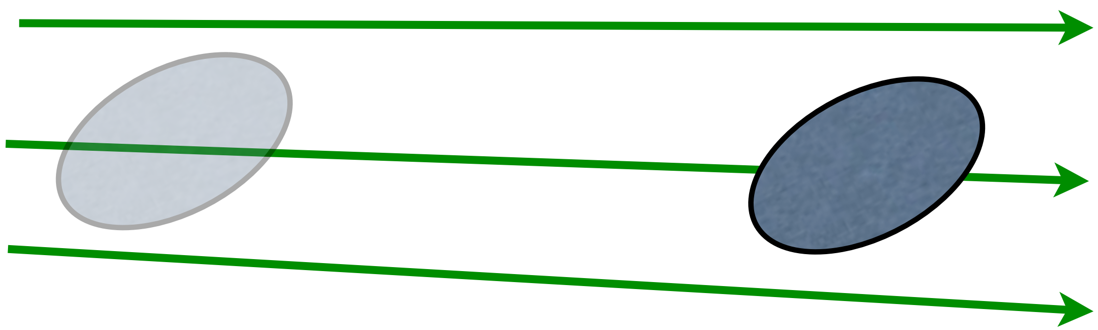

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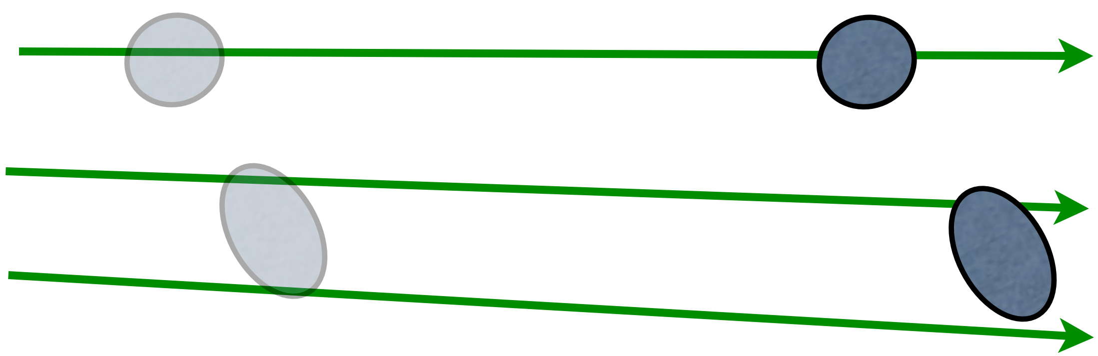

two overdensities will move (almost) identically, cancellation in equal-time correlators

IR safety of TSPT

TSPT deals directly with equal-time correlators

All Γ_n are finite for soft momenta

$$\lim_{\epsilon \to 0} \Gamma_n(k_1, \ldots, k_l, \epsilon q_1, \ldots, \epsilon q_{n-l}) < \infty$$

➡ no IR divergences in the individual loop diagrams

Related to the equivalence principle through Ward identities

Physical IR effect: smearing of the BAO feature in the correlation functions

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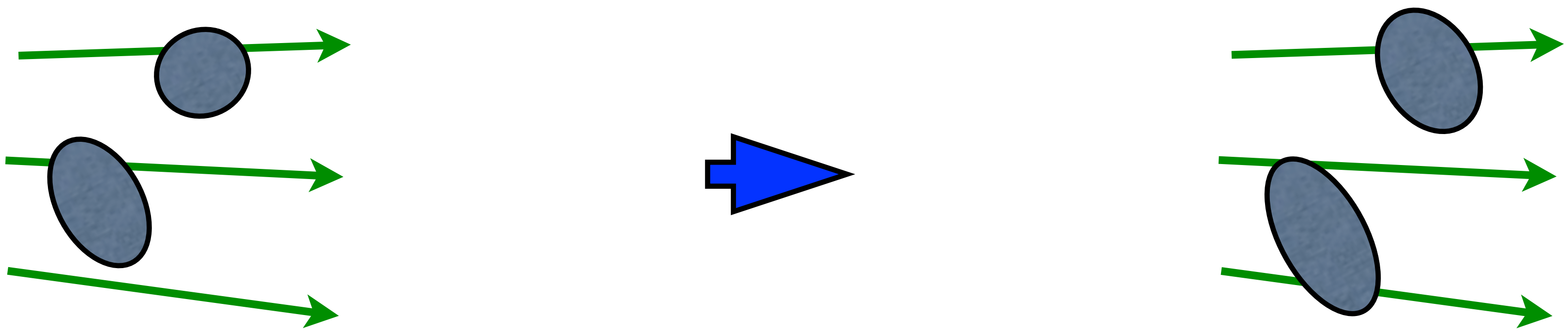

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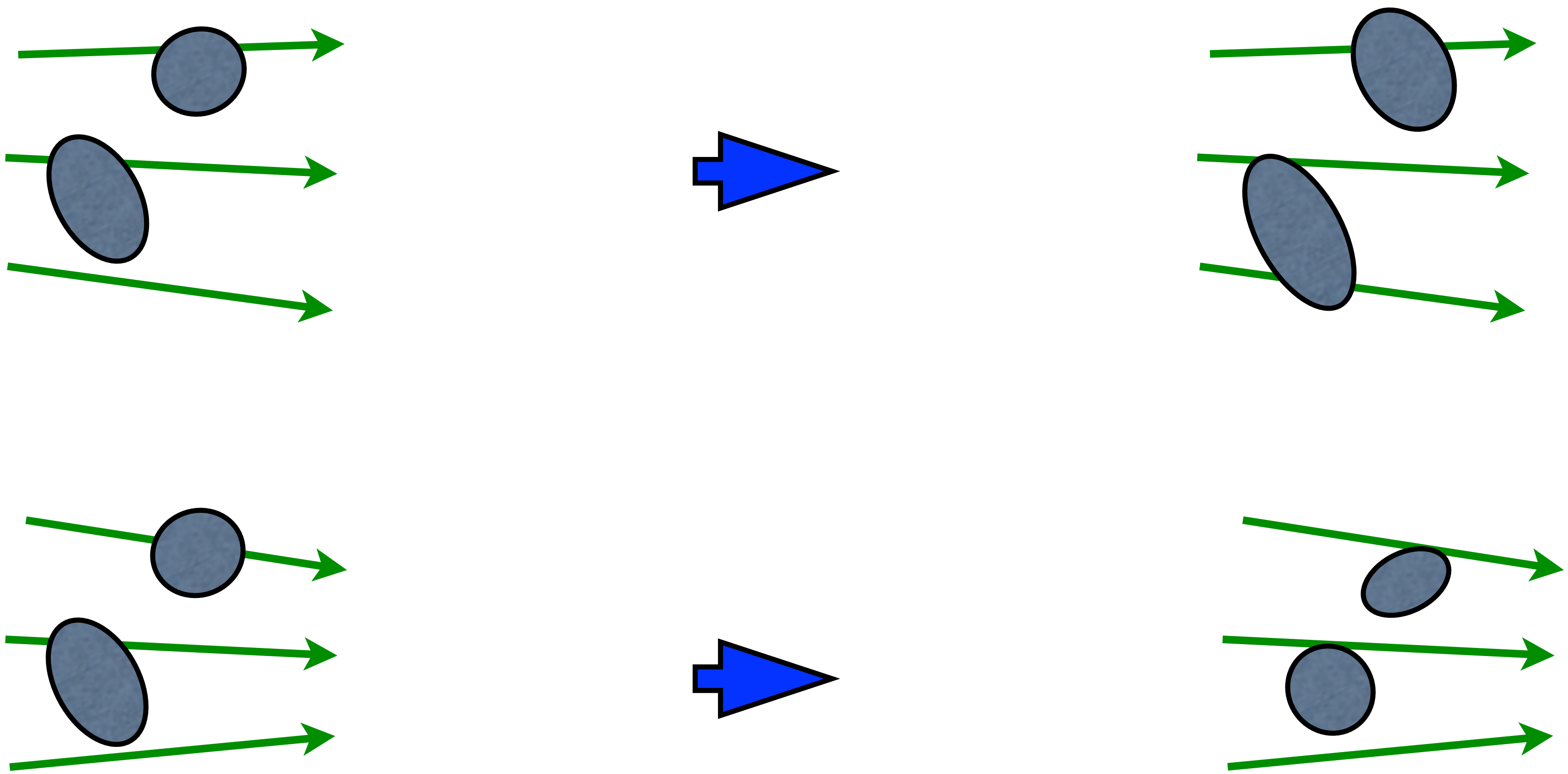
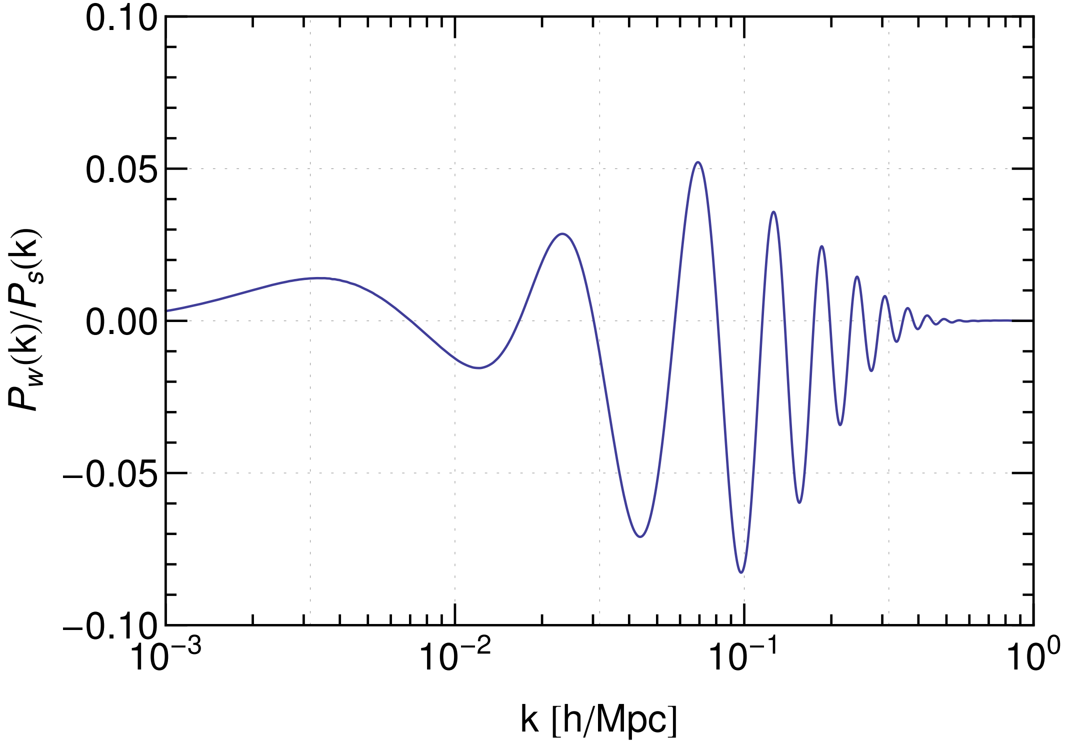


IR resummation

In TSPT large IR contributions can be systematically resummed

Step 1: smooth + wiggly decomposition

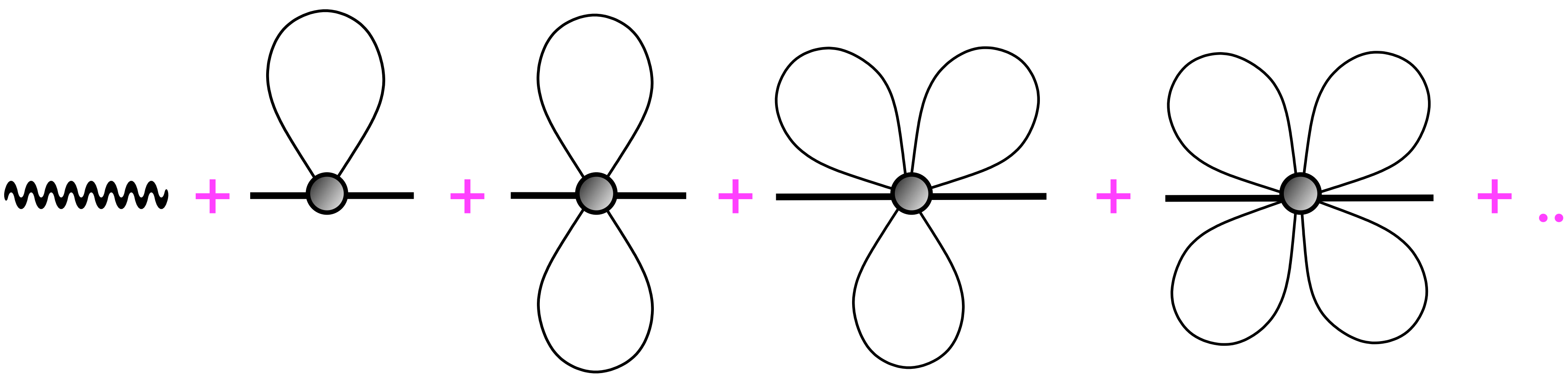
$$P_L(k) = P_{Ls}(k) + P_{Lw}(k) \quad \Longrightarrow \quad \Gamma(k) = \Gamma_s(k) + \Gamma_w(k)$$



Step II: identify leading diagrams correcting the wiggly part

➡ daisies

$$P_{Lw}^{\text{dressed}} =$$



$$g^2 \frac{k^2}{k_{osc}^2} \qquad \left(g^2 \frac{k^2}{k_{osc}^2}\right)^2 \qquad \left(g^2 \frac{k^2}{k_{osc}^2}\right)^3 \qquad \left(g^2 \frac{k^2}{k_{osc}^2}\right)^4$$

Step III: resummation

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$$\Sigma_L^2 = \frac{4\pi}{3}\int_0^{k_L} dq\, P_{Ls}(q)\big(1 - j_0(qr_s) + 2j_2(qr_s)\big)$$

BAO wavelength

Baldauf et al. (2015)

Blas, Garny, Ivanov, S.S. (2016)

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separation between hard and IR momenta

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NB. $\Sigma_L \sim \sigma_v$ for $k_L > 1/r_s$, but the integrand differs at small q

Step IV: add the smooth part

$$P_L^{\text{dressed}} = P_{Ls}(k) + \mathrm{e}^{-k^2\Sigma_L^2} P_{Lw}(k)$$

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Step IV: use $P_L^{\text{dressed}}(k)$ instead of $P_L(k)$ in all computations, i.e. higher correlation functions and hard loop corrections (with appropriate adjustment to avoid double-counting)

example:

$$B^{\text{IR}-\text{resummed}}(k_1, k_2, k_3) = B(k_1, k_2, k_3)\Big|_{P_L \mapsto P_L^{\text{dressed}}}$$

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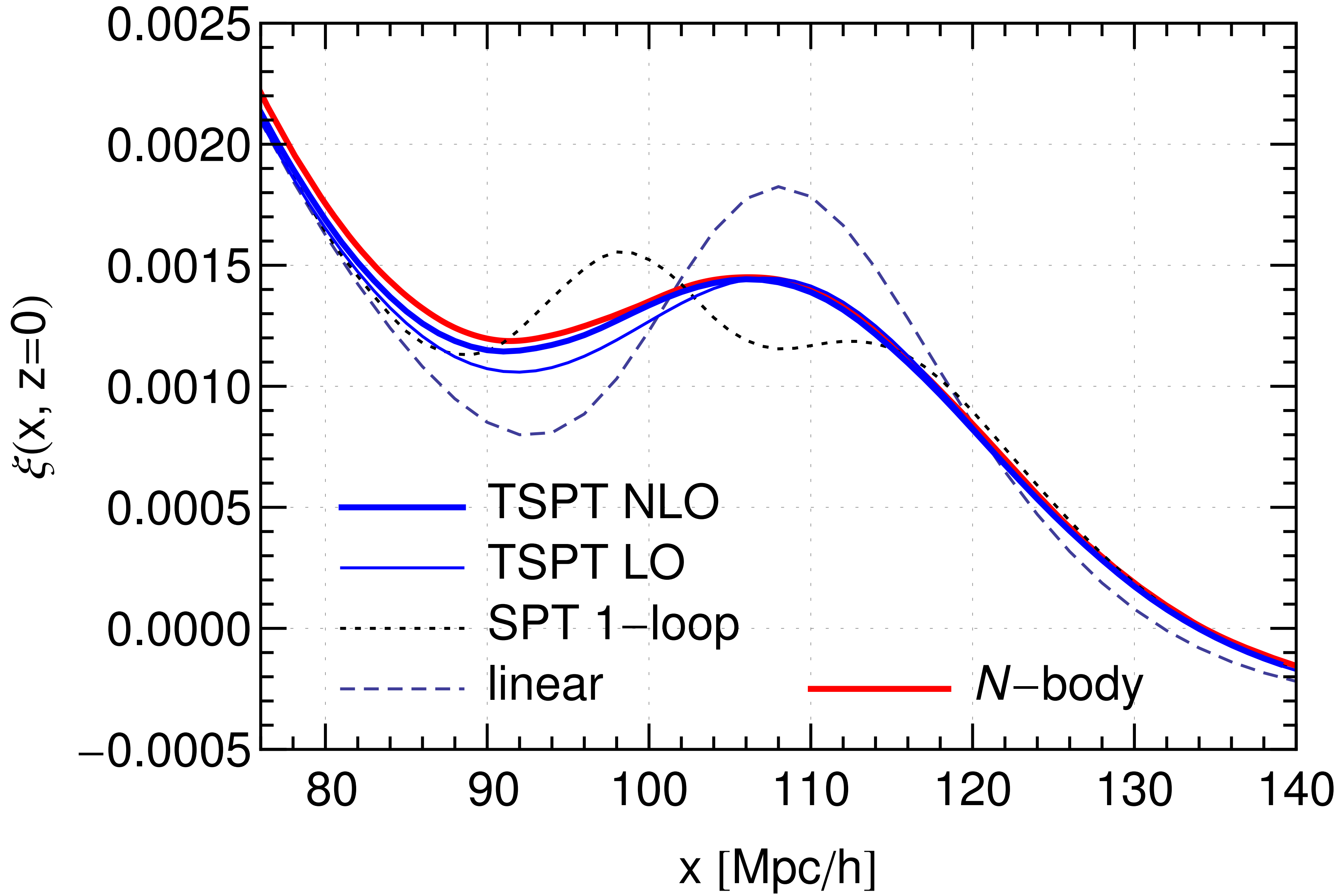
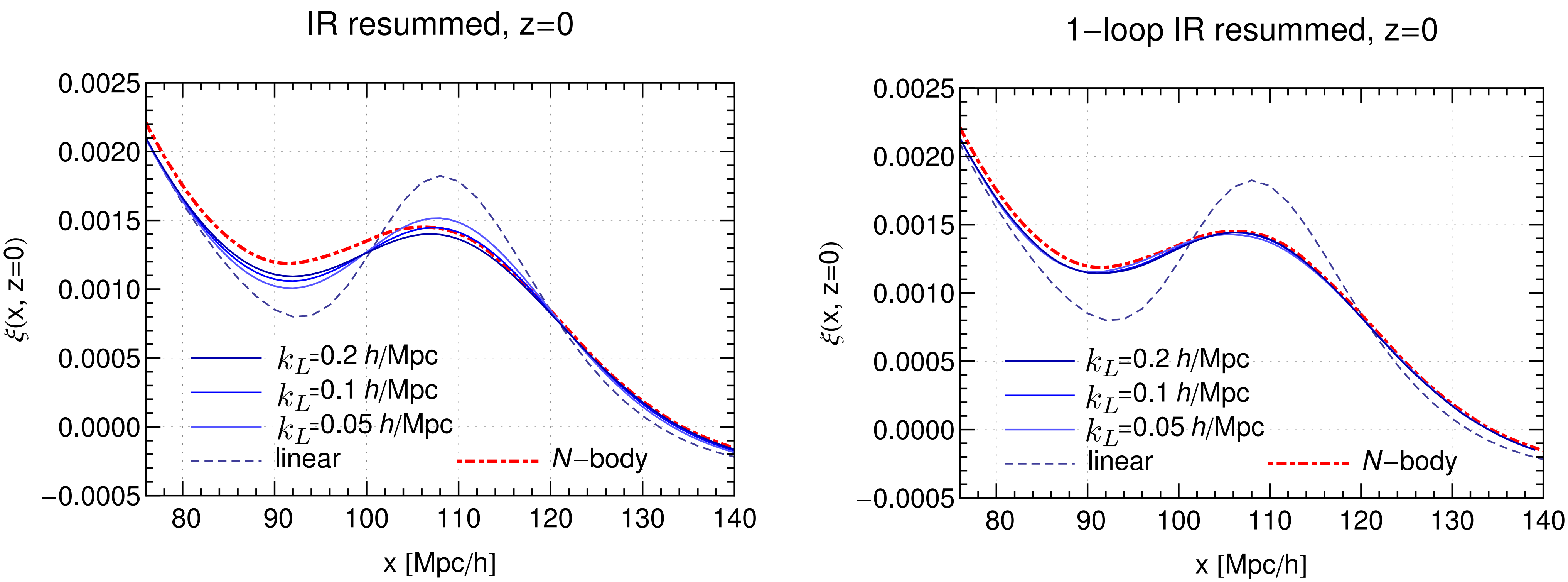
Further developments:

- NLO IR corrections (shift of BAO peak)

Comparison with N-body



Sensitivity to the IR separation scale: LO vs NLO

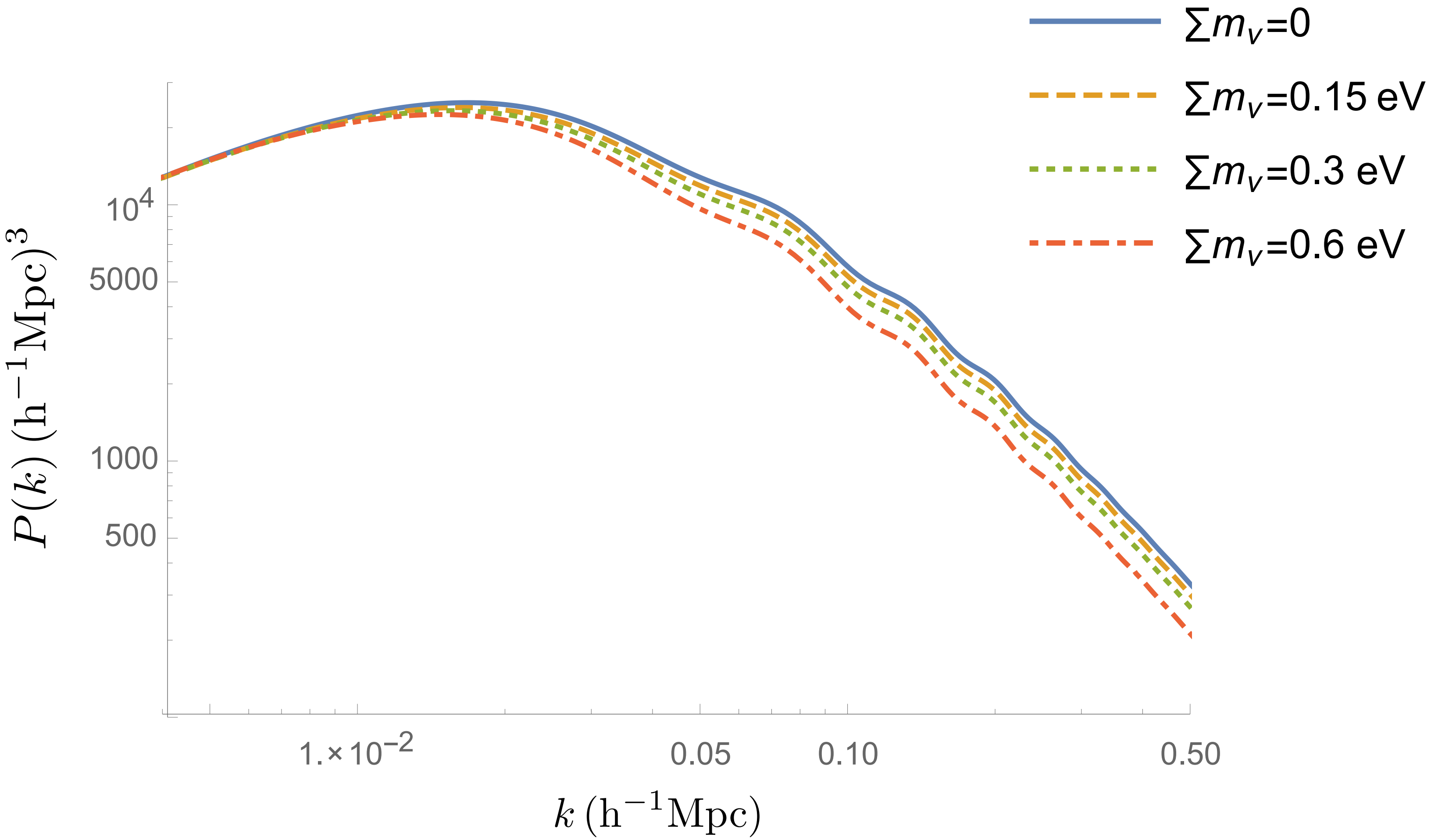


dependence on k_L decreases with the loop order

Residual dependence gives an estimate of the error ~ 2% in the BAO range

BAO and the neutrino mass

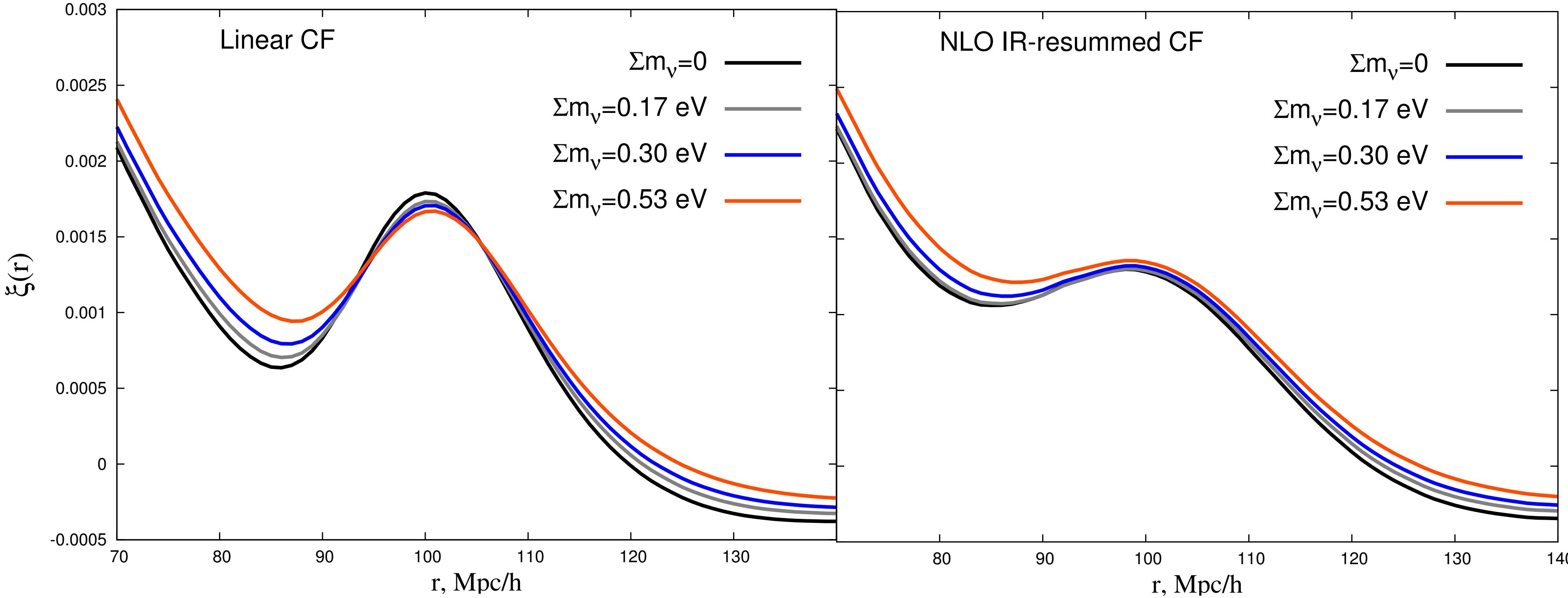
Effect on linear PS:



At $k > 0.05\ \mathrm{h}^{-1}\mathrm{Mpc}$ degenerate with the overall normalization

BAO and the neutrino mass

Non-linear effects remove the degeneracy



A probe of m_ν alternative to CMB and Lyα ?

BAO and the neutrino mass

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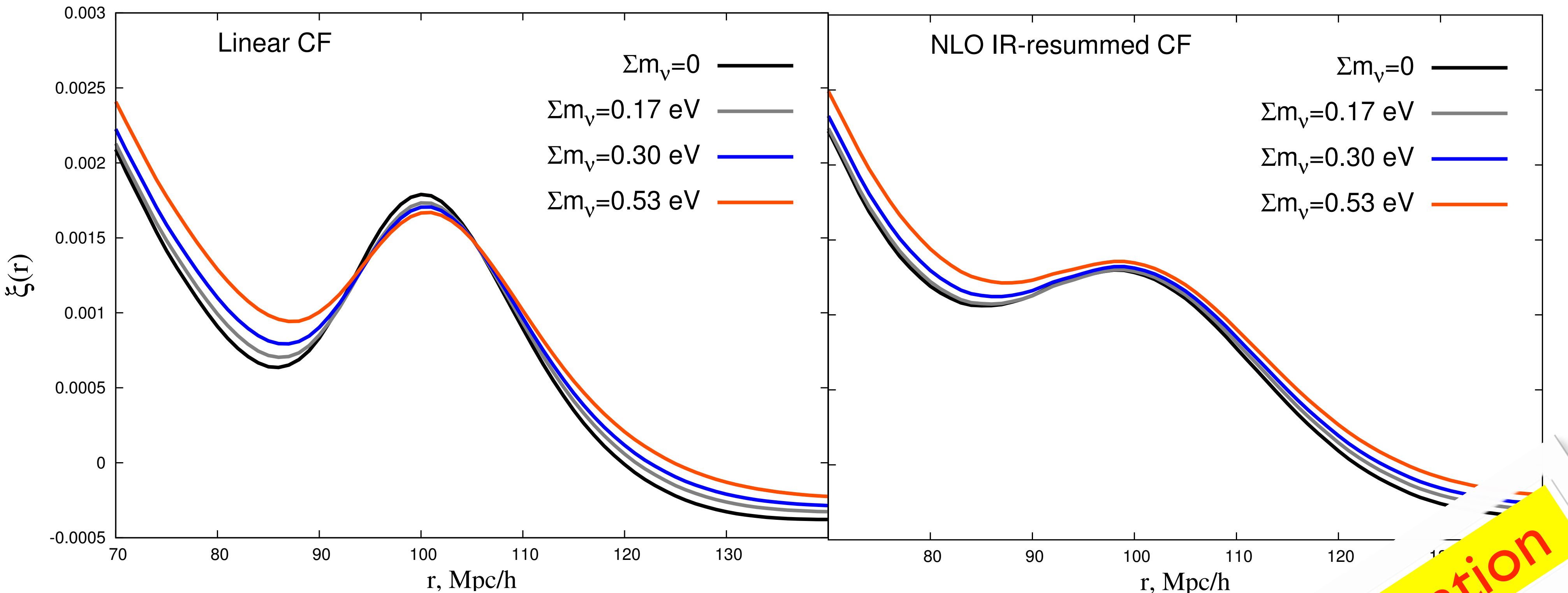

A probe of m_ν alternative to CMB and Lyα f

under investigation

Ultraviolet problem

Loop integrals diverge at large momenta if $P_L(k) \propto k^n, \quad n \geq -1$

Reason: they include very short modes that are not decribed by fluid, but virialize and decouple

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EFT of LSS

Baumann, Nicolis, Senatore, Zaldarriaga (2010)
Carrasco, Hertzberg, Senatore (2012)
Pajer, Zaldarriaga (2013)

+ follow up's

$$\dot{u}^i + \mathcal{H}u^i + u^j\nabla_j u^i + \nabla\phi = -\frac{1}{\rho}\partial_j\tau^{ij}$$

$$\tau^{ij}_{vis} + \tau^{ij}_{stoch}$$

$$\tau^{ij}_{vis} = -c_s^2\delta^{ij}\delta_\rho + \tilde{c}\delta^{ij}\Delta\delta_\rho + c_1\delta^{ij}(\Delta\phi)^2 + c_2\partial^i\partial^j\phi\Delta\phi + c_3\partial^i\partial_k\phi\partial^j\partial_k\phi + \ldots$$

coefficients c_i must be marginalized over or matched from N-body simulations

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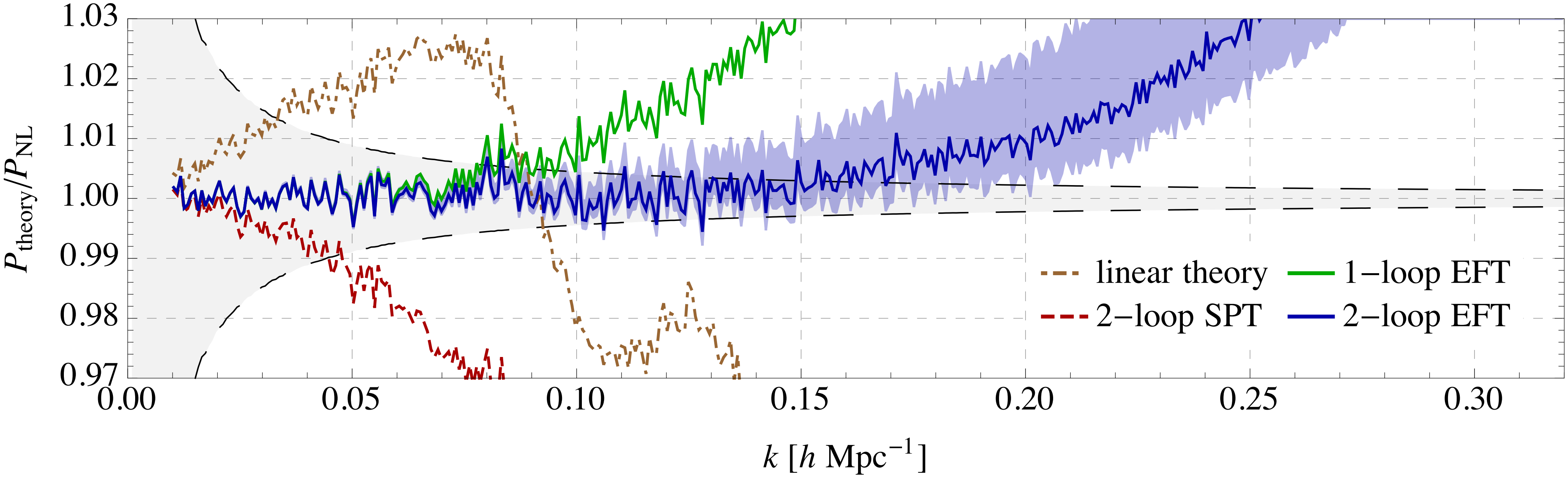
from Foreman, Perrier, Senatore (2015)



Issues:

- proliferation of free parameters at higher orders

- coefficients of the counterterms must have non-local time-dependence for consistency of the perturbative assumption

Abolhasani, Mirbabayi, Pajer (2015)

$$c_s^2 \delta_\rho \mapsto \int dt'\, c_s^2(t,t')\, \delta_\rho(t')$$, etc.

- treatment of stochastic terms is complicated

UV renormalization in TSPT

Introduce a cutoff:

$$P(k) \mapsto P^{\Lambda}(k) = \begin{cases} P(k), & k < \Lambda \\ 0, & k > \Lambda \end{cases}$$

$$\Gamma_n \mapsto \Gamma_n^{\Lambda}$$

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Wilsonian renormalization group:

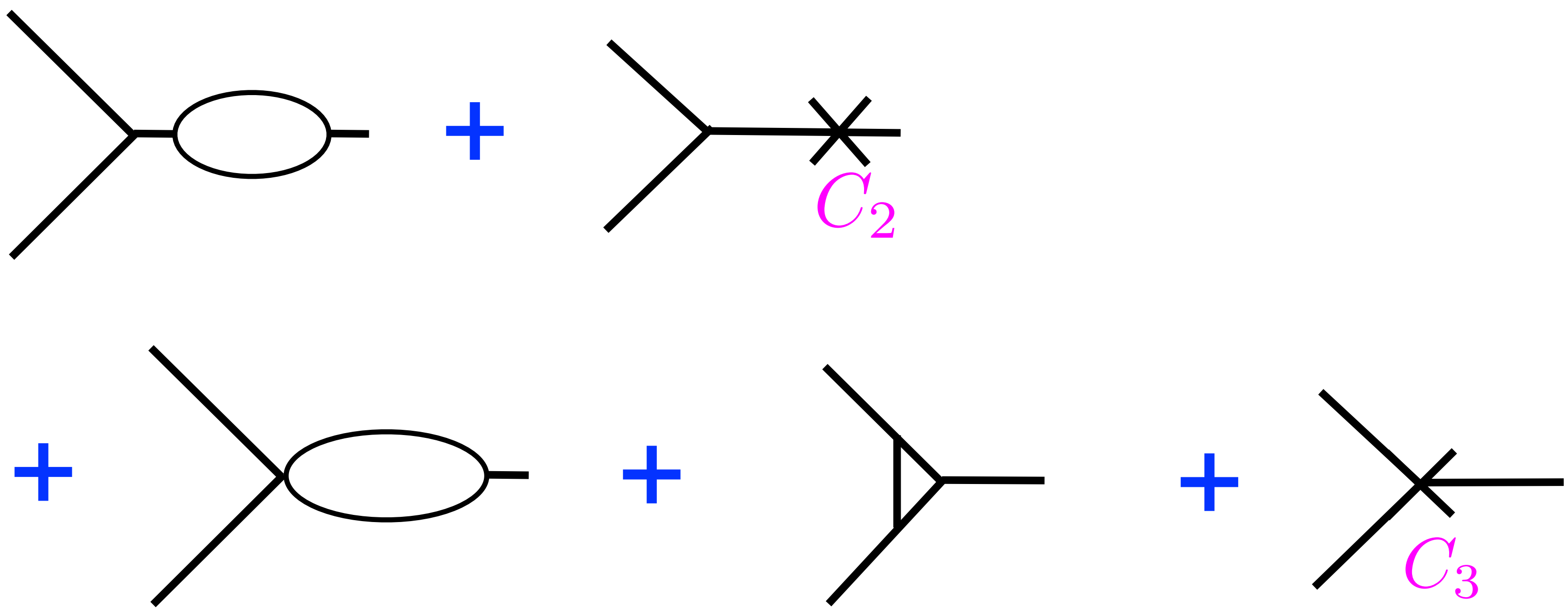
$$\frac{d\Gamma_n^{\Lambda}}{d\Lambda} = \mathcal{F}_n[P^{\Lambda}, \Gamma^{\Lambda}]$$

Boundary conditions = counterterms C_n encapsulating the effects of short modes

UV renormalization in TSPT

+ $C_n(\{k\},\tau)$ local in time by construction

+ clear separation between PR and PI counterterms



+ stochastic contributions are at the same footing as viscous ones

Structure of counterterms

a) Use τ^{ij}_{EFT} ➡ $C_2(c_s^2)\,,\quad C_3(c_s^2, c_1, c_2, c_3)$

b) At $\Lambda \gg k$ the RG eqs. factorize: $\dfrac{d\Gamma_n^\Lambda}{d\Lambda} = \mathcal{F}_n(\{k\})\beta_n(\Lambda)$

$O\big((k/k_{NL})^2\big)$

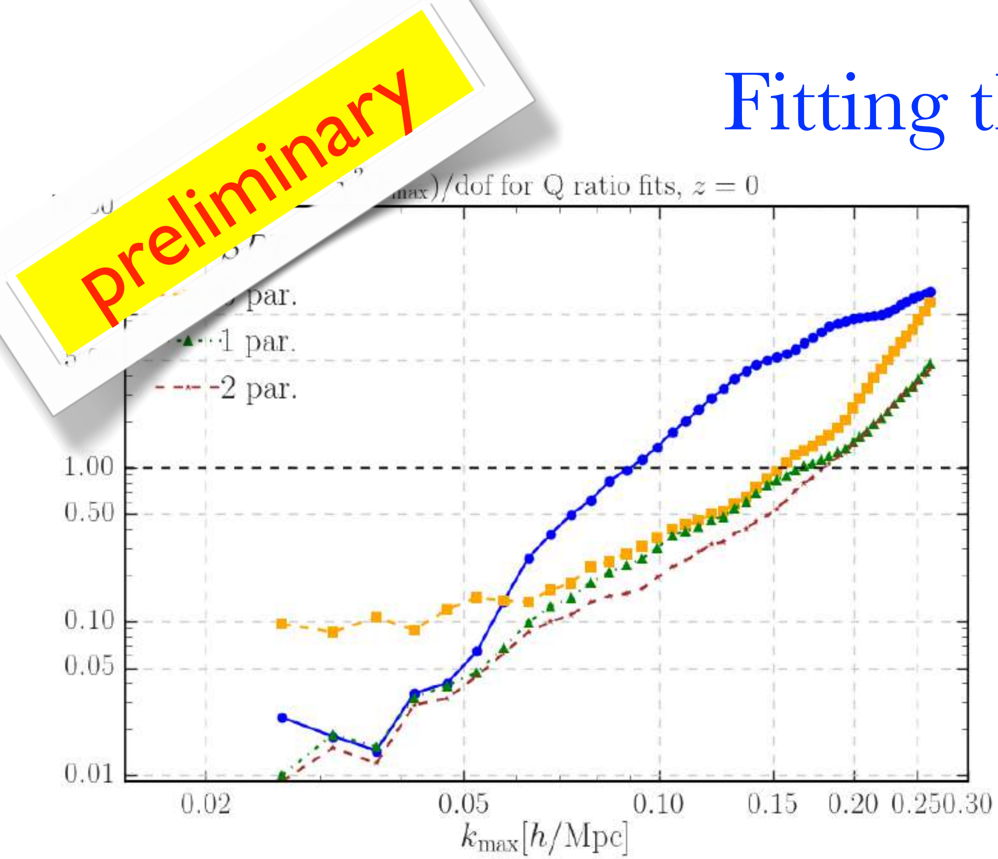
➡ It is mathematically consistent to choose

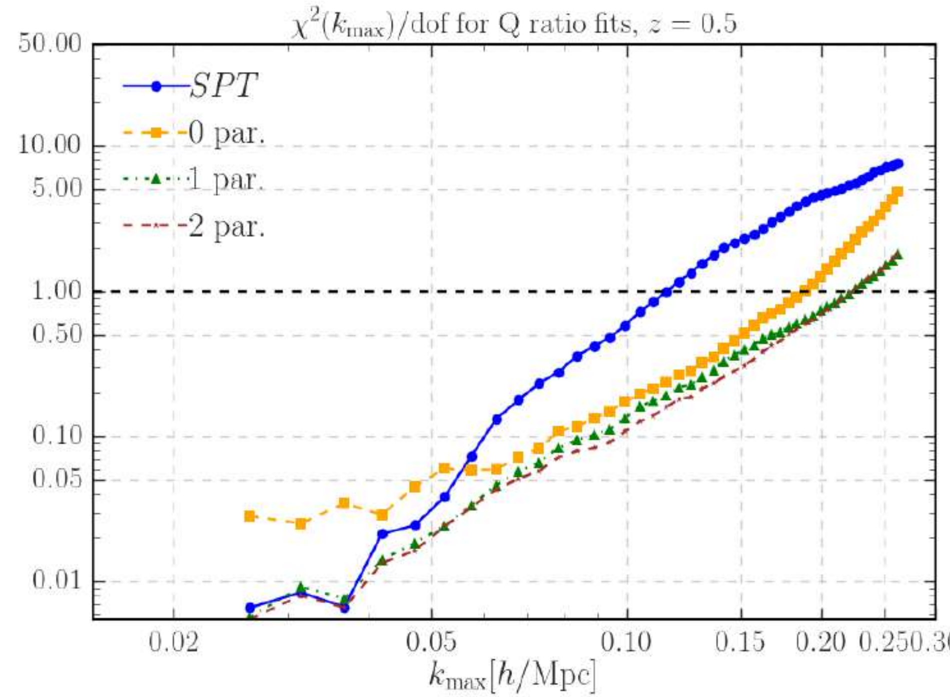
$C_n = \mathcal{F}_n(\{k\})\, C_n^{(0)}(\tau)$

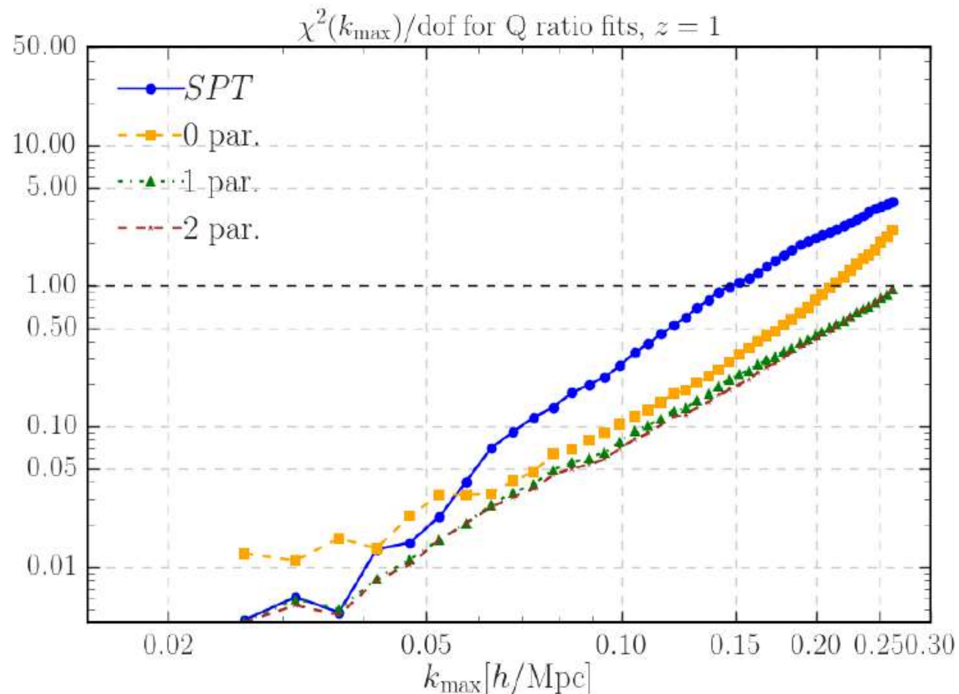
- sufficient to cancel the UV divergences
- stable under RG (absorbs the cutoff dependence)

Reduces the number of free parameters by a factor of 3
for 1-loop bispectrum, more for higher orders

Fitting the bispectrum







another reason for reduction: all EFT contributions into

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cf. Bertolini, Solon (2016)

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CiC statistics

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$$\delta_W = \frac{1}{\rho_0} \int d\mathbf{x}\, W(\mathbf{x})(\rho(\mathbf{x}) - \rho_0)$$

$$\frac{3}{4\pi R^3}\theta(R - |\mathbf{x}|)$$

$$P(\delta_W)$$

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$$\boxed{P(\delta_W)}$$

CiC in TSPT

many people, see C. Uhleman's talk

$$P(\delta_W) = \int [D\delta(\mathbf{x})] e^{-\Gamma[\delta(\mathbf{x})]/g^2} \; \delta^{(1)}\left[\int d\mathbf{x} W(\mathbf{x})\delta(\mathbf{x}) - \delta_W \right]$$

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Summary and Outlook

- perturbative methods are essential to fully exploit the potential of LSS surveys (m_ν, f_{NL} , properties of DM and DE)

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- classification of UV counterterms

- inclusion of "astrophysical" effects (biases, redshift space distortion, baryons)

- comparison with the data, searches for new physics