Direct numerical simulation of an early universe phase transition

- Ingredients:
  - Higgs field $-\ddot{\phi} + \nabla^2\phi - \frac{\partial V}{\partial \phi} = \eta W(\dot{\phi} + V^i\partial_i\phi)$ Ignatius et al (1994), Kurki-Suonio, Laine (1996)
    - η coupling to fluid (models energy transfer)
  - Relativistic fluid

$$\dot{E} + \partial_i(EV^i) + P[\dot{W} + \partial_i(WV^i)] - \frac{\partial V}{\partial \phi}W(\dot{\phi} + V^i\partial_i\phi) = \eta W^2(\dot{\phi} + V^i\partial_i\phi)^2.$$

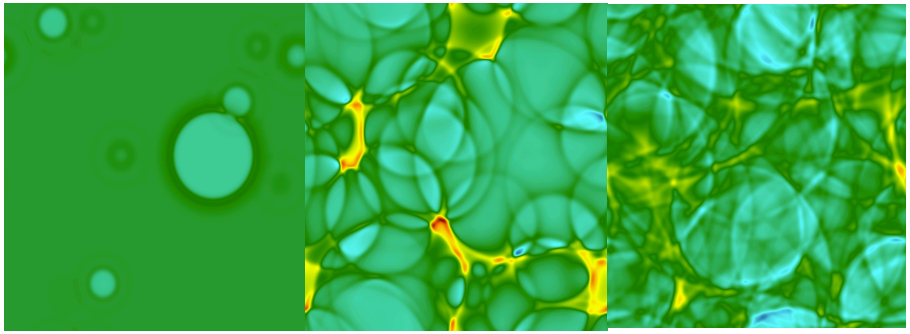
$$\dot{Z}_i + \partial_j(Z_iV^j) + \partial_i P + \frac{\partial V}{\partial \phi}\partial_i\phi = -\eta W(\dot{\phi} + V^j\partial_j\phi)\partial_i\phi.$$

    - E energy density, Z_i momentum density, V_i velocity, W γ-factor
  - Discretisation — Wilson & Matthews (2003); Different approach: Giblin, Mertens (2013)
  - Metric perturbation

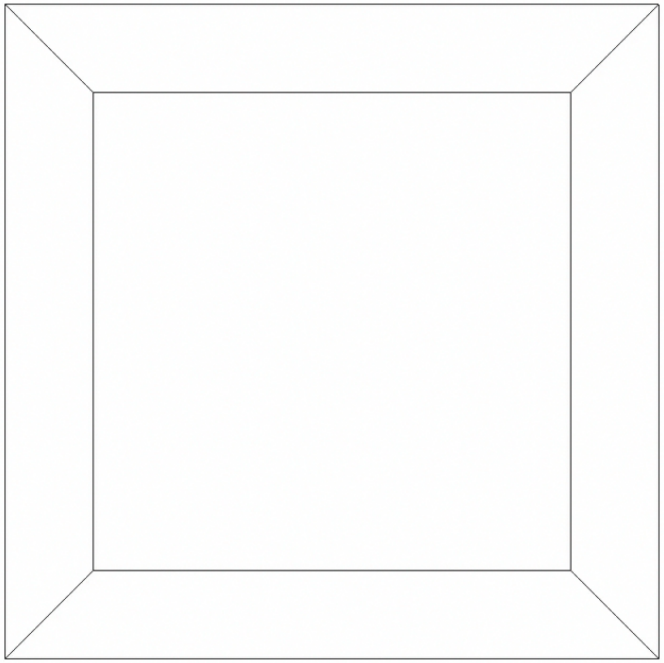
$$\ddot{h}_{ij} - \nabla^2 h_{ij} = 16\pi G T^{\mathrm{TT}}_{ij}$$ Garcia-Bellido, Figueroa, Sastre (2008)

Hydrodynamics of phase transitions

- Latent heat of transition goes into fluid compression waves – **sound** Hogan (1986)
- New: sound waves source gravitational waves long after transition is complete
- Duration ~ Hubble time - much longer than transition time



Fluid energy density as transition proceeds



Fluid kinetic energy density

Hindmarsh, Huber, Rummukainen, Weir (2013)
See also Giblin, Mertens (2013)

David Weir, Helsinki University

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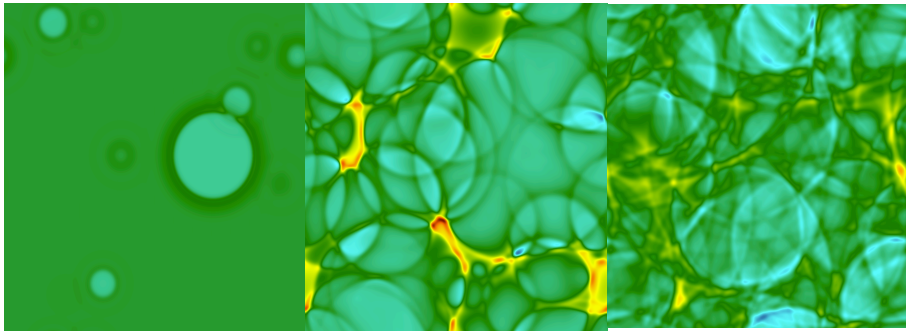

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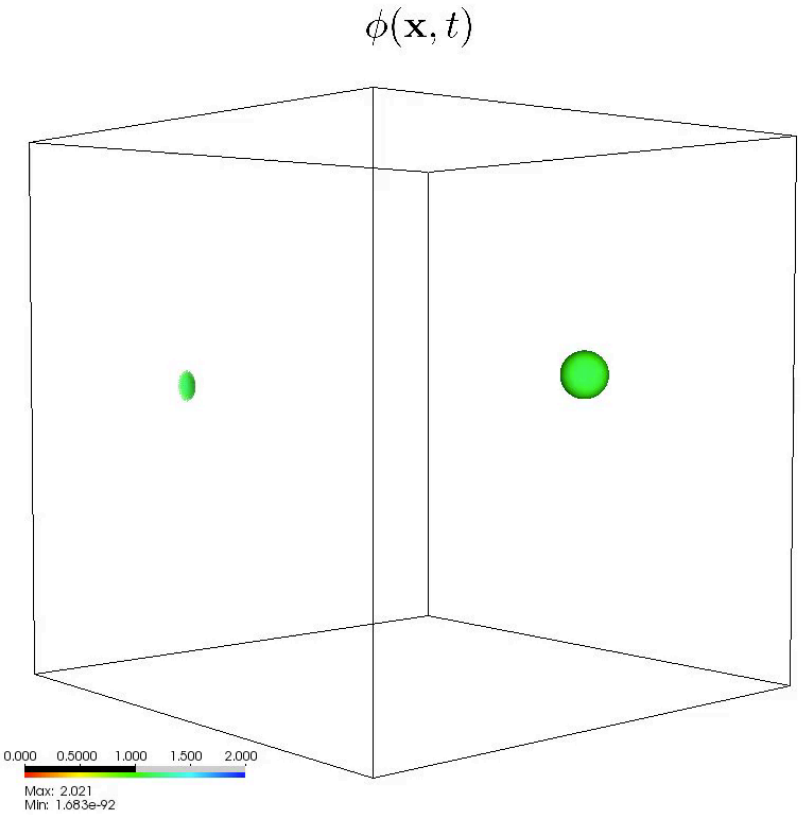


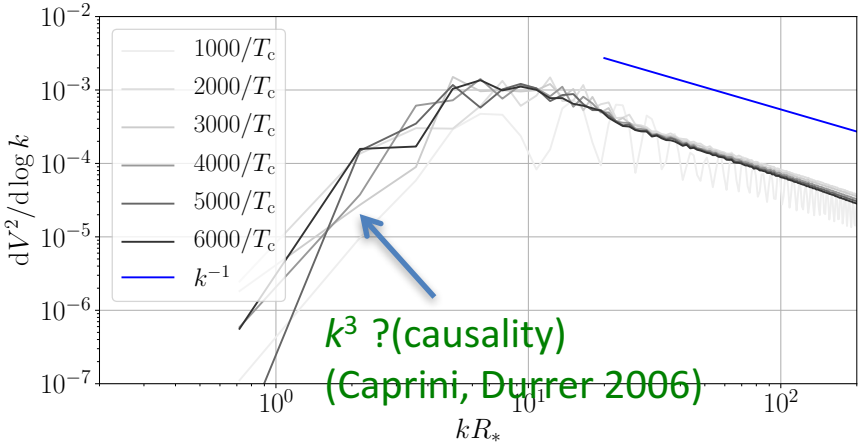
Higgs field

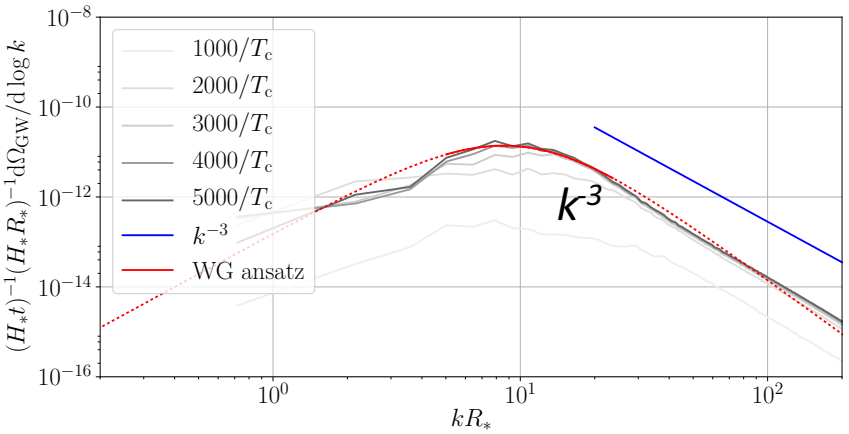
David Weir, Helsinki University

Hindmarsh, Huber, Rummukainen, Weir (2013)
See also Giblin, Mertens (2013)

Recent work

- Power laws in the GW power spectrum?
- 2015/6: 17M CPU-hours PRACE Tier-0
- 4200^3 lattice on 24k cores
- Parameters:
  - α = 0.012
  - v_w = 0.92
  - $R_* = 1920 T_c$
- Nucleate bubbles at the same time: clear bubble separation scale $R_* = (V/N_b)^{1/3}$
- Power laws above peak (detonations):
  - Velocity: k^{-1}
  - Gravitational waves: k^{-3}
  - Power laws for deflagrations steeper





Hindmarsh, Huber, Rummukainen, Weir 2017

GW energy density

$$\rho_{\rm GW} = \frac{1}{32\pi G}\langle \dot{h}_{ij}^2 \rangle .$$

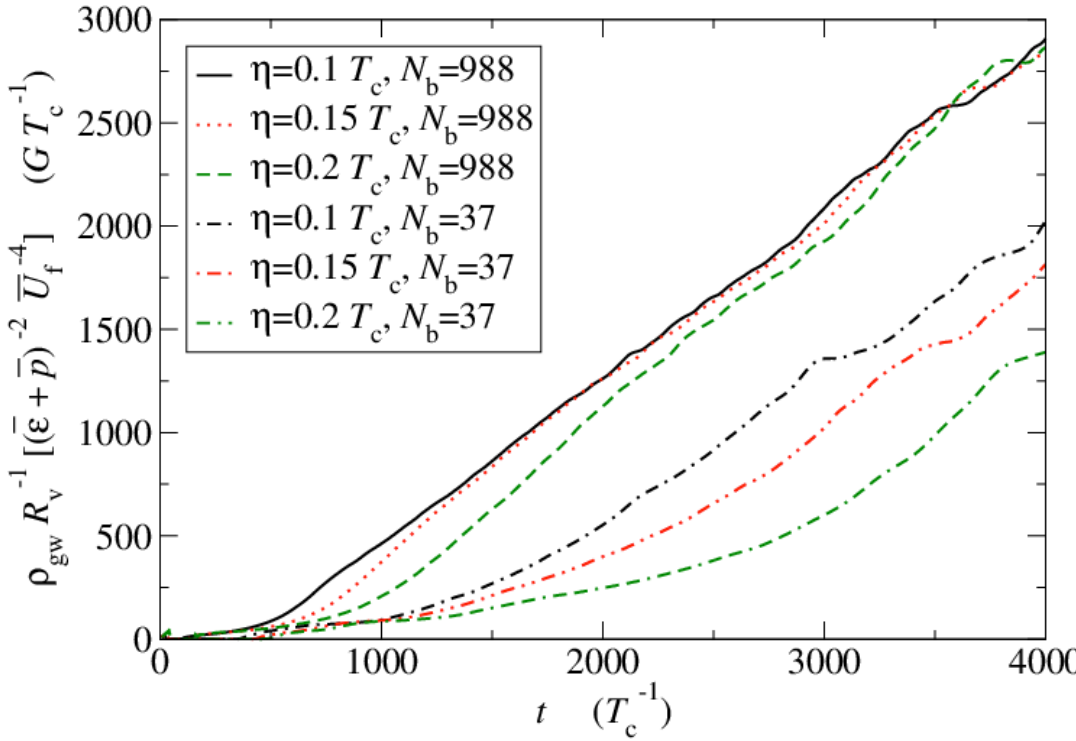
- Fluid sources GWs continuously:

$$T_{ij} = (\bar{\epsilon} + \bar{p}) V_i V_j |^{TT}$$

- GW energy density grows with time

$$\rho_{\rm GW} \propto t\, G L_{\rm f} [(\bar{\epsilon} + \bar{p}) \bar{U}_{\rm f}^2]^2$$

- GW energy density depends on
  - Lifetime of source $\tau_{\rm v}$
  - Flow length scale $L_{\rm f}$
  - Kinetic energy ($\epsilon\, v^2$)



Hindmarsh, Huber, Rummukainen, Weir (2015)

Sound shell model of GW production

- Fluid flow = sound waves
- Two length scales
  - bubble separation
  - sound shell thickness
- GWs produced by colliding shells of sound waves

David Weir

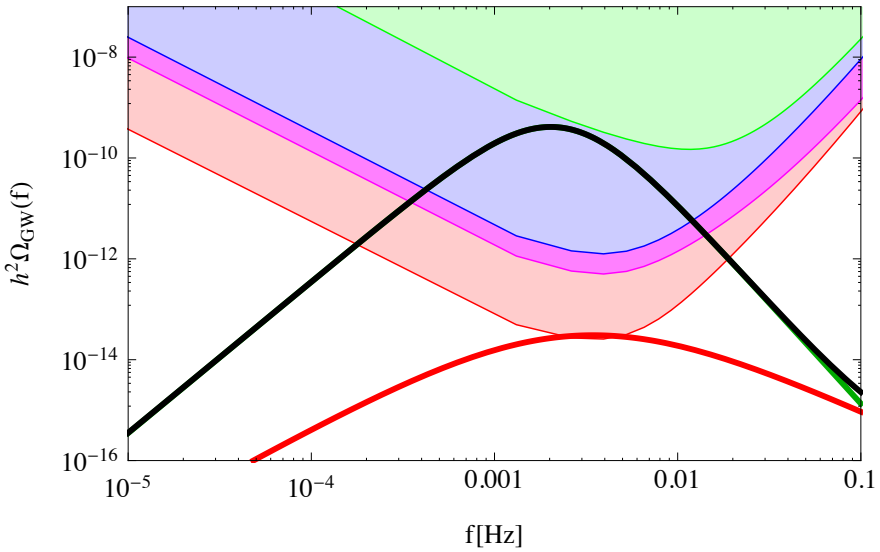
Hindmarsh 2017

(e)LISA prospects for electroweak-scale phase transitions

- 4 mission configurations

Name	C1	C2	C3	C4
Full name	N2A5M5L6	N2A1M5L6	N2A2M5L4	N1A1M2L4
# links	6	6	4	4
Arm length [km]	5M	1M	2M	1M
Duration [years]	5	5	5	2
Noise level	N2	N2	N2	N1

- e.g. strong detonation
  $\alpha = 0.5$, $v_w = 0.95$
- $1/(H_* R_*) = 100$
  (horizon distance/bubble separation)



Black: acoustic production
Red: turbulence model
Warning:
turbulence not properly understood

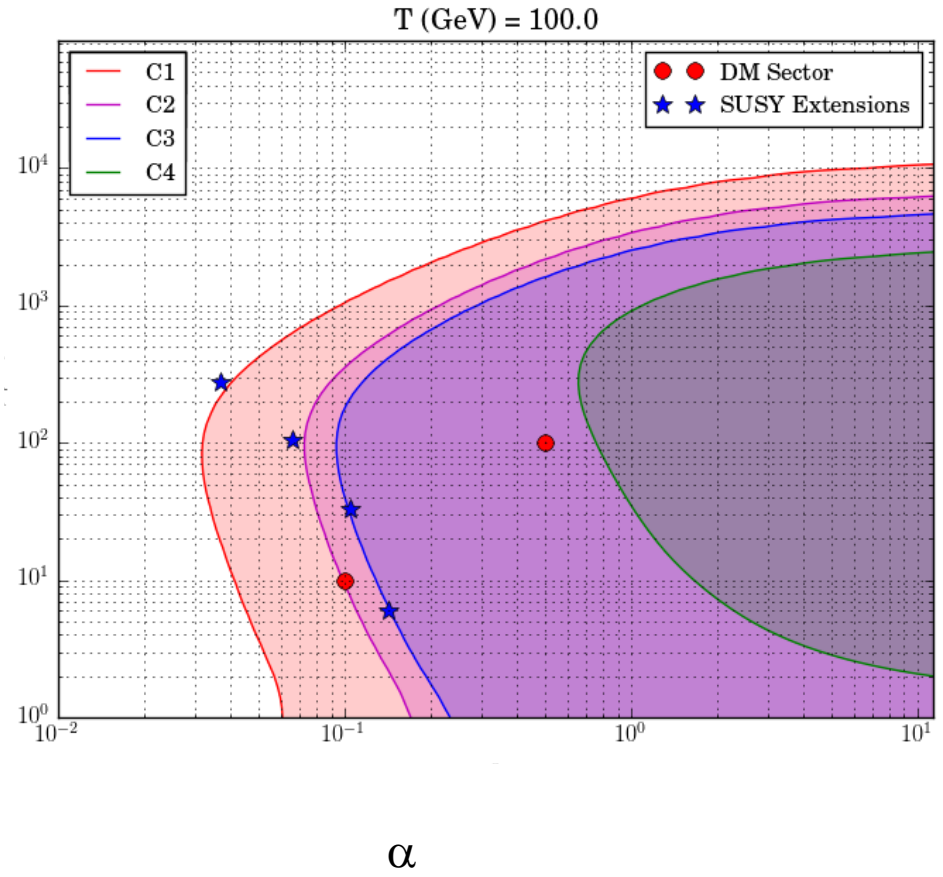
Caprini et al (2015) [eLISA Cosmology Working Group]

(e)LISA GW detection prospects

Caprini et al (2015) [eLISA Cosmology Working Group

Name	C1	C2	C3	C4
Full name	N2A5M5L6	N2A1M5L6	N2A2M5L4	N1A1M2L4
# links	6	6	4	4
Arm length [km]	5M	1M	2M	1M
Duration [years]	5	5	5	2
Noise level	N2	N2	N2	N1

- Need long arms, long mission and low noise
- C1 reaches down to $\alpha \sim 0.03$
- ESA proposal 2017
  - 6 links
  - 2.5M km arms
  - 4 year mission (extendable to 10 yr)



Summary

- GW astronomy has begun (LIGO/VIRGO)
- 1st order phase transitions in the early universe can produce gravitational waves
  - Source: sound waves from the nucleating droplets of the low temperature phase
  - Significantly larger than previous estimates
  - GW spectrum contains information about fluid velocity field
- LISA 2028 … 2034 (Taiji, DECIGO …)
  - Could detect an electroweak phase transition at t = 10 picoseconds
  - Measure latent heat, phase boundary speed, bubble nucleation rate?
  - Probe new physics beyond the reach of LHC