



University of  
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Entanglement in  
Strongly Correlated Systems

Benasque  
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# Higher-order Topological Insulators

A paradigm for topological states of matter

$$\partial(\partial M) = \emptyset$$

(the boundary of a boundary is empty)

... works for sufficiently smooth manifolds only.



**Crystals have no smooth surface!**

# Outline

- 1. First-order topological insulators**
- 2. Higher-order topological insulators**
- 3. Models for second-order 3D TIs**



Titus Neupert  
(U Zurich)



Ashley Cook  
(U Zurich)



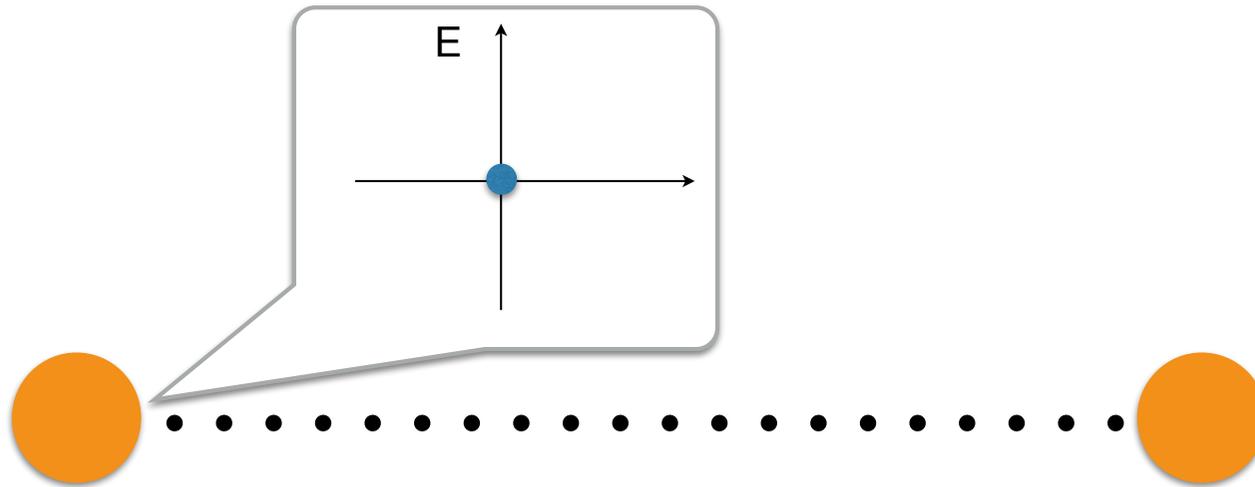
B. Andrei Bernevig  
(U Princeton)



Maia G. Vergniory  
(Basque U)

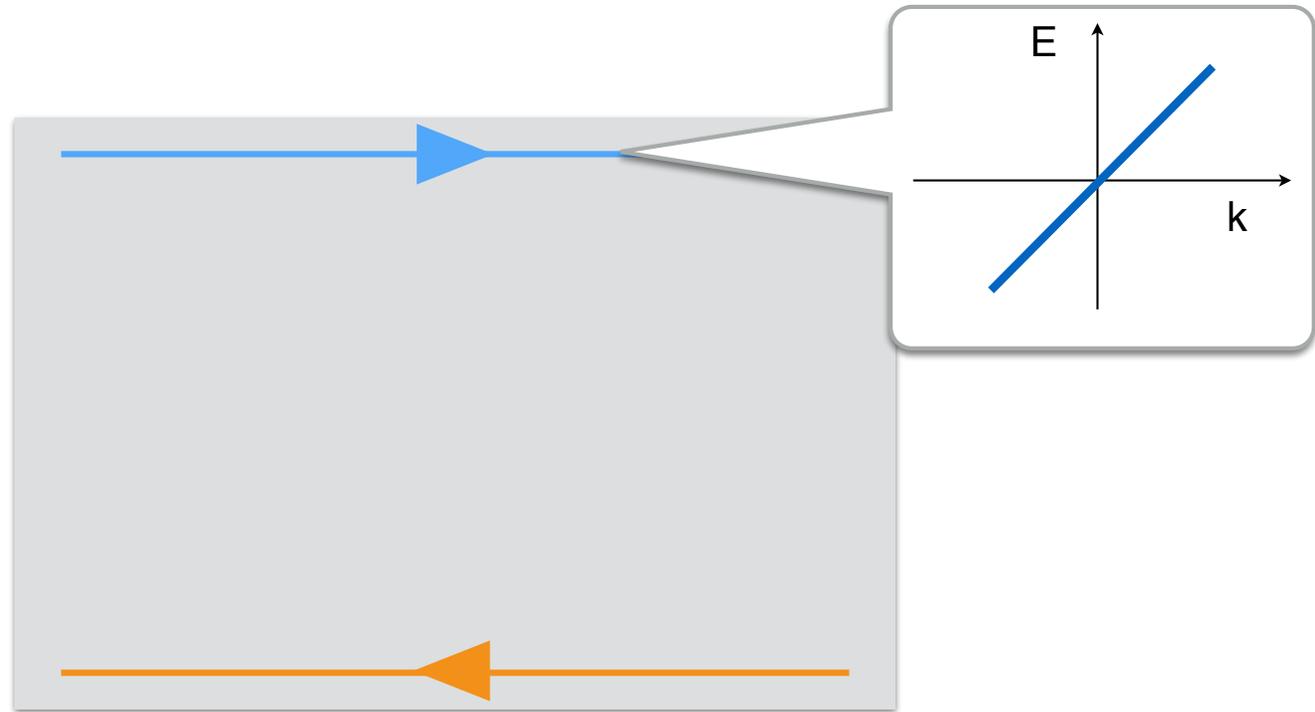
# 1. First-order Topological Insulators

# 1D: Su-Schrieffer-Heeger Chain



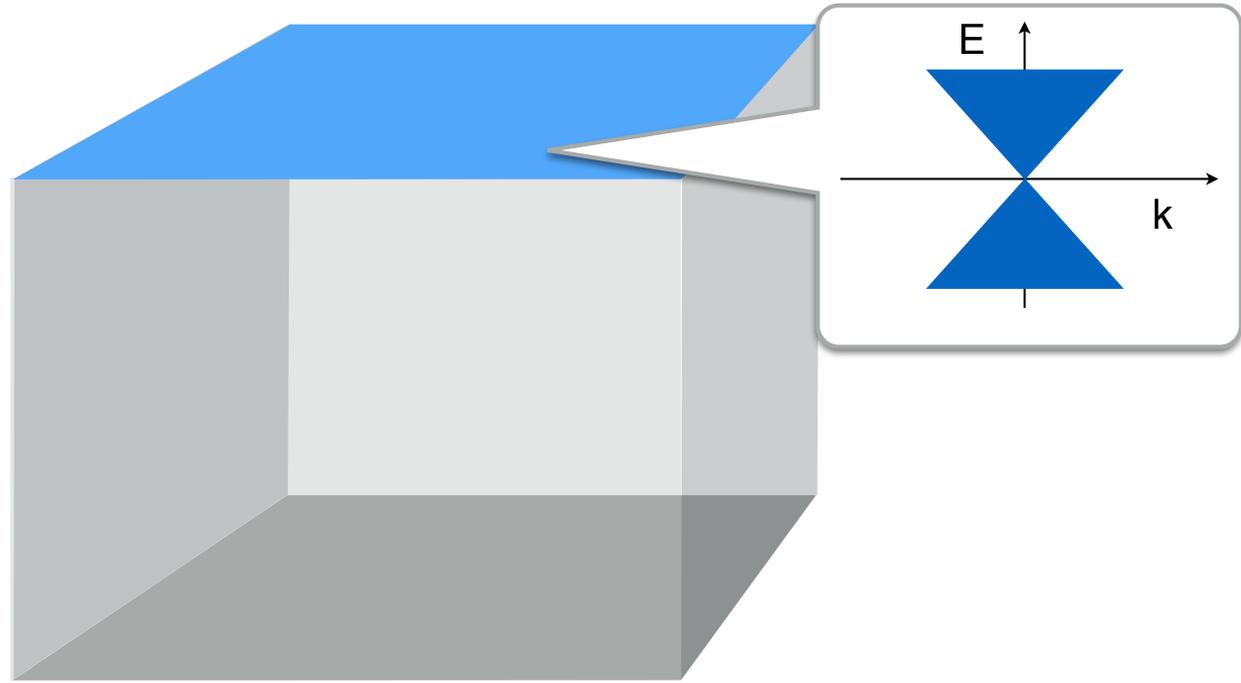
Protected by chiral symmetry,  
 $\mathbb{Z}$  classification

# 2D: Quantum Hall Effect



Protected without symmetry,  
 $\mathbb{Z}$  classification

# 3D: Topological Insulator (TI)



Protected by time-reversal symmetry,  
 $\mathbb{Z}_2$  classification

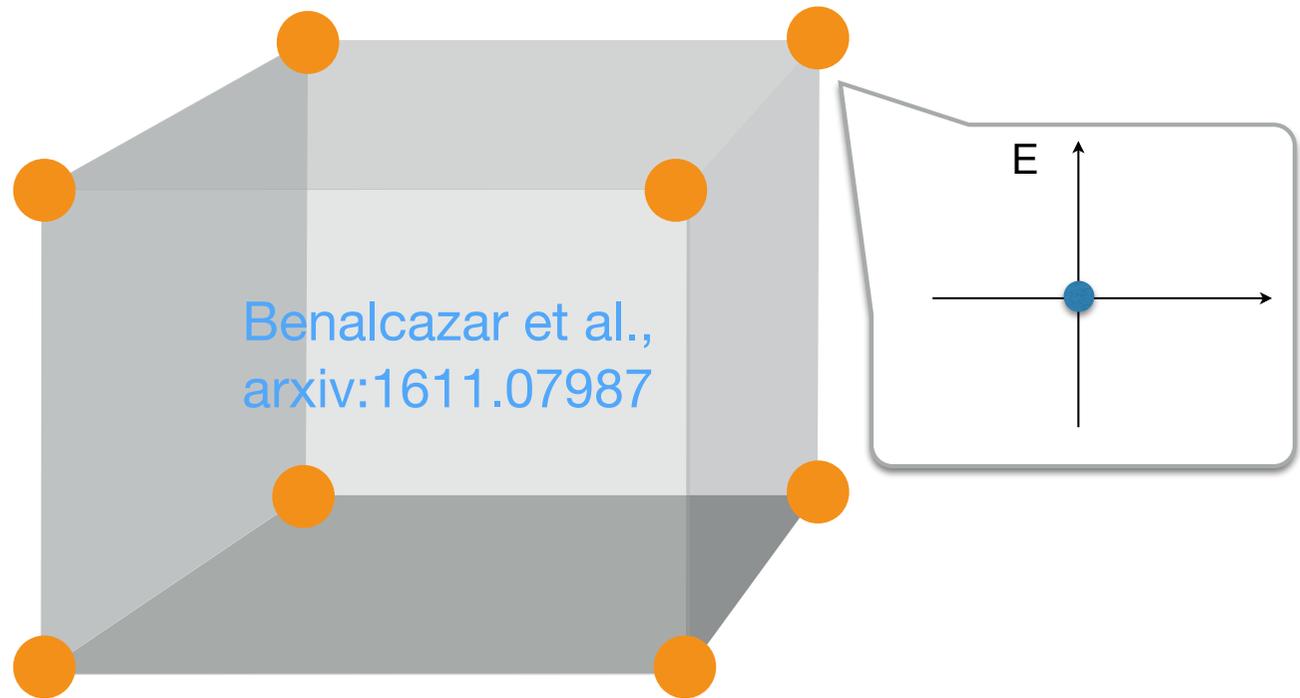
## **2. Higher-order Topological Insulators**

# Second-order 2D TI



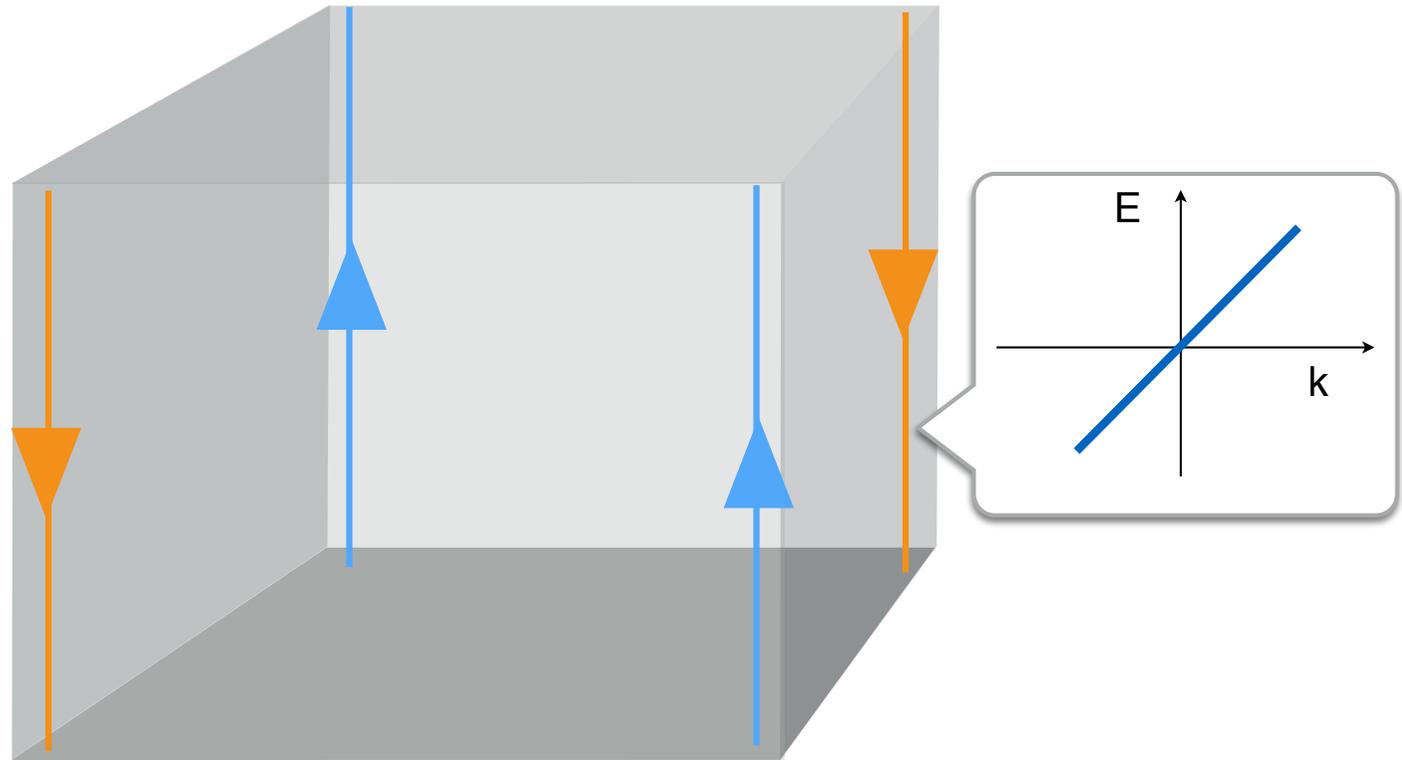
Protected by mirror symmetries,  
 $\mathbb{Z}_2 \times \mathbb{Z}_2$  classification

# Third-order 3D TI



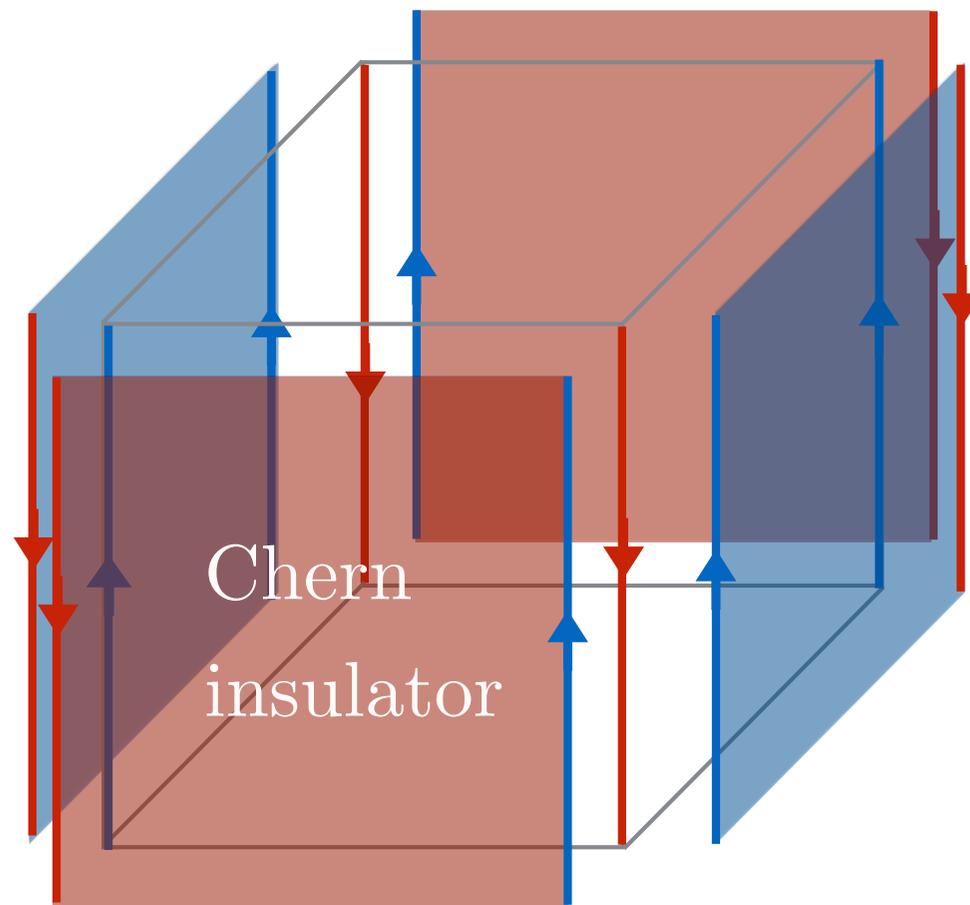
Protected by mirror symmetries,  
 $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$  classification

# Second-order 3D TI: TRB case

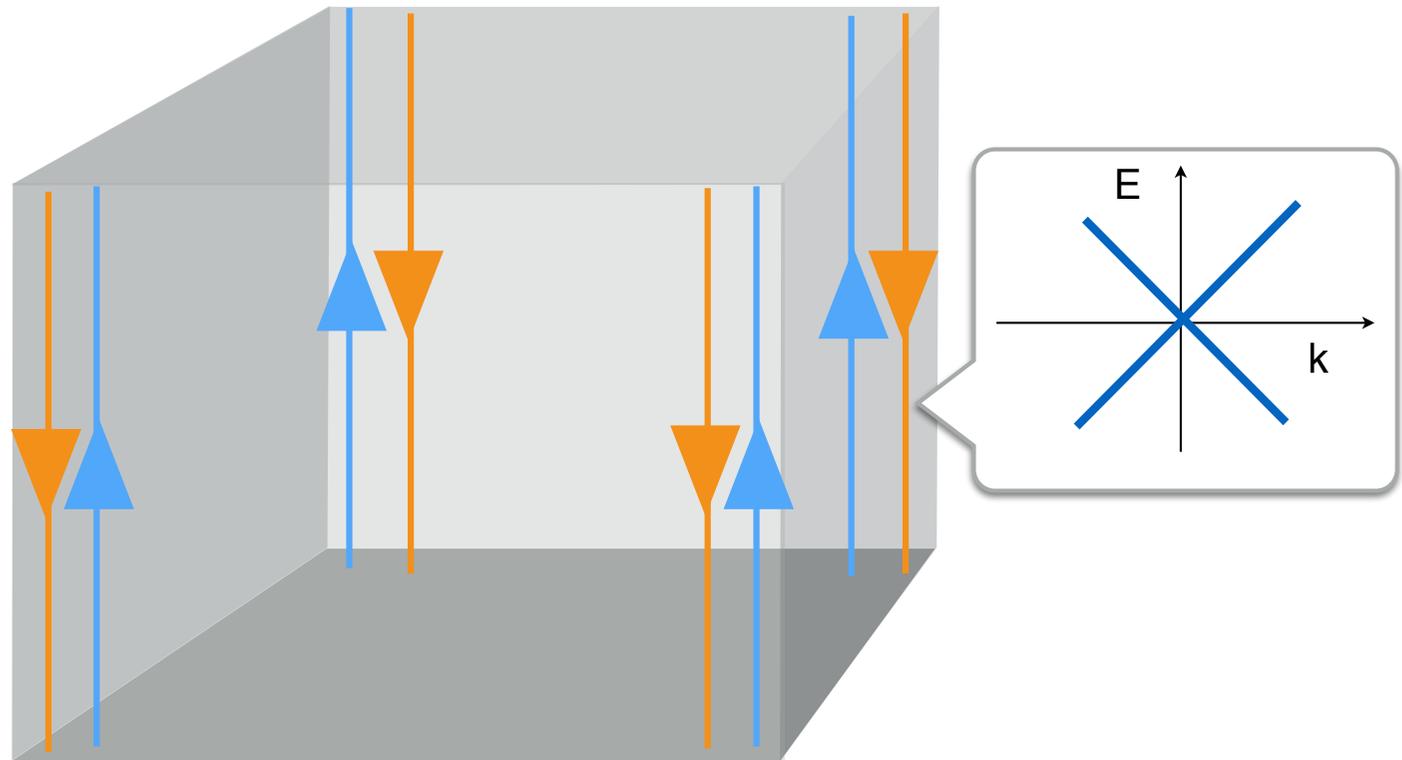


Protected by  $C_4T$  symmetry,  
 $\mathbb{Z}_2$  classification

# $\mathbb{Z}_2$ Classification

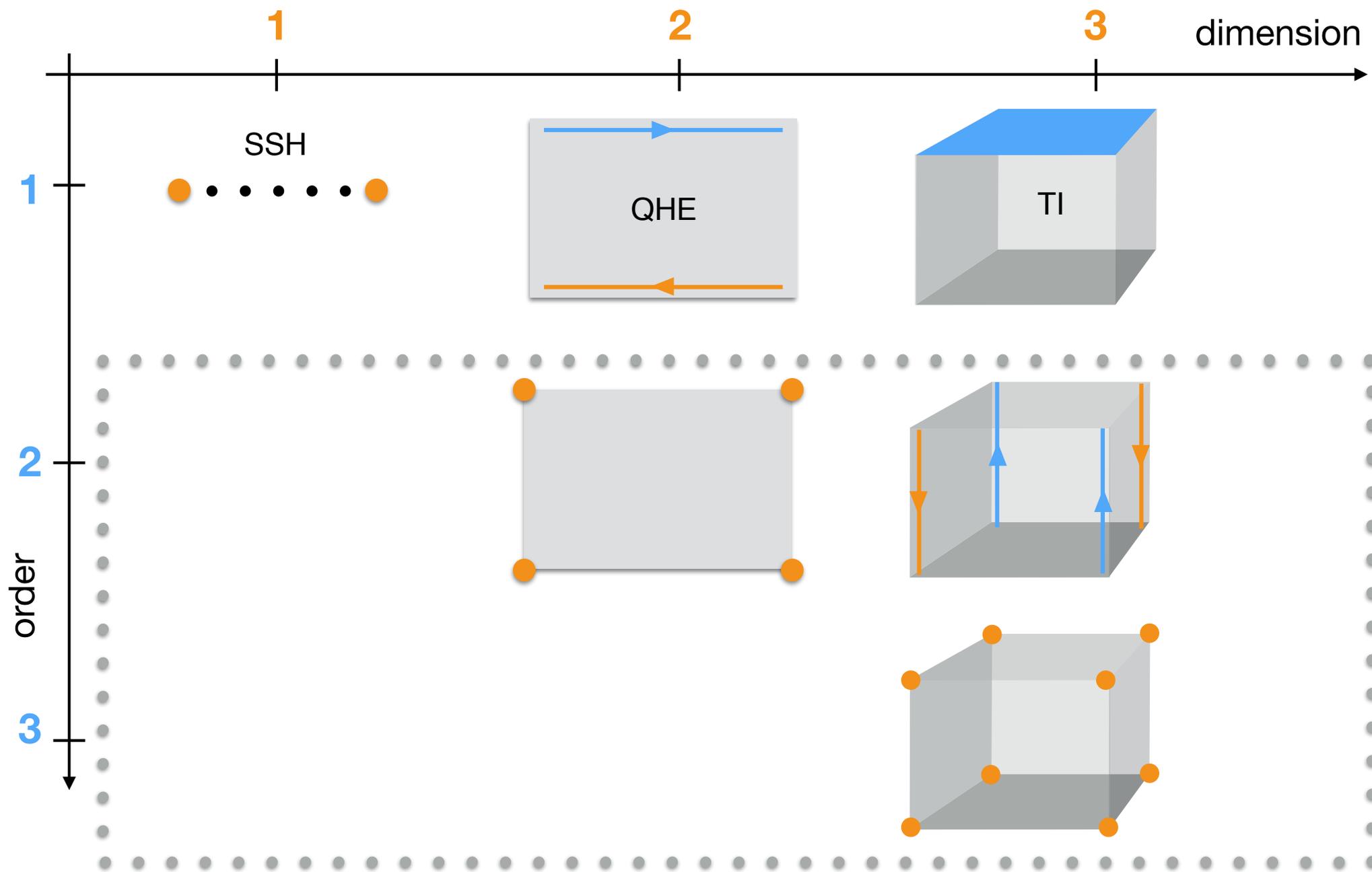


# Second-order 3D TI: TRS case



Protected by mirror symmetries,  
 $\mathbb{Z}_2$  classification

# Higher-order topological insulators



# **3. Models for second-order 3D Topological Insulators**

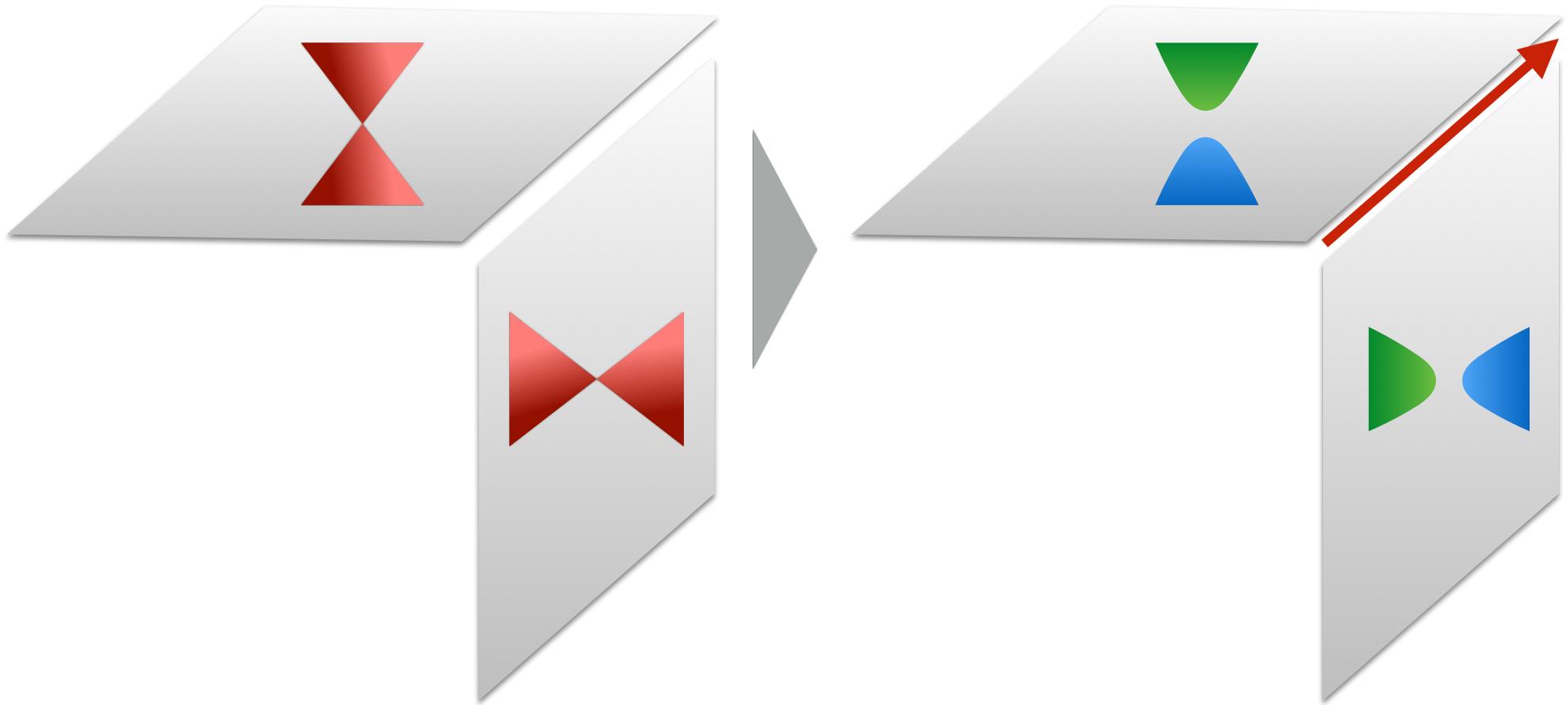
# Construction of a TRB second-order order 3D TI

**Toy model** with only  $C_4T$  in z-direction

$$H_4(\vec{k}) = \underbrace{\left( M + \sum_i \cos k_i \right) \tau_z \sigma_0 + \Delta_1 \sum_i \sin k_i \tau_y \sigma_i}_{\text{3D TI}} + \underbrace{\Delta_2 (\cos k_x - \cos k_y) \tau_x \sigma_0}_{\text{T, } C_4 \text{ breaking term}}$$

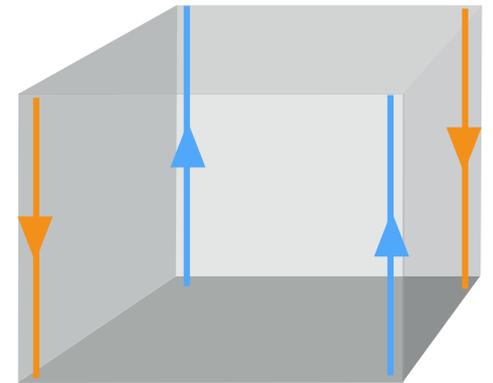
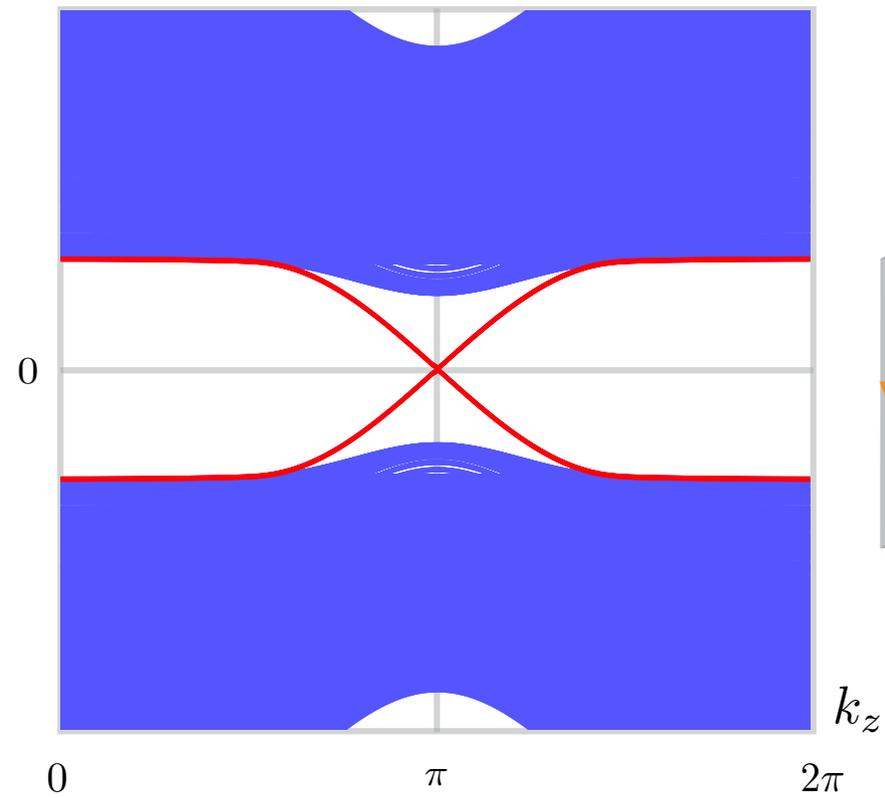
**$C_4T$  is still preserved!**

# Construction of a TRB second-order order 3D TI



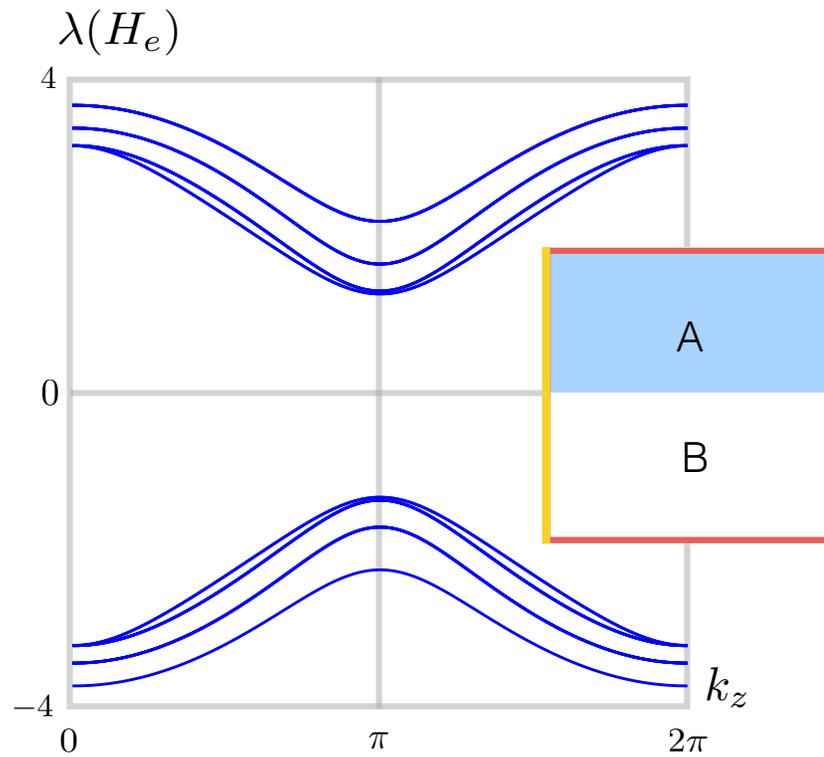
# Spectrum in column geometry

$\lambda(H_C)$



# Nested entanglement spectrum

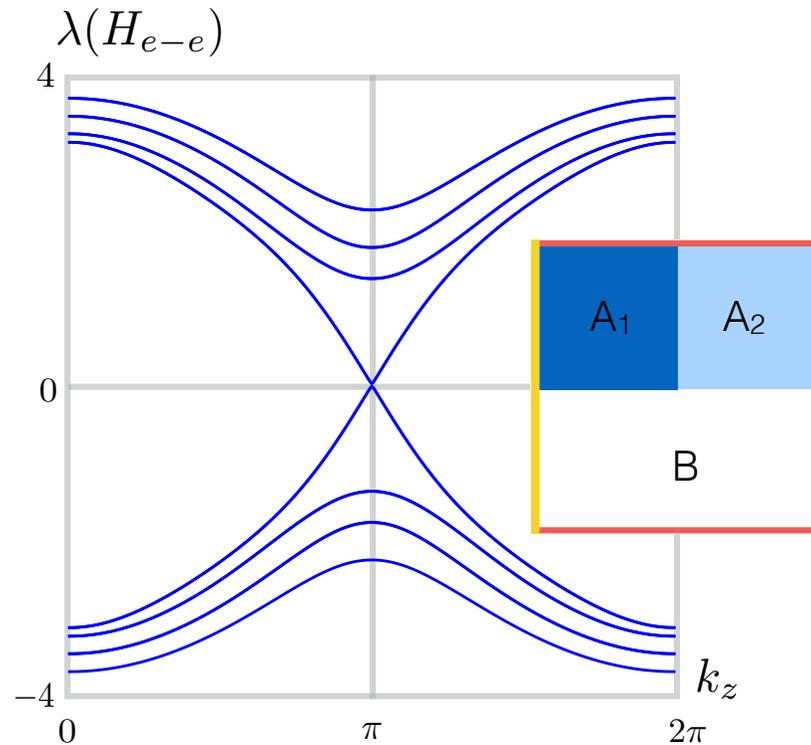
$$\rho_A = \text{Tr}_B |\Psi\rangle\langle\Psi| \equiv \frac{1}{Z_e} e^{-H_e}$$



# Nested entanglement spectrum

Define: entanglement spectrum of entanglement Hamiltonian

$$\rho_{e;A_1} = \text{Tr}_{A_2} |\Psi_e\rangle\langle\Psi_e| \equiv \frac{1}{Z_{e-e}} e^{-H_{e-e}}$$



# Take-Home Message

**new paradigm for hierarchical topological phases protected by spatial symmetries**

- edge modes protected by a 3D bulk invariant
- single edge has same properties as that of QSHE or QHE
- possibility of lossless transport in an intrinsically 3D system

