1. Relativistic Boltzmann equation

Show

$$\left(p^\mu \partial_\mu + mF^\mu \frac{\partial}{\partial p^\mu}\right) \theta(p^0)\delta(p^2 + m^2) = 0$$

provided that either $F^\mu p_\mu = 0$ or $F^\mu = -\partial^\mu m = 0$

Starting from the fluid-fleld coupling model

$$\partial_\mu T_{\mathrm{f}}^{\mu\nu} + \partial^\nu\phi \frac{\partial V_T(\phi)}{\partial\phi} = \tilde{\eta}\frac{\phi^2}{T}(U\cdot\partial\phi)\partial^\nu\phi$$

show that the entropy current ($S^\mu = sU^\mu$, $s = dp/dT$) satisfies

$$\partial_\mu S^\mu = \tilde{\eta}(\beta\phi)^2(U\cdot\partial\phi)^2$$

Don't forget that $T^{\mu\nu} = wU^\mu U^\nu + pg^{\mu\nu}$, $w = Ts$, and $p = g_{\mathrm{eff}}\frac{\pi^2}{90}T^4 - V_T(\phi)$.

3. Junction conditions

Starting from EM conservation across the bubble wall:

$$w_-\gamma_-^2 v_- = w_+\gamma_+^2 v_+, \quad w_-\gamma_-^2 v_-^2 + p_- = w_+\gamma_+^2 v_+^2 + p_+$$

show

$$v_+ v_- = \frac{p_+ - p_-}{e_+ - e_-}, \quad \frac{v_+}{v_-} = \frac{e_- + p_+}{e_+ + p_-}$$

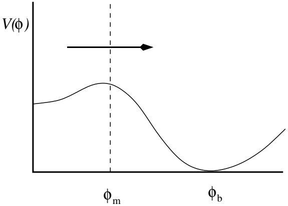
and hence

$$v_+ v_- = \frac{1-(1-3\alpha_+)r}{3-3(1+\alpha_+)r}, \frac{v_+}{v_-} = \frac{3+(1-3\alpha_+)r}{1+3(1+\alpha_+)r}$$

where

- $\epsilon_\pm = \frac{1}{4}(e_\pm - 3p_\pm)$, $\epsilon = \epsilon_+ - \epsilon_-$
- $\alpha_+ = \frac{4(\epsilon_+ - \epsilon_-)}{3w_+}$
- $r = w_+/w_-$

4. Quantum tunnelling vs. thermal activation



- Position ϕ
- momentum π
- Hamiltonian $H = \frac{1}{2}\pi^2 + V(\phi)$

Think about the quantum version of the thermal activation problem.

- *Quantum-Statistical metastability*, I. Affleck, PRL **46**, 388 (1981)
- *The uses of instantons*, S. Coleman, in *Aspects of Symmetry*

$$\Gamma = \frac{2}{\hbar}\mathrm{Im}E_0$$

where the imaginary part of the ground state energy is roughly

$$\mathrm{Im}E_0 \sim \frac{\hbar\omega}{2}e^{-S_0/\hbar}$$

and S_0 is the extremal value of the functional

$$S = \int d\tau \left(\frac{1}{2}\left(\frac{d\phi}{d\tau}\right)^2 + V(\phi)\right)$$