A simple tensor network algorithm for 2d steady states

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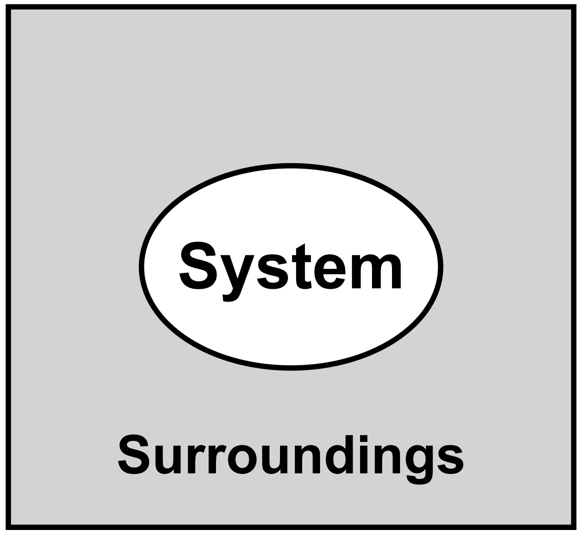
Benasque 10.02.2017

CENTRO DE CIENCIAS DE BENASQUE PEDRO PASCUAL

A. Kshetrimayum, H. Weimer, R. Orus, arXiv:1612.00656

Motivation



Most systems, in nature, are not isolated! They are usually affected by environment.

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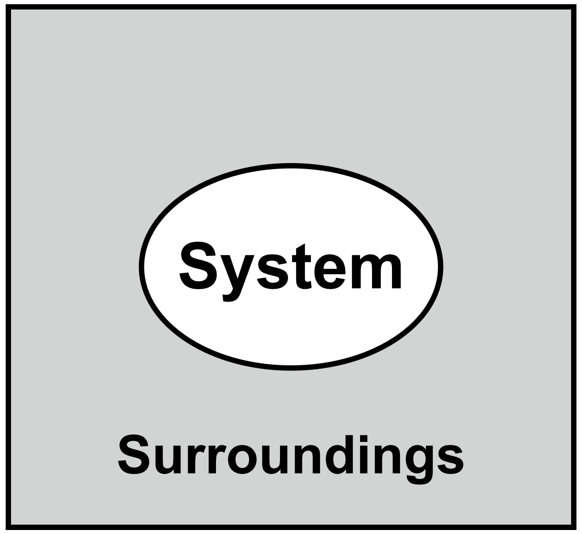

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Examples of such systems include heat transfer, quantum decoherence, etc.

Heat transfer

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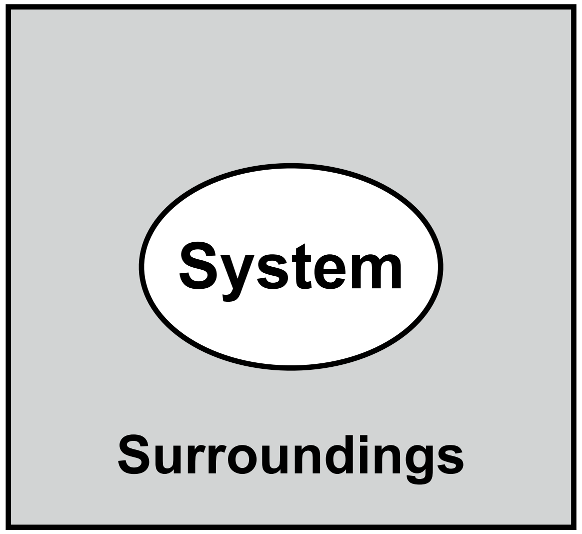

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Examples of such systems include heat transfer, quantum decoherence, etc.

Heat transfer

Systems interacting with an environment are known as open systems. Such interactions usually lead to dissipation.

Motivation

In the context of quantum many-body systems, dissipation often leads to many interesting phenomenon such as

Decoherence of complex wave functions
Quantum Thermodynamics
Engineering Topological order through dissipation
Driven-dissipative universal quantum computation
Dissipative phase transitions
.....

M. Schlosshauer, Rev. Mod. Phys. 76 1267 (2005)
S. Vinjanampathy and J. Anders, Con. Phys. 57, 3, 1-35 (2016).
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1. Exhibit the same computational complexity class as equilibrium system ($O(d^{2N})$)
2. Physical constraints of a density matrix that can be used to represent such systems

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2. Physical constraints of a density matrix that can be used to represent such systems

Efficient numerical tools for study of open quantum many body systems is still lacking and continues to be a challenge!

Content of this talk

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An important concept: Choi isomorphism

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Finding steady states: parallelism with imaginary time evolution

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Tensors Networks

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Steady States with TNs:

- Recent advances in 1d systems
- Applications in 2d systems

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Benchmark results: Dissipative spin-1/2 quantum Ising model in 2d

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Conclusions & Outlook

Choi isomorphism

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Simply turning a bra into a ket!

$$|i\rangle\langle j| \equiv |i\rangle \otimes |j\rangle$$

(Vectorization)

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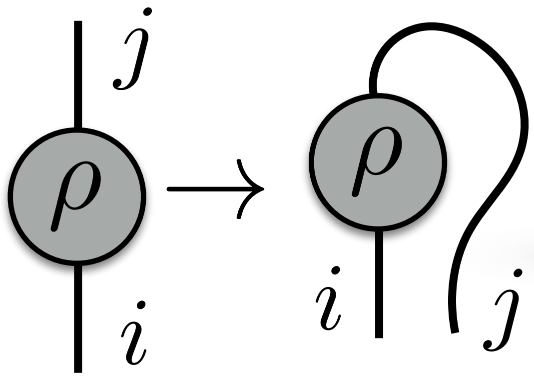
In the context of a reduced density matrix

$$\rho \rightarrow |\rho\rangle_\sharp$$



Understanding the coefficients of ρ as those of a vector $|\rho\rangle_\sharp$

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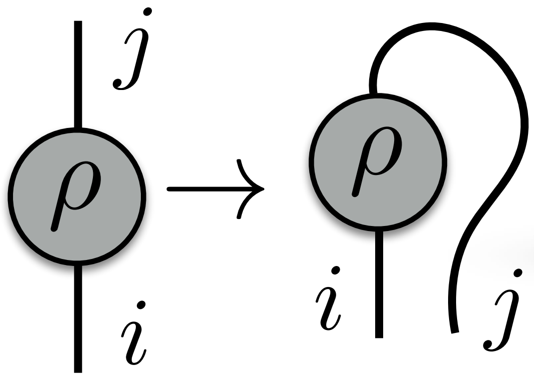

Understanding the coefficients of ρ as those of a vector $|\rho\rangle_\sharp$

Assuming Markovian evolution, the dynamics of an open quantum system can be described by the following Lindbladian Master equation

$$\frac{d}{dt}\rho = \mathcal{L}[\rho] = -i\,[H,\rho] + \sum_\mu \left(L_\mu \rho L_\mu^\dagger - \frac{1}{2} L_\mu^\dagger L_\mu \rho - \frac{1}{2}\rho L_\mu^\dagger L_\mu \right)$$

$\mathcal{L}$ is the Liouvillian operator, H the Hamiltonian of the system and $\{L_\mu, L_\mu^\dagger\}$ the jump/Lindblad operators describing the dissipation.

Choi isomorphism

Using the Choi isomorphism, one can then write the vectorized form of this equation as

$$\frac{d}{dt}|\rho\rangle_\sharp = \mathcal{L}_\sharp |\rho\rangle_\sharp$$

where the vectorized Liouvillian operator is given by

$$\mathcal{L}_\sharp \equiv -i\left(H \otimes \mathbb{I} - \mathbb{I} \otimes H^T\right) + \sum_\mu \left(L_\mu \otimes L_\mu^* - \frac{1}{2} L_\mu^\dagger L_\mu \otimes \mathbb{I} - \frac{1}{2}\mathbb{I} \otimes L_\mu^* L_\mu^T\right).$$

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we can now integrate the above equation to get

$$|\rho(T)\rangle_\sharp = e^{T\mathcal{L}_\sharp}|\rho(0)\rangle_\sharp$$

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we can now integrate the above equation to get

$$|\rho(T)\rangle_\sharp = e^{T\mathcal{L}_\sharp}|\rho(0)\rangle_\sharp$$

we can then obtain the steady state for very large times

$$|\rho_s\rangle_\sharp \equiv \lim_{T\to\infty}|\rho(T)\rangle_\sharp \quad \text{i.e.} \quad \frac{d}{dt}|\rho_s\rangle_\sharp = \mathcal{L}_\sharp|\rho_s\rangle_\sharp = 0$$

Parallelism with imaginary time evolution

Assume a nearest-neighbor Liouvillian:

where the sum is over nearest-neighbor terms

$$\mathcal{L}[\rho] = \sum_{\langle i,j \rangle} \mathcal{L}^{[i,j]}[\rho]$$

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We find a parallelism with finding the ground state of a Hamiltonian using imaginary time evolution

Ground States	**Steady States**
$H = \sum_{\langle i,j \rangle} h^{[i,j]}$	$\mathcal{L}_\sharp = \sum_{\langle i,j \rangle} \mathcal{L}_\sharp^{[i,j]}$
e^{-TH}	$e^{T\mathcal{L}_\sharp}$
$\lvert e_0 \rangle$	$\lvert \rho_s \rangle_\sharp$
$\langle e_0 \lvert H \rvert e_0 \rangle = e_0$	${}_\sharp\langle \rho_s \lvert \mathcal{L}_\sharp \rvert \rho_s \rangle_\sharp = 0$
Imaginary time	Real time

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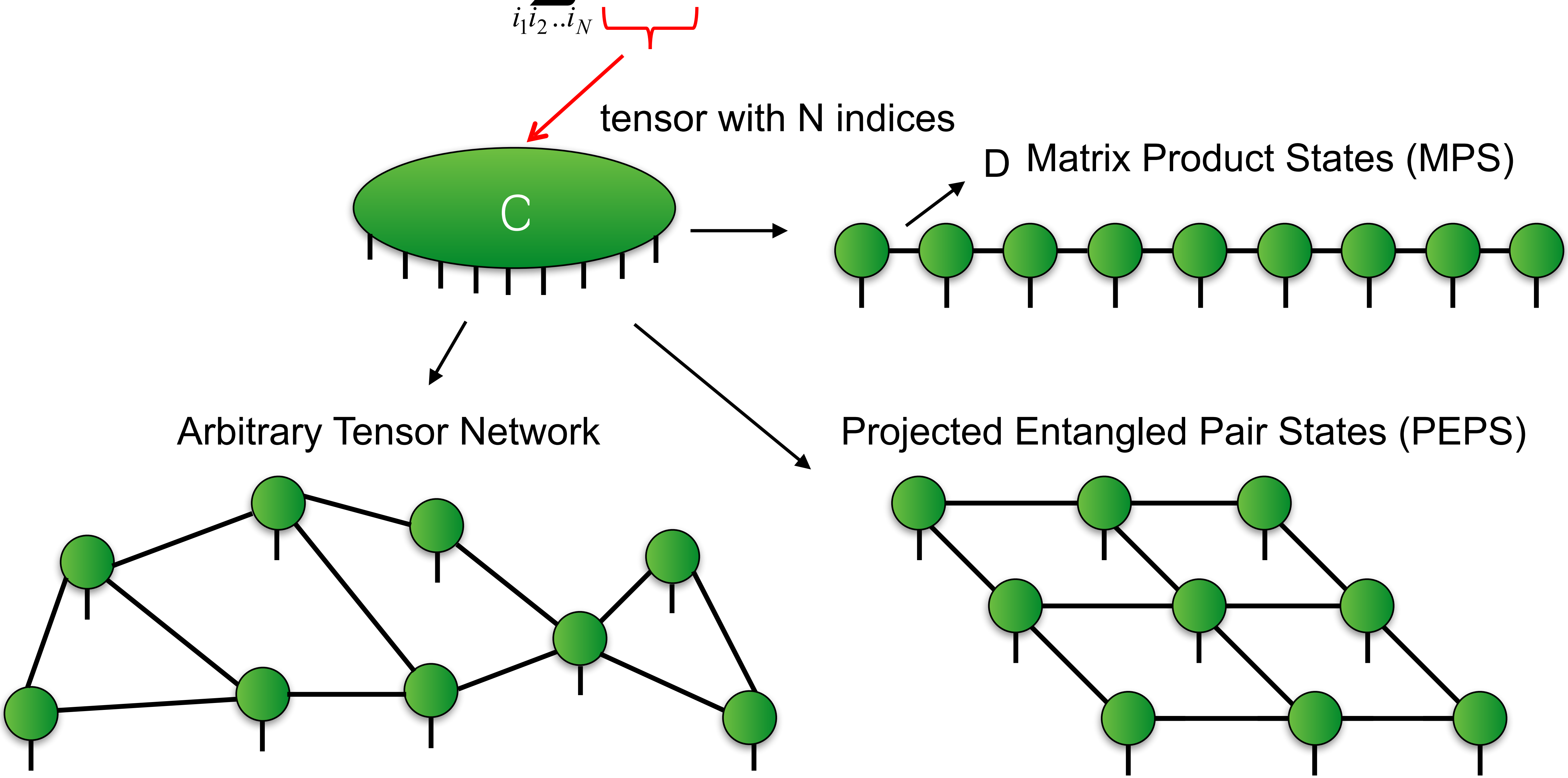
$\|e_0\rangle$	$\|\rho_s\rangle_\sharp$
$\langle e_0 \| H \| e_0 \rangle = e_0$	${}_\sharp\langle \rho_s \| \mathcal{L}_\sharp \| \rho_s \rangle_\sharp = 0$
Imaginary time	Real time

Use the usual TN algorithms to obtain the steady states??

Tensor Networks

A quantum many-body wave function can be written as

$$|\psi\rangle = \sum_{i_1 i_2 .. i_N} C_{i_1 i_2 .. i_N} |i_1\rangle \otimes |i_2\rangle \otimes \ldots \otimes |i_N\rangle$$



Steady States with TNs:1d

Recent advances in 1D systems

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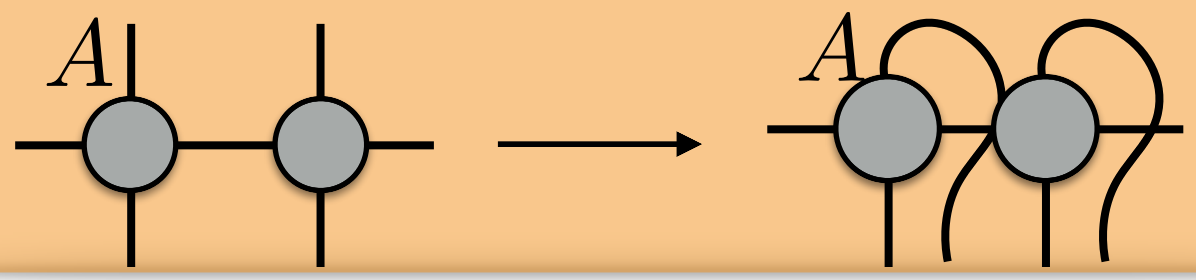
Choi isomorphism + MPOs

M. Zwolak and G. Vidal, Phys. Rev. Lett. 93, 207205 (2004)



Simple & efficient

Issue of positivity!!

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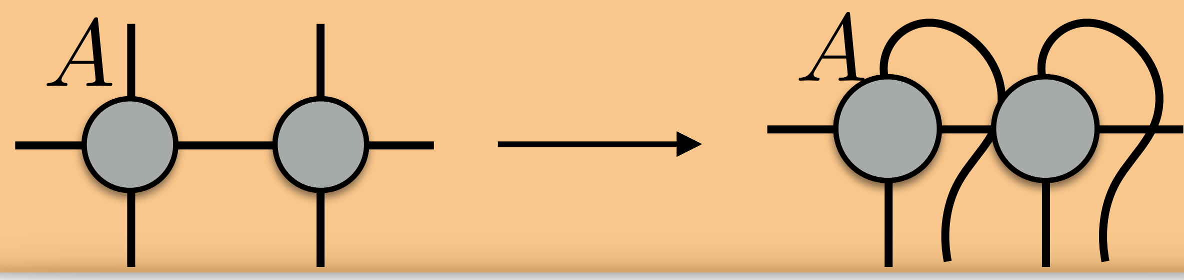

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Other techniques using MPDOs, disentanglers, etc

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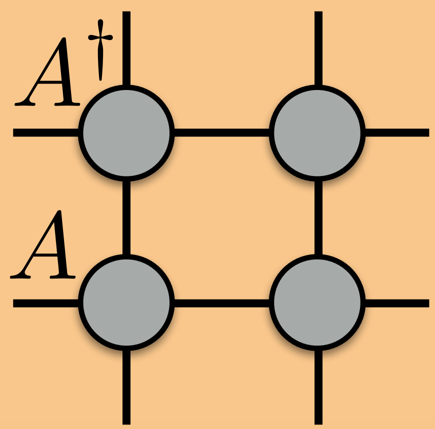

Positive by construction

Large purification may be required!!

Very costly!!

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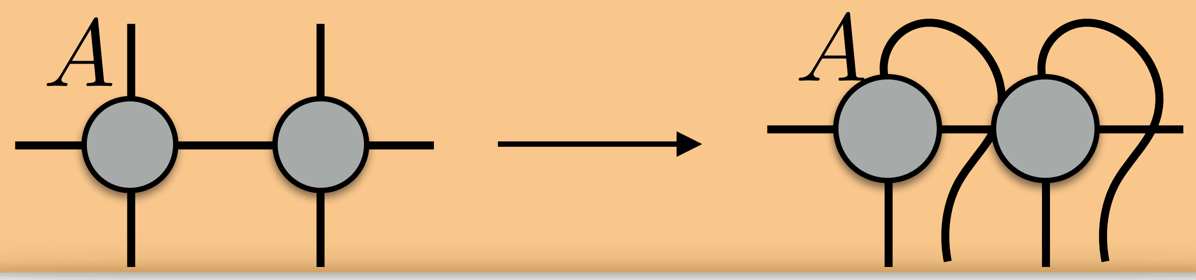

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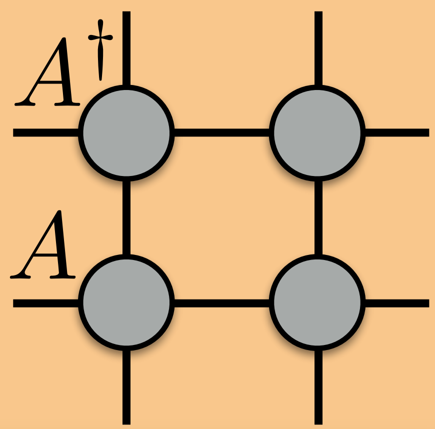

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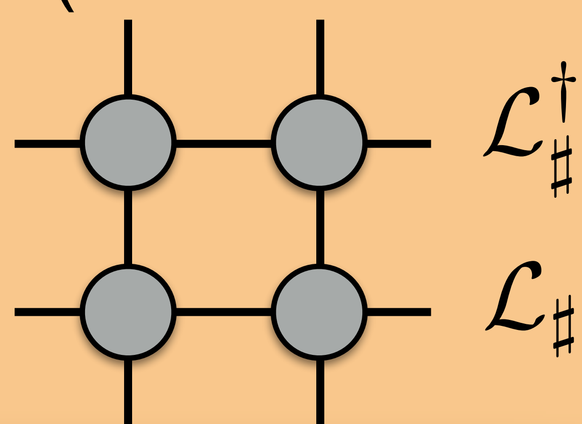
Target the ground state of $\mathcal{L}_\sharp^\dagger \mathcal{L}_\sharp$ (hermitian & positive semi-definite)

A. A. Gangat, T. I and Y.-Jer Kao, arXiv:1608.06028; E. Mascarenhas, H. Flayac, and V. Savona, Phys. Rev. A 92, 022116 (2015);J. Cui, J. I. Cirac, and M. C. Bañuls, Phys. Rev. Lett. 114, 220601 (2015)



Perform imaginary time evolution & get direct convergence with TEBD or DMRG

Interactions no more local!!

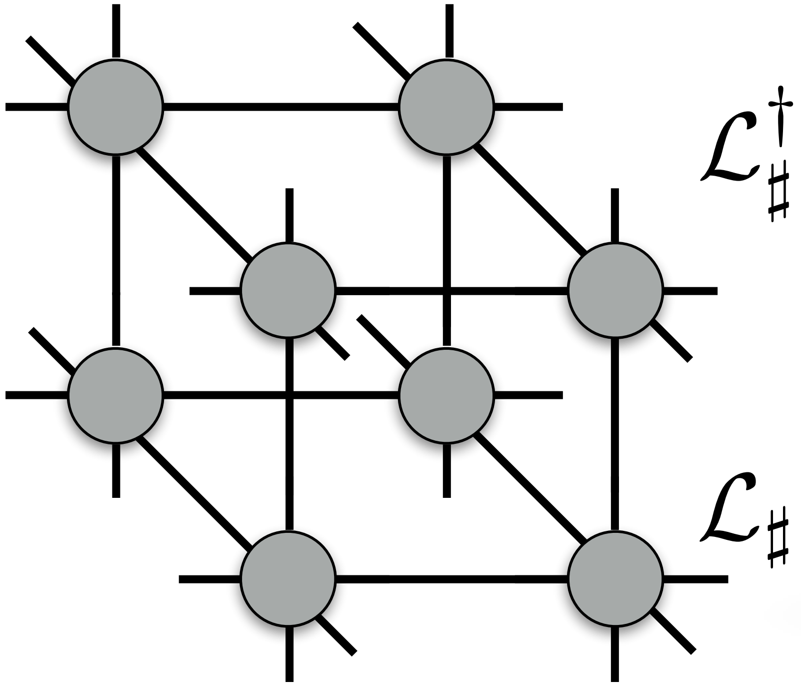
Bond dimension of the MPOs get squared!!

(still manageable in 1d)

Steady States with TNs:2d

Applications in 2d systems??

Targeting the ground state of $\mathcal{L}_\sharp^\dagger \mathcal{L}_\sharp$ is extremely difficult here



Can (in principle) perform imaginary time evolution & get direct convergence with TEBD

Interactions in $\mathcal{L}_\sharp^\dagger \mathcal{L}_\sharp$ no more local!!

Bond dimension of the PEPOs get squared!!

(very difficult in 2d!!)

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Use real time evolution with $\mathcal{L}_\sharp$ until steady state is reached

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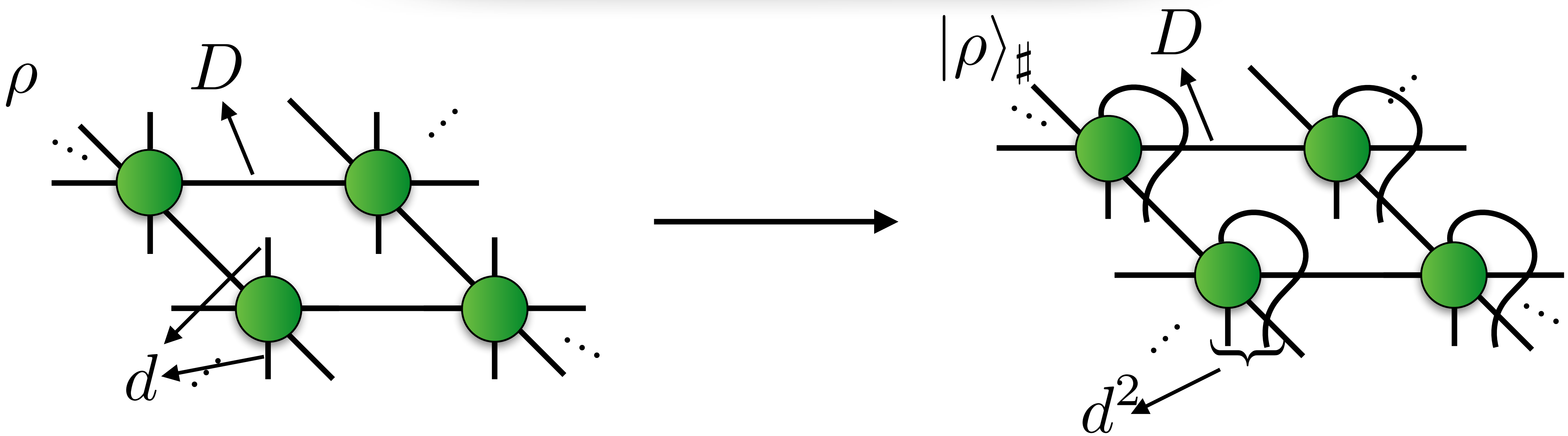

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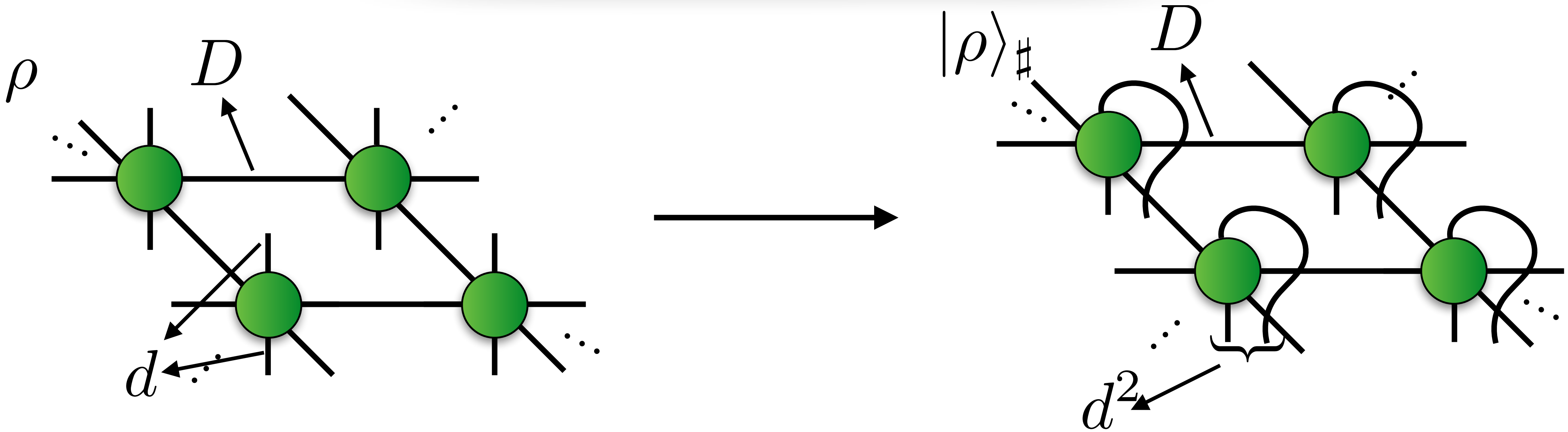

Simple and efficient
Growth of entanglement??
Not necessarily positive!

Growth of entanglement in 2D may be slow compared to the fixed point attractor
Very good accuracy and small positivity error

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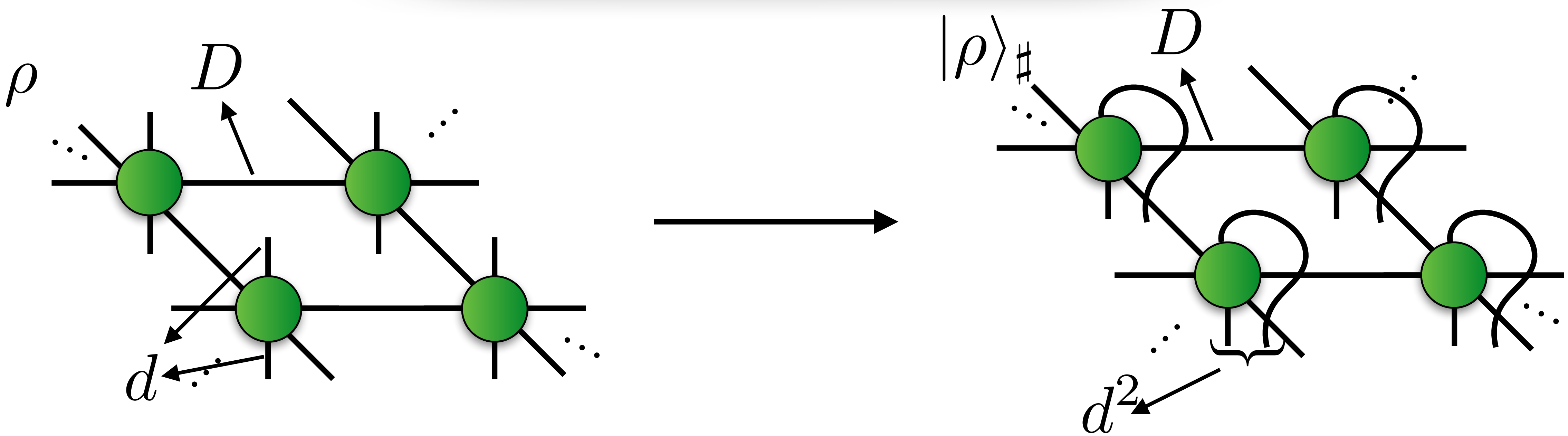

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So, one can apply the usual iPEPS algorithm in the vectorized form.

Benchmark results

The 2d dissipative quantum Ising model

$$H = \frac{V}{4} \sum_{\langle i,j \rangle} \sigma_z^{[i]} \sigma_z^{[j]} + \frac{h_x}{2} \sum_i \sigma_x^{[i]} + \frac{h_z}{2} \sum_i \sigma_z^{[i]}, \text{ with } L_\mu = \sqrt{\gamma}\, \sigma_-^{[\mu]}$$

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Interesting for a number of reasons

This model is relevant for experiments of ultracold gases of Rydberg atoms

F. Letscher et al, arXiv:1611.00627; N. Malossi et al, Phys. Rev. Lett. 113, 023006 (2014)

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Phase diagram of the steady state of this model is still controversial:

Existence of a bistable phase in the steady state??

Tony E. Lee, H. Häfner and M. C. Cross, Phys. Rev. A 84, 031402 (2011); M. Marcuzzi et al, Phys. Rev. Lett. 113, 210401 (2014); M. F. Maghrebi and A. V. Gorshkov, Phys. Rev. B 93, 014307 (2016); J. J. Mendoza-Arenas et al, Phys. Rev. A 93, 023821 (2016)

1st order phase transition?

H. Weimer, Phys. Rev. A 91, 063401 (2015); H. Weimer, Phys. Rev. Lett. 114, 040402 (2015)

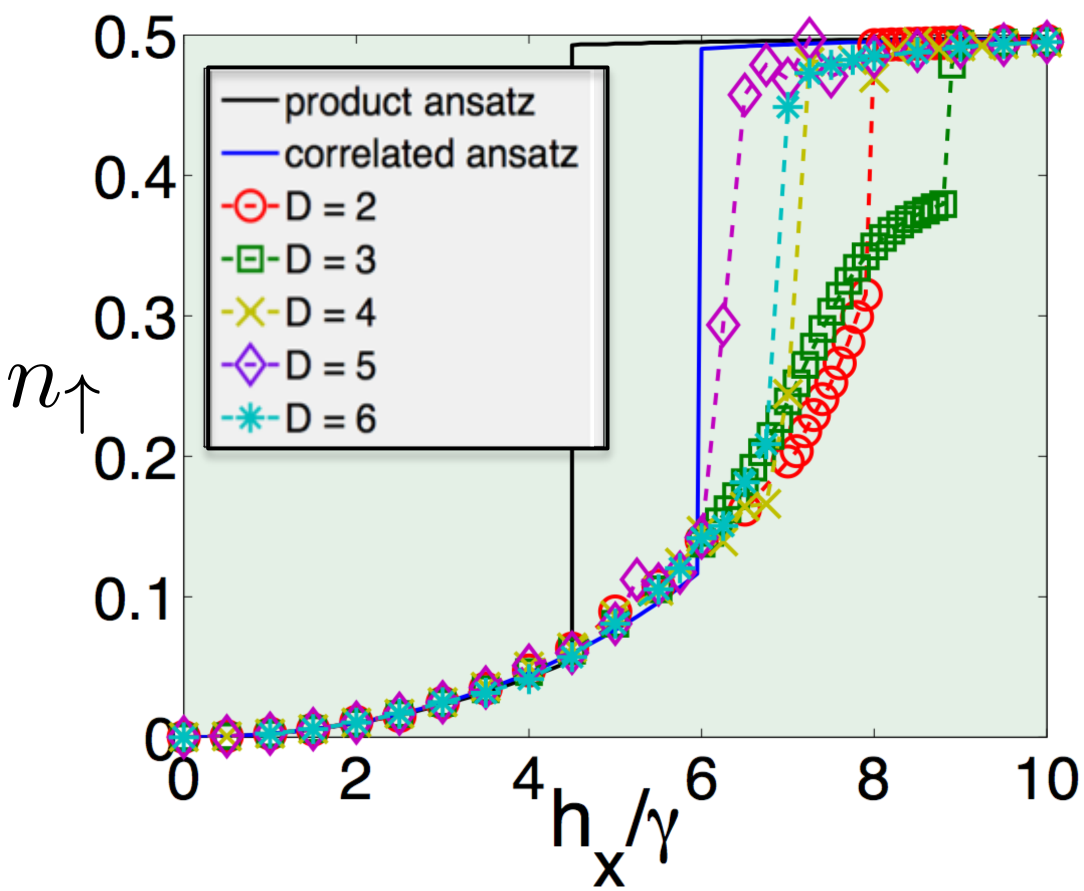
Antiferromagnetic phase in the steady state?

Tony E. Lee, H. Häfner and M. C. Cross, Phys. Rev. A 84, 031402 (2011); M. Höning et al, Phys. Rev. A 87, 023401 (2013)

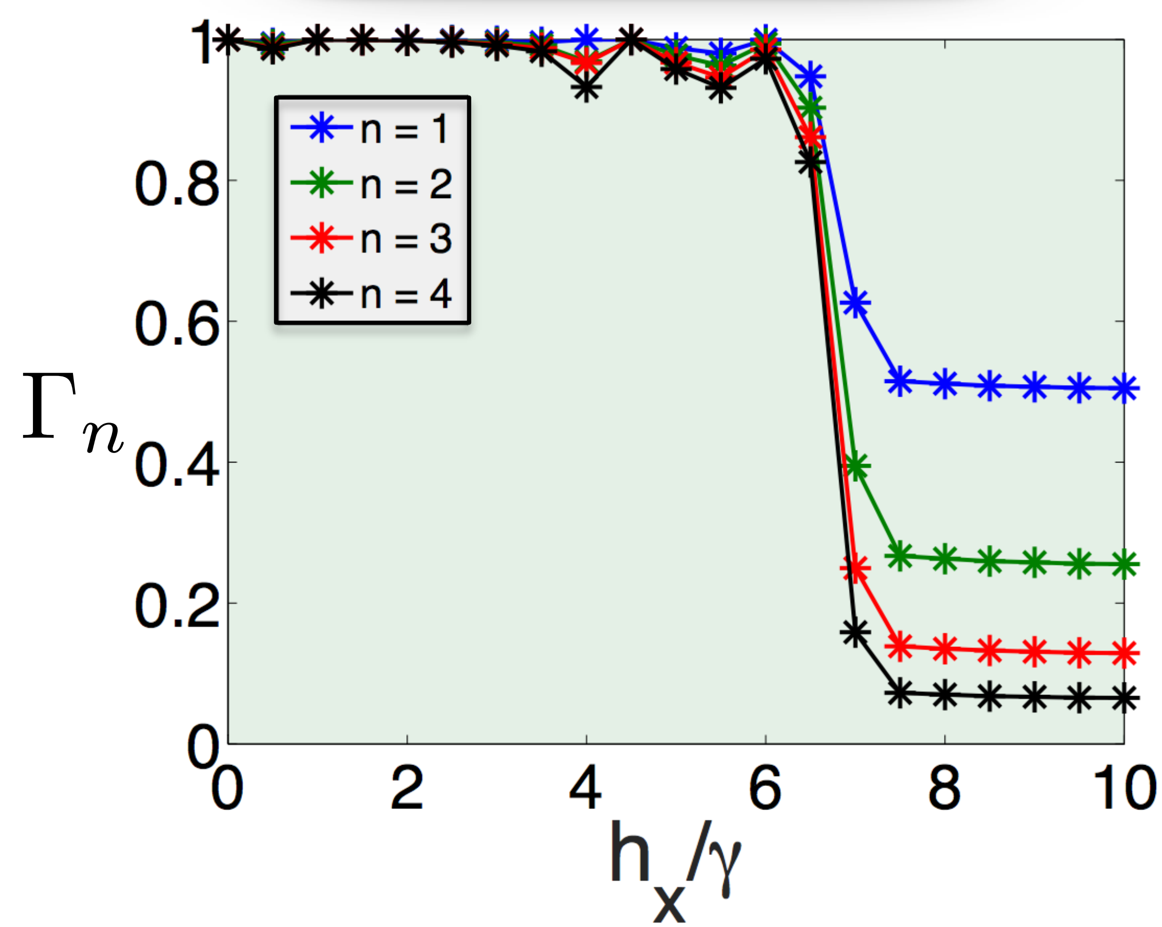
Benchmark results

$$V = 5\gamma, \gamma = 0.1, h_z = 0$$

spin-up density



purity of n-sites



$$n_\uparrow = \frac{1}{2N} \sum_{i=1}^{N} \left\langle \left(1 + \sigma_z^{[i]} \right) \right\rangle$$

$$\Gamma_n = \mathrm{tr}(\rho_n^2)$$

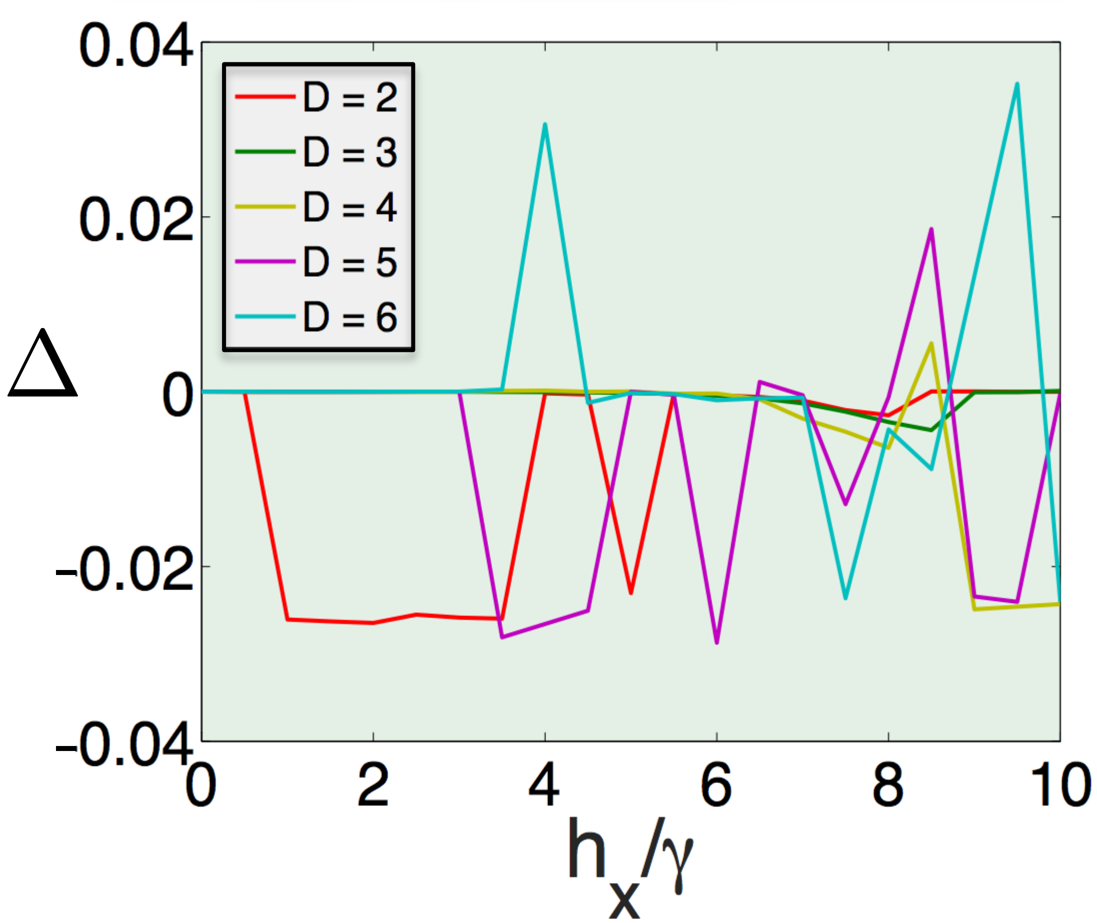
Good agreement with the correlated variational ansatz
1st order transition
No bi-stability

H. Weimer, Phys. Rev. A 91, 063401 (2015); H. Weimer, Phys. Rev. Lett. 114, 040402 (2015)

Benchmark results

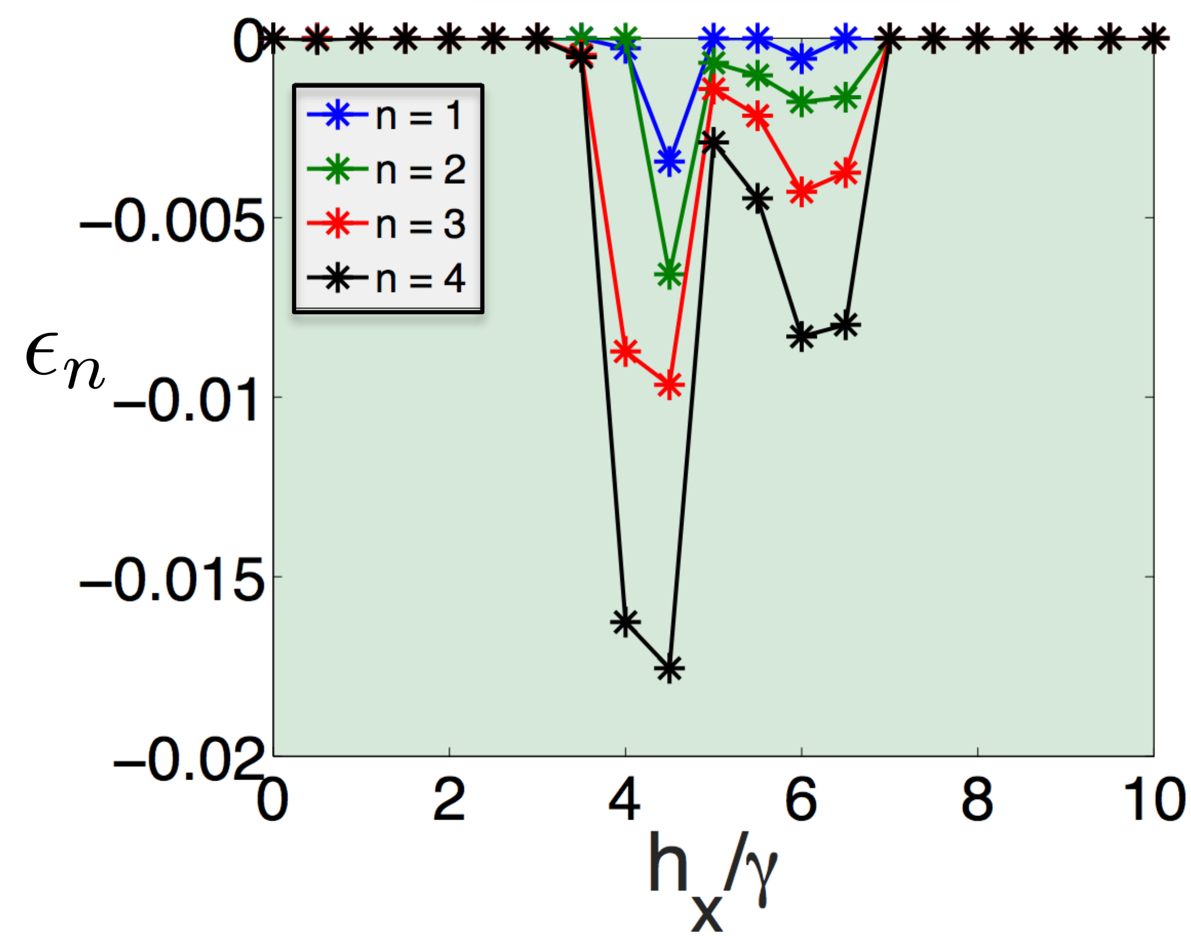
$$V = 5\gamma, \gamma = 0.1, h_z = 0$$

steady-state approximation



$$\Delta =_\sharp \langle \rho_s | \mathcal{L}_\sharp | \rho_s \rangle_\sharp$$

positivity error



$$\epsilon_n = \sum_{i|\nu_i<0} \nu_i(\rho_n)$$

Very good accuracy
error due to positivity: not very large

Benchmark results

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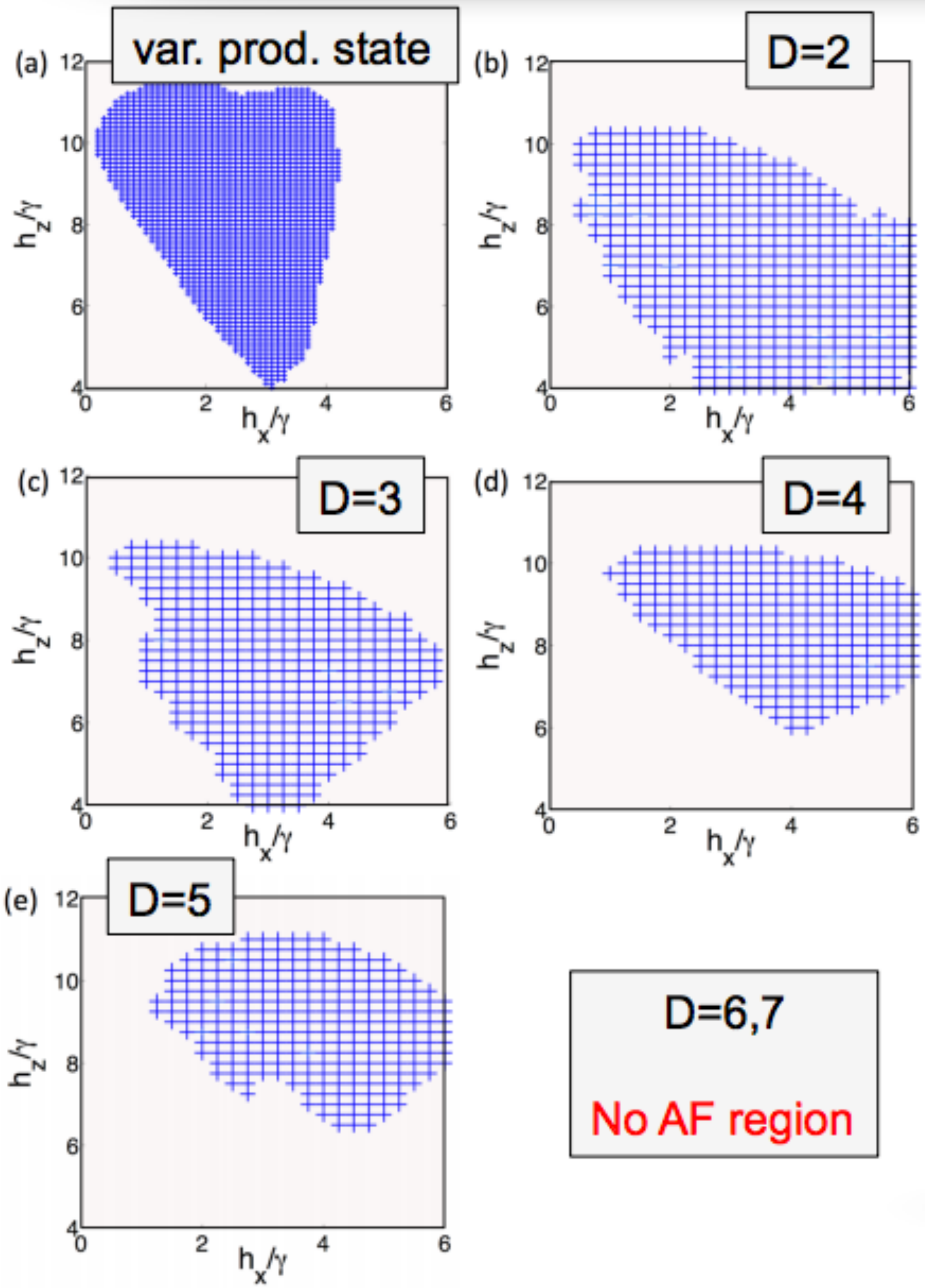

$$V = 5\gamma, \gamma = 0.1$$

Turning on the longitudinal field, previous studies have found the existence of AF region in the steady state phase diagram of h_x/γ vs h_z/γ

Tony E. Lee, H. Häfner and M. C. Cross, Phys. Rev. A 84, 031402 (2011)
M. Höning et al, Phys. Rev. A 87, 023401 (2013)

Using our techniques, the AF region is found to shrink from D = 2-5 and finally disappear for D = 6,7. This suggests that the AF region may not be there after all for these parameter regimes.

Conclusions & Outlook

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- Proposed a simple TN algorithm to approximate the steady states of 2d dissipative systems of infinite size.
  - very accurate results and relatively small errors induced.

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  - no bistable region in its steady state.
  - first order phase transition
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- Investigate into dissipative QPTs, Topological order by dissipation, etc?
- Connections to area-laws for rapidly-mixing dissipative systems?

Conclusions

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Hendrik Weimer,
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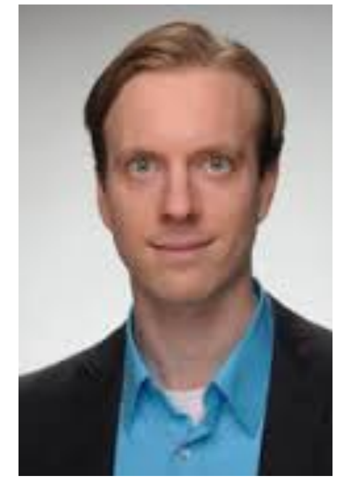

Hendrik Weimer,
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Thank you!