Momentum signatures of the 3D Anderson metal-insulator transition

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Overview

1. Motion in disordered media *(crash course)*
a) From Diffusion to Weak and Strong localization
b) One-parameter hypothesis and finite size scaling

2. Localization signatures in momentum space
a) Numerical scenario
b) CBS & CFS peaks

3. Critical properties of the 3D Anderson transition
a) Width of the CBS peak
b) Contrast of the CFS peak (smoking gun of AT)

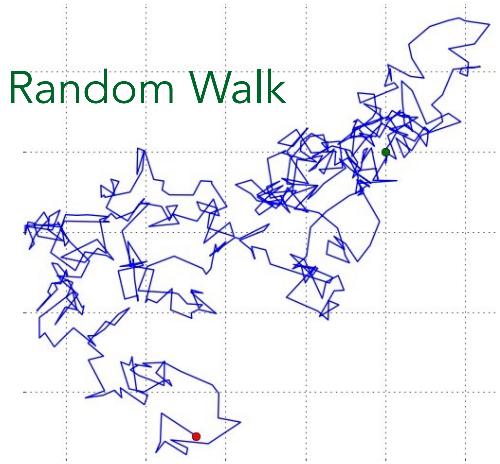
4. Perspectives (multi-fractality)

5. Summary

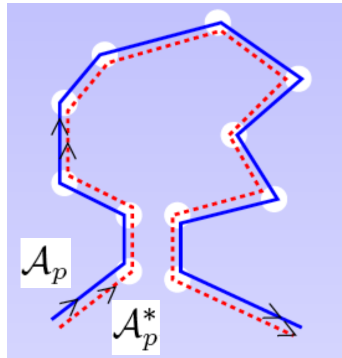
Waves in disordered media in a nutshell

- ***Pinball Wizard***

Discard interference
Drude/Boltzmann model
Large-scale motion = Diffusion
Conductance $g=\sigma L^{d-2}$ (Ohm's law),
L = sample size.



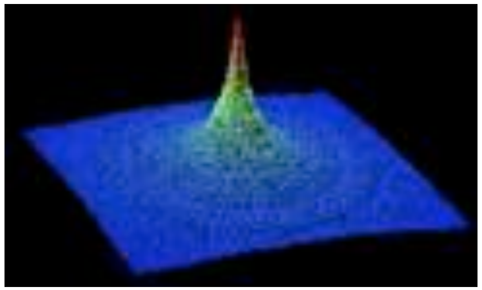
Ladder



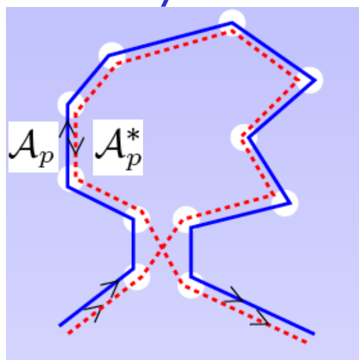
- ***There has to be a twist*** LOOPS

Interference DO play a role
(quasi) loops
Coherent backscattering (CBS)
Weak localization corrections

CBS

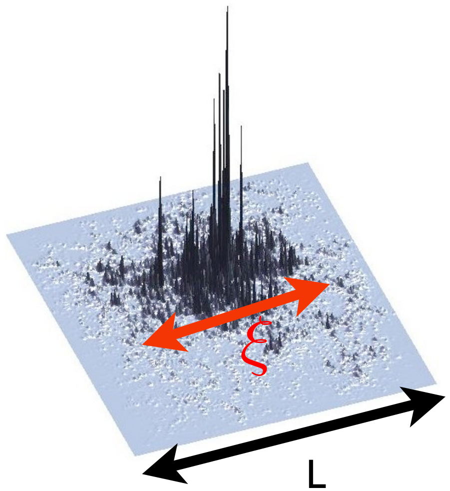


Maximally-Crossed



- ***Sure plays mean pinball***

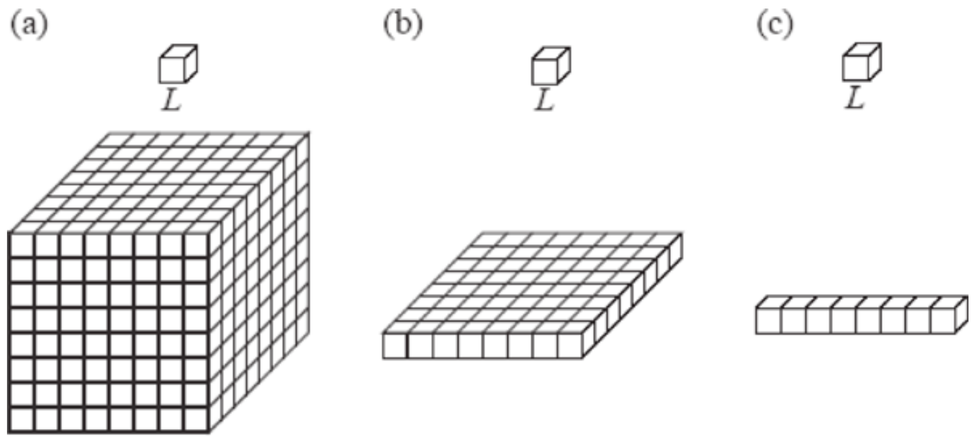
Interference breaks transport
Strong localization
Disorder-driven Metal-Insulator transition



Milestones:
Anderson Localization (58)
Scaling Theory of Localization (79)

One-parameter hypothesis (Gang of 4)

How does the dimensionless conductance g change when the system size L increases?



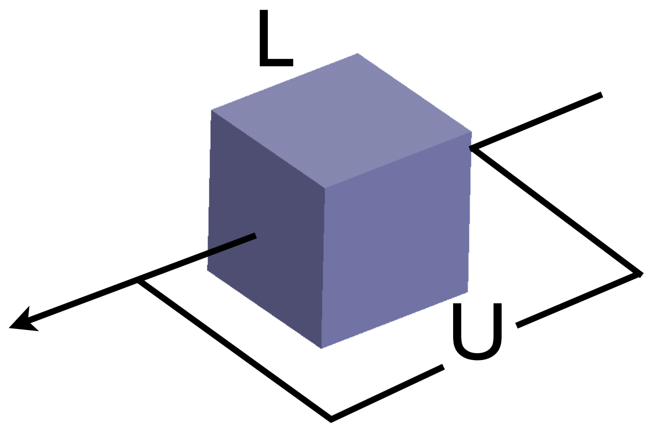
- **One-parameter scaling hypothesis:** $$\boxed{\frac{\partial \ln g}{\partial \ln L} = \beta(L, E, \ell_s, \ldots) \equiv \beta(g) = \frac{d \ln g}{d \ln L}}$$ scaling function

- **Equivalently:** $$\boxed{g(L, E, \ell, \ldots) \equiv F\left[\frac{L}{\xi_{cor}(E, \ell, \ldots)}\right]}$$ For L large enough

All microscopic lengths subsumed into a single **correlation length**

Scaling function in metallic and localized regimes

Dimensionless conductance g and $\beta(g)$



Metallic regime (diffusion process)

$g \gg 1$

$$I = jS = L^{d-1}\, j$$

$$j = \sigma E$$

$$E = \frac{U}{L} \qquad \sigma = \frac{2mD}{\hbar} = \frac{2}{d} k\ell$$

$$G = \frac{I}{U} = L^{d-2}\,\sigma \quad \text{Ohm's law}$$

$$\boxed{g = (kL)^{d-2}\sigma = \frac{2}{d}\,(kL)^{d-2}\,k\ell}$$

$$\boxed{\beta(g) = d-2}$$

Localized regime

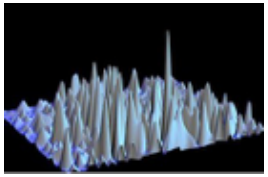
$g \ll 1$

$$\boxed{g = \exp(-L/2\ell)}$$

$$\boxed{\beta(g) = \ln g}$$

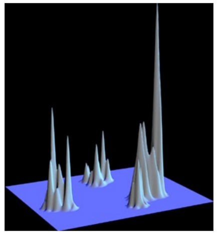
Match behaviors assuming monotony

Anderson Metal-Insulator transition



Metal: $\beta(g) > 0$
$g \uparrow$ as $L \uparrow$

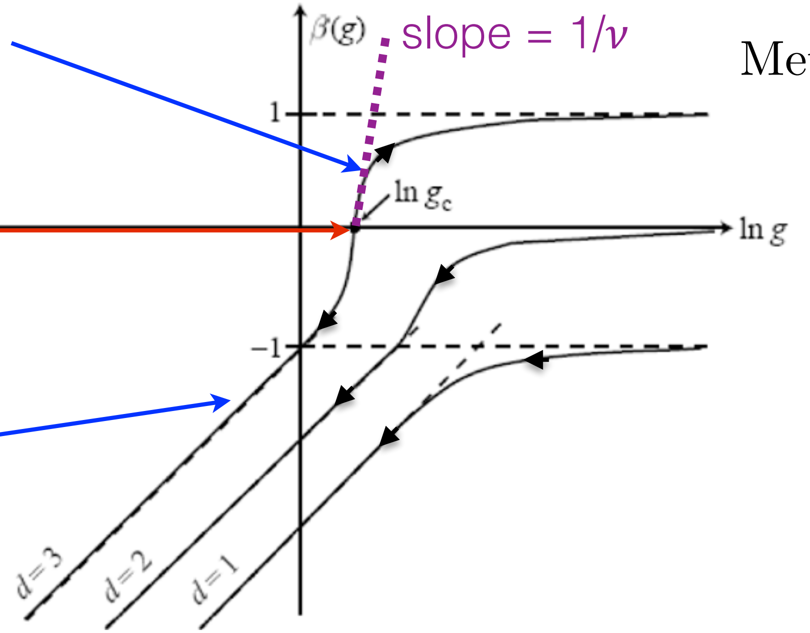
Metal-Insulator transition in 3D $\beta(g) = 0$



Insulator: $\beta(g) < 0$
$g \downarrow$ as $L \uparrow$

$L > \xi$

1D & 2D: Localization is the rule $\beta(g) < 0$



Metallic side: $g \propto L^{d-2}$

$\boxed{\beta(g) = d - 2}$

smooth monotonic interpolation

Insulating side: $g \sim g_0 \exp(-L/\xi)$ $\boxed{\beta(g) = \ln(g/g_0)}$

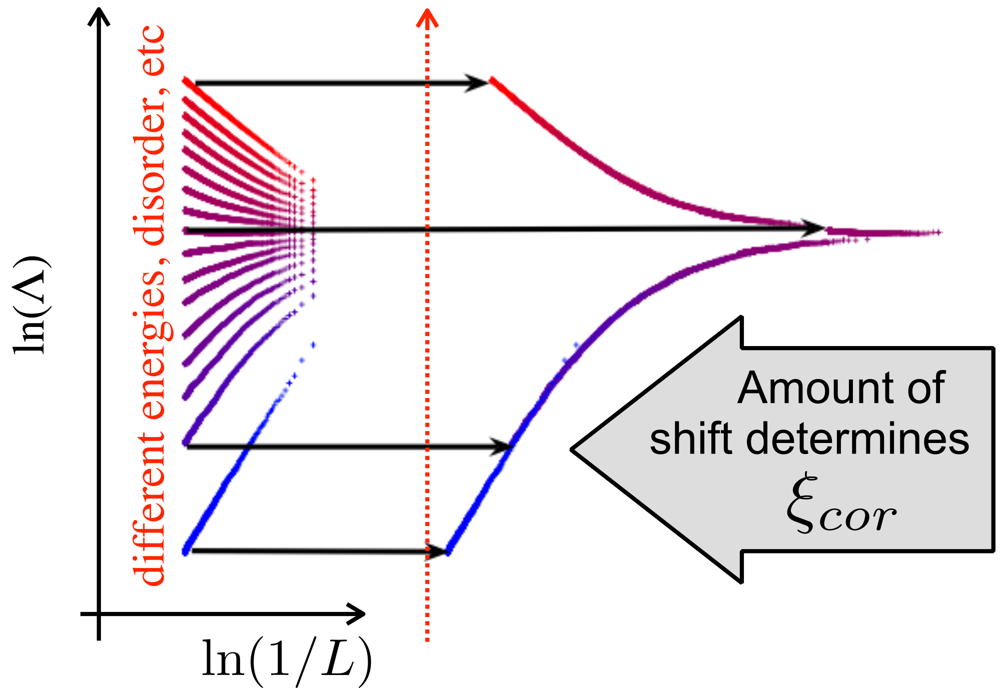
Correlation length & Finite size scaling

$$\boxed{g(L,E,\ell,...) \equiv F\left[\frac{L}{\xi_{cor}(E,\ell,...)}\right]}$$

For L large enough

Any critical physical observable *Λ(E,L, …)* is a function of $x = L/\xi_{cor}$ alone around the mobility edge with **same** critical exponents.

How to construct the scaling function?



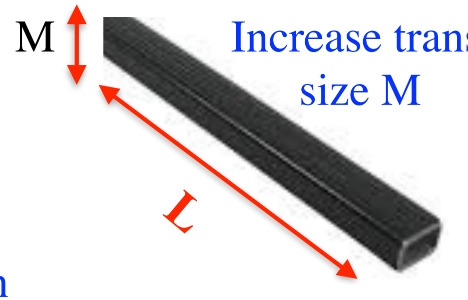
$$\ln(\xi_{cor}/L) = \ln(1/L) + \ln(\xi_{cor})$$

Translation by $\ln(\xi_{cor})$

- **How to compute ξ_{cor} ?** Quasi-1D: all states are localized. Extrapolate to 3D.

$$\frac{1}{\lambda(E,M)} = -\lim_{L\to\infty}\frac{1}{2L}\ln \mathrm{Tr}[G_L^\dagger(E,M)G_L(E,M)]$$

Lyapunov exponent *via* Transfer-Matrix method



Increase transverse size M

Localized side: $\lambda(E,M)\to\xi(E)$ 3D localization length

Mobility edge: $\lambda(E,M)\sim M$ scales with M

Diffusion constant

Metallic side: $\lambda(E,M)\to(\#\text{ tranverse channels})\times\xi_{1D}(E) = 2\pi\hbar\overline{\rho}(E)D(E)\,M^2$

Av-DoS per unit volume

$$\boxed{\text{1-parameter hypothesis} \Rightarrow \Lambda = \frac{\lambda(E,M)}{M}\text{is a scaling quantity} \Rightarrow \Lambda \equiv \Lambda(x)} \qquad x = \frac{M}{\xi_{cor}(E)}$$

By identification:

Localized side $\xi_{cor}(E)\propto\xi(E)\sim(E_c-E)^{-\nu}$ near the critical point

(must DV at critical point)

Metallic side $\xi_{cor}(E)\propto\dfrac{1}{2\pi\hbar\overline{\rho}(E)D(E)}\sim(E-E_c)^{-s}$ near the critical point

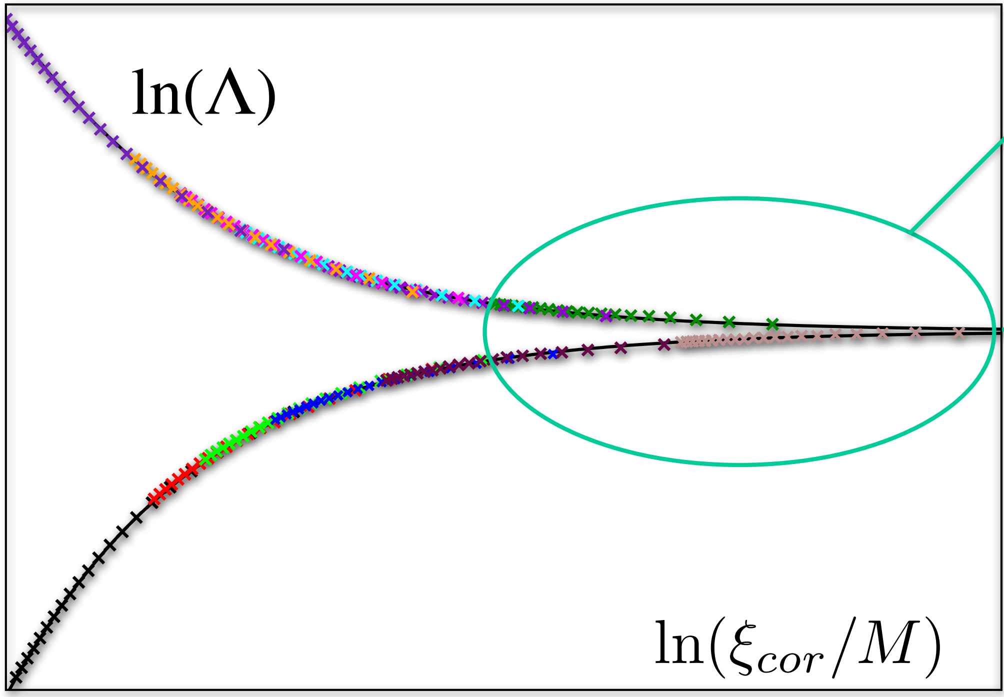
Wegner's scaling law
$s=(d-2)\nu=\nu$ (3D)

$$\boxed{\xi_{cor}(E)\sim|E-E_c|^{-\nu}}$$

critical exponent

mobility edge

Extracting the mobility edge and critical exponents



Critical exponent ν

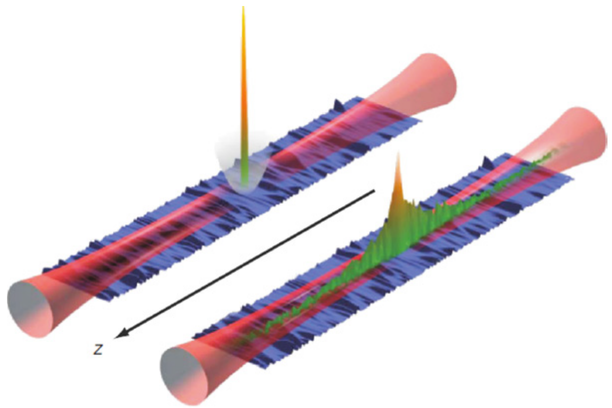
Slevin's fit method (double Taylor expansion, corrections to scaling)

Mobility edge E_c

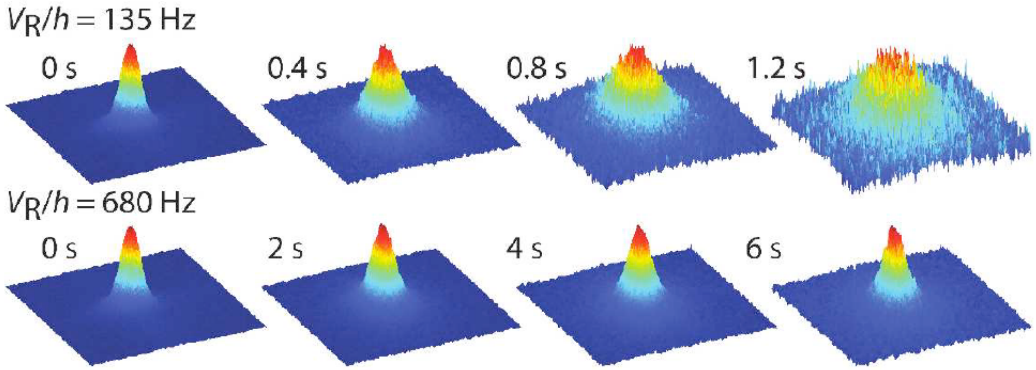
Localization in spatial disorder: momentum space signatures

Anderson localization with cold atoms so far: **real-space signatures**

Cold atoms - 1D Speckle (2008)



Cold atoms - 3D Speckle (2011)



Momentum distribution easily measurable for cold atom experiments

Are there any signatures of Anderson localization in momentum space?

Numerical road map to the momentum distribution

Hamiltonian $\hat{H} = \frac{\hat{p}^2}{2m} + \boxed{V(\mathbf{r})}$ Random potential

Optical laser Speckle,
Uncorrelated box distribution,
Continuous or lattice model, *etc*

Initial cloud with a kick	Time evolution in random medium	Experimentally feasible	"Time-of-flight" measurement
(1) Start with an initial plane wave	(2) Propagate with the Hamiltonian ***H***	(3) Filter around energy ***E***	(4) Compute the momentum density
$\lvert\psi(t=0)\rangle = \lvert\boldsymbol{k}_0\rangle$	$\lvert\psi(t)\rangle = e^{-i\hat{H}t/\hbar}\lvert\mathbf{k}_0\rangle$	$\lvert\psi_E(t)\rangle = \hat{F}_\sigma(E)\lvert\psi(t)\rangle$ $\hat{F}_\sigma(E) = e^{-\frac{(E-\hat{H})^2}{2\sigma^2}}$	$\lvert\langle\mathbf{k}\lvert\psi_E(t)\rangle\rvert^2$

(5) Repeat over many disorder configurations

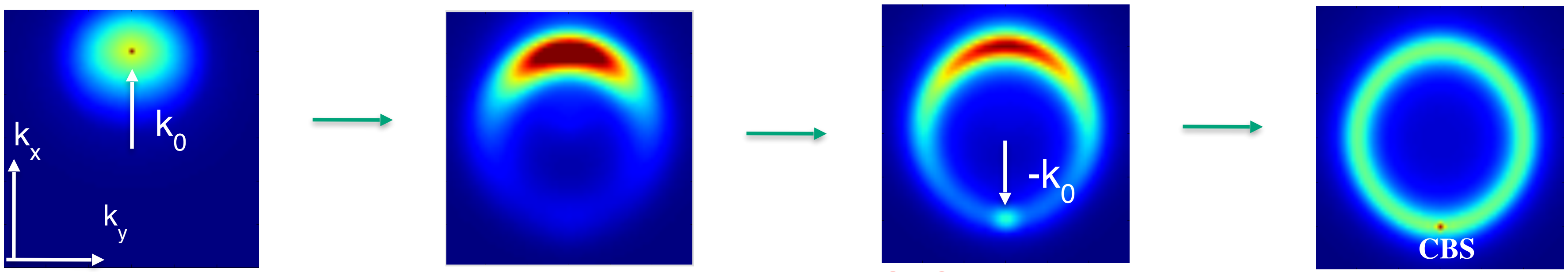
$$n_E(\mathbf{k},t) = \overline{|\langle\mathbf{k}|\psi_E(t)\rangle|^2}$$

In presence of V(***r***), initial wave spreads over a broad range of energies E (cf ***spectral function***). As a result, properties depending sharply on E are blurred by the energy spread. Need for *energy-filtering*.

2D numerical experiment: What do we see?

- **“Early times”** (in units of the correlation time) Isotropization and CBS

Time = 1 Time = 4 Time = 8 Time = 20

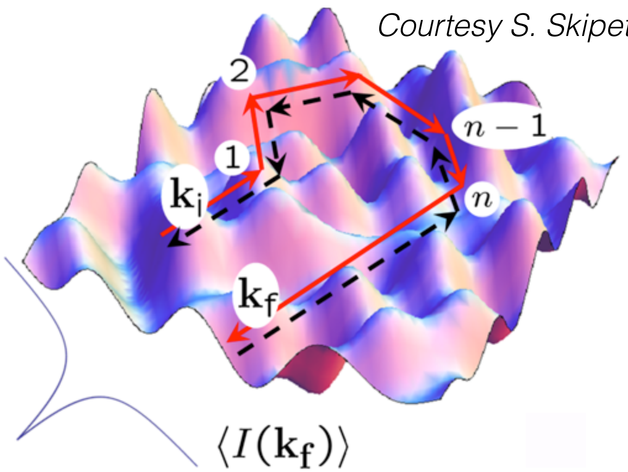


Few scatterings

CBS starts to appear at $-\mathbf{k}_0$

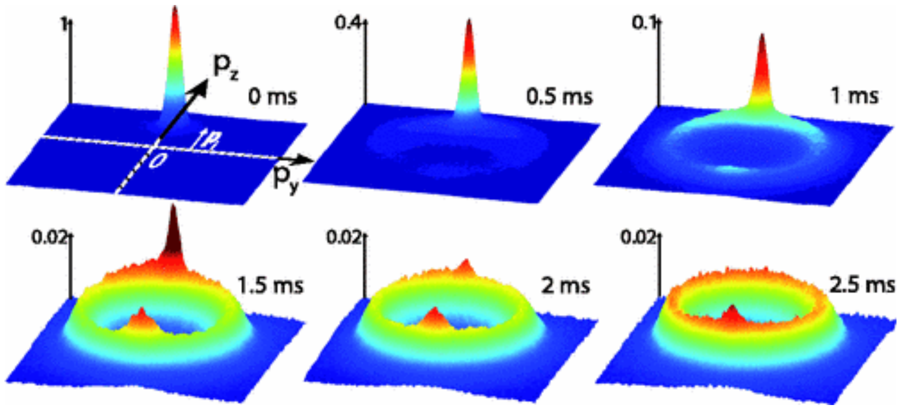
CBS peak on top of diffusive background

Courtesy S. Skipetrov



- CBS is a 2-path interference effect
- Signature of phase coherence

Observed in cold-atomic system



*N. Cherroret et al., PRA **85**, 011604(R) (2012)*
*F. Jendrzejewski et al., PRL **109**, 195302 (2012)*
*G. Labeyrie et al., EPL **100**, 66001 (2012)*

An important object: the spectral function

Retarded Green's function

Spectral function

$$G^R(E) = \frac{1}{E - H + \mathrm{i}\epsilon} \Rightarrow A(\mathbf{k}, E) = -2\langle \mathbf{k}|\mathrm{Im}\overline{G^R(E)}|\mathbf{k}\rangle = \lim_{L\to\infty}\left(\frac{2\pi}{L}\right)^d \overline{\sum_n |\langle \mathbf{k}|\varphi_n\rangle|^2 \delta(E - \varepsilon_n)}$$

Eigenspectrum for a given disorder realization, system size L.

$$\boxed{\int \frac{d\mathbf{k}}{(2\pi)^d} A(\mathbf{k}, E) = 2\pi\nu(E)}$$

$\nu(E)$ = density of states per unit volume

$$\boxed{\int \frac{dE}{2\pi} A(\mathbf{k}, E) = 1}$$

$\frac{A(\mathbf{k}, \mathbf{E})}{2\pi\nu(E)} \equiv$ average momentum distribution at energy E

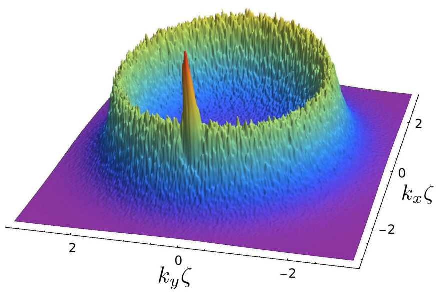
A(**k**,E) = probability density to have energy E at momentum **k**.

$$n_D(\mathbf{k}) = \lim_{t\to\infty} n_D(\mathbf{k}, t) = \int \frac{dE}{2\pi} \boxed{\frac{A(\mathbf{k}, E)}{2\pi\nu(E)}}\,\boxed{A(\mathbf{k_0}, E)}$$

Exact result (ergodicity)

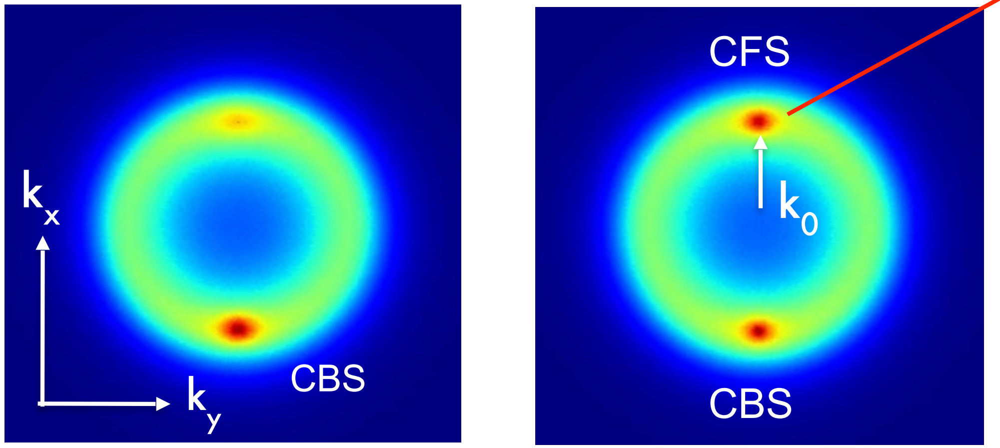
No sign of Localization in there.

Describes the isotropic ring structure of the diffusive background



Is that *ALL*? Coherent Forward Scattering peak!

- Dynamics at **"Long times"**



Time=100 Time=1000

A pleasant surprise!
New peak emerges at $+\boldsymbol{k}_0$

T. Karpiuk et al., PRL **109**, 190601 (2012)
K. L. Lee et al., PRA **90**, 043605 (2014)
S. Ghosh et al., PRA **90**, 063602 (2014)

Relevant time scale is the **Heisenberg time**

$$\tau_H = 2\pi\hbar\,\overline{\rho}(E)\,\xi^3(E)$$

CFS is a genuine signature of Anderson localization in the bulk!

For time-reversal invariant systems (GOE), CBS and CFS become **twin peaks** in the long-time limit (localized regime).

$$n_E(\mathbf{k}_0, t) = \int \frac{d\omega}{2\pi}\, e^{-i\omega t}\, \overline{\rho(\mathbf{k}_0, E)\rho(\mathbf{k}_0, E-\hbar\omega)}$$

$$\rho(\mathbf{k}_0, E) = 2\pi \sum_n |\psi_n(\mathbf{k}_0|^2\ \delta(E-\epsilon_n)$$ LDOS in momentum space (*aka* spectral function)

CFS related to LDOS-LDOS correlations in momentum space

Relation to Form Factor

- **Spectral decomposition:**

$$|\psi(t)\rangle = e^{-iHt/\hbar}|k_0\rangle \;=\sum_n \phi_n^*(k_0)e^{-iE_nt/\hbar}|\phi_n\rangle$$

Eigenenergies Eigenfunctions

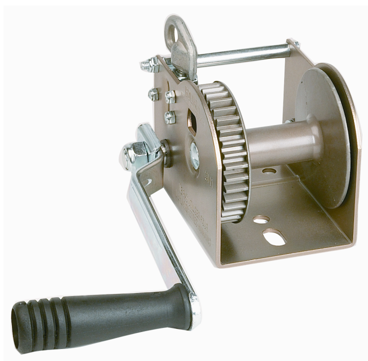
- **Momentum distribution in the forward direction:**

$$n(\mathbf{k}_0,t) = \overline{|\langle \mathbf{k}_0|\psi(t)\rangle|^2} = \overline{\sum_{n,m} |\phi_n(\mathbf{k}_0)|^2\, |\phi_m(\mathbf{k}_0)|^2\, e^{-i(E_n-E_m)t/\hbar}}$$

$$n_E(\mathbf{k}_0,t) = \int \frac{d\omega}{2\pi}\, e^{-i\omega t}\,(2\pi)^2\overline{\sum_{n,m} |\phi_n(\mathbf{k}_0)|^2\, |\phi_m(\mathbf{k}_0)|^2 \quad \delta(E-E_n)\delta(E-\hbar\omega-E_m)}$$

RMT-inspired decoupling

$$n(\mathbf{k}_0,t) = \int \frac{dE}{2\pi}\, n_E(\mathbf{k}_0,t)$$



$$(\ldots)$$

CFS contrast: $\boxed{\Lambda(t) = 2\pi\hbar\, L^d\, \overline{\rho}(E)\, K_E(t)} = 1 \quad \text{for } t/\tau_H \to \infty$

Form factor

Heisenberg time

$$\boxed{\tau_H(E) = 2\pi\hbar\,\overline{\rho}(E)\,\xi^d(E)}$$

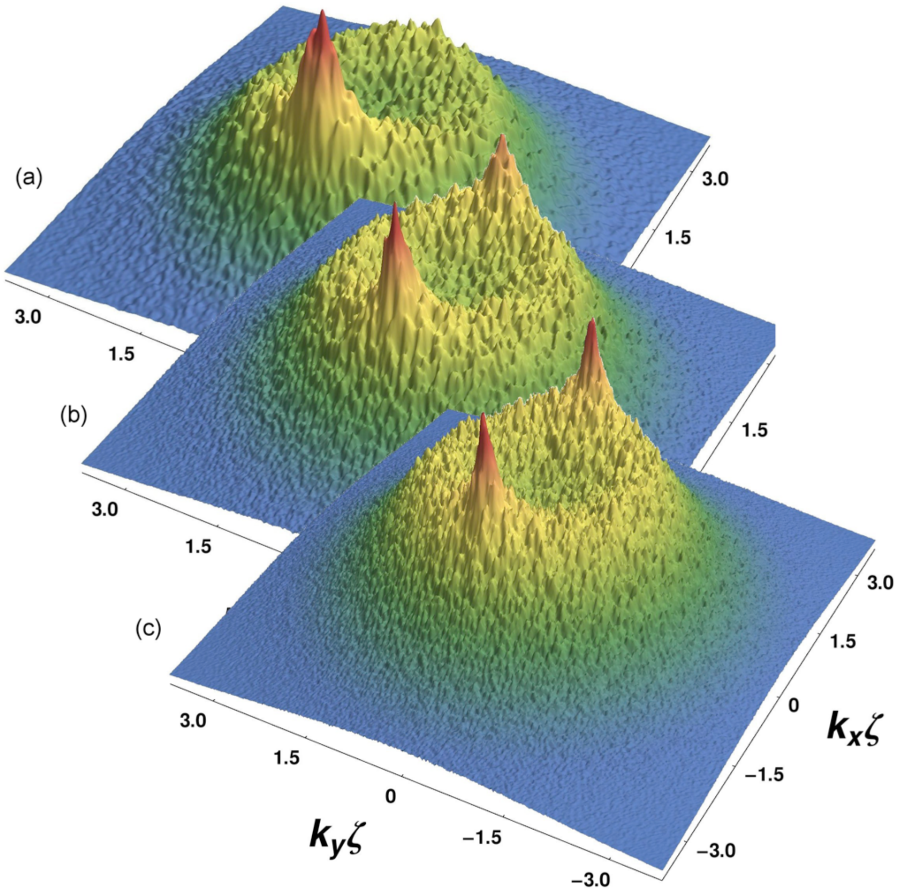
$\xi(E) \equiv \text{localization length}$

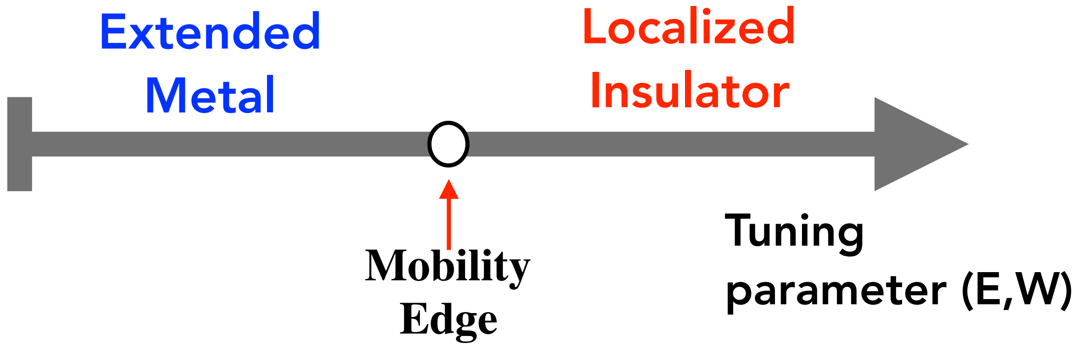
$$\overline{\rho}(E) = \frac{1}{L^d}\overline{\sum_n \delta(E - E_n)}$$ **av-DOS per unit volume**

$$K_E(t) = \int \frac{d\omega}{2\pi}\, K_E(\omega)\, e^{-i\omega t}$$

$$K_E(\omega) = \frac{\overline{\rho(E)\rho(E - \hbar\omega)}}{\overline{\rho}^2(E)} - 1$$ DOS-DOS correlator

Critical properties of the 3D AT from CBS & CFS





CBS & CFS arise because of interference mechanisms leading ultimately to localization: ***they must be sensitive to the nature, localized or extended, of the eigenstates:***

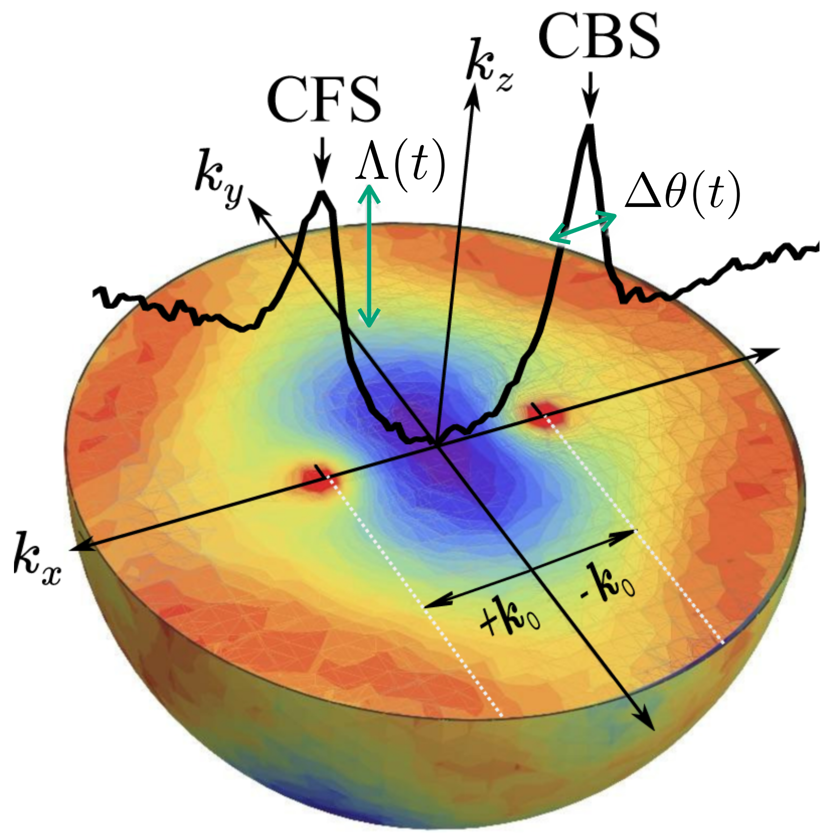
CLAIM

The critical properties of the 3D Anderson transition are encoded in:

1) Dynamics of the CBS width

2) Dynamics of the CFS contrast

- How to prove this?
- How to extract critical quantities?



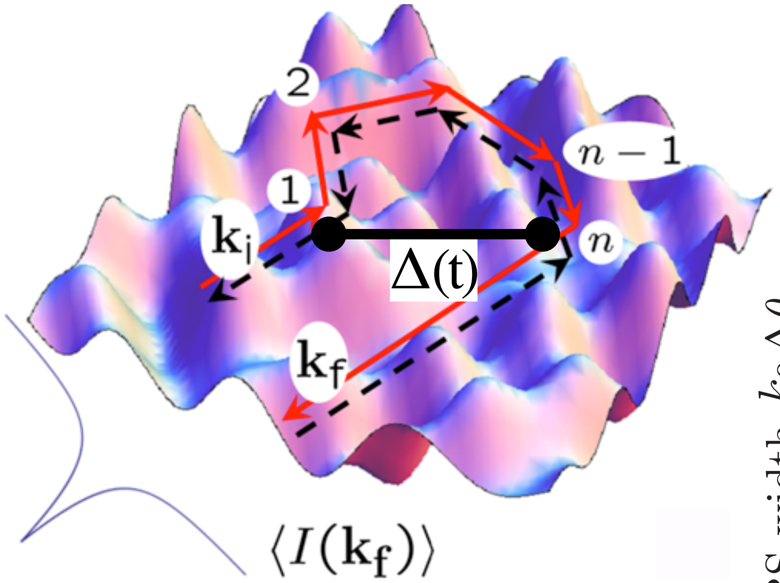
Momentum distribution
(3D Anderson lattice model)

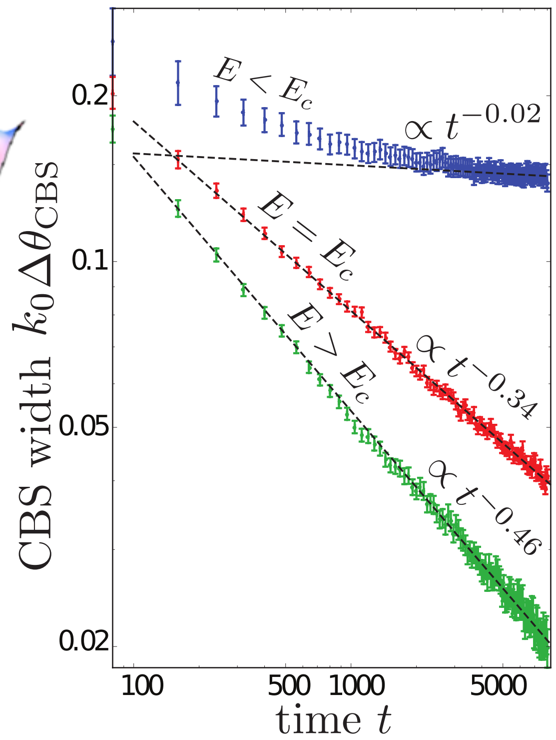
☞ **Show that CBS width and CFS contrast satisfy the 1-parameter scaling hypothesis**

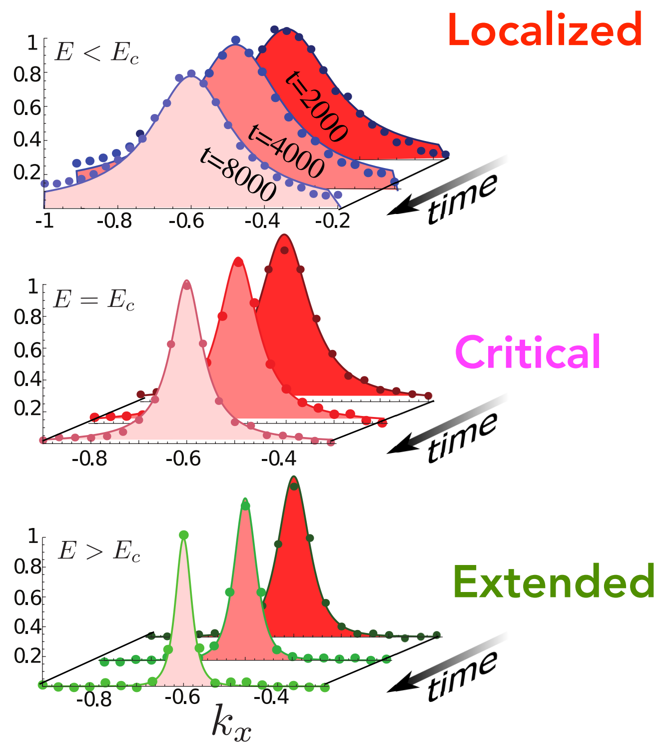
Use finite time scaling to extract the mobility edge and critical exponent

3D CBS peak width reveals the Anderson transition

Speckle at fixed disorder strength W, tuning parameter = E







"Double slit"

$$\text{Width} = \Delta\theta_{\rm CBS} \sim \frac{1}{\Delta(t)}$$

Diffusion

$$\Delta(t) \sim \sqrt{Dt}$$
$$\Delta\theta_{CBS}(t) \sim 1/\sqrt{Dt}$$

Subdiffusion

$$\Delta(t) \sim t^{1/3}$$
$$\Delta\theta_{\rm CBS}(t) \sim t^{-1/3}$$

Localization

$$\Delta(t) \sim \xi$$
$$\Delta\theta_{\rm CBS}(t) \sim 1/\xi$$

*S. Ghosh et. al. PRL **115**, 200602 (2015)*

Finite time scaling of the CBS width

Define length L through $\boxed{t = 2\pi\hbar\overline{\rho}(E)L^3}$

(Heisenberg time associated to linear size L)

Define $\boxed{\Lambda(E,t) = \frac{1}{k_0 L \Delta\theta_{\mathrm{CBS}}}}$

$\Lambda(E,t) \sim \sqrt{2\pi\hbar\overline{\rho}(E)D(E)L} \equiv \sqrt{L/\xi_{cor}(E)}$ Diffusive side

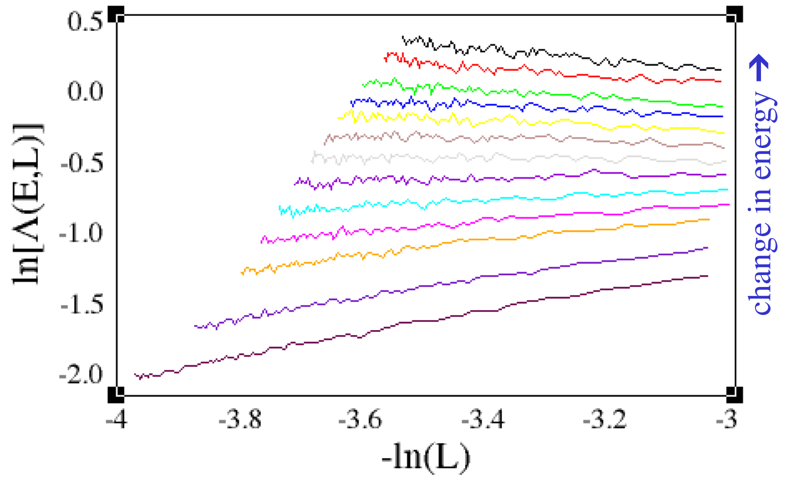
$\Lambda(E,t) \sim \xi(E)/L \equiv \xi_{cor}(E)/L$ Localized side

$\Lambda(E,t) \sim \text{Constant}$ Critical point

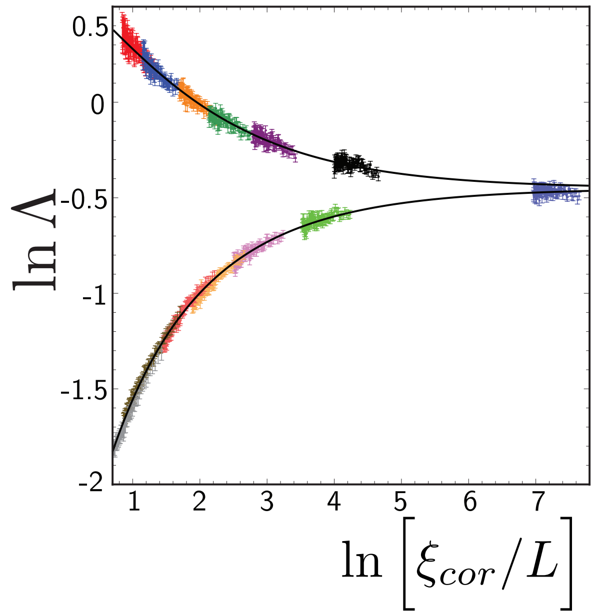
Claim:

$\boxed{\Lambda(E,t) \equiv \Lambda(x)}$

is a scaling observable.



"Finite size" (L) scaling



$\boxed{E_c = -0.479 \pm 5.10^{-4} \text{ and } \nu = 1.61 \pm 0.02}$

S. Ghosh et. al. PRL **115**, 200602 (2015)

TM method: $E_c = -0.478 \pm 7.10^{-4}$ and $\nu = 1.62 \pm 0.03$.

Delande & Orso (PRL, 2014): $E_c \approx -0.43$

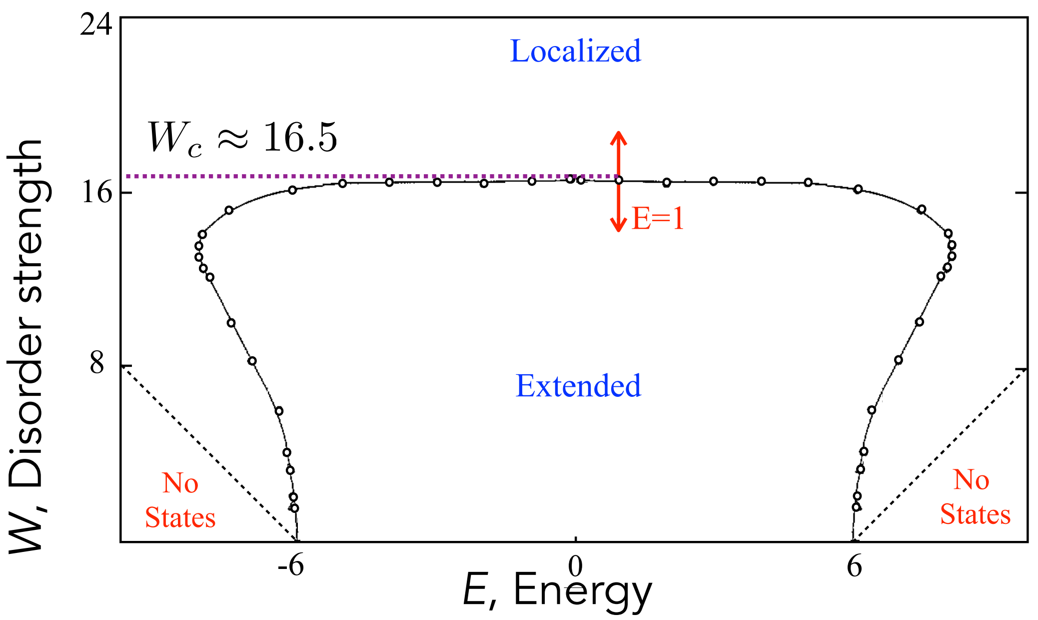
CFS dynamics across the Anderson transition

3D cubic lattice Model:

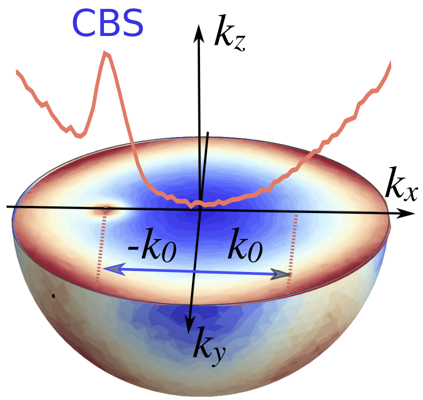
$$H = -J \sum_{\langle i,j \rangle} \left(C_i^\dagger C_j + C_j^\dagger C_i \right) + \sum_i W_i C_i^\dagger C_i$$

W_i : onsite *i.i.d.* random potentials, uniformly distributed *in* [-W/2,+W/2].

Phase diagram



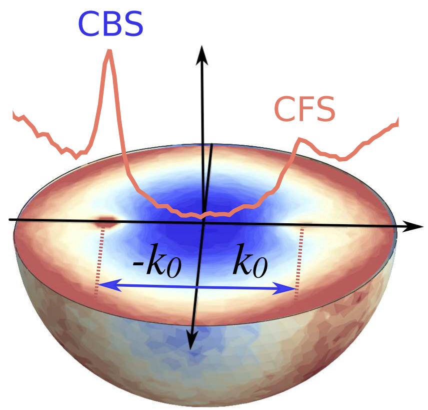
- We choose E=1.
- *W* is the tuning parameter.



- **Metal**

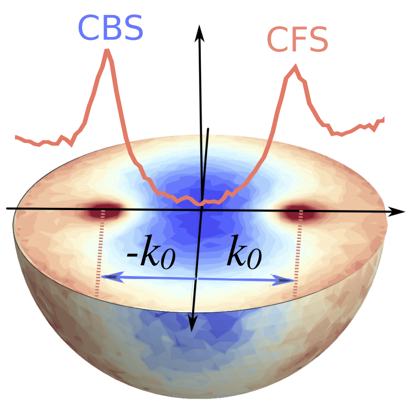
Only the CBS is visible

No CFS!



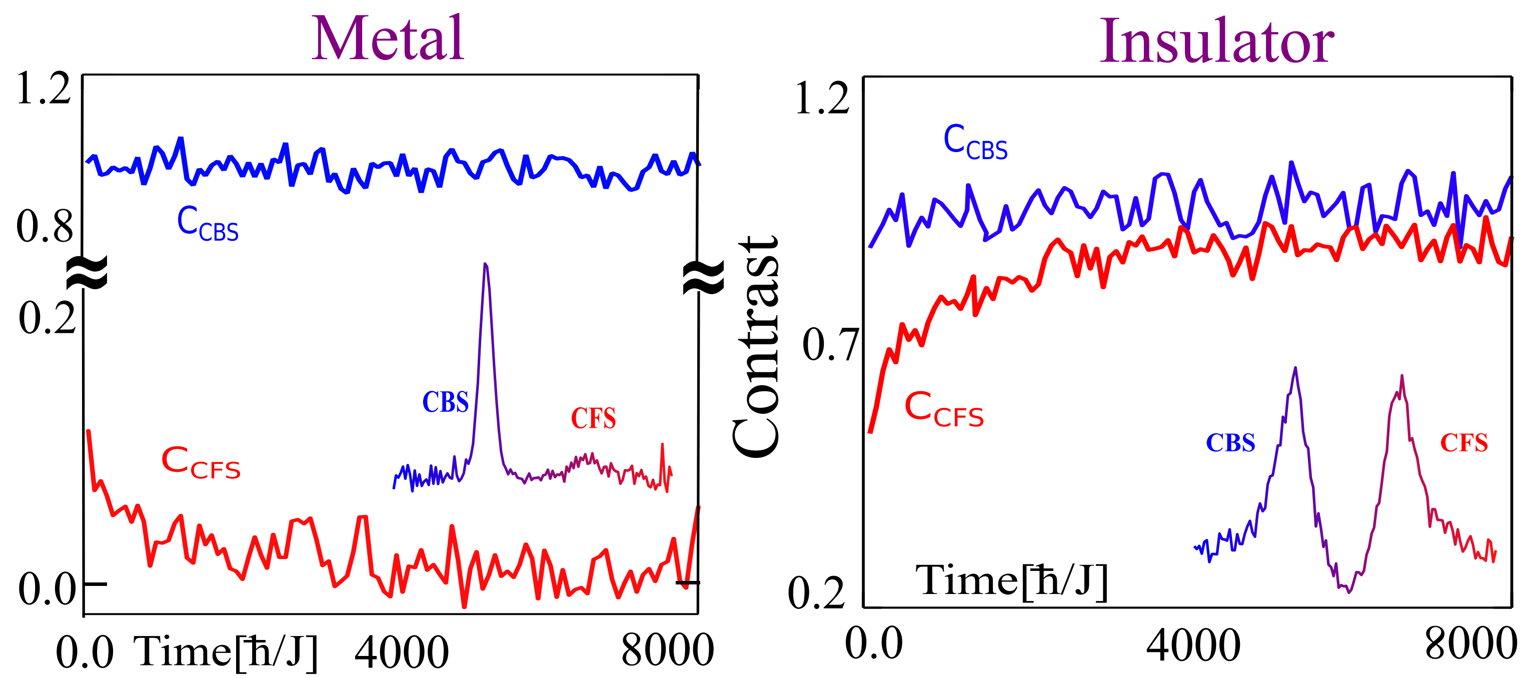
- **Critical**

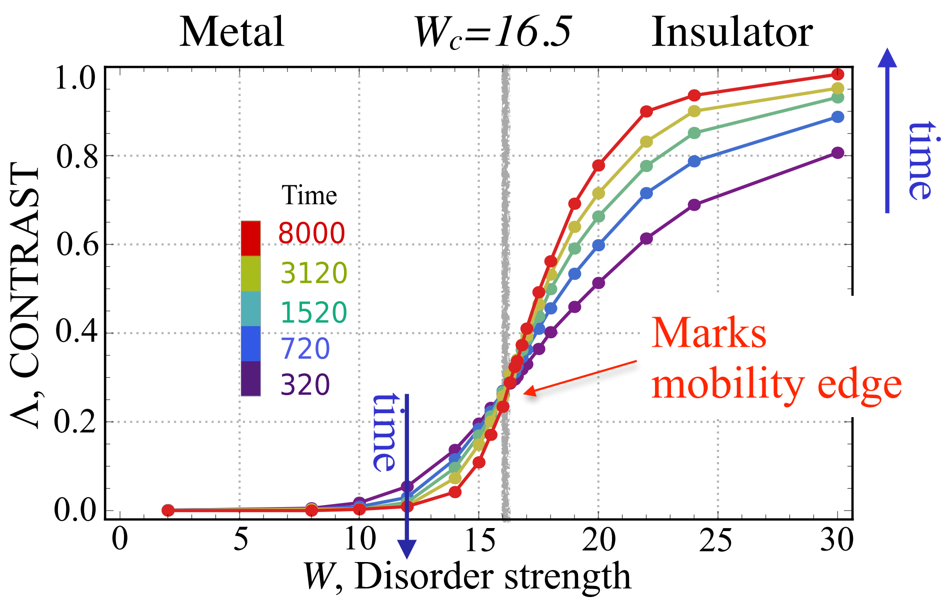
A "critical" CFS peak



- **Insulator**

CBS & CFS are both visible





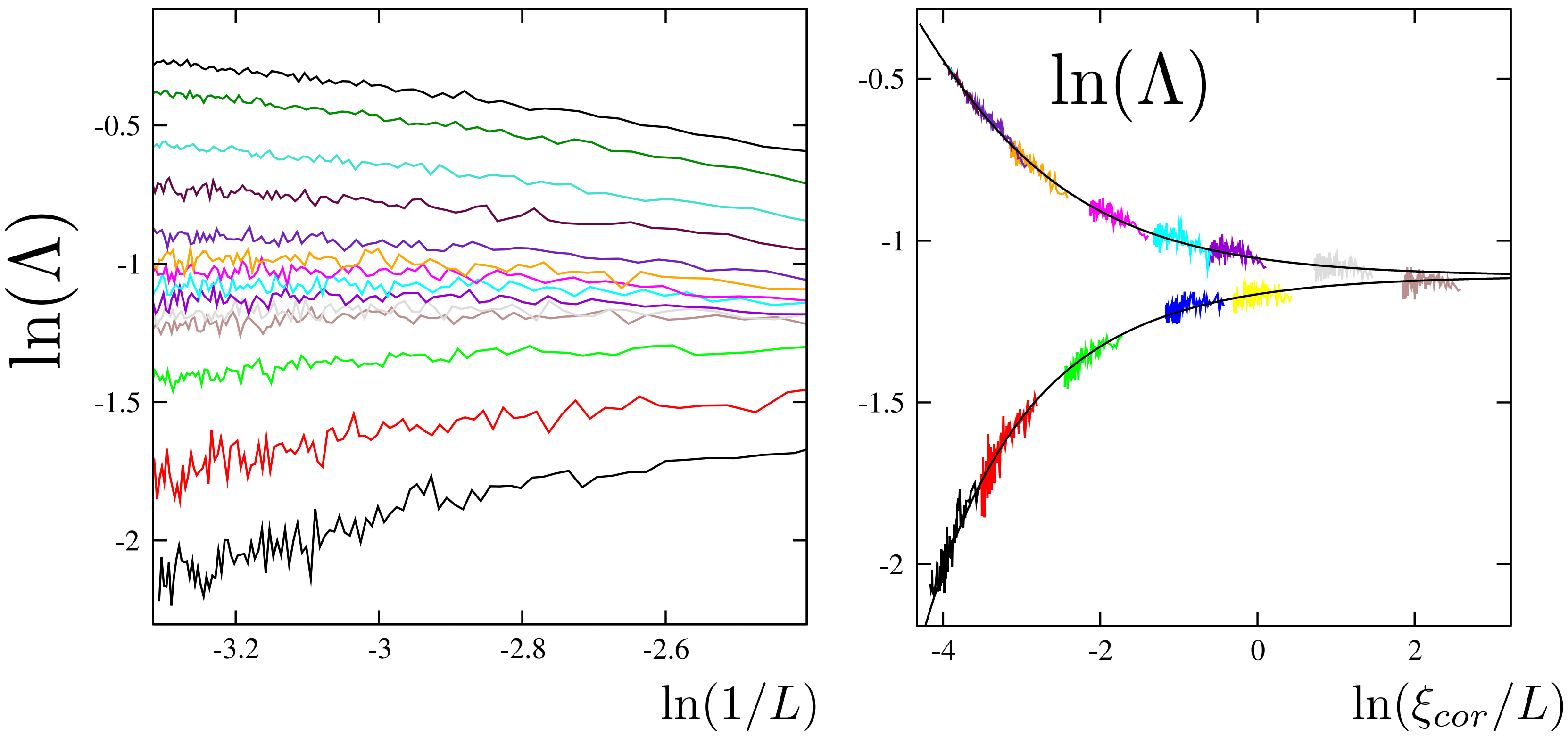
Smoking gun of 3D AT!

- Jumps from 0 to 1 across the transition.
- Crossing locates ME.

S. Ghosh et al., PRA **95**, 041602 (R) (2017)

Finite time scaling of the CFS contrast

Choose Λ(W, t) ≡ Λ(W, L) = CFS contrast as the scaling quantity $t = 2\pi\hbar\overline{\rho}L^3$



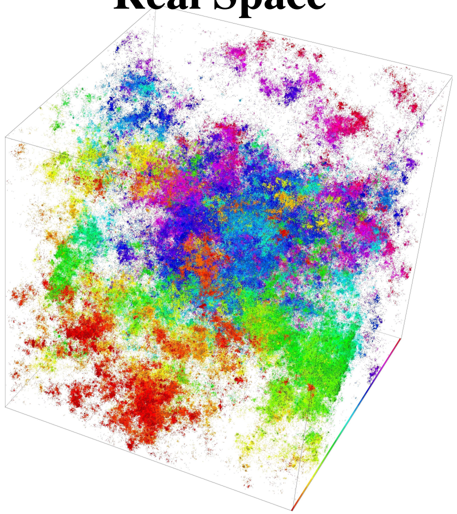
Extracted critical exponent: $\nu = 1.51 \pm 0.07$

Extracted mobility edge: $W_c = 16.53 \pm 0.03$

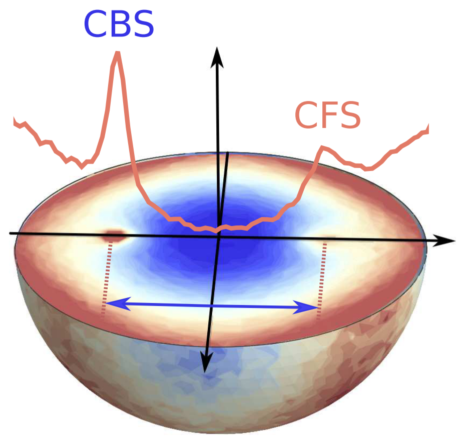
"Best" TM results (Slevin & Ohtsuki, NJP, 2014): $\nu = 1.573$ $W_c = 16.536$

Perspectives: CFS and multi-fractality

Real Space



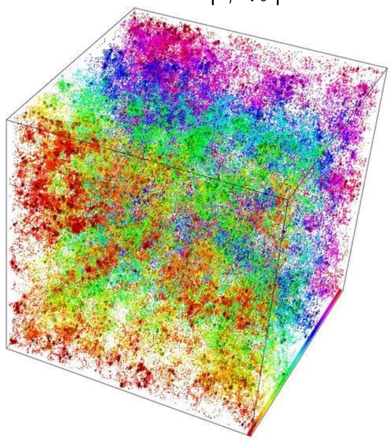
Momentum Space



Multi-fractal nature of eigenstates at the critical point

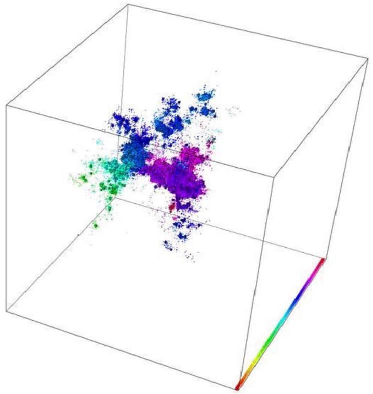
- **Metal:**
Extended eigenfunctions

$|\psi_n|^2 \sim L^{-d}$

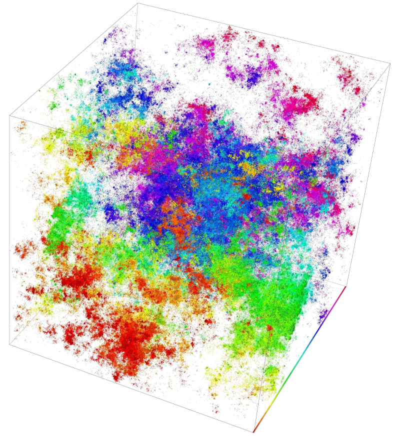


- **Insulator:**
Localized eigenfunctions

$|\psi_n|^2 \sim \xi^{-d} e^{-|\vec{r}-\vec{r}_0|/\xi}$



- **Mobility Edge:**
Fractal eigenfunctions



Strong fluctuations: Regions where the eigenfunction is exceptionally large, regions where it is exceptionally small

Rodriguez et. al. PRB 84, 134209 (2011)

$$P_q = \int_{L^d} d\vec{r}\, |\psi_n(\vec{r})|^{2q} \sim L^{-D_q(q-1)}$$

Multifractal dimension

$$0 \leq D_q \leq d$$

Localized eigenfunctions

Diffusive eigenfunctions

$$\int_{L^d} d\vec{r}\, |\psi_n(\vec{r})|^2 \ln|\psi_n(\vec{r})|^2 \sim D_1 \ln L$$

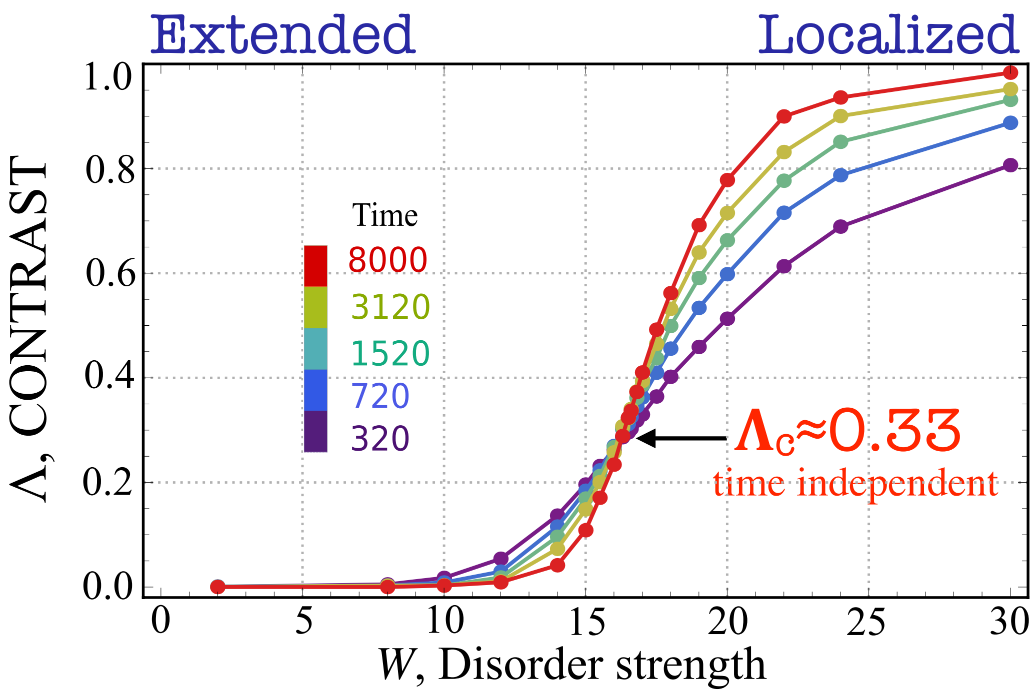
Information dimension

Bogomolny-Giraud conjecture (2010): $\boxed{\chi + D_1/d = 1.}$

$$\chi = 2\pi\hbar\, L^d\, \overline{\rho}(E)\, K_E(t \to 0)$$

Spectral Compressibility

Multi-fractal signatures in CFS



Localized side:

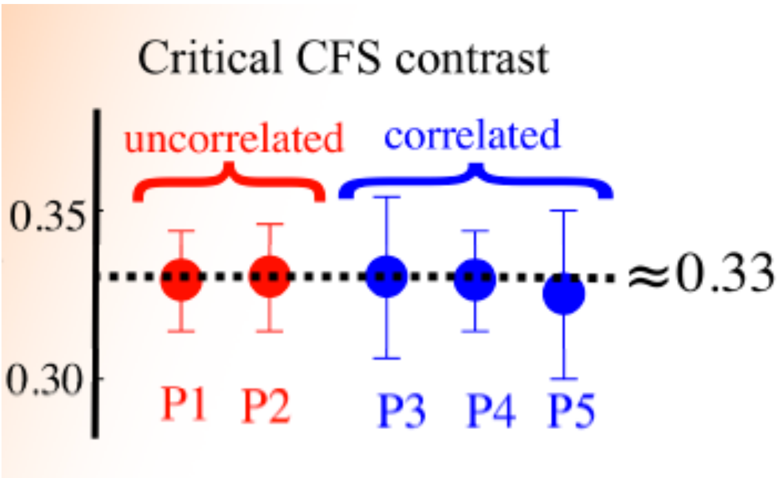
$$\Lambda(t) = 2\pi\hbar\, L^d\, \overline{\rho}(E)\, K_E(t)$$

At the MIT point

$$\Lambda_c = \mathrm{C}^{\mathrm{te}} \equiv 2\pi\hbar\, L^d\, \overline{\rho}(E)\, K_E(t \to 0)$$

$$\boxed{\Lambda_c = \chi_c \quad \text{at the MIT point}}$$

Spectral compressibility at ME



The CFS contrast at the critical point seems to be robust against disorder

Bogolmony-Giraud conjecture in 3D:

$$\boxed{\chi_c = 1 - D_1/3}$$

Information dimension

$$\sum_{j=0}^{N} |\psi_j|^2 \ln |\psi_j|^2 \sim D_1 \ln N$$

Combining with $\Lambda_c = \chi_c$

$$\boxed{D_1 = 3(1 - \Lambda_c)}$$

Our work: $\Lambda_c \approx 0.329 \;\Rightarrow D_1 \approx 2.013$

To be compared to: $D_1 \approx 1.97$ *as found from L. J. Vasquez, PhD thesis (2010)*

☞ **The CFS contrast at critical point seems to be a measure of multi-fractal character of the eigenstates.**

Summary (take-home messages)

- **CBS width and CFS contrast obey the one-parameter hypothesis (scaling). The critical properties of the transition are encoded in their time dynamics.**

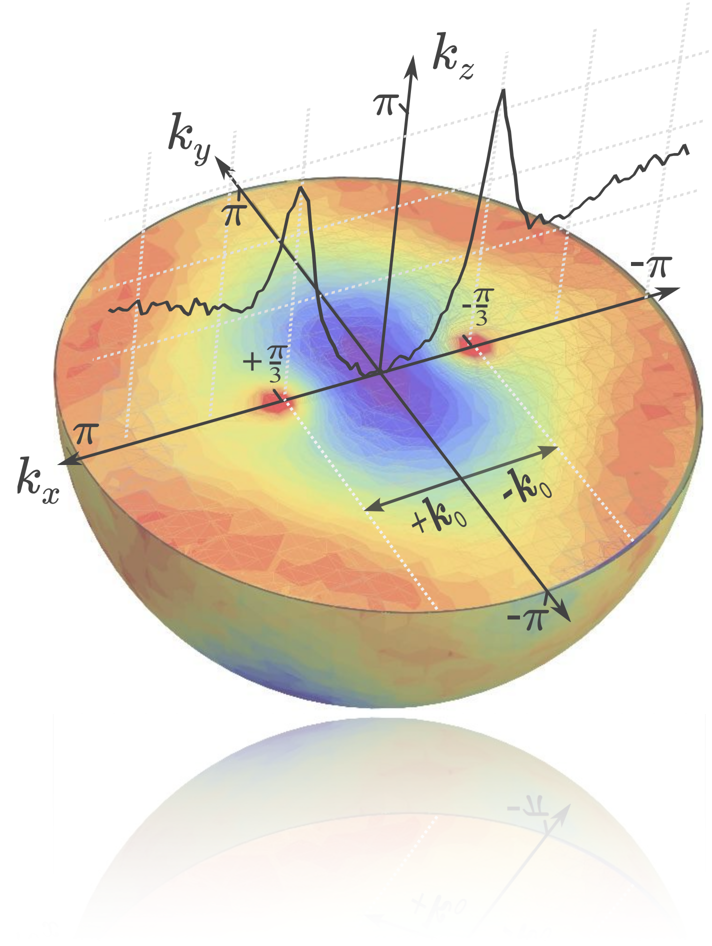
(Already observed for CBS in acoustic experiments by John Page's group and collaborators)

- **CFS is *robust*: it exists in other symmetry classes than just GOE.**

- **CFS contrast jumps from zero to a finite value across the Anderson transition point: measurable *critical quantity* of ATs.**

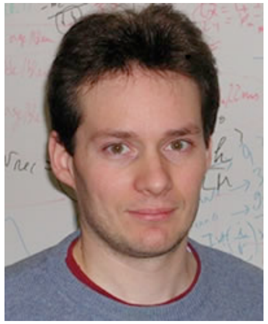
- **CFS contrast at critical point encodes valuable information on the *multi-fractal* properties of the eigenstates.**

- **Pending: How to observe it for classical waves?**



My Precious Collaborators

S. Ghosh (PhD)



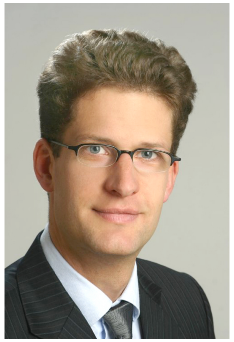
B. Grémaud

N. Cherroret

D. Delande

K. L. Lee

T. Karpiuk



C. Müller

David Guéry-Odelin

Gabriel Lemarié

Our papers on the subject

N. Cherroret, T. Karpiuk, C.A. Müller, B. Grémaud, C. Miniatura
PRA **85**, 011604 (2012)

T. Karpiuk, N. Cherroret, K. L. Lee, B. Grémaud, C.A. Müller, C. Miniatura
PRL **109**, 190601 (2012)

K. L. Lee, B. Grémaud, C. Miniatura, PRA **90**, 043605 (2014)

S. Ghosh, N. Cherroret, B. Grémaud, C. Miniatura, D. Delande, PRA **90**, 063602 (2014)

S. Ghosh, D. Delande, C. Miniatura, and N. Cherroret, PRL **115**, 200602 (2015)

S. Ghosh, C. Miniatura, N. Cherroret, and D. Delande, PRA **95**, 041602(R) (2017)

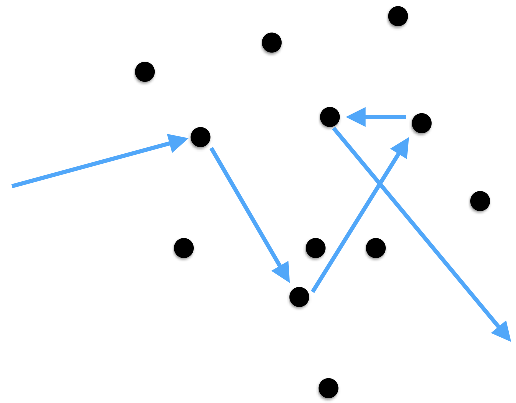
G. Lemarié, C. A. Mueller, D. Guéry-Odelin, and C. Miniatura, PRA **95**, 043626 (2017)

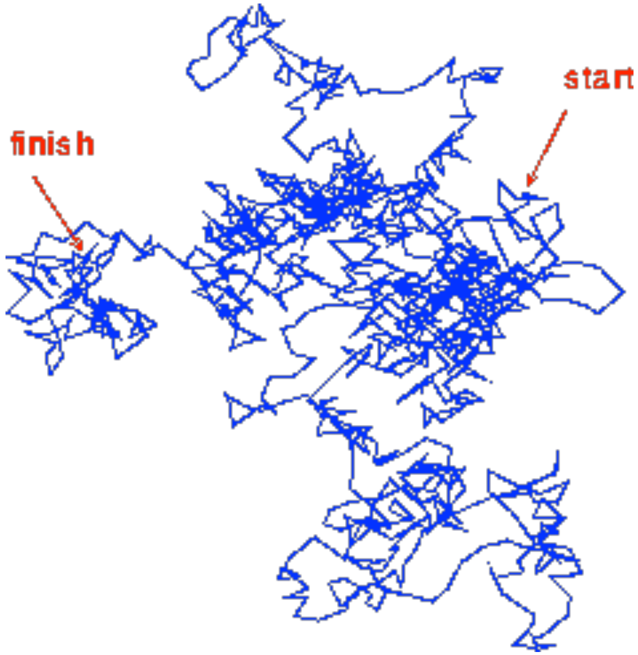
Appendix

Particles' picture: Conduction is a Pinball game

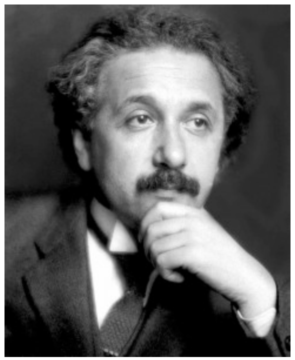
In a ***disordered*** landscape, particles do not move along straight lines, but experience a ***series of scatterings off obstacles sitting at random positions***.

As a net result of this pinball game, the particle undergoes
a ***random walk*** and its large-scale motion is ***diffusive*** rather than ballistic. Very intuitive behavior.





Drude model for a Metal
→ Conductance $g \sim 1/L$ (Ohm's law)
L = sample size



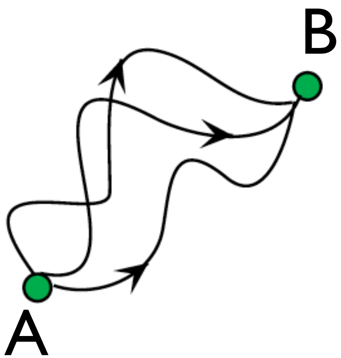
Quantum Mechanics: Particles are Waves

What's new? Interference!

C***onductance*** G between A and B ~ probability P to propagate from A to B.

Classical physics assumes that this probability is just the ***sum of the probabilities of all possible paths connecting the two points***. Wave physics tells us to sum up **amplitudes**!

What does it change? Nothing (!?)

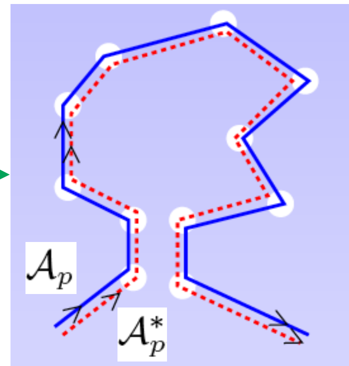


$$\mathcal{A}(A \to B) = \sum_{paths\ p} \mathcal{A}_p = \sum_{paths\ p} \sqrt{I_p} e^{i\phi(p)}$$

Average over disorder randomizes the phases

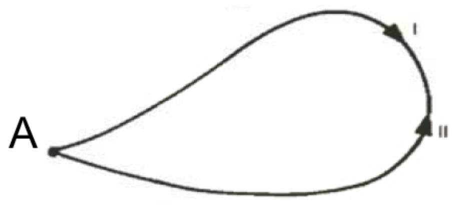
$$\langle |\mathcal{A}|^2 \rangle \approx \sum_{paths\ p} \langle \mathcal{A}_p^* \mathcal{A}_p \rangle$$

→ Ladder Diagrams



→ Classical Diffusion

Wait…



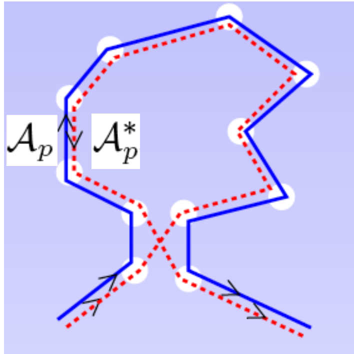
A randomly chosen **loop** from A to A interfere perfectly with its time-reversed counterpart.

$$P_{quantum}(A \to A) = 2\ P_{classical}(A \to A)$$

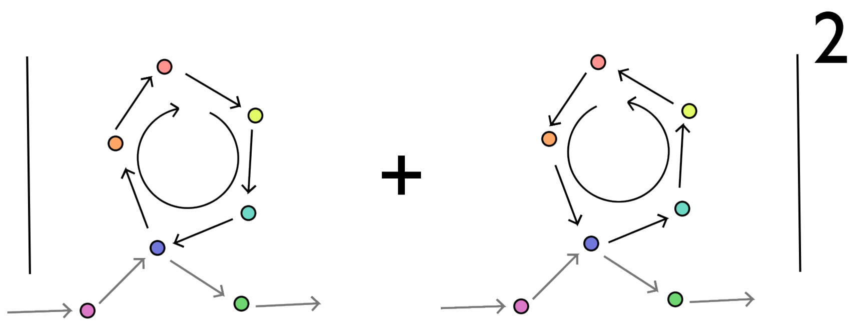
Wave «prefers to stay at A»

Weak Localization corrections!

Maximally-Crossed Diagrams



Weak localization is due primarily to (quasi) self-intersecting scattering paths (loops)
It is a precursor of strong localization.

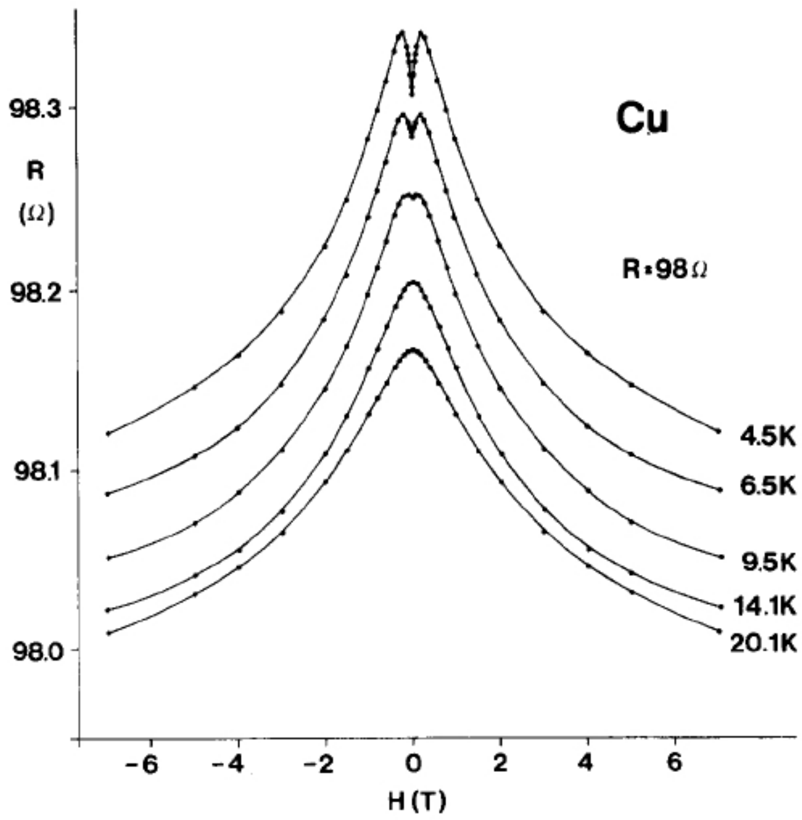


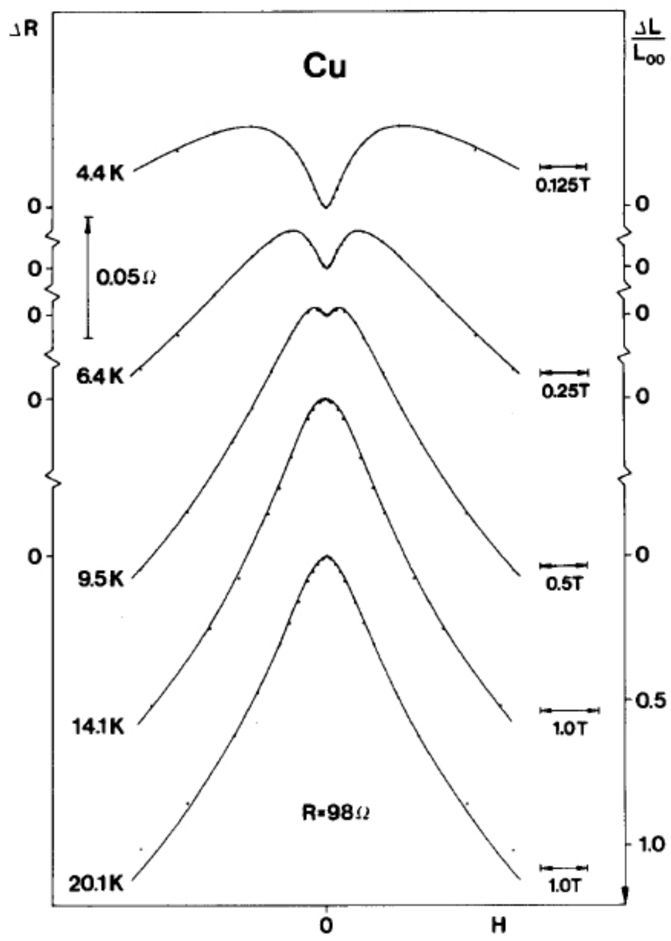
1-loop effect: diffusion is maintained but diffusion constant D *decreases*

$\boxed{\text{G } \textit{decreases}}$

Related phenomena: Coherent Backscattering (CBS)

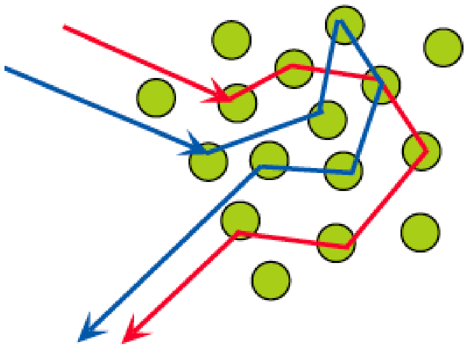
Negative/positive magneto-resistance experiments (Bergmann, 1984)





Coherent Backscattering Effect (CBS)

Light scatterers

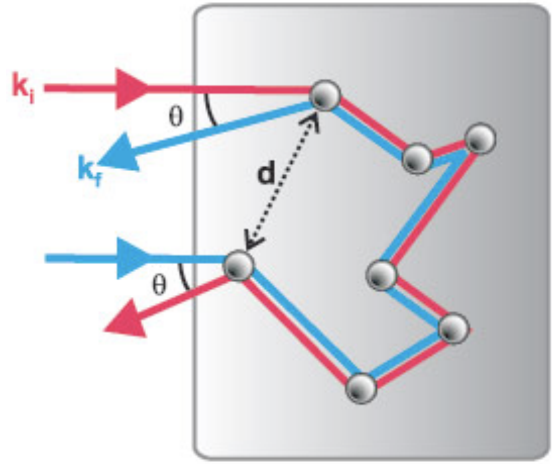


Spatial average do scramble interference between different scattering paths ....

... but cannot scramble interference originating from reversed paths **!**

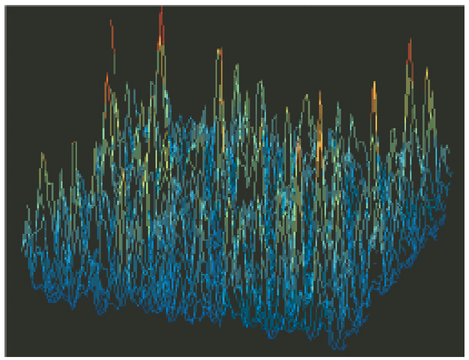
(Cf. loop interference)

Cf Young slits with separation *d*.



The larger d, the smaller the interfringe.

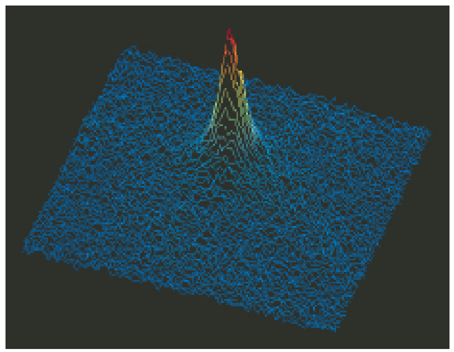
Angular intensity distribution of far-field *multiply* scattered light



Speckle pattern

Disorder average

Angular intensity distribution of configuration-averaged speckle patterns

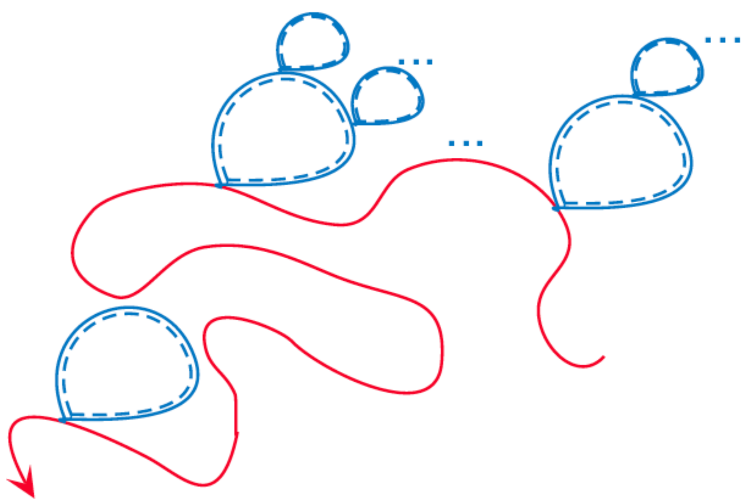


CBS Peak

Backscattering direction

Is that all? No! ***Absence of diffusion for certain random lattices, Anderson ('58)***

Cumulative effect of interference within disorder can bring transport to a halt!

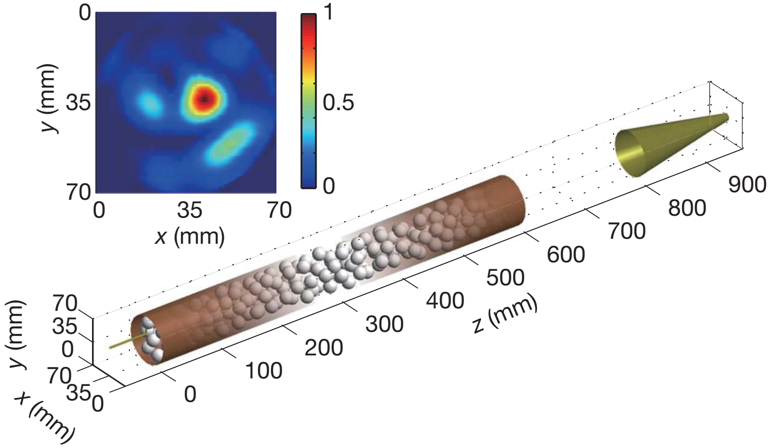


Self-consistent Localization Theory (Vollardt-Wölfle, mean-field)

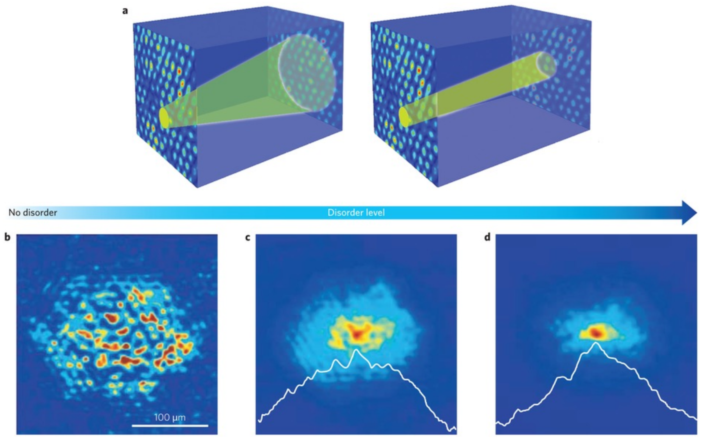
Cumulative effect of ***nested*** loops → Strong Localization D=0

Anderson localization (AL) is the *complete suppression* of *diffusion* of waves in a *disordered* medium due to *destructive interference between the many scattering amplitudes*.

Microwaves (2011)

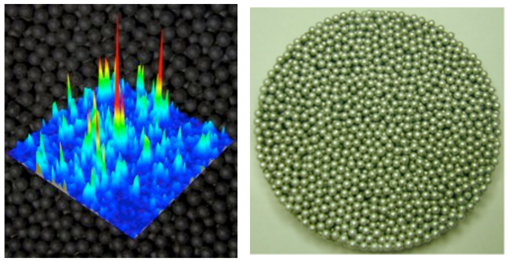


2D photonic band gap material (2013)

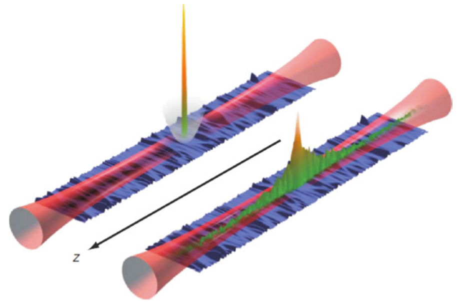


Light???
The story continues
(a real challenge!)

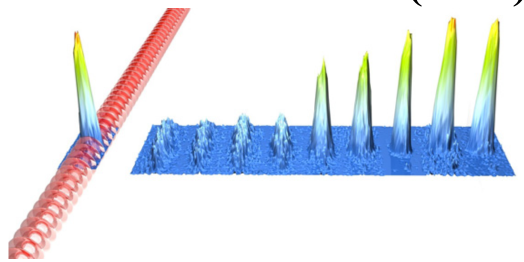
Acoustics (2012, 2016)



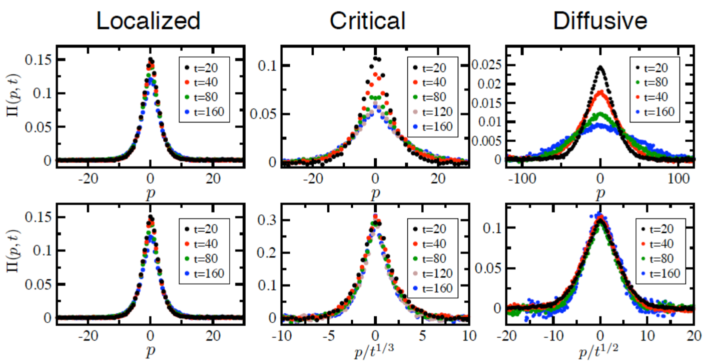
Cold atoms - 1D Speckle (2008)



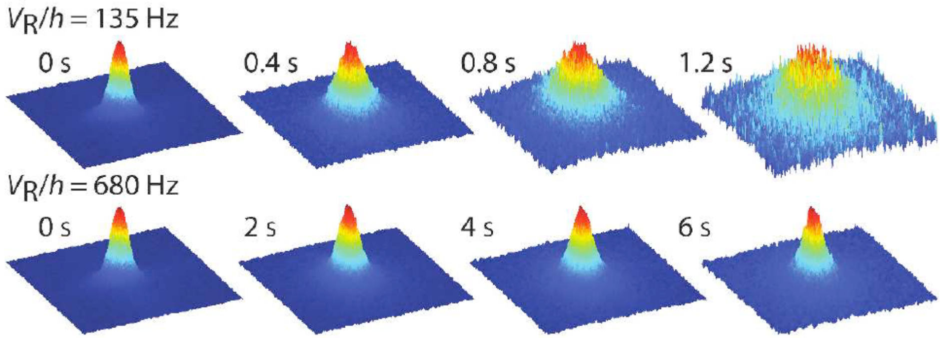
Cold atoms - quasi-periodic lattice (2008)



Cold atoms - Kicked Rotor (1995, 2008, 2010)



Cold atoms - 3D Speckle (2011)



A scaling function β(g) for CFS contrast

Define effective dimensionless conductance

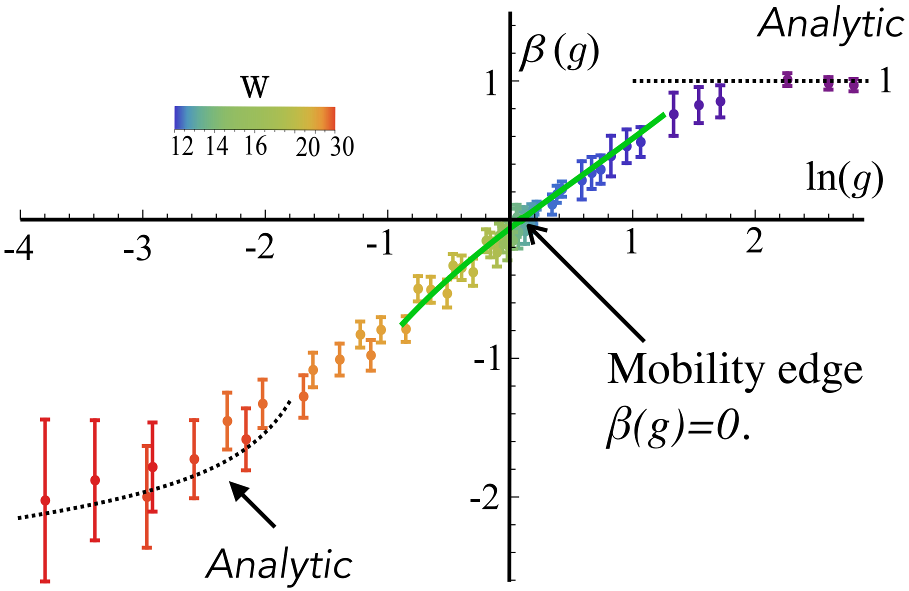
(with Λ(W, t) ≡ Λ(W, L) = CFS contrast)

$$g(W, L) = \Lambda^{-2/3} - 1$$

$g(W, L) \propto L$ Diffusive side (Ohm's law)

$g(W, L) \to 0$ Localized side

Compute (numerically): $\beta(W, L) = \dfrac{d \ln g}{d \ln L}$



- Data for different time and disorder follows same universal curve
- CFS contrast obeys the one-parameter scaling! β(W, L) = β(g)

Extracted critical exponent: $\nu = 1.51 \pm 0.07$

(inverse of the slope at origin)