Extracting universal data from many-body states: momentum polarization and Klein twist

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Entanglement in Strongly Correlated Systems
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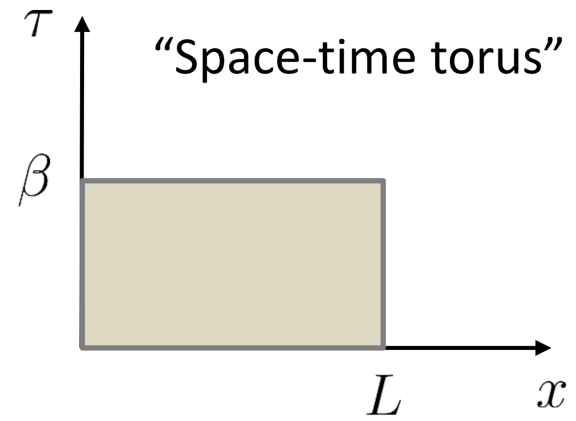
Outline

- One slide review of what we obtained yesterday on chiral CFT
- Momentum polarization for 2d chiral topological states
- Klein twist for 1d critical states (and hopefully for 2d non-chiral topological states soon)
- Summary

Calculating chiral character of CFT

$$\begin{aligned}\chi_a(q) &= \mathrm{Tr}_a(e^{-\beta H}) \\ &= \mathrm{Tr}_a(q^{L_0 - c/24}) \\ &= q^{h_a - c/24}(1 + d_1 q + d_2 q^2 + \ldots)\end{aligned}$$



$$q = e^{-2\pi\frac{\beta}{L}} \to 0 \quad \text{for } L \ll \beta$$

$$q = e^{-2\pi\frac{\beta}{L}} \to 1 \quad \text{for } L \gg \beta$$

Modular transformation:

$$\begin{aligned}\chi_a(q) &= \sum_b S_{a,b}\chi_b(q') \\ &\simeq S_{a,I}\chi_I(q' \to 0) \\ &\simeq S_{a,I}(q')^{-c/24} \\ &= \boxed{S_{a,I}e^{\frac{\pi c}{12}\frac{L}{\beta}}}\end{aligned} \qquad q' = e^{-2\pi\frac{L}{\beta}} \to 0 \quad \text{for } L \gg \beta$$

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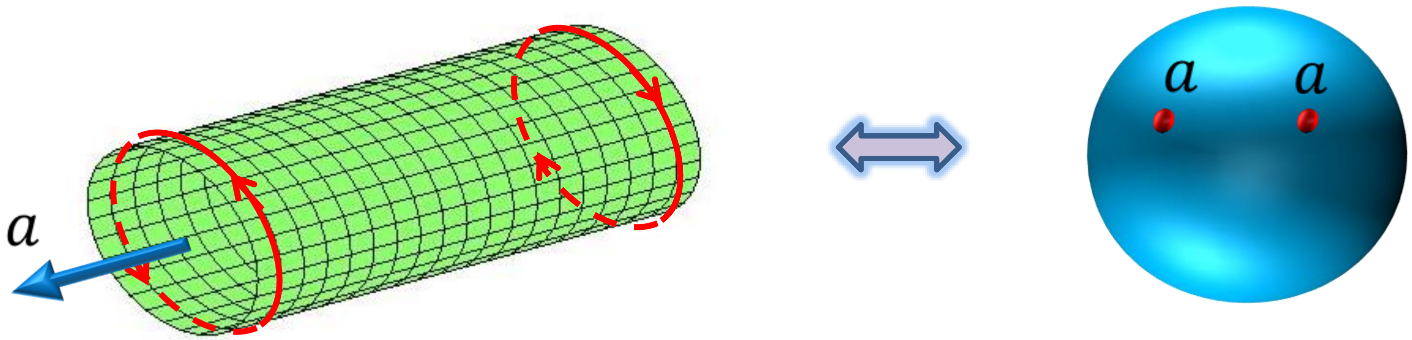
Topological spin of anyons



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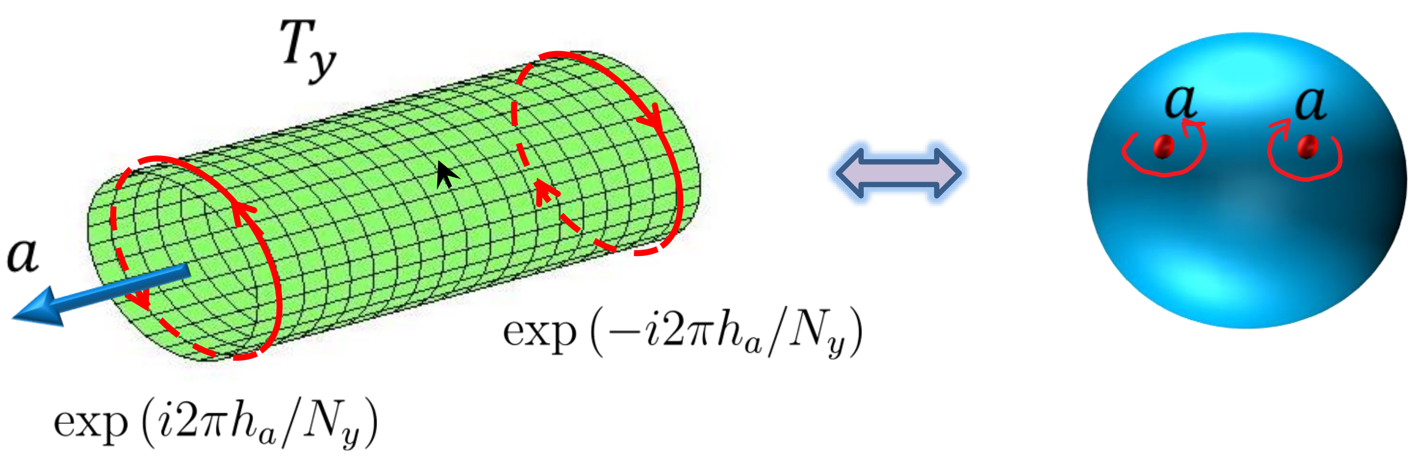

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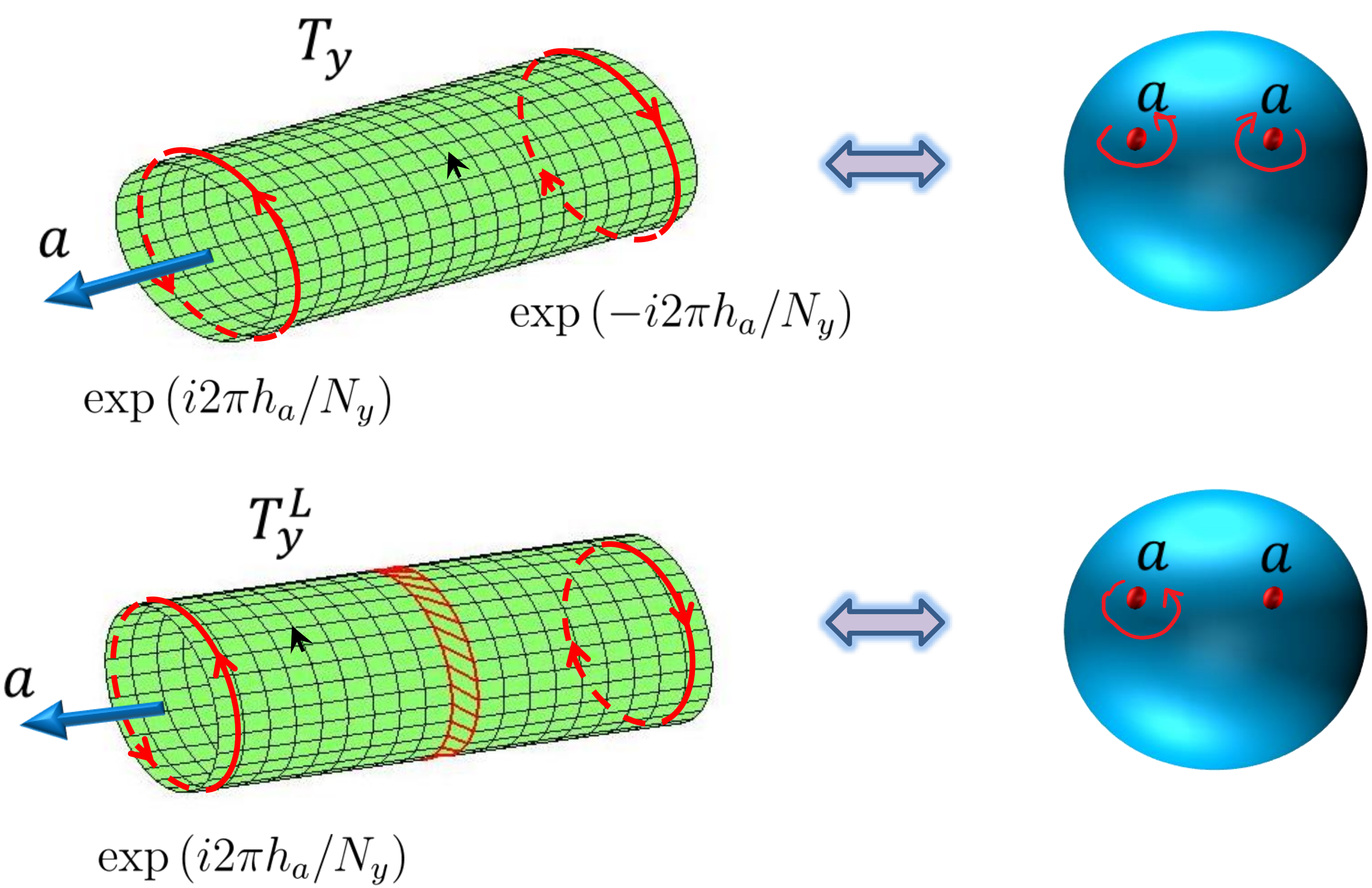

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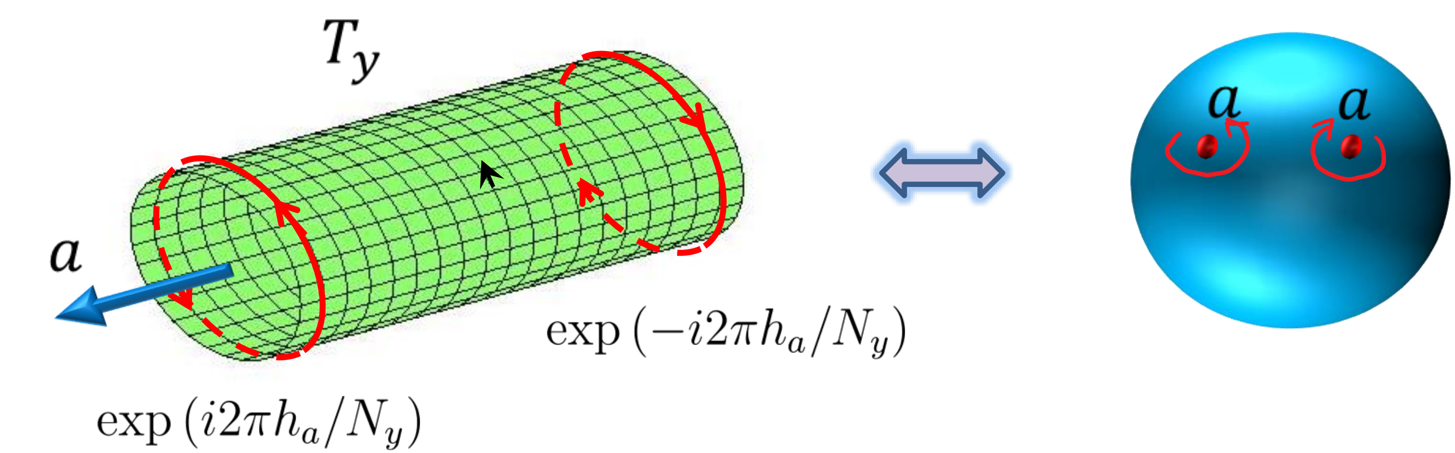
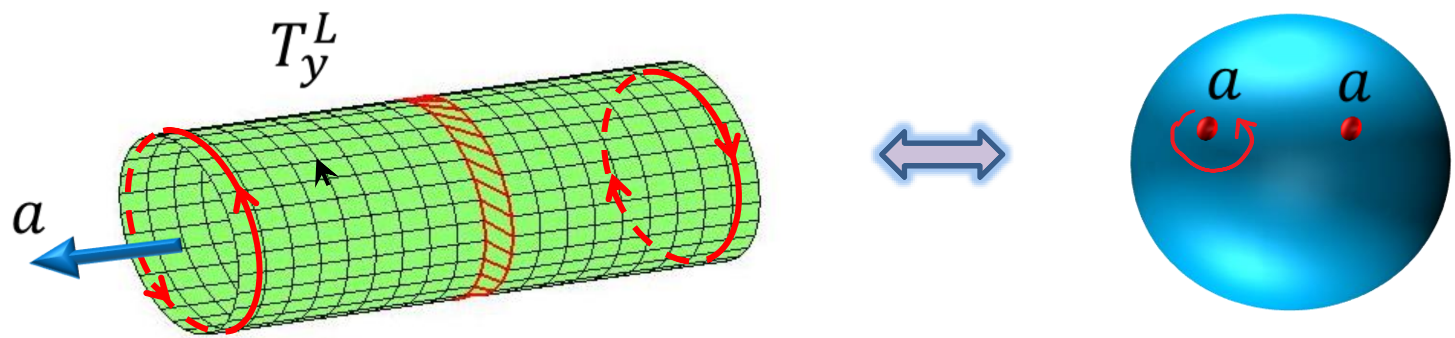


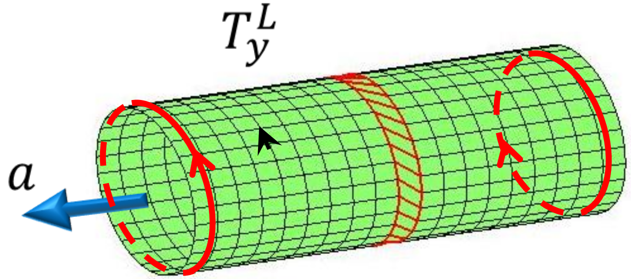


A naive expectation!

Momentum polarization

$|G_a\rangle$: minimally entangled basis

$$\langle G_a | T_L^y | G_a \rangle = \exp\left[i \frac{2\pi}{N_y} (h_a - \frac{c}{24}) - \alpha N_y \right]$$



HHT, Y. Zhang & X.-L. Qi, ’13

Zaletel, Mong & Pollmann, ’13

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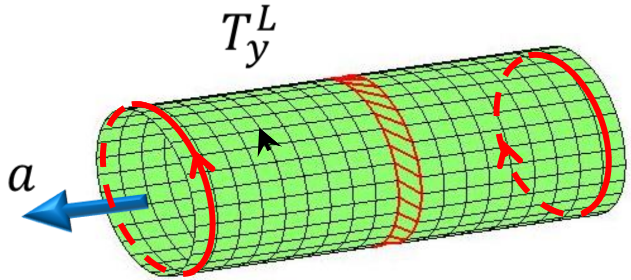

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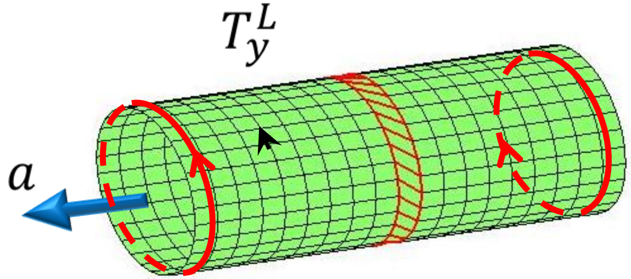

$$\langle G_a|T_L^y|G_a\rangle = \exp\left[i\frac{2\pi}{N_y}(h_a - \frac{c}{24}) - \alpha N_y\right]$$

Momentum polarization

$$\frac{\langle G_a|T_L^y|G_a\rangle}{\langle G_I|T_L^y|G_I\rangle} = \exp\left[i\frac{2\pi}{N_y}(h_a - h_I)\right] = \exp\left(i\frac{2\pi}{N_y}h_a\right)$$

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Momentum polarization

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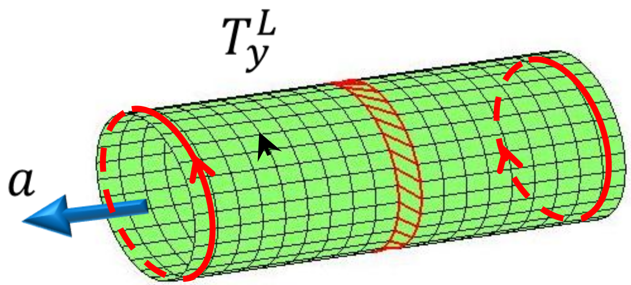

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Momentum polarization

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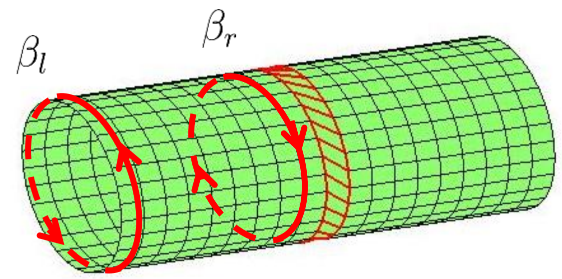
- The ground states in different sectors satisfy $\langle G_a|T_L^y|G_b\rangle \propto \delta_{ab}$

HHT, Y. Zhang & X.-L. Qi, '13

Zaletel, Mong & Pollmann, '13

Derivation based on chiral CFT

$$\begin{aligned}\langle G_a|T_L^y|G_a\rangle &= \mathrm{Tr}(|G_a\rangle\langle G_a|T_L^y) \\ &= \mathrm{Tr}_L(\rho_{La}T_L^y)\end{aligned}$$



- ρ_{La} describes the thermal state of two chiral CFTs:

$$\rho_{La} \propto e^{-(\beta_l H_l + \beta_r H_r)}\big|_a \qquad \beta_l \to \infty \quad \beta_r = \text{finite}$$

X.-L. Qi, Katsura & Ludwig '12

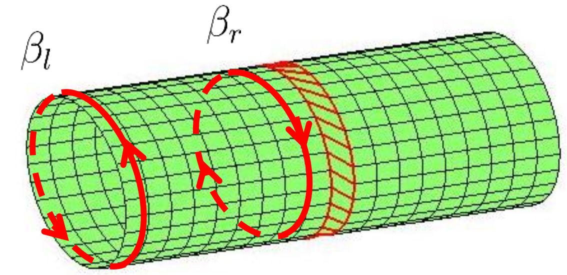
- Partial translation operator

$$T_y^L = e^{i(P_l+P_r)} = e^{i(H_l - H_r)}$$

$$\Rightarrow \quad \langle G_a|T_L^y|G_a\rangle = \frac{\mathrm{Tr}_a[e^{-(\beta_l - i)H_l}]}{\mathrm{Tr}_a(e^{-\beta_l H_l})} \cdot \frac{\mathrm{Tr}_a[e^{-(\beta_r + i)H_r}]}{\mathrm{Tr}_a(e^{-\beta_r H_r})}$$

Derivation based on chiral CFT

$$\langle G_a|T_L^y|G_a\rangle = \frac{\mathrm{Tr}_a[e^{-(\beta_l - i)H_l}]}{\mathrm{Tr}_a(e^{-\beta_l H_l})} \cdot \frac{\mathrm{Tr}_a[e^{-(\beta_r + i)H_r}]}{\mathrm{Tr}_a(e^{-\beta_r H_r})}$$



First term (almost temperature):

$$\begin{aligned}
\frac{\mathrm{Tr}_a[e^{-(\beta_l - i)H_l}]}{\mathrm{Tr}_a(e^{-\beta_l H_l})} &= \frac{\mathrm{Tr}_a[e^{-(\beta_l - i)\frac{2\pi}{N_y}(L_0 - c/24)}]}{\mathrm{Tr}_a(e^{-\beta_l \frac{2\pi}{N_y}(L_0 - c/24)})} \\
&= \frac{\mathrm{Tr}_a[(q')^{L_0 - c/24}]}{\mathrm{Tr}_a(q^{L_0 - c/24})} \\
&= \frac{\chi_a(q')}{\chi_a(q)} \\
&= (q'/q)^{h_a - c/24} \\
&= \exp\left[i\frac{2\pi}{N_y}(h_a - \frac{c}{24})\right]
\end{aligned}$$

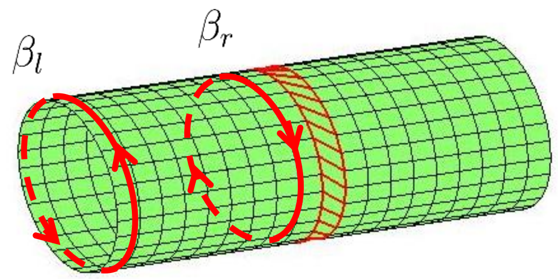
$$\beta_l \to \infty$$

$$q' = e^{-2\pi\frac{\beta_l - i}{N_y}} \to 0$$
$$q = e^{-2\pi\frac{\beta_l}{N_y}} \to 0$$

$$\begin{aligned}
\chi_a(q) &= q^{h_a - c/24}(1 + d_1 q + \ldots) \\
&\simeq q^{h_a - c/24}
\end{aligned}$$

Derivation based on chiral CFT

$$\langle G_a | T_L^y | G_a \rangle = \frac{\mathrm{Tr}_a[e^{-(\beta_l - i)H_l}]}{\mathrm{Tr}_a(e^{-\beta_l H_l})} \cdot \frac{\mathrm{Tr}_a[e^{-(\beta_r + i)H_r}]}{\mathrm{Tr}_a(e^{-\beta_r H_r})}$$



Second term (finite temperature):

$$\begin{aligned}
\frac{\mathrm{Tr}_a[e^{-(\beta_r + i)H_r}]}{\mathrm{Tr}_a(e^{-\beta_r H_r})} &= \frac{\mathrm{Tr}_a[e^{-(\beta_r + i)\frac{2\pi}{N_y}(L_0 - c/24)}]}{\mathrm{Tr}_a(e^{-\beta_r \frac{2\pi}{N_y}(L_0 - c/24)})} \\
&= \frac{\mathrm{Tr}_a[(q')^{L_0 - c/24}]}{\mathrm{Tr}_a(q^{L_0 - c/24})} \\
&= \frac{\chi_a(q')}{\chi_a(q)} \\
&= \exp(-\alpha N_y)
\end{aligned}$$

$$\beta_r = \text{finite} \ll N_y$$

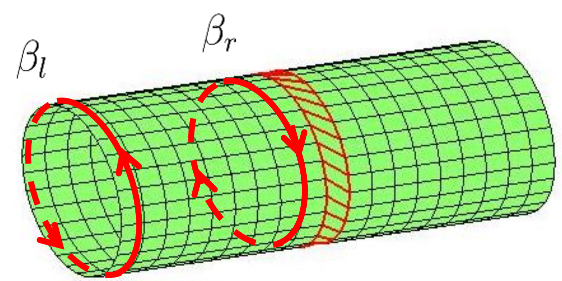
$$q' = e^{-2\pi \frac{\beta_r + i}{N_y}} \to 1$$

$$q = e^{-2\pi \frac{\beta_r}{N_y}} \to 1$$

$$\begin{aligned}
\chi_a(q') &= S_{a,I} \exp\left(\frac{\pi c}{12} \frac{N_y}{i + \beta_r}\right) \\
\chi_a(q) &= S_{a,I} \exp\left(\frac{\pi c}{12} \frac{N_y}{\beta_r}\right)
\end{aligned}$$

Derivation based on chiral CFT

$$\langle G_a | T_L^y | G_a \rangle = \frac{\mathrm{Tr}_a[e^{-(\beta_l - i)H_l}]}{\mathrm{Tr}_a(e^{-\beta_l H_l})} \cdot \frac{\mathrm{Tr}_a[e^{-(\beta_r + i)H_r}]}{\mathrm{Tr}_a(e^{-\beta_r H_r})}$$



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$$\boxed{\langle G_a | T_L^y | G_a \rangle = \exp\left[i \frac{2\pi}{N_y}(h_a - \frac{c}{24}) - \alpha N_y \right]}$$

Momentum polarization

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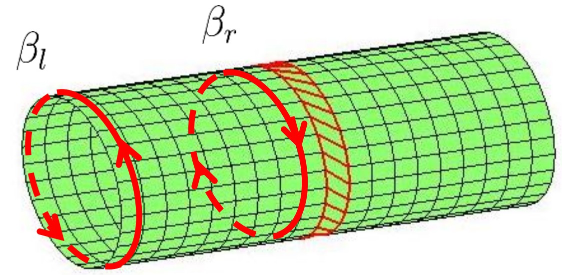


Further remarks:

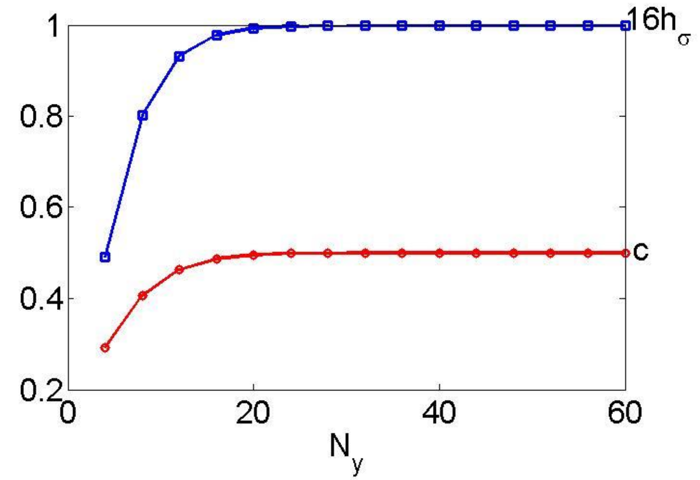
- In the continuum FQH case, Zaletel et al. showed that $\mathrm{Im}(\alpha)$ contains universal information of the Hall viscosity.
- Some numerical results give indication that the topological spin could still be extracted even when the edge CFT assumption fails.
- For trial wave functions, it's convenient to compute the momentum polarization by Monte Carlo.

$$\langle \psi | T_y^L | \psi \rangle = \sum_\alpha |\psi(\alpha)|^2 \frac{\psi(T_y^L \alpha)}{\psi(\alpha)}$$

Kitaev's honeycomb model and ½ Laughlin state

Kitaev's honeycomb model:

- $h_\sigma = 1/16$ and $c = 1/2$ accurately obtained in numerics.



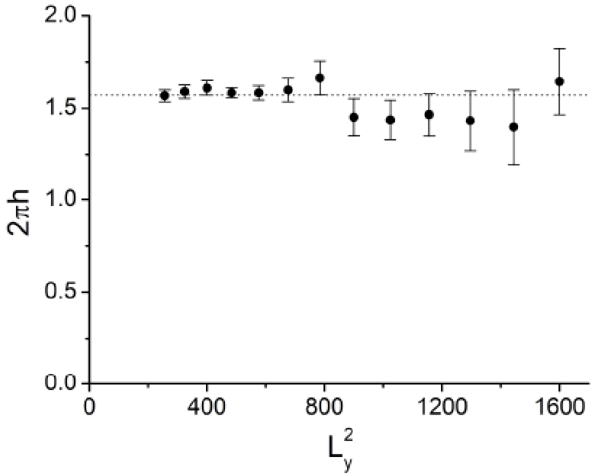
½ Laughlin state:

$$c = 1.078 \pm 0.091$$

$$h_s = 0.2521 \pm 0.0063$$

$$c = 1$$

$$h_s = 1/4$$



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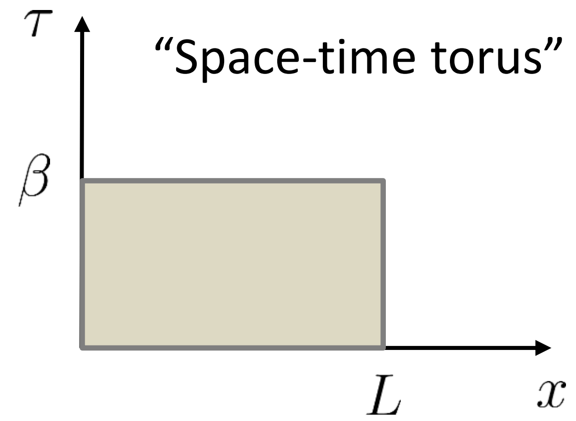
Non-chiral CFT

- Non-chiral CFT: gapless left and right movers with linear dispersion



$$
\begin{aligned}
H &= \frac{2\pi}{L}(L_0 - \frac{c}{24}) + \frac{2\pi}{L}(\bar{L}_0 - \frac{c}{24}) \\
P &= \frac{2\pi}{L}(L_0 - \bar{L}_0)
\end{aligned}
$$

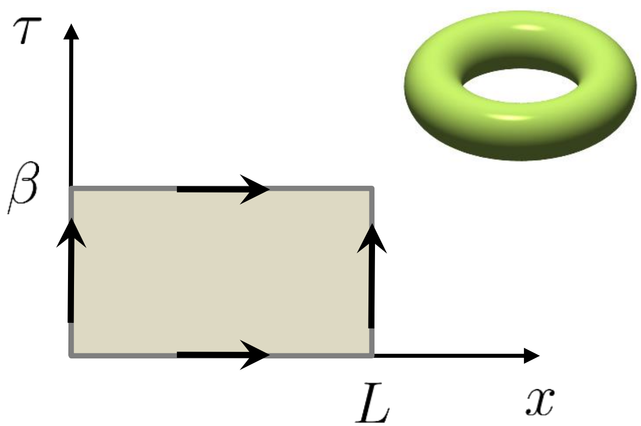
Eigenvectors: $L_0|\alpha,\bar{\gamma}\rangle = h_\alpha|\alpha,\bar{\gamma}\rangle \qquad \bar{L}_0|\alpha,\bar{\gamma}\rangle = h_\gamma|\alpha,\bar{\gamma}\rangle$

Partition function: $Z^{\mathcal{T}} = \mathrm{Tr}_{\mathcal{H}\times\bar{\mathcal{H}}}(q^{L_0-c/24}\bar{q}^{\bar{L}_0-c/24})$

- ➢ Non-chiral CFTs describe 1d critical chains and gapless edge states of some 2d time-reversal invariant topological states.

Torus versus Klein bottle

Torus

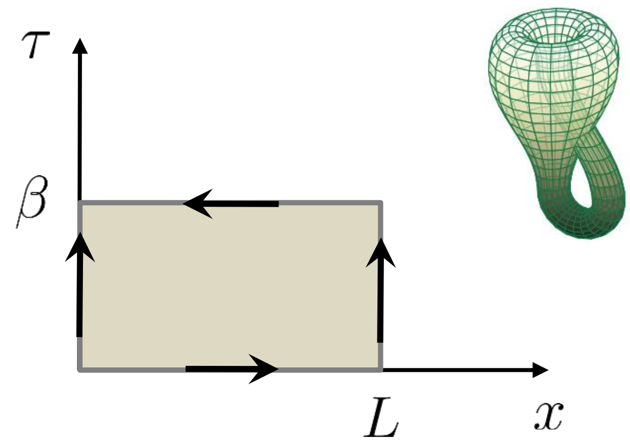


$$Z^{\mathcal{T}} = \mathrm{Tr}_{\mathcal{H}\times\bar{\mathcal{H}}}(q^{L_0-c/24}\bar{q}^{\bar{L}_0-c/24})$$

Diagonal partition function:

$$Z^{\mathcal{T}} = \sum_a \chi_a(q)\bar{\chi}_a(\bar{q})$$

Klein bottle



$$Z^{\mathcal{K}} = \mathrm{Tr}_{\mathcal{H}\times\bar{\mathcal{H}}}(\Omega q^{L_0-c/24}\bar{q}^{\bar{L}_0-c/24})$$

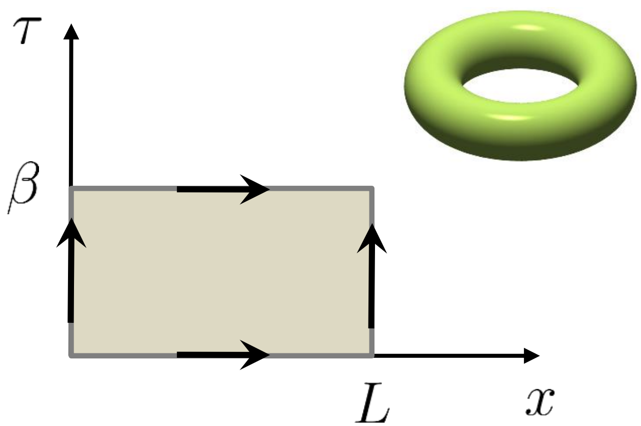
$$\Omega|\alpha,\bar{\gamma}\rangle = |\gamma,\bar{\alpha}\rangle$$

$$\begin{aligned} Z^{\mathcal{K}} &= \mathrm{Tr}_{\mathcal{H}}(q^{2L_0-c/12}) \\ &= \sum_a \chi_a(q^2) \end{aligned}$$

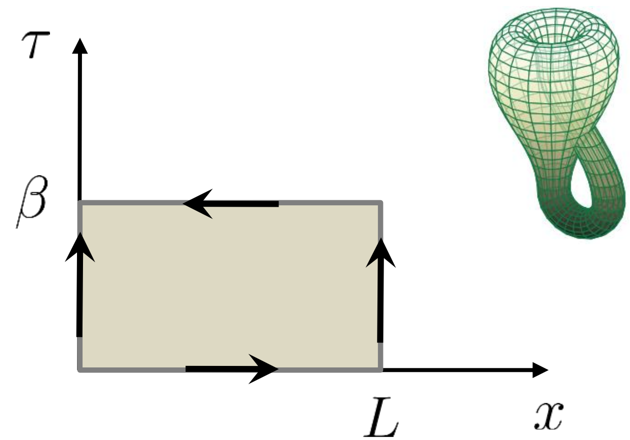
HHT, Dubail & Bondesan, 17’

Torus versus Klein bottle

Torus



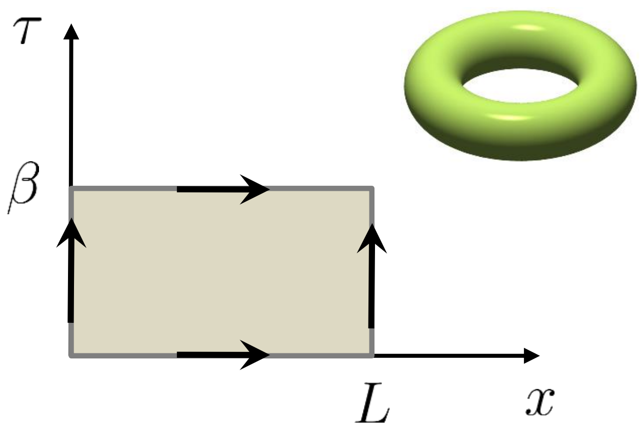
Klein bottle



$$
\begin{aligned}
Z^{\mathcal{T}}(L,\beta) &\simeq \exp\left[-f_0\beta L + \frac{\pi c}{6\beta v}L + \mathcal{O}(1/L)\right] \\
Z^{\mathcal{K}}(L,\beta) &\simeq \left(\sum_a S_{a,1}\right)\exp\left[-f_0\beta L + \frac{\pi c}{24\beta v}L + \mathcal{O}(1/L)\right]
\end{aligned}
$$

Torus versus Klein bottle

Torus



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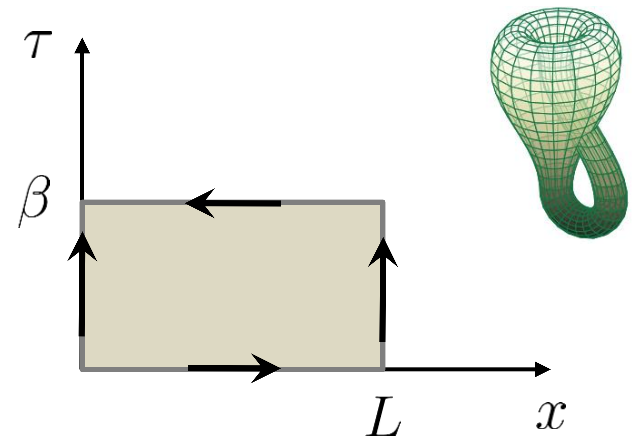

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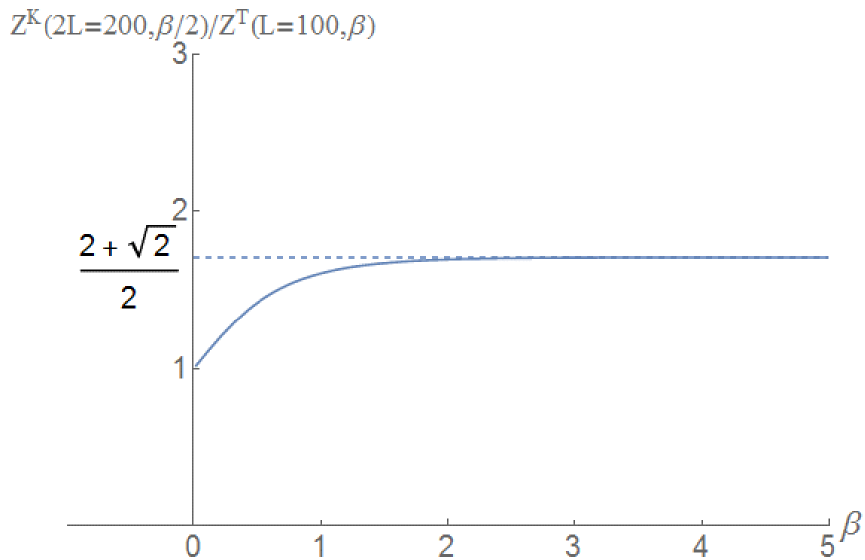
\end{aligned}
$$

$$
\boxed{\frac{Z^{\mathcal{K}}(2L,\frac{\beta}{2})}{Z^{\mathcal{T}}(L,\beta)} = \sum_a S_{a,1} = \frac{1}{\mathcal{D}}\sum_a d_a}
$$

“Boundary entropy” without boundary!

1d critical Ising chain

$$H = -\sum_{i=1}^{L} \sigma_i^x \sigma_{i+1}^x - \sum_{i=1}^{L} \sigma_i^z$$



Klein bottle partition function:

$$Z^{\mathcal{K}} = \mathrm{Tr}(Pe^{-\beta H})$$

$$P|\sigma_1, \sigma_2, \ldots, \sigma_{L-1}, \sigma_L\rangle = |\sigma_L, \sigma_{L-1}, \ldots, \sigma_2, \sigma_1\rangle$$

$$\frac{Z^{\mathcal{K}}(2L, \frac{\beta}{2})}{Z^{\mathcal{T}}(L, \beta)} = \frac{1}{\mathcal{D}} \sum_{a=I,\sigma,\psi} d_a = \frac{2+\sqrt{2}}{2}$$

Summary

- Momentum polarization provides a tool to extract topological spin and chiral central charge from the ground-state wave functions of 2D chiral topological phases.

- Klein twist provides a tool to obtain information of quantum dimensions from the thermal states of 1d critical systems (and hopefully also for 2d topological models with non-chiral CFT edge states).

- The underlying theoretical tool for deriving them is the modular transformation properties of the chiral CFT characters.

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Thank you for your attention!