Synthetic topology and manybody physics in synthetic lattices

ICFO
Institut
de Ciències
Fotòniques

Alessio Celi

EU STREP EQuaM

May 16th, 2017
Atomtronics - Benasque

Plan

- Integer Quantum Hall systems and Edge states
- Cold atom realizations: synthetic gauge field
  - Real space
  - Synthetic lattice (Extradimension)

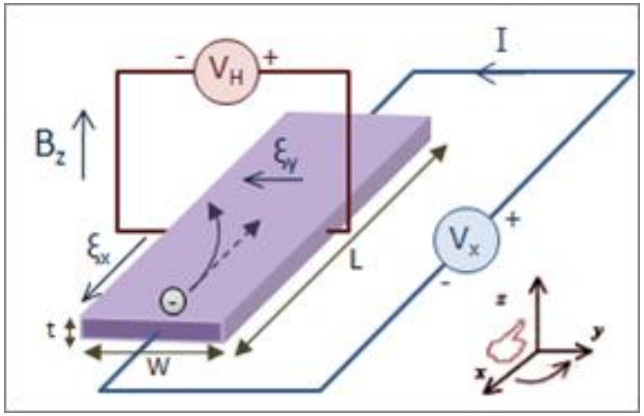
    Topology in narrow strips
  - Further applications: non-trivial topology

    interacting system

    ....
- Dimerized interacting ladder
  - Meissner/Vortex phase (in analogy to type II superconductors)
  - Effect of the dimerization
  - Prospects

Quantum Hall effect

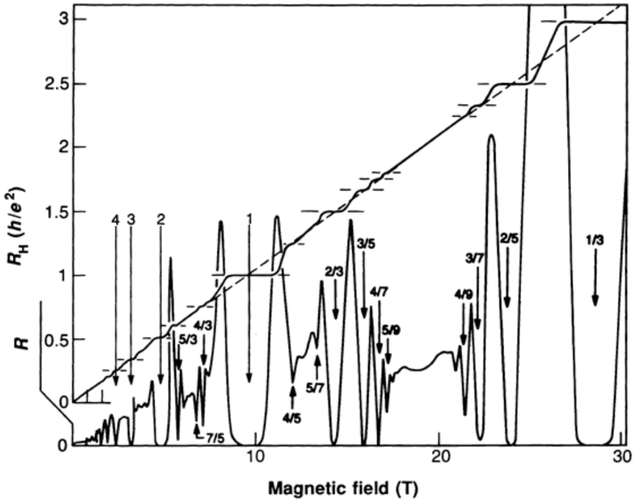
1879: Classical Hall effect (consequence of Lorentz force)



E. Hall

1980: Quantum Hall effect: Electric conductivity quantized

$$\sigma = \frac{I_{\text{channel}}}{V_{\text{Hall}}} = \nu \frac{e^2}{h}$$



K. Von Klitzing

Integer Quantum Hall effect on lattice

IQH explained in terms of single particle physics (Landau level filling)

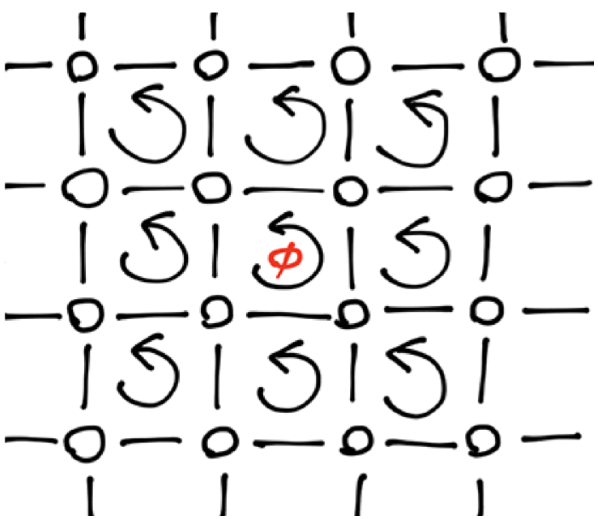
$$H = -\sum_{n,m} (J a^\dagger_{n+1,m} + J' e^{i\Phi n} a^\dagger_{n,m+1}) a_{n,m} + h.c.$$

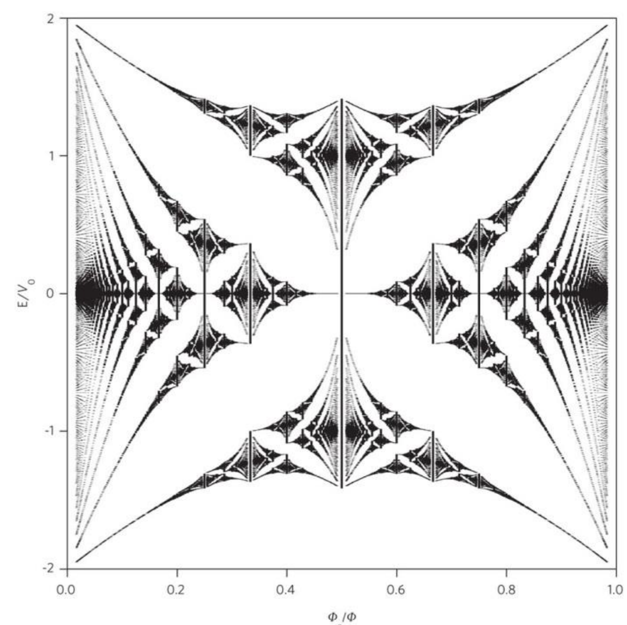
Quantization determined by topology of filled bands (1-Chern number)

Bulk-boundary correspondence (Topological insulator prototype)

Edge states determined by spectrum of periodic system

On square lattice simple formulation: Hofstadter model





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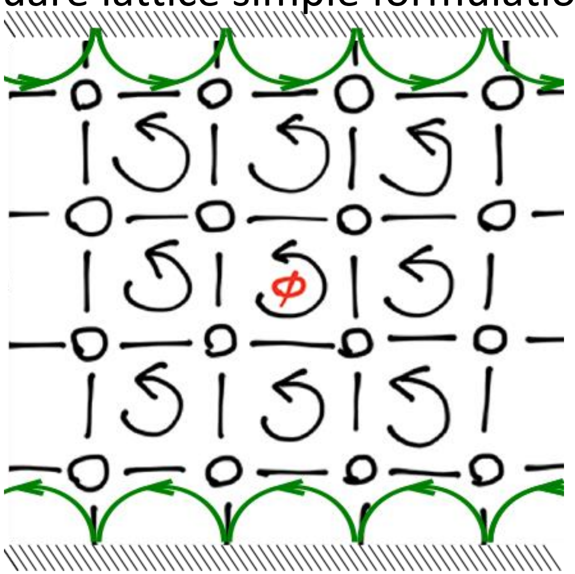
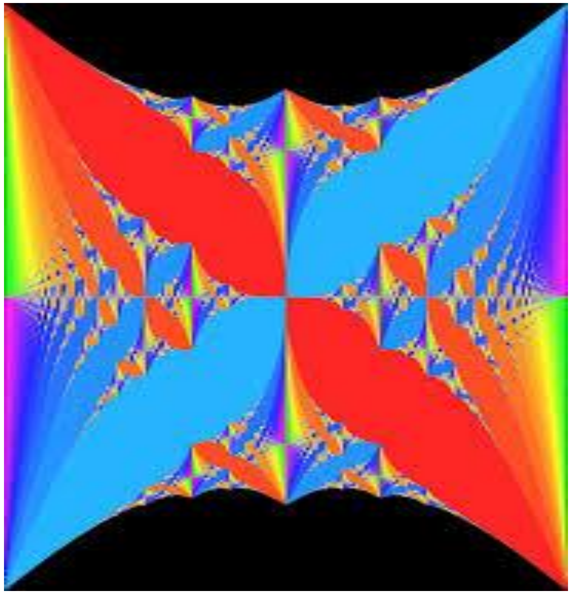
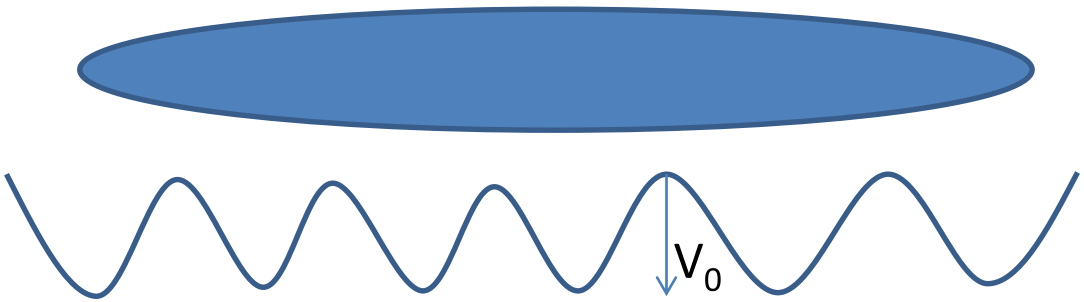
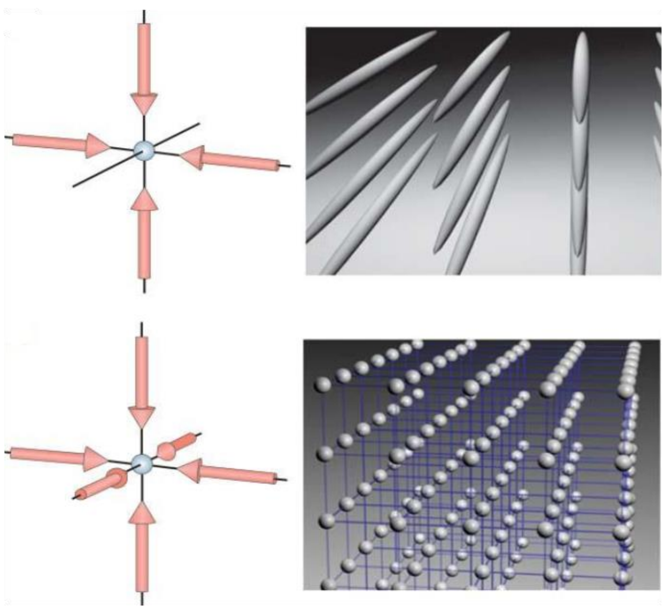
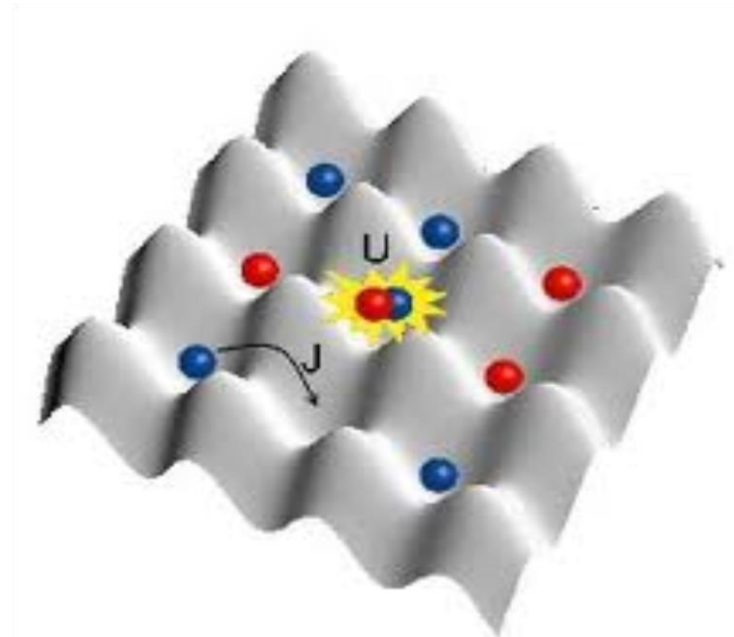




How? Cold atoms in optical lattices as ideal electrons in metals







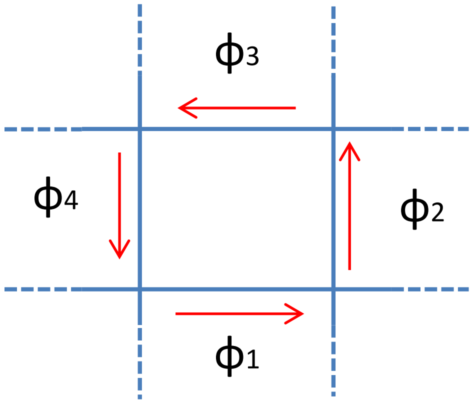
Mott-superfluid phase transition (predicted 1998- observed 2002)

How? C. atoms in OL as charged electrons in external fields

Increasing complexity OL simulator:

- Modify Hopping & connectivity
- Tune interaction (intensity, range)
- Distinguish internal states of atoms

Hopping with phases → Synthetic magnetic field for neutral atoms



Synthetic Aharonov-Bohm effect

$\phi = \Sigma_i \phi_i$ = magnetic flux

How? C. atoms in OL as charged electrons in external fields

Many ways of generating magnetic fluxes on the lattice

- Rotation (= constant magnetic field)
- Shaking (also non-Abelian *PRL 109 145301 (2012)…*)

Exp. collaboration with Hamburg: search for
Spin liquid phases in frustrated antiferromagnets with *Bosons*

How? C. atoms in OL as charged electrons in external fields

Many ways of generating magnetic fluxes on the lattice

- Rotation (= constant magnetic field)
- Shaking
- Raman laser + 2D superlattice

Theory : Jaksch & Zoller *NJP* ***5*** *56* (2003)

…..

Experiments: *PRL 107 255301 (2012), PRL 111 185301 (2013)* (I.Bloch group)
PRL 111 185302 (2013) (W.Ketterle group)

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- Raman laser + *"Extradimension"*

How? C. atoms with in OL as particles in Extradimension

[Boada,AC,… PRL 108, 133001 (2012)] Highlighted in *Physics*

In OL 3D Hubbard model → 1D,2D by tuning optical potential

And > 3D?

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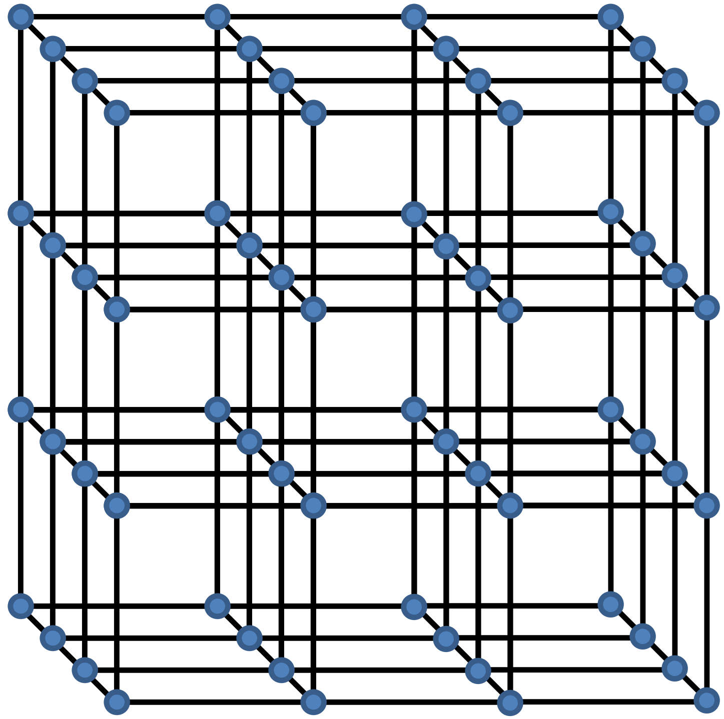
Hopping in *D*+1 hypercubic lattice as



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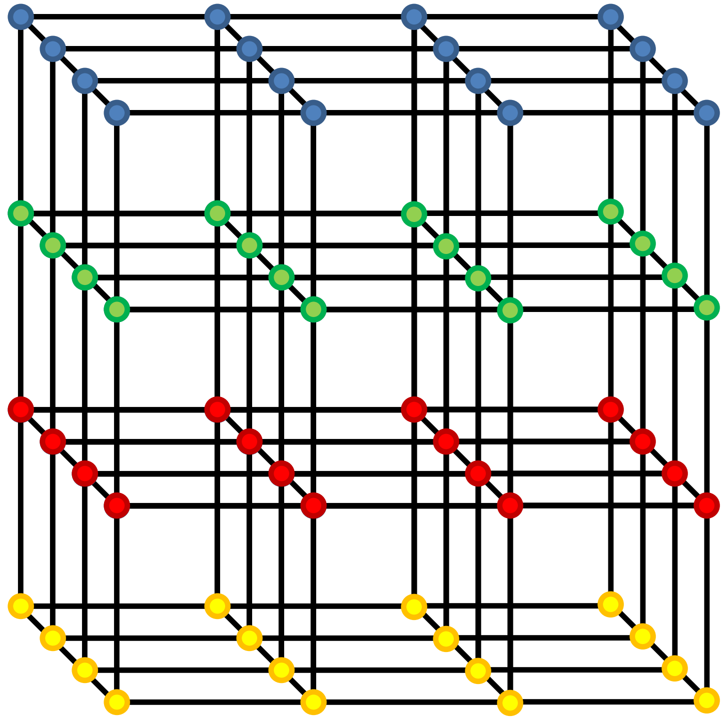

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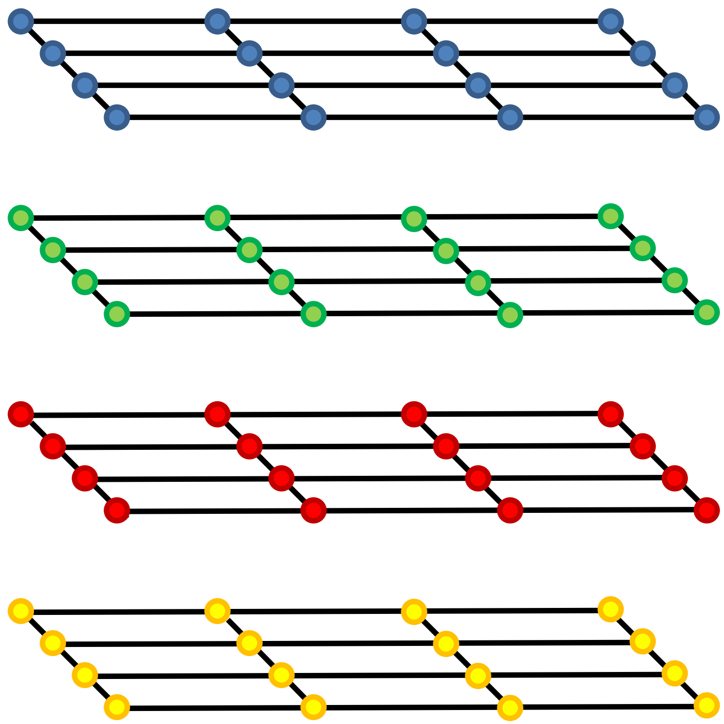

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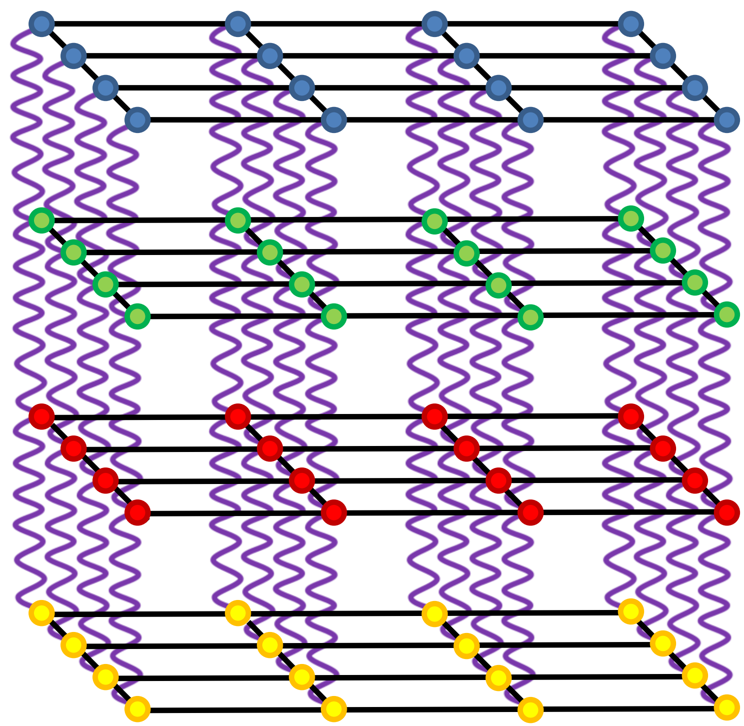
Coupled atomic states hopping in *D*-lattices



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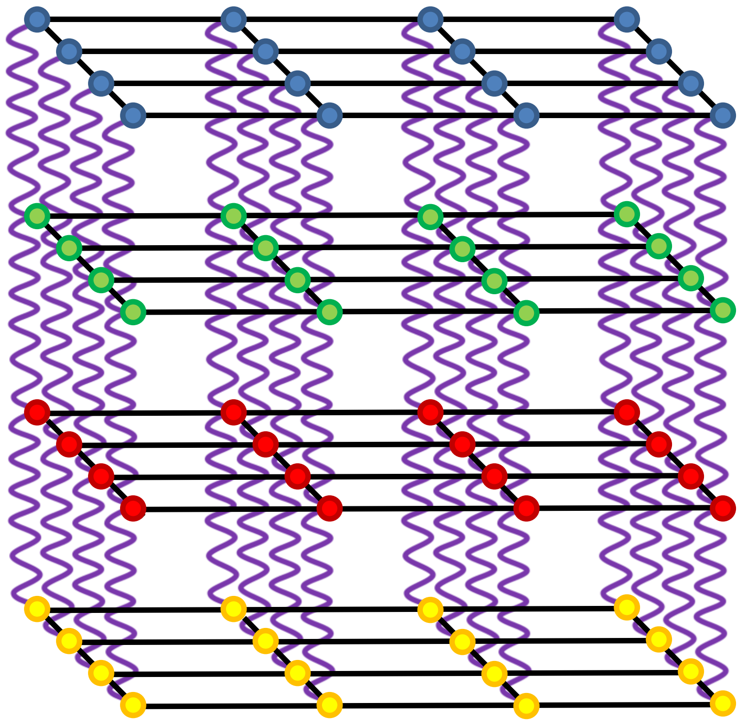


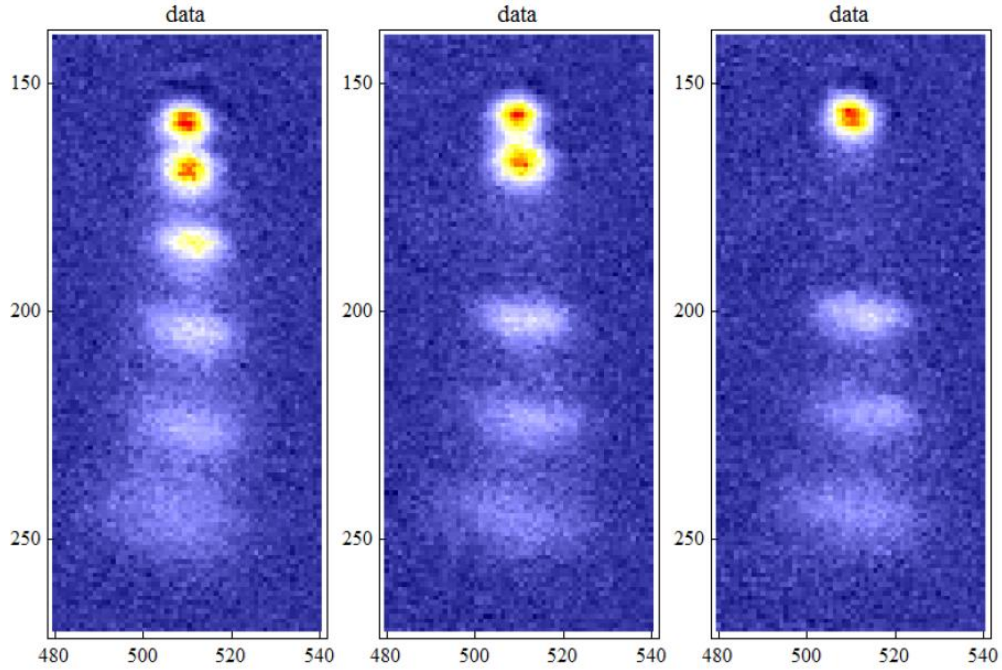
Alternative to cold atoms
e.g. photonic crystal
Jukic & Buljan PRA (2013)
cold molecules
Wall, Maeda & Carr
New J. Phys. 17 025001 (2015)
....
Ring resonators
Opt. Lett. 41, 741-744 (2016)

How? C. atoms with in OL as particles in Extradimension

[Boada,AC,… PRL 108, 133001 (2012)] Highlighted in *Physics*

Observables 4D behavior:
density of states,
scaling of Entropy

Example: Lens ^{173}Yb
(F=5/2, 6 states)



Applications: 4D-Q.Hall, *S.-S.-C. Zhang and J. Hu, Science 294, 823 (2001)*

Edge,Tworzydlo, Beenakker PRL 109, 135701 (2012)

Price,Zilberberg,Ozawa,Carusotto,Goldman PRL 115, 195303 (2015)

…

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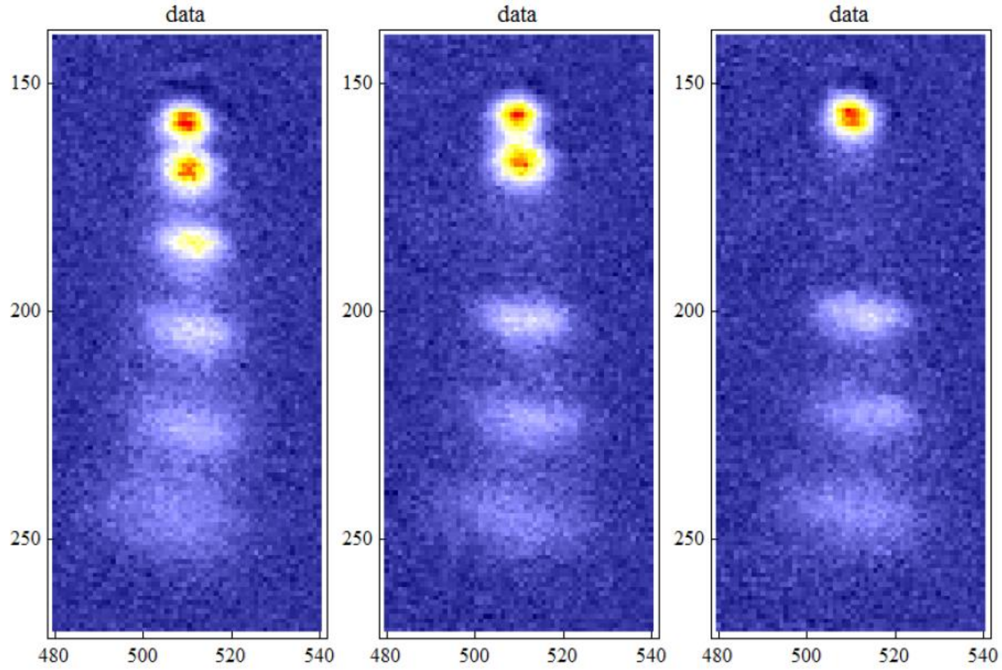

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Topological observable:
Linear and quadratic response

$$j_l = \frac{e^2}{h} \sum_{\varepsilon_\alpha < \varepsilon_F} \frac{1}{(2\pi)^4} E_m \int_{BZ} F^\alpha_{lm}\, d^4k + \frac{C_2(\varepsilon_F)}{4\pi^2} \epsilon_{lmno}\, E_m\, B_{no},$$

$$C_2(\varepsilon_F) = \sum_{\varepsilon_\alpha < \varepsilon_F} \frac{1}{32\pi^2} \int_{BZ} d^4k\ \epsilon_{lmno}\, F^\alpha_{lm}(\mathbf{k}) F^\alpha_{no}(\mathbf{k})$$

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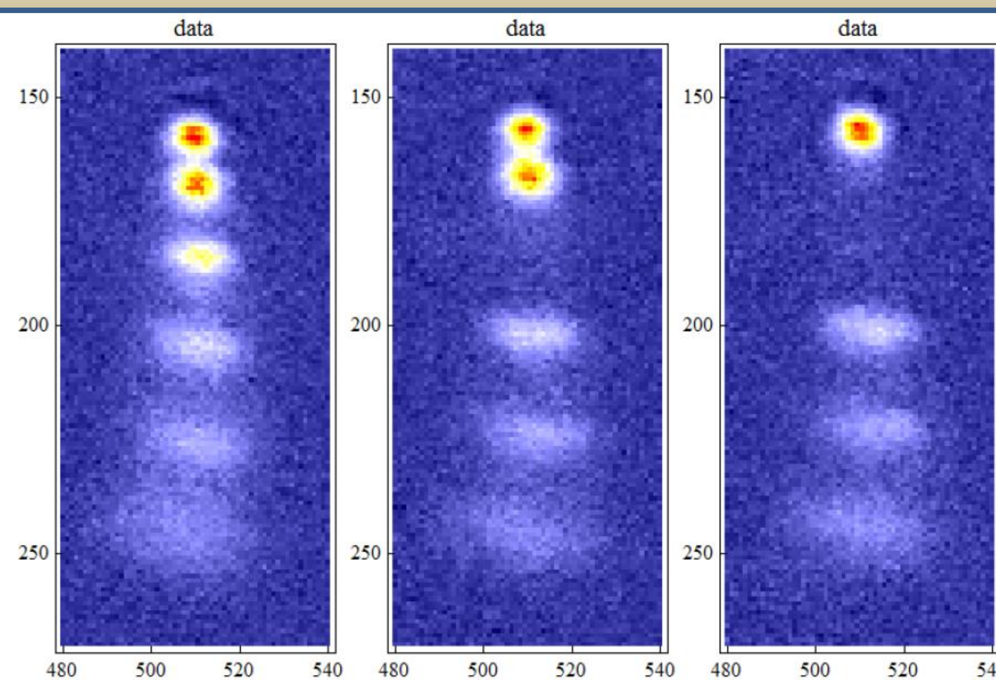

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Efficient algorithm for C_2: M. Mochol-Grzelak, A. Dauphin, AC, M. Lewenstein, *to appear*

How? C. atoms in OL as charged electrons in external fields and synthetic dimension [AC *et al* PRL 112 , 043001 (2014)]

Or also…

Science 14 February 2014:
Vol. 343 no. 6172 p. 711
DOI: 10.1126/science.343.6172.711-b

EDITORS' CHOICE

PHYSICS

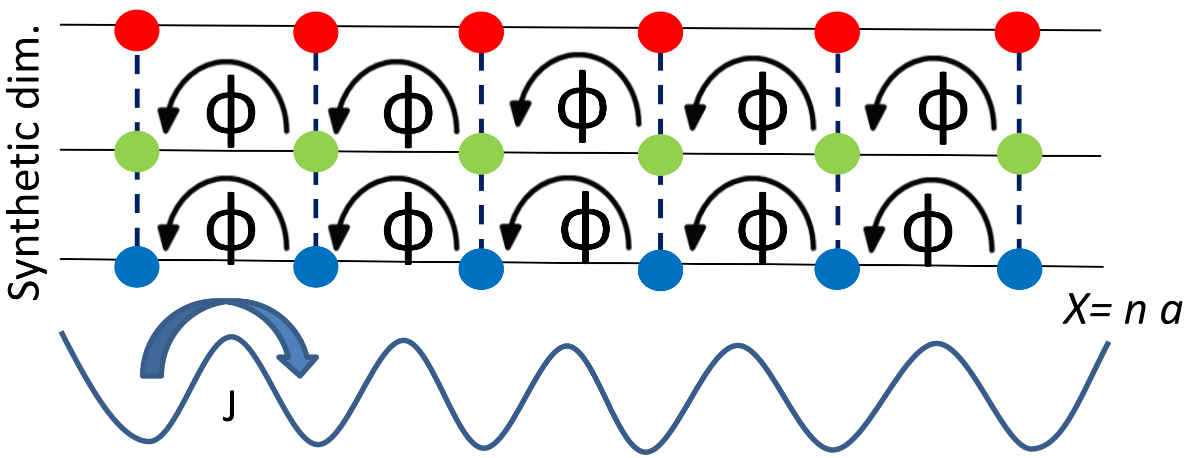
A Semisynthetic Lattice

Jelena Stajic

Atomic vapors at very low temperatures are useful for the quantum simulation of solid-state systems, because their properties can be finely controlled and tuned. These neutral atoms are not, however, completely analogous to the charged carriers in solids; for instance, an external magnetic field causes electrons to move in circular orbits but has no such effects on neutral atoms. Celi *et al.* propose a simple method for creating a uniform magnetic flux in a one-dimensional (1D) optical lattice that, if realized, might be used to observe exotic phenomena such as Hofstadter-butterfly–like fractal spectra or the dynamics of topological edge states. The method is based on synthetically extending the 1D lattice into the second dimension of internal atomic states (spin) by coupling those states using a pair of Raman laser beams that are directed at an angle with respect to the optical lattice; the required amount of the Raman laser light is substantially smaller than in existing schemes. The resulting band structure supports edge states in the spin variable whose dynamics should be observable through spin-sensitive density measurements.

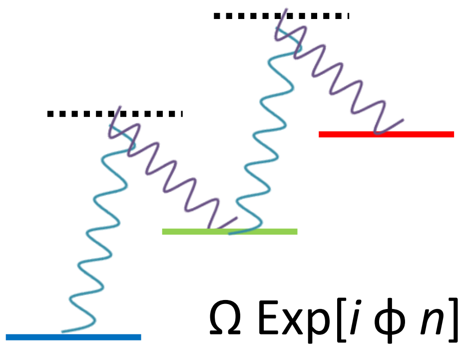
Phys. Rev. Lett. 112, 043001 (2014).

How? C. atoms in OL as charged electrons in external fields and synthetic dimension [AC *et al* PRL 112 , 043001 (2014)]



Constant magnetic flux ϕ!

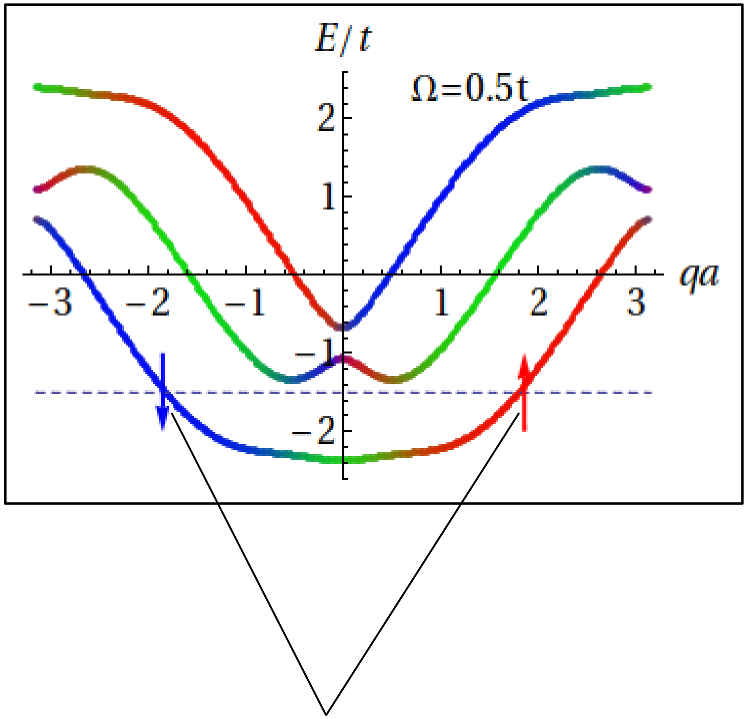
1d-lattice loaded e.g. with ^{87}Rb (F=1, m=-1,0,1)
+
Raman dressing



Sharp Boundaries ➡ Edge currents (hard to get in real 2d lattice)
signal of Topological nature of Q. Hall (bulk-boundary corr.)

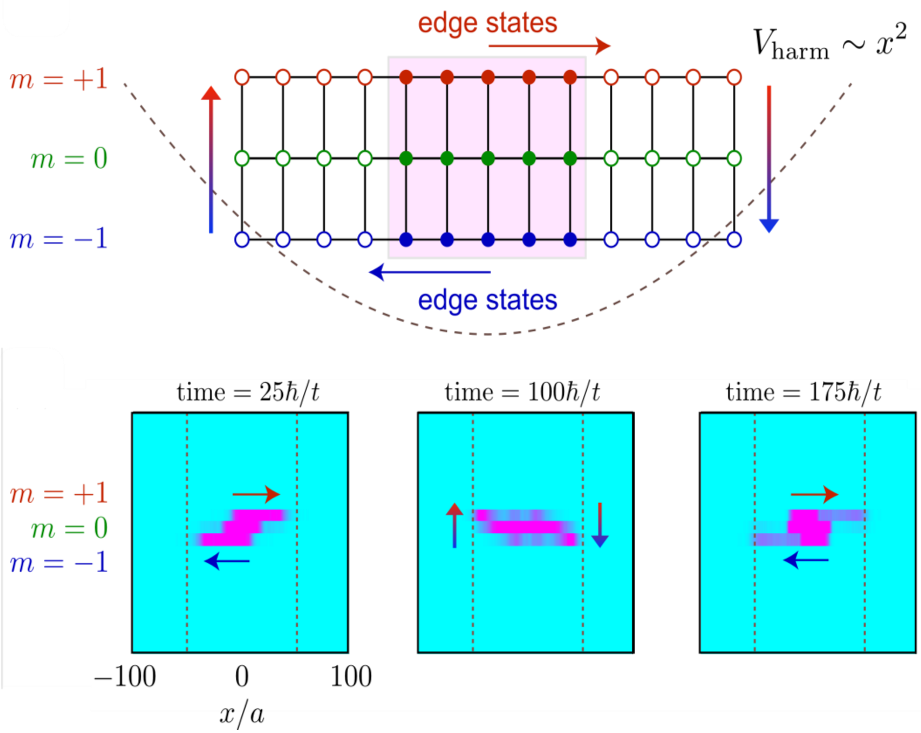
How? C. atoms in OL as charged electrons in external fields and synthetic dimension [AC *et al* PRL 112 , 043001 (2014)]

Spectrum



Edge states

Evolution



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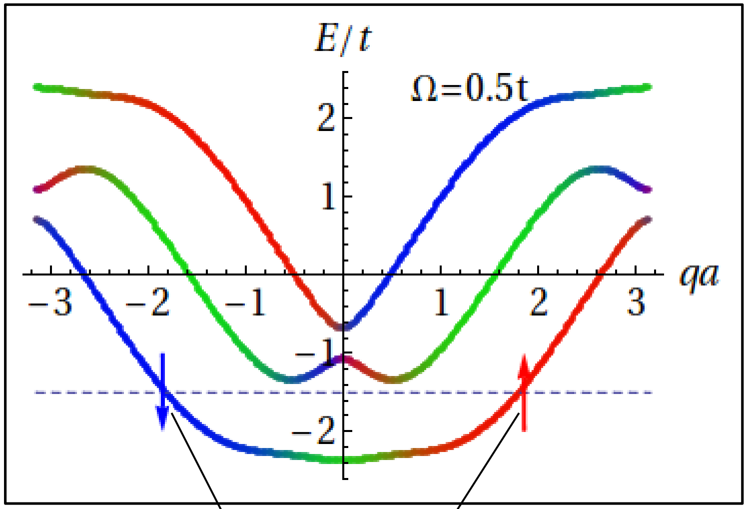

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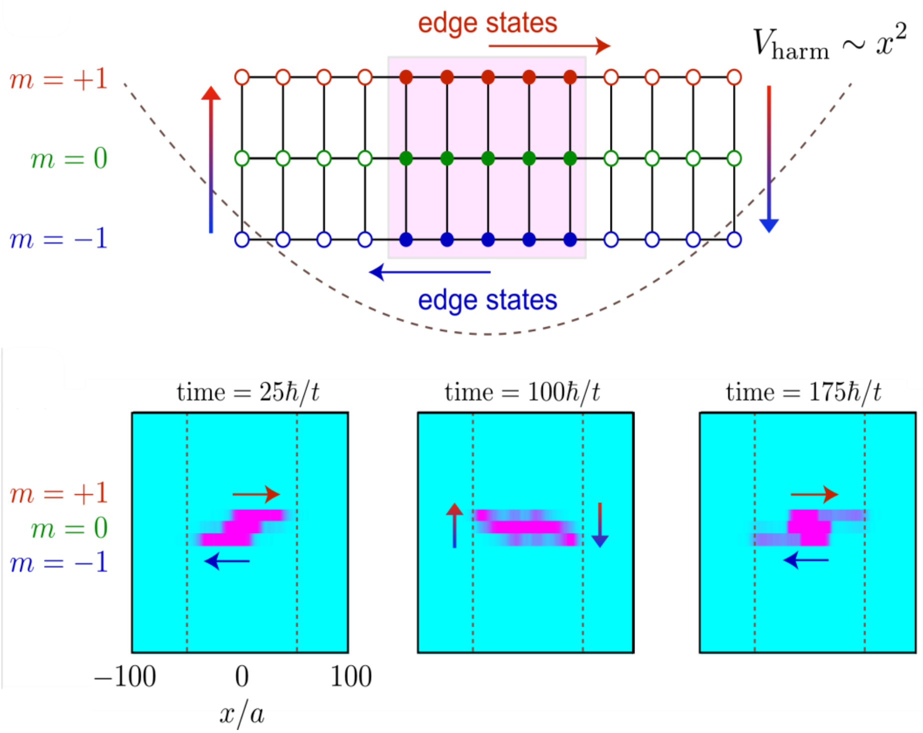


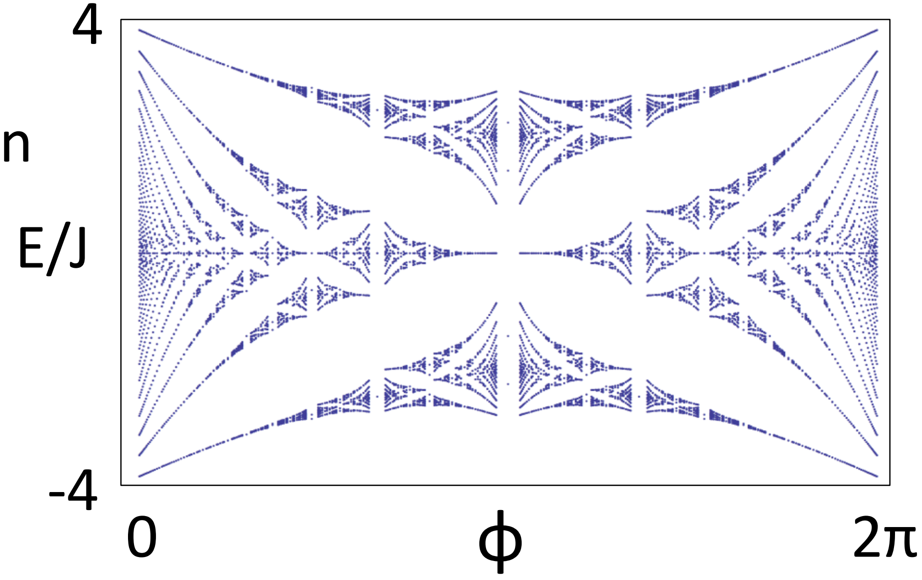
Alternative realization: real-space ladder

Hügel & Paredes PRA, Atala et al. Nat. Phys. (2014)

How? C. atoms in OL as charged electrons in external fields and synthetic dimension [AC *et al* PRL 112 , 043001 (2014)]

Wrapping the synthetic dimension

Hofstadter butterfly spectrum



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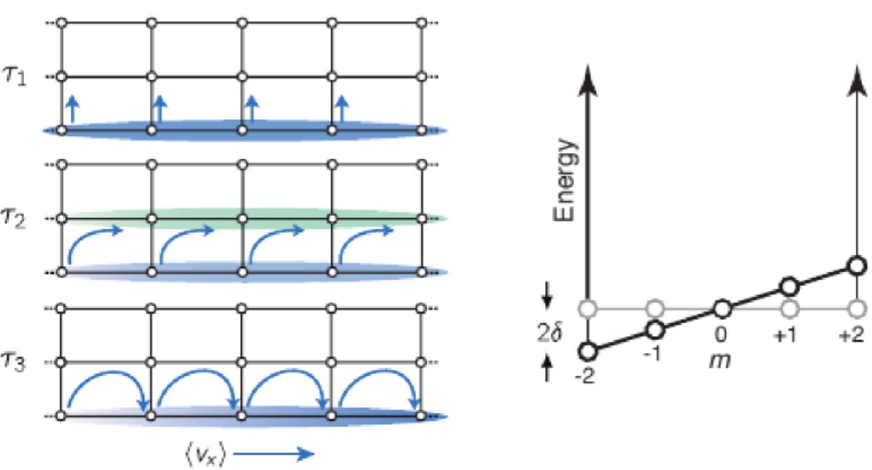
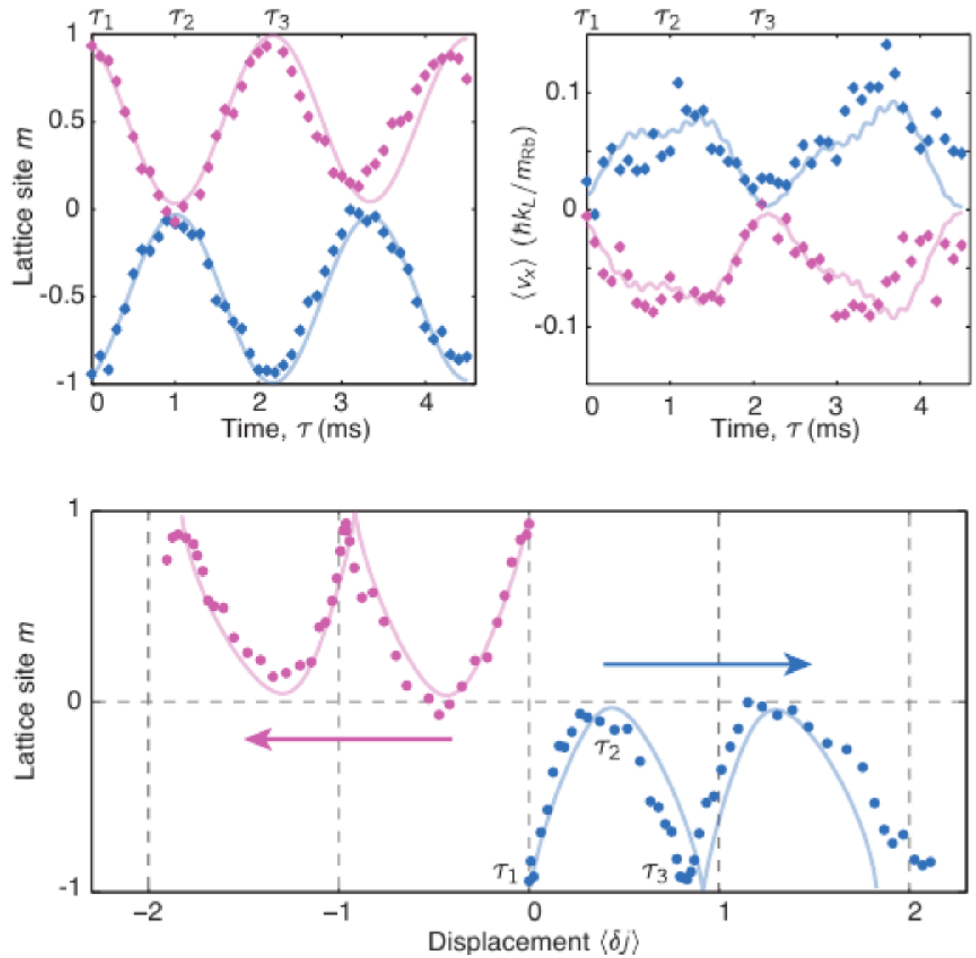
Experimental Realizations: **I)** NIST I.B. Spielman group ^{87}Rb

Edge & bulk states visualization [*Science* 349 1514-1518 (2015)]

$\phi \approx 2\pi/3$





AC, Tarruell *Probing the edge with cold atoms*
Science 349 1450-1451 (2015) (Perspective)

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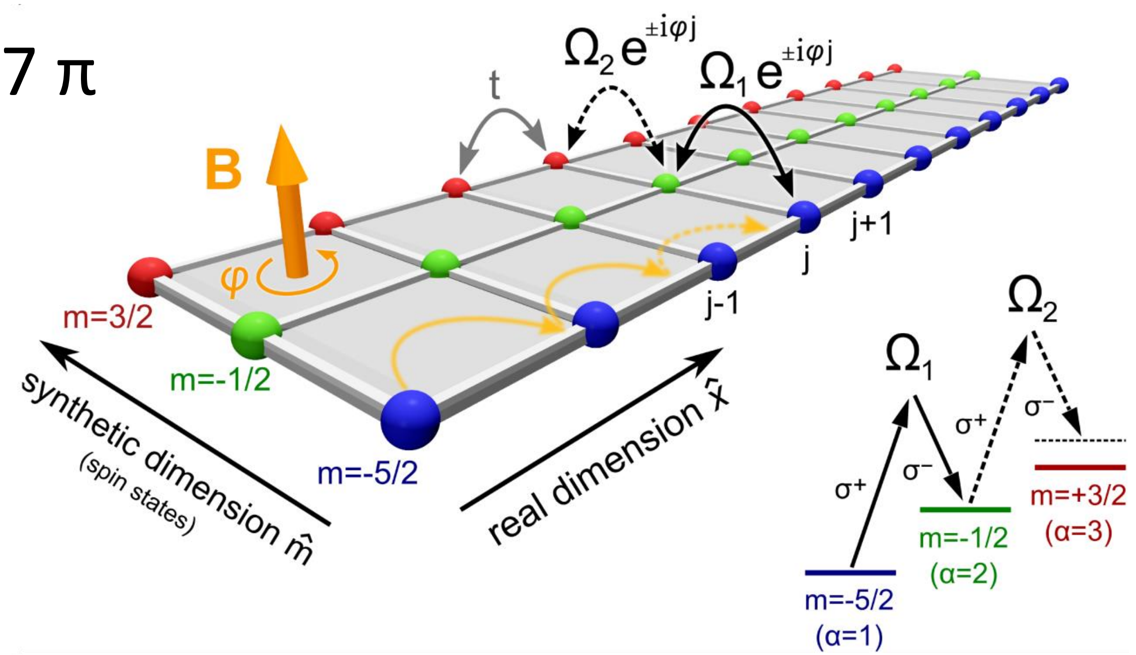
Experimental Realizations: **II)** LENS Fallani group ^{173}Yb

Chiral Edge in Fermionic gas [*Science* 349 1510-1513 (2015)]

$\phi \approx 0.37\,\pi$



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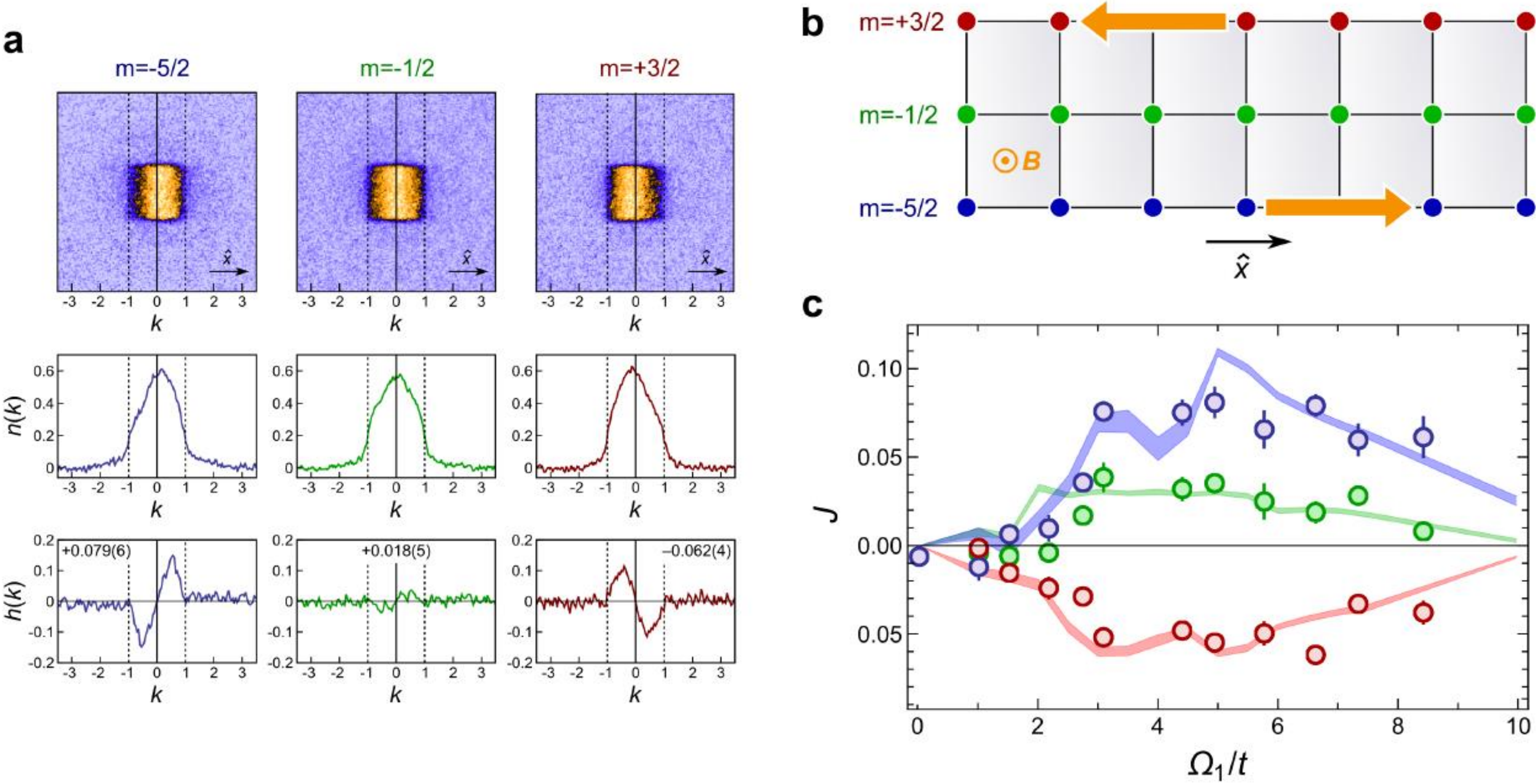
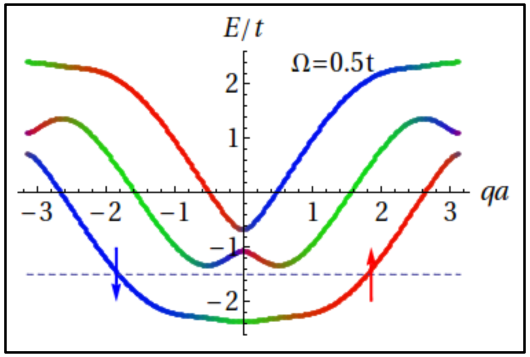


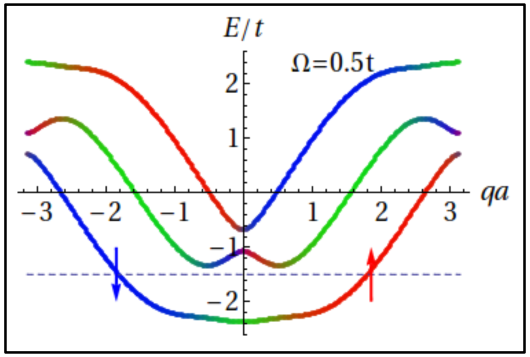
Topology in narrow strips



Narrow Hofstadter strips have edge states

What about the “bulk”?

Topology in narrow strips



Narrow Hofstadter strips have edge states

What about the “bulk”?

How big should it be to display topological properties?

Is there some reminiscence of open/closed boundary correspondence?

Is Chern number defined? / Can we measure it?

Measuring Chern numbers in (narrow) Hofstadter strips, S.Mugel,….AC *arXiv:1705.04676*

Pragmatic approach: measure transverse displacement to a force after a Bloch oscillation, Laughlin pump argument

Cf. Thouless pump [Lohse *et al* Nat. Phys. (2015)]
[Nakajima *et al* Nat. Phys (2016)]

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"Usual" semiclassical approach

$$\mathbf{k}(t) = \mathbf{k}_0 + \frac{t}{\hbar d} F_x \, \mathbf{e}_x \longrightarrow \mathbf{v}_j(\mathbf{k}) = \frac{1}{\hbar}\partial_{\mathbf{k}} E_j(\mathbf{k}) + \frac{F_x}{\hbar d}\mathcal{F}_j(\mathbf{k})\mathbf{e}_y$$

Filled band:

$$\langle \mathbf{v}(t) \rangle = \frac{2\pi F_x}{\hbar d} \mathcal{C}_j \mathbf{e}_y$$

$$\mathcal{C}_j = \frac{1}{2\pi}\int_{BZ} \mathcal{F}_j$$

Ex. Thermal gas
[Dauphin *et al* PRL (2013)]
[Aidelsburger *et al* Nat. Phys (2014)]

We consider a wave packet localized in *y* (narrow dim) and extended in *x* (large dim) in the lowest band

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Wave packet:

$$|\psi(\mathbf{k})|^2 \sim \frac{1}{A_{BZ}} \delta(k_x - k_x(t))$$

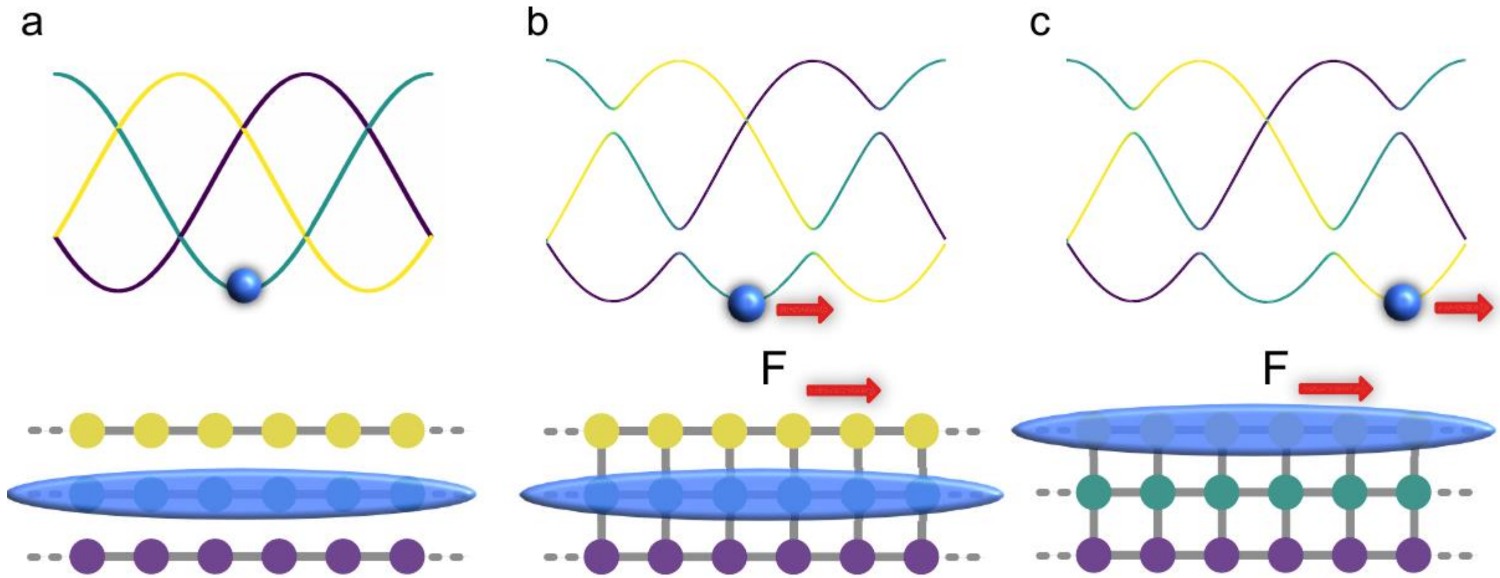
$$\langle \mathbf{r}(T) - \mathbf{r}(0) \rangle = \int_0^T \langle \mathbf{v}(t) \rangle = \frac{\hbar d}{|F_x|} \int_{BZ} \mathbf{v}(\mathbf{k}) = \text{sgn}(F_x) \mathcal{C} \, d \, \mathbf{e}_y$$

State easy to prepare if the coupling $J_y \ll J_x$

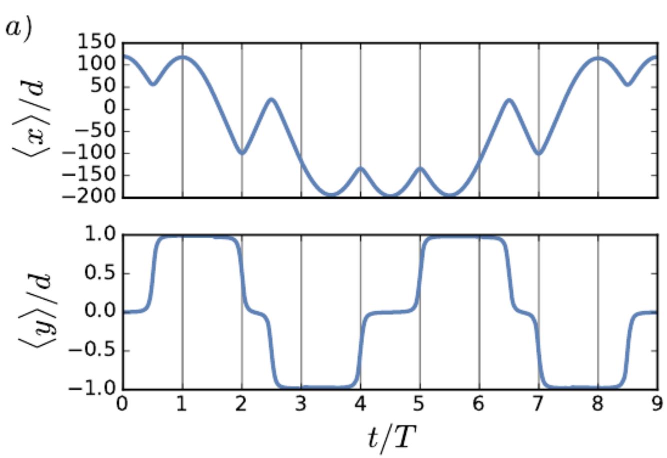
Different than geometric pumping of Lu *et al.* PRL (2016)

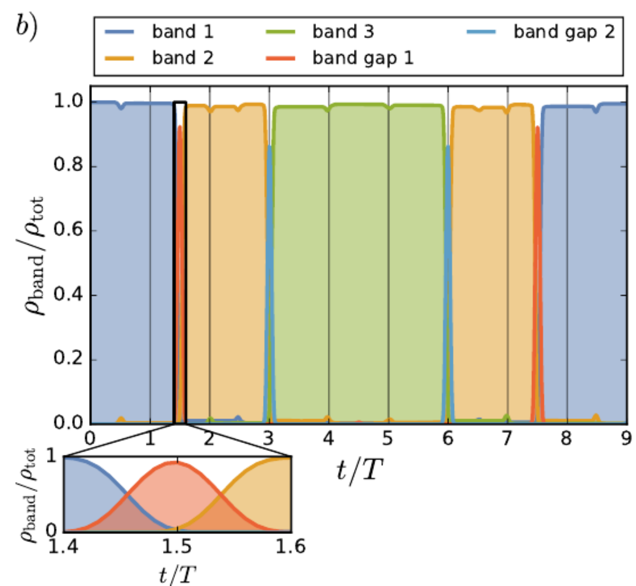
Measuring Chern numbers in (narrow) Hofstadter strips, S.Mugel,….AC *arXiv:1705.04676*

Scheme:



Results: $J_y = \frac{1}{5} J_x, \quad \Phi = \frac{2\pi}{3} \quad N_y = 3$





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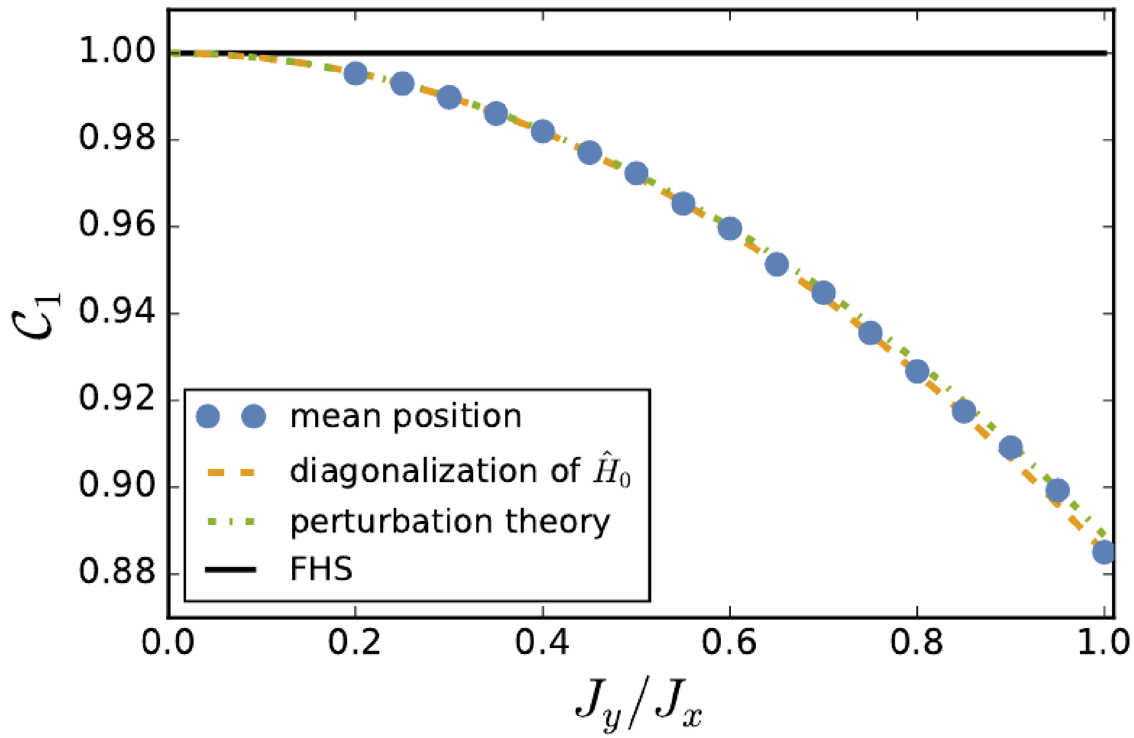
Why does it work? Perturbative argument also for edge states:

- Gap linear in J_y/J_x
- Hybridization spin states (spreading in y) quadratic in J_y/J_x



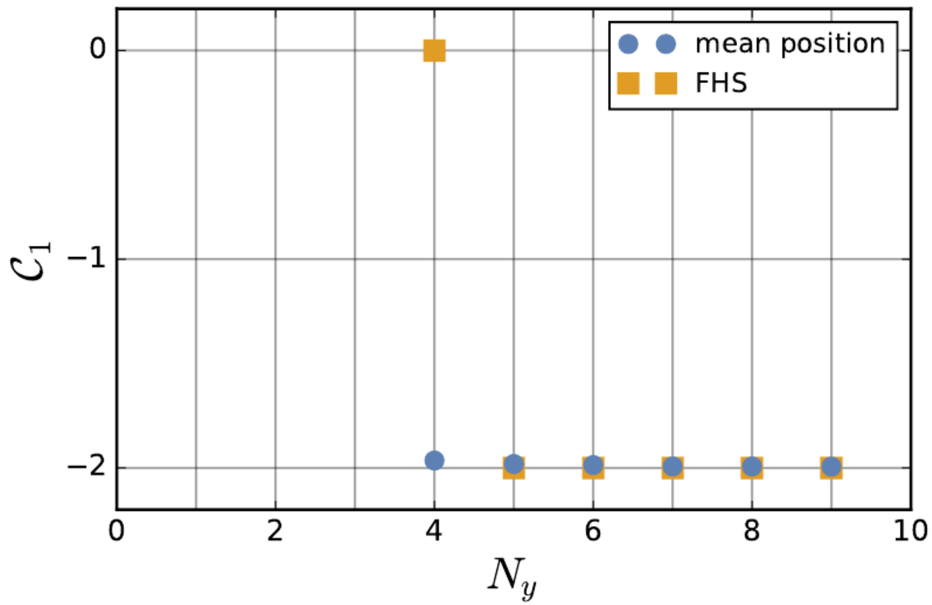
Quadratic degradation of the measurement

Measuring Chern numbers in (narrow) Hofstadter strips, S.Mugel,….AC *arXiv:1705.04676*

Higher $\mathcal{C}$ possible for $N_y \geq \mathcal{C}+2$

Ex: $\Phi = \frac{4\pi}{5} \rightarrow \mathcal{C}_1 = -2$

"Better" than Fukui-Hatsugai-Suzuki algorithm J. Phys. Soc. Jpn. (2005)

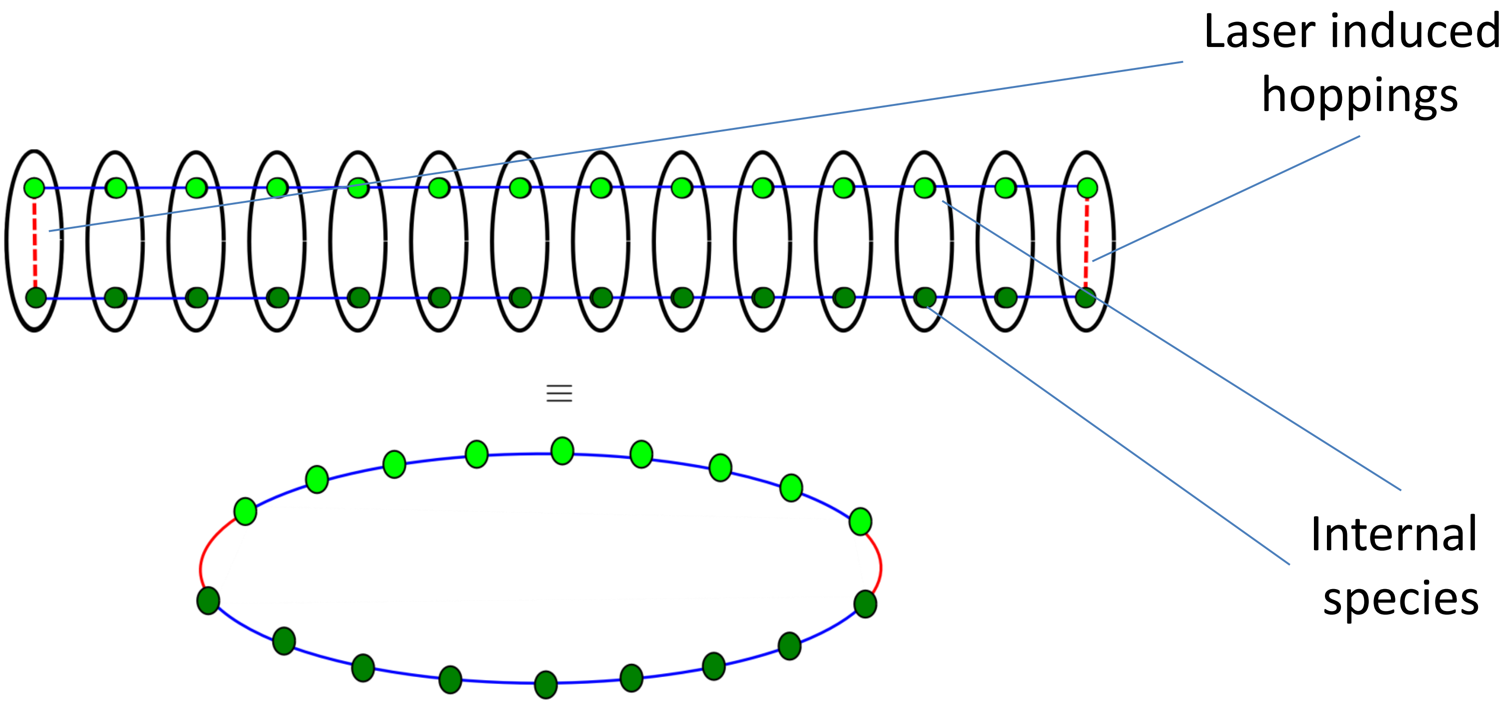


Robust to disorder (< gap) and typical harmonic confinement

Interactions? Gap small (although may hold thought adiabatic argument, see later)

How? C. atoms in OL as charged electrons in synthetic non-trivial topologies [Boada,AC *et al* NJP 17 (2015) 045007]

Circle from a line *on the lattice*



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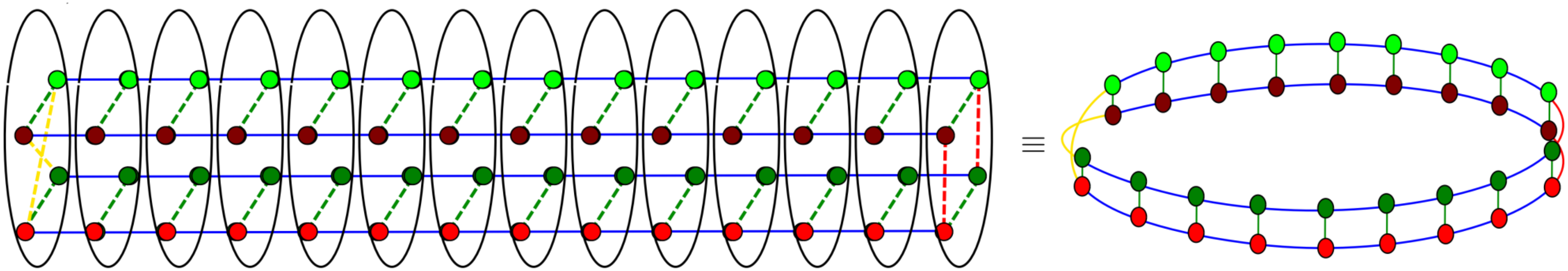
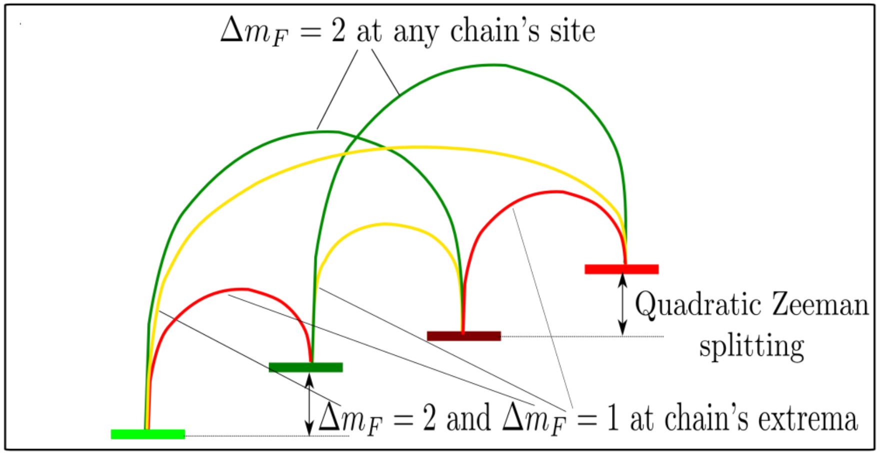
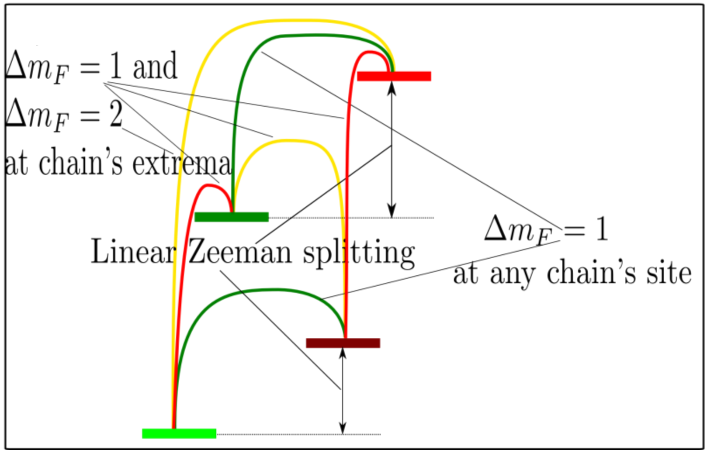
Möbius strip by *twisting the cylinder*







How? C. atoms in OL as charged electrons in synthetic non-trivial topologies [Boada,AC *et al* NJP 17 (2015) 045007]

Observables:

Single particle: ex. IQH in cylinder vs Möbius

→ Edge current stops

See Morais-Smith's group works PRB 89, 235112 (2014), SSC 215, 27 (2015)

Many Body: interactions synthetic lattice long-range!

Topology induced changes in Mott-superfluid transition
in extended Hubbard model (& spin system)

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General feature of synthetic lattices

Interactions in synthetic lattices

Let's go back to the "simple" synthetic ladder…

Bosons with *N* internal states, e.g. ^{87}*RB N=3, ≈SU(3) inv. inter.*

synthetic hopping + density-density interactions + SU(N)-breaking terms

= Novel effective Hamiltonians & phases

Two possibilities: *Grass, AC & Lewenstein PRA 90 043628 (2014)*

- SU(N)-breaking from spatial hopping $\sum_{<ij>,\sigma} J_\sigma a_i^{\dagger\sigma} a_j^{\sigma'} + h.c.$
  e.g using spin-dependent lattices + shaking (*cf.* PRL 109 145301 (2012))

-> Antiferromagnetic phase e.g. *N=2*

- SU(N)-breaking from interactions $U_d \sum_{i,\sigma\sigma'} \hat{n}_i^\sigma n_i^{\sigma'}$
  Rich Mott structure! different effective Hamiltonians
  e.g. $0 < \frac{U_d}{U} < 1$(for right chem. pot.) N-state Potts-like Hamiltonians

Interactions in synthetic lattices

Interesting experimentally motivated question:

Synth. IQH in synthetic lattices + interactions -> Fractional QH effect?!

Question addressed very recently by various collaborations for Fermions

"Magnetic crystals and helical liquids in alkaline-earth fermionic gases"
Nat. Commun. 6, 8134 (2015), (NJP 18, 035010 (2016)) (Fazio group)

"Charge Pumping of Interacting Fermion Atoms in the Synthetic Dimension"
Tian-Sheng Zeng, Ce Wang, Hui, PRL 115, 095302 (2015)

Charge pumping as detection method

"Adiabatic Control of Atomic Dressed States for Transport and Sensing",
N.R. Cooper and A.M. Rey, PRA 92 021401 (2015)

"Topological fractional pumping with alkaline-earth(-like) ultracold atoms",
(Fazio group), arXiv:1607.07842

Majorana Fermions

"Majorana Quasi-Particles Protected by Z_2 Angular Momentum Conservation",
arXiv:1702.04733 (Innsbruck-ICTP-LENS)

……

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Interesting experimentally motivated question:

Synth. IQH in synthetic lattices + interactions -> Fractional QH effect?!

and for Bosons… mainly real space slabs

“Bosonic Mott insulator with Meissner currents”, A. Petrescu and K.Le Hur, Phys. Rev. Lett. 111 150601 (2013)

“Strongly interacting bosons on a three-leg ladder in the presence of a homogeneous flux”, F. Kolley, M. Piraud, I.P. McCulloch, U. Schollwöck, F. Heidrich-Meisner, NJP 17 092001 (2015) (PRA A 94, 063628 (2016))

“Persisting Meissner state and incommensurate phases of hard-core boson ladders in a flux”, M.Di Dio, S. De Palo, E. Orignac, R. Citro, M.L. Chiofalo, Phys. Rev. B 92, 060506 (2015) (NJP 18 055017 (2016)) (arXiv:1703.07742)

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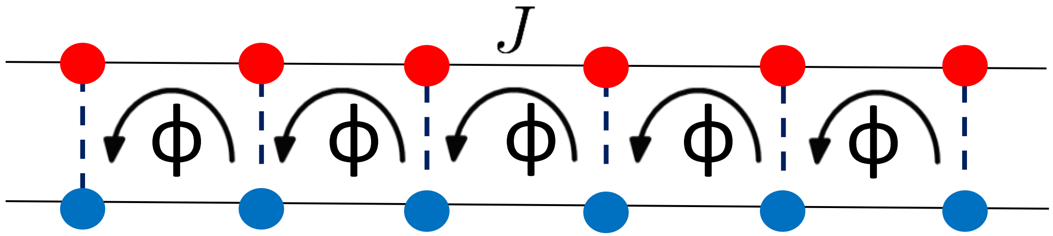
Vortex Melting in dimerized synthetic ladder,
E. Tirrito, R. Citro, M.Lewestein, AC, *soon*

Meissner/Vortex phase in flux ladder

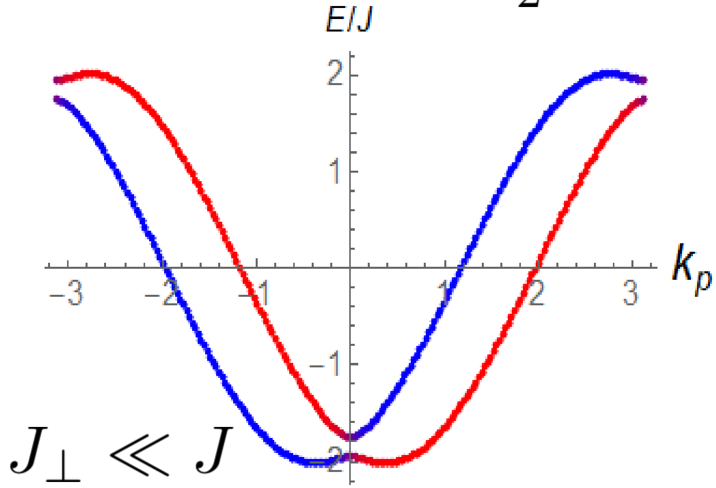


[Cf. Haug et al. 1612.09109]…

No interactions: real = synthetic ladder

Weak interleg (Raman) coupling:

2 minima, $k_m \sim \pm\frac{\phi}{2}$



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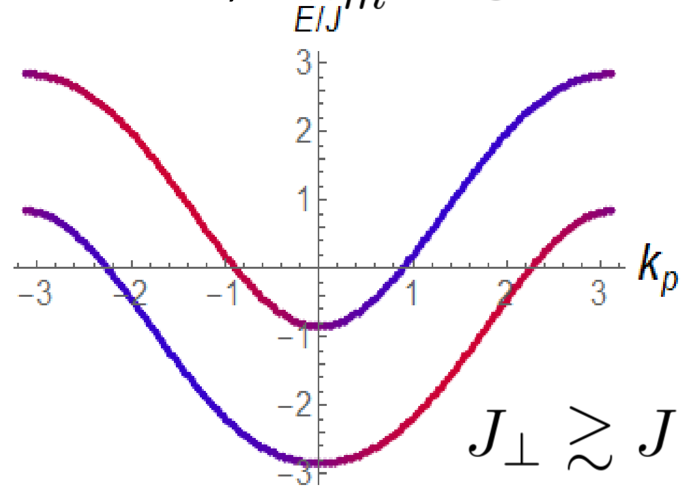

[Orignac,Giamarchi, PRB 2001] Analogous to type II, also in presence interactions

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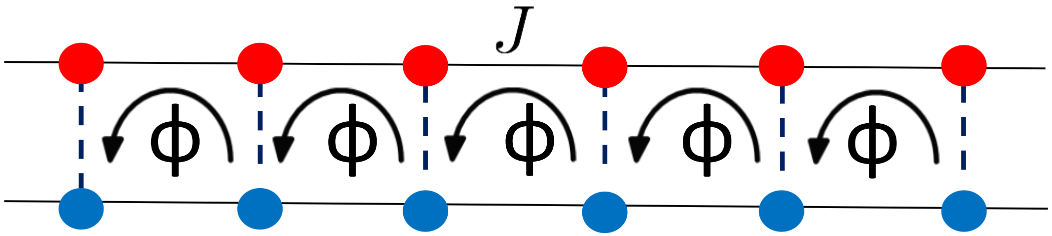

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Observables

$$J_c(j,m) = i\,\langle \hat{a}^\dagger_{j+1,m}\hat{a}_{j,m}\rangle + H.c.$$

$$J_\perp(j) = i\,\langle \hat{a}^\dagger_{j,1/2}\hat{a}_{j,-1/2}\rangle + H.c.$$

$J_\perp \ll J$

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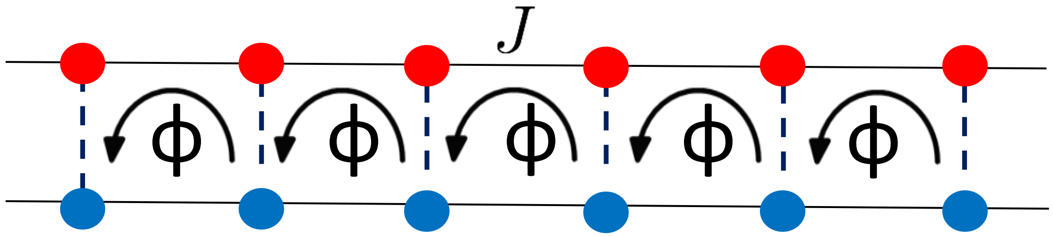

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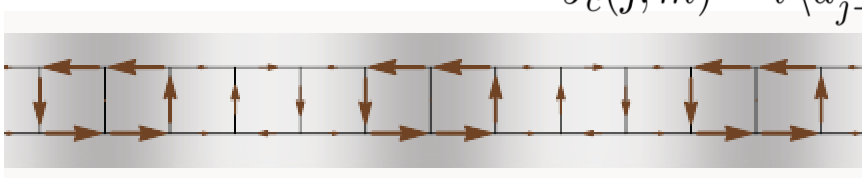

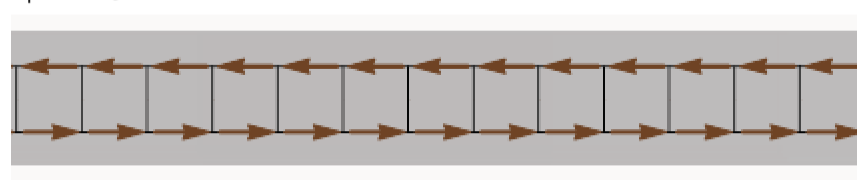

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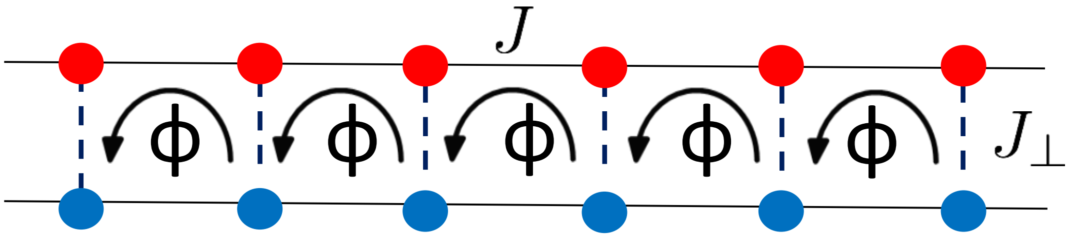
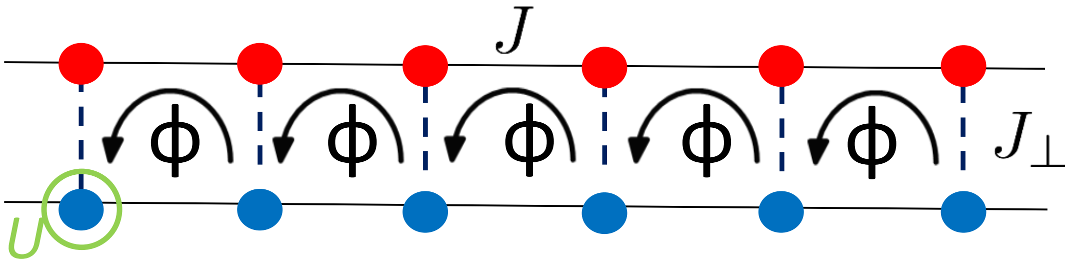


Effect of interactions: suppress vortex phase

Meissner/Vortex phase in flux ladder



Effect of interactions: suppress vortex phase

Real ladder: vortex phase survives in the hard-cord limit for ϕ large

more phases at $U \neq \infty$

see [Petrescu, Le Hur,PRL 2013]
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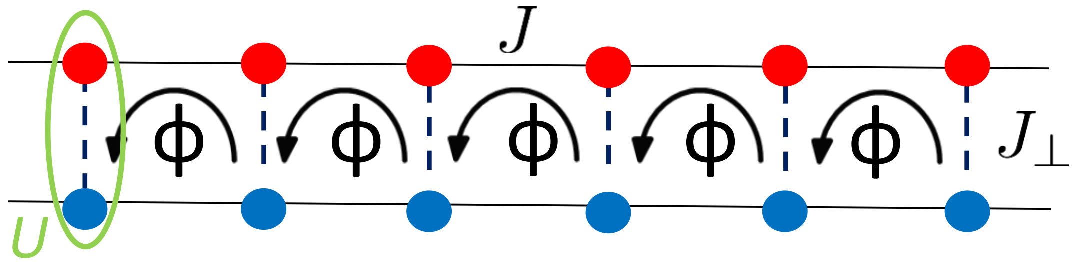

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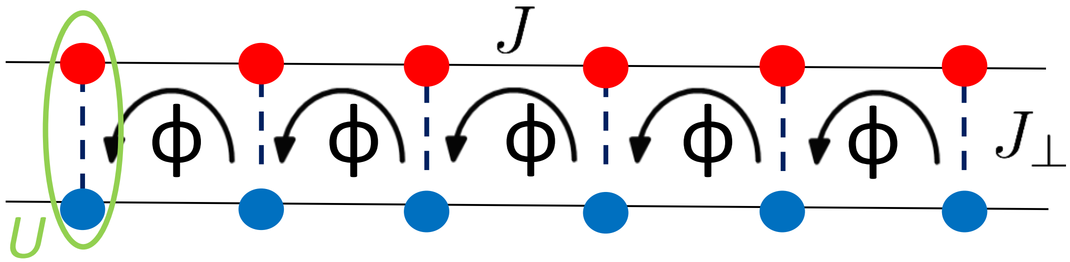

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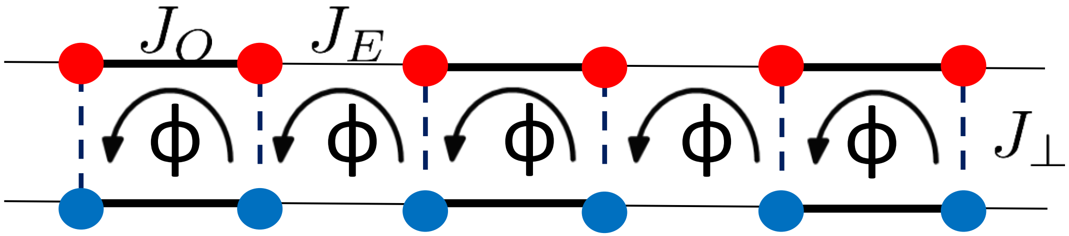
Idea: enucleate vortices by dimerizing the lattice ("easy" exp. handle)

Vortex Nesting and Melting in synthetic ladders, E. Tirrito, R. Citro, M.Lewenstein, AC *in progress*



Effect of dimerization: new handle $\Delta = \frac{J_O - J_E}{2}$

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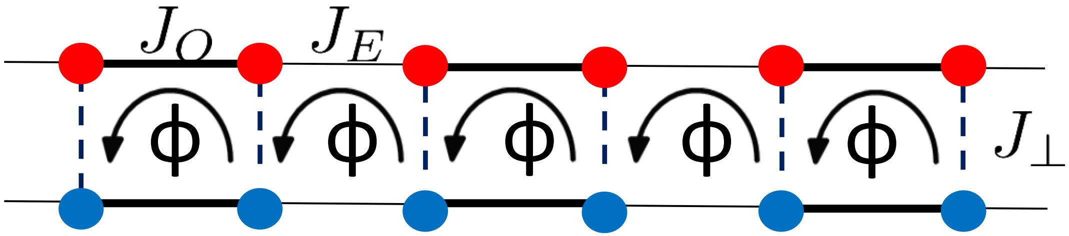

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No interactions: 4 bands

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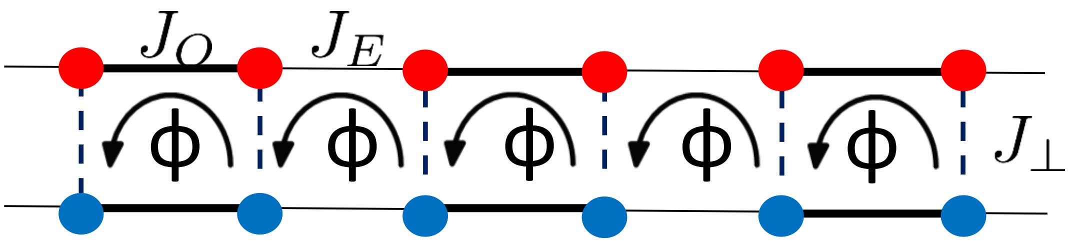

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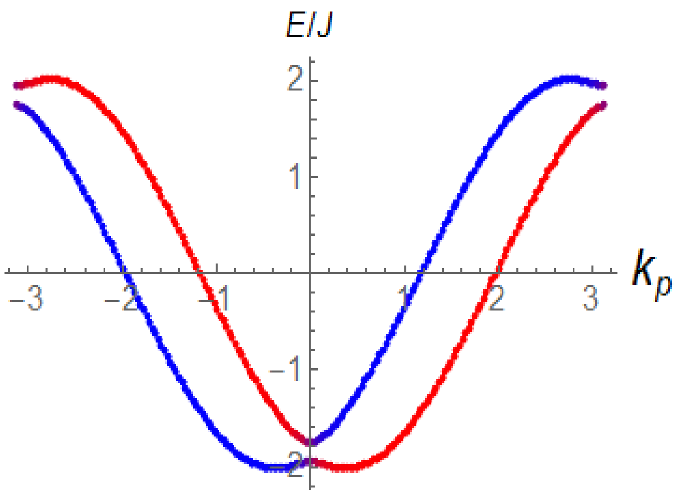
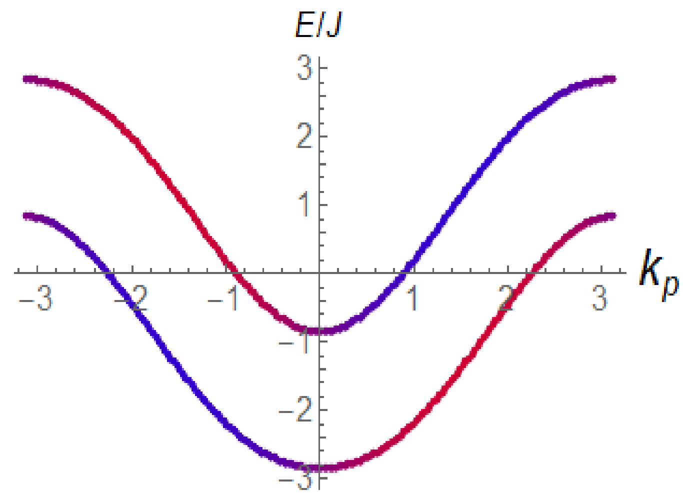


$J_\perp \ll J$

$\Delta = 0$

Just band folding



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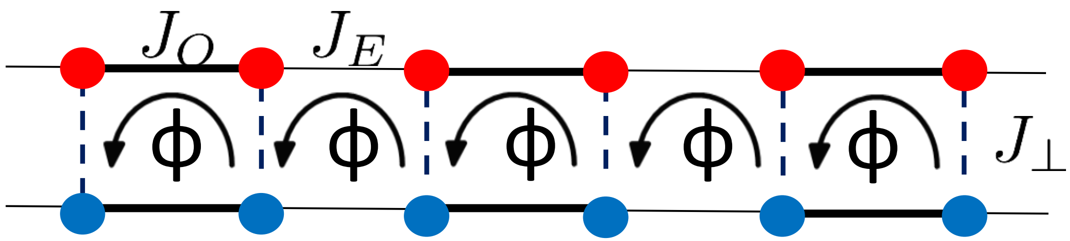

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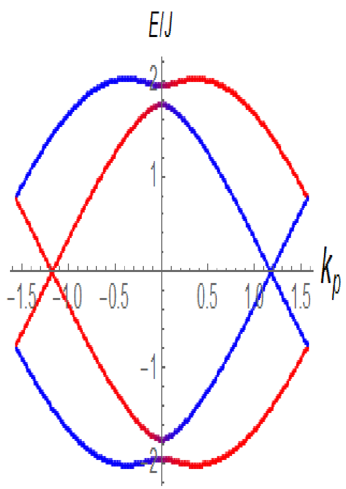

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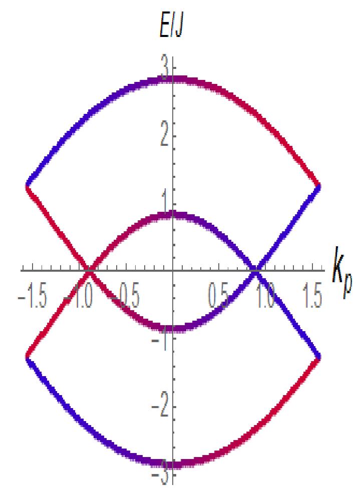

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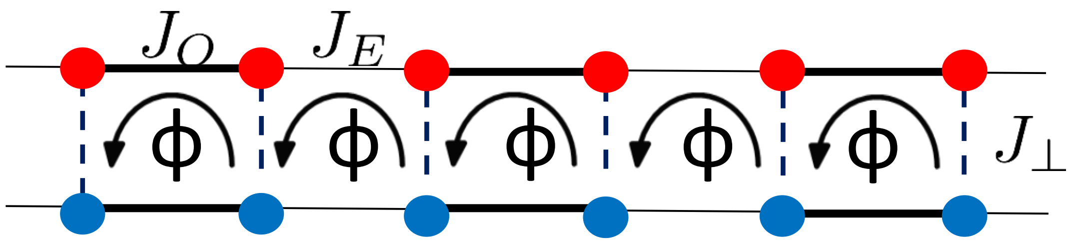

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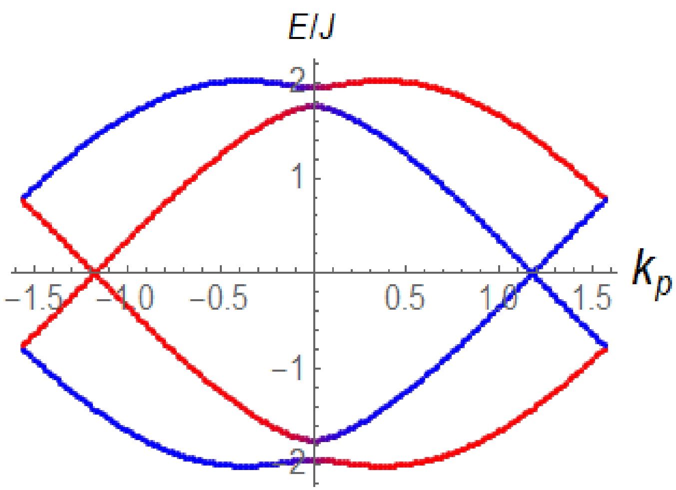

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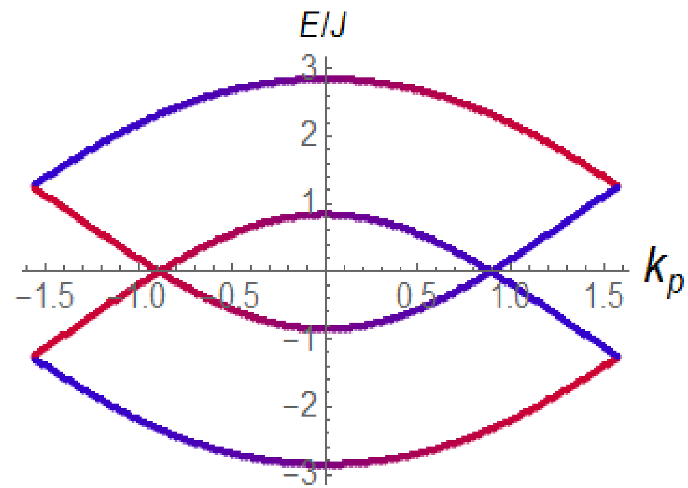

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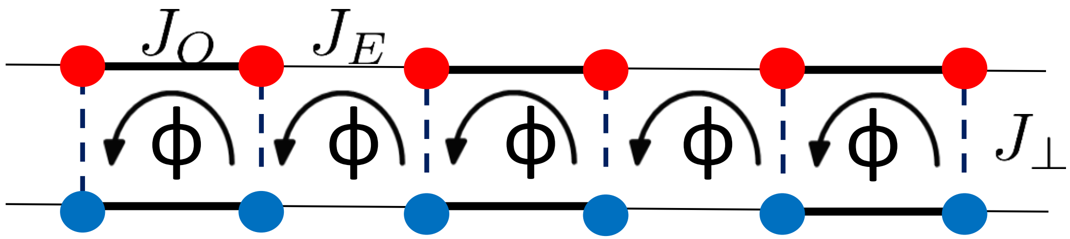

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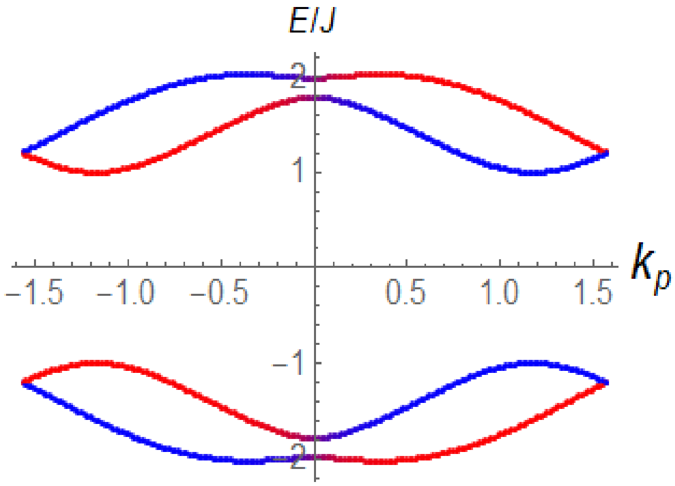
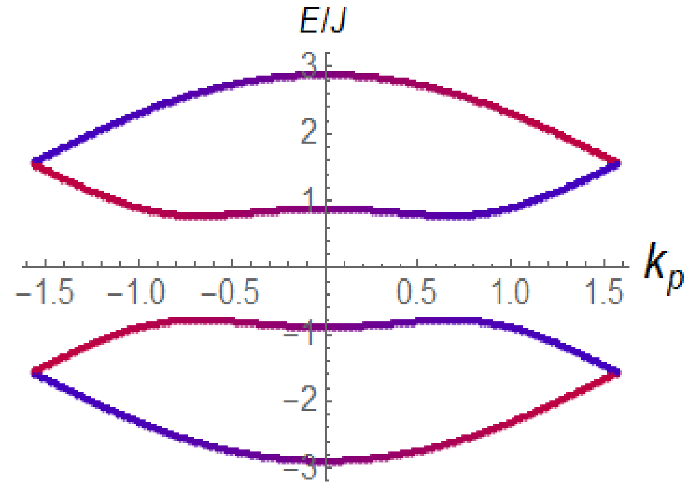


$J_\perp \ll J$

$\Delta = 0.5J$

Bands deform & mix



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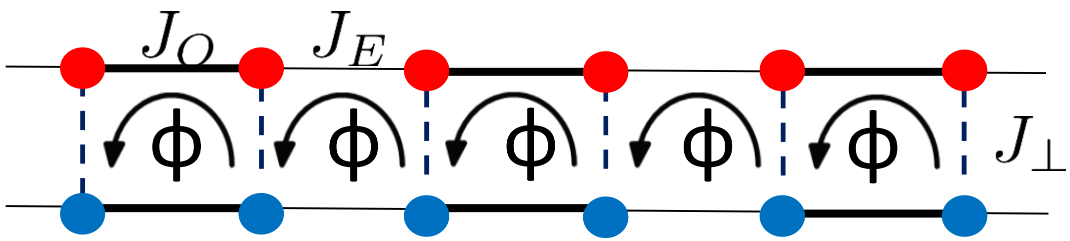

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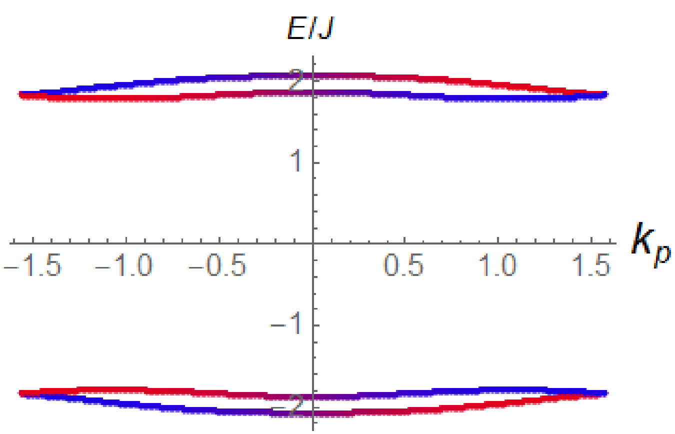
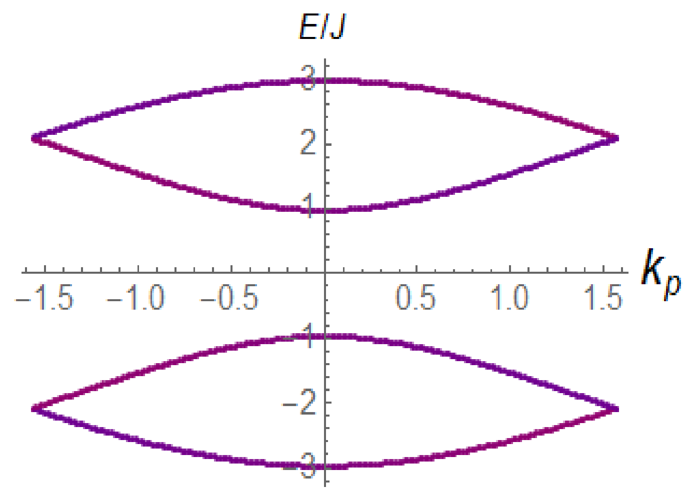


$J_\perp \ll J$

$\Delta = 0.9J$

Bands become "flat"

Both regimes Meissner



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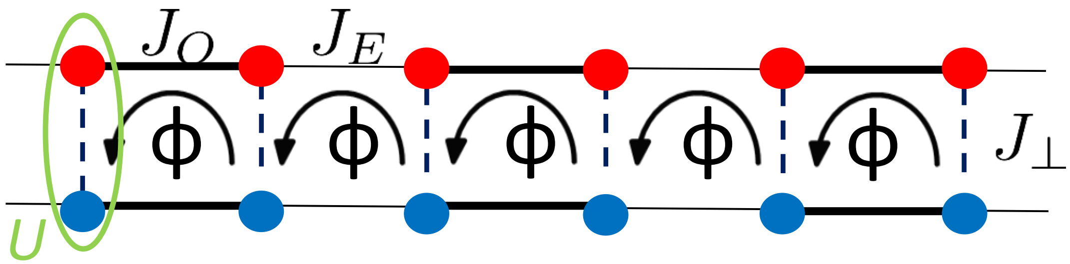

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Interactions: $U \to \infty$ 3 states per rang

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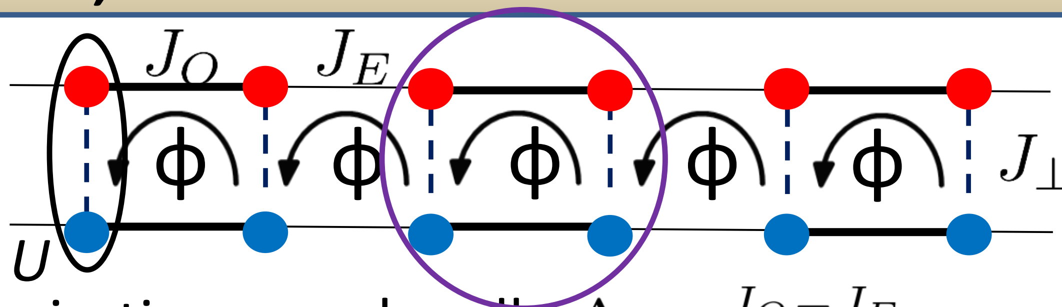

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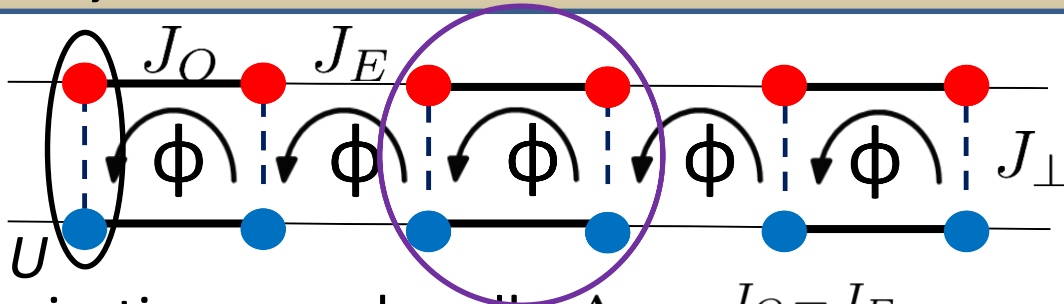

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	1 *n=0,*	4 *n=1,*	4 *n=2*
Spectrum plaquette	0	$\pm J_\perp \sqrt{1 + \left(\frac{J_O}{J_\perp}\right)^2 - 2\cos\frac{\phi}{2}\left(\frac{J_O}{J_\perp}\right)}$	$\pm J_\perp$
		$\pm J_\perp \sqrt{1 + \left(\frac{J_O}{J_\perp}\right)^2 + 2\cos\frac{\phi}{2}\left(\frac{J_O}{J_\perp}\right)}$	± 0

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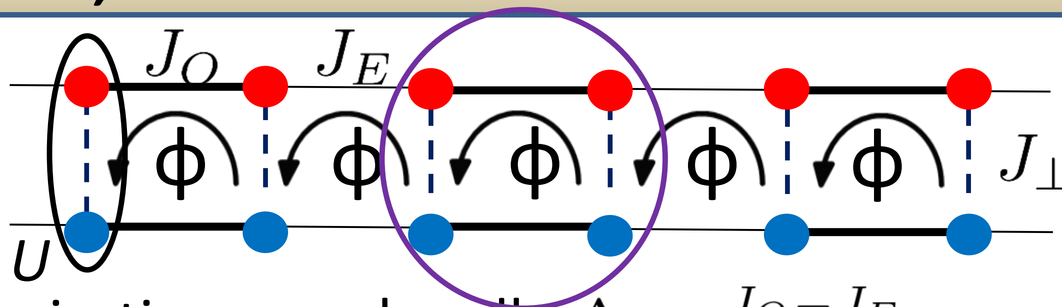

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$J_\perp \gtrsim J_O$ Plaquette in *n=2* → Band insulator

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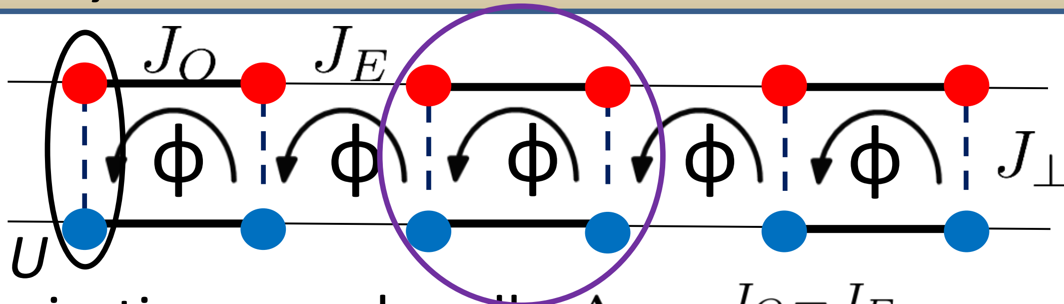

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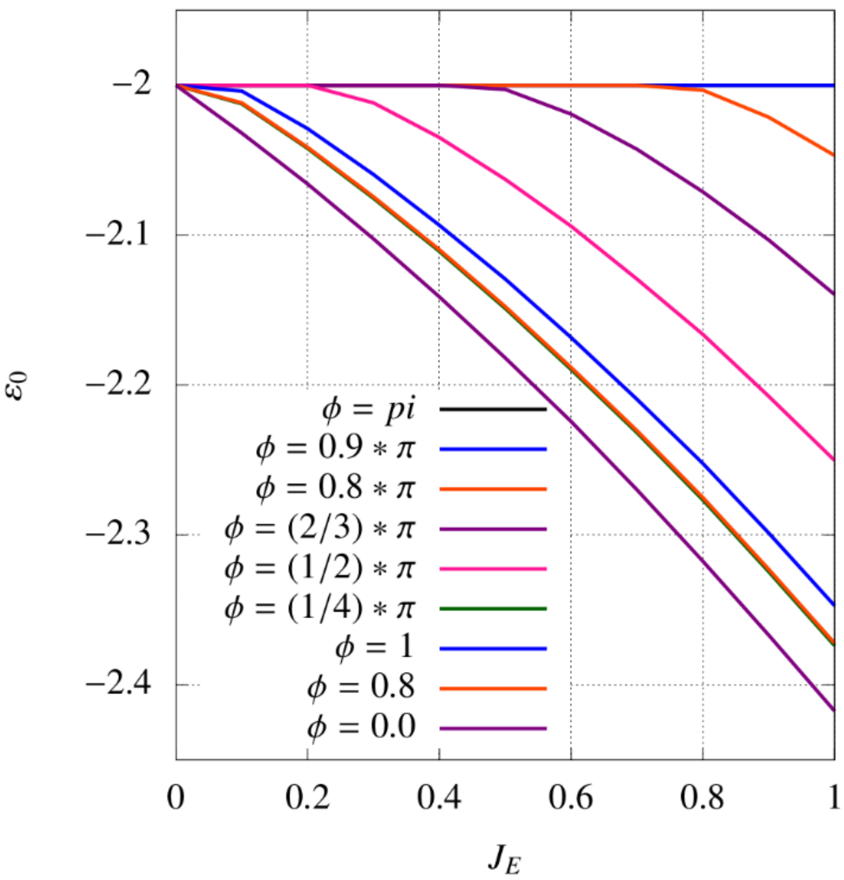
Spectrum plaquette	0	$\pm J_\perp \sqrt{1 + \left(\frac{J_O}{J_\perp}\right)^2 - 2\cos\frac{\phi}{2}\left(\frac{J_O}{J_\perp}\right)}$	$\pm 2J_\perp$
		$\pm J_\perp \sqrt{1 + \left(\frac{J_O}{J_\perp}\right)^2 + 2\cos\frac{\phi}{2}\left(\frac{J_O}{J_\perp}\right)}$	± 0

$J_\perp \gtrsim J_O$ Plaquette in *n=2* → Band insulator

$J_\perp < J_O$ Plaquette in *n=1* → Imprinted vortex

Vortex Nesting and Melting in synthetic ladders, E. Tirrito, R. Citro, M.Lewenstein, AC *in progress*

DMRG calculations confirm perturbative expectations Ex. $J_\perp = J_O = 1$



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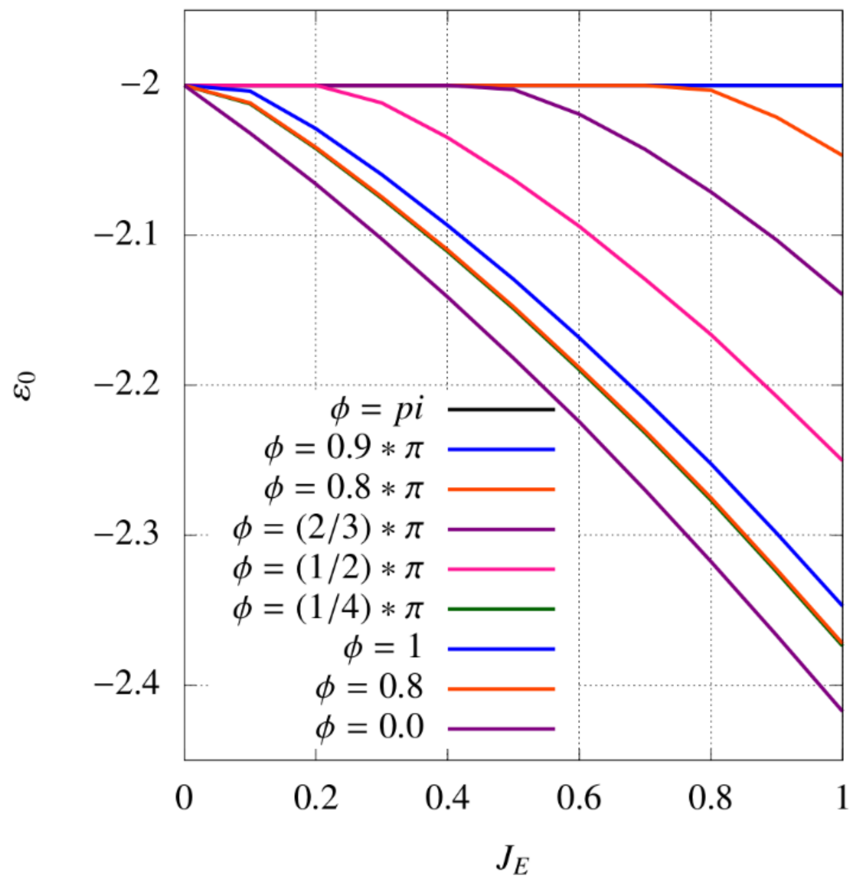
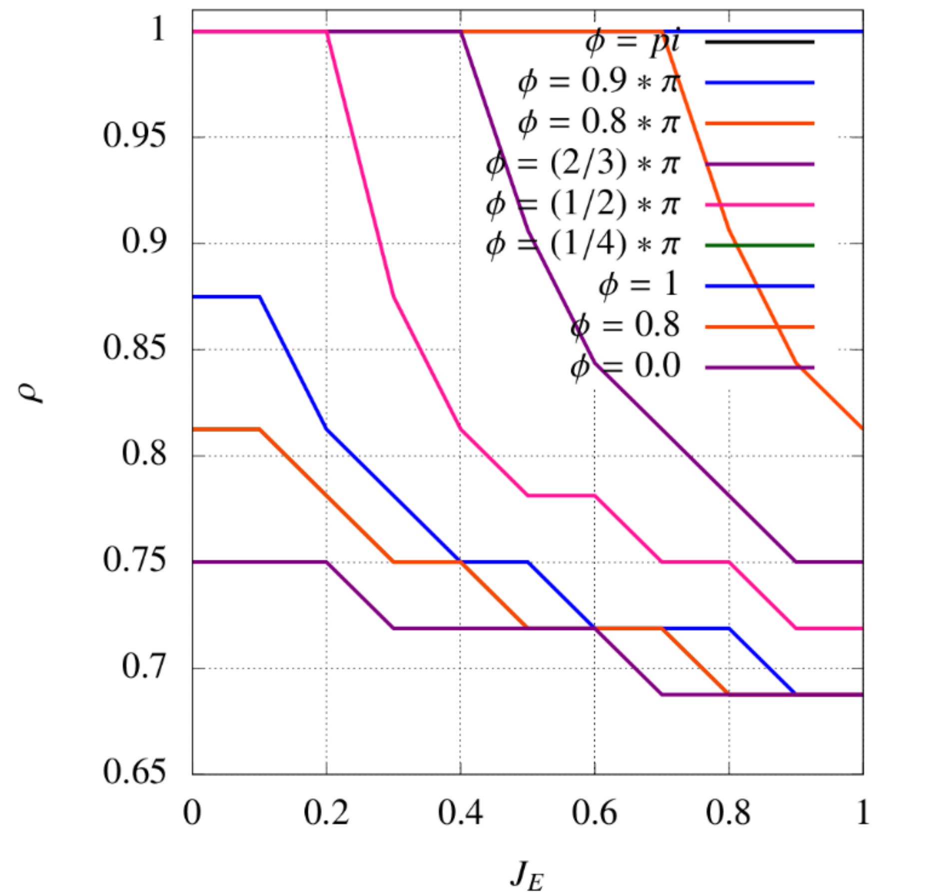
DMRG calculation confirms perturbative expectations   Ex. $J_\perp = J_O = 1$





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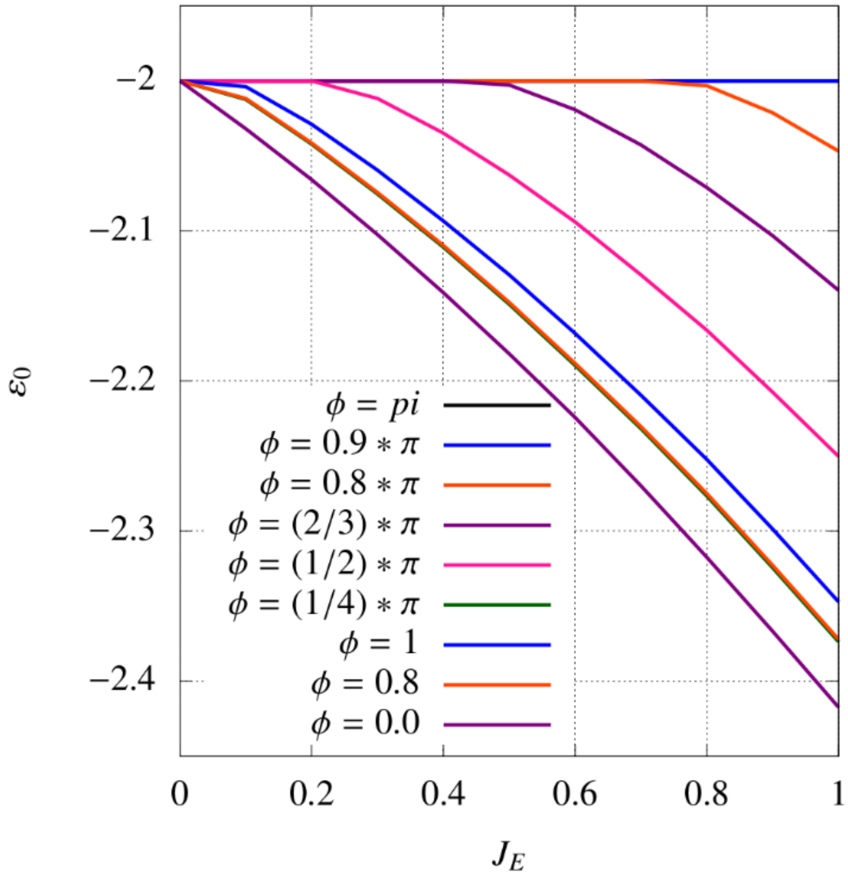
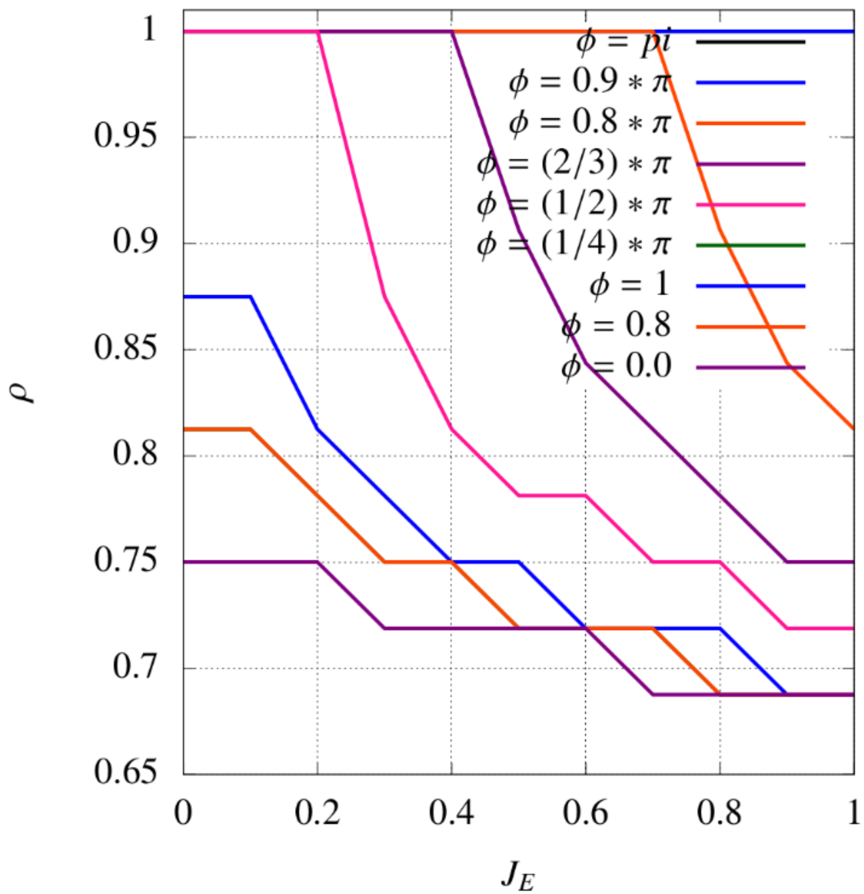
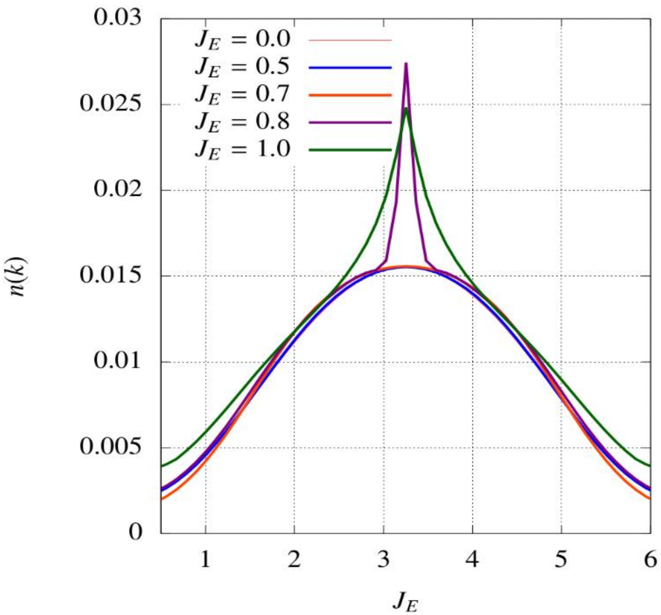


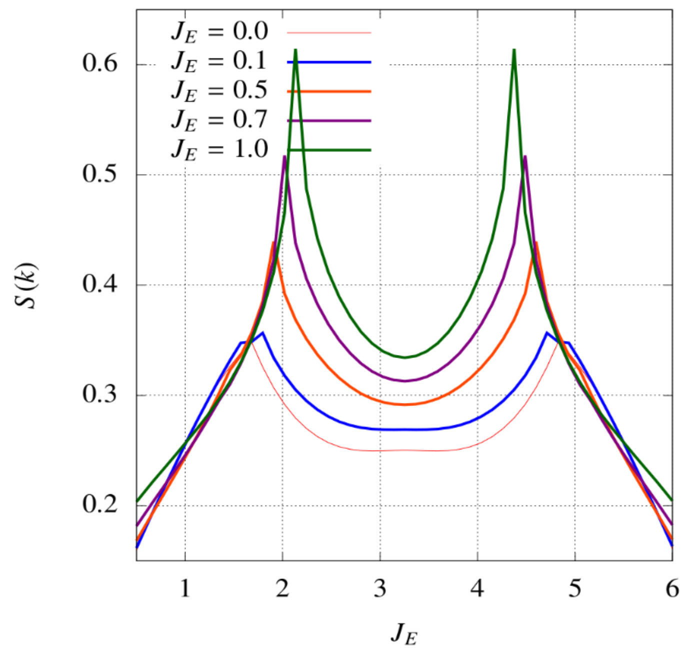


Phase diagram through calculation of currents and structure factors

Vortex Nesting and Melting in synthetic ladders, E. Tirrito, R. Citro, M.Lewenstein, AC *in progress*

Moderate $J_\perp$ Ex. $\phi = \pi/3$





$0 \longrightarrow J_E \longrightarrow J_O$

Melted vortex → Meissner charge density wave → Meissner

Vortex Nesting and Melting in synthetic ladders, E. Tirrito, R. Citro, M.Lewenstein, AC *in progress*

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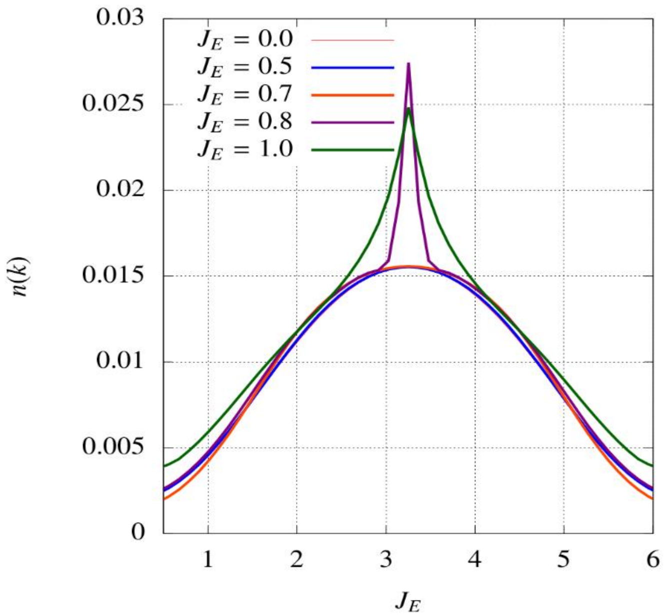
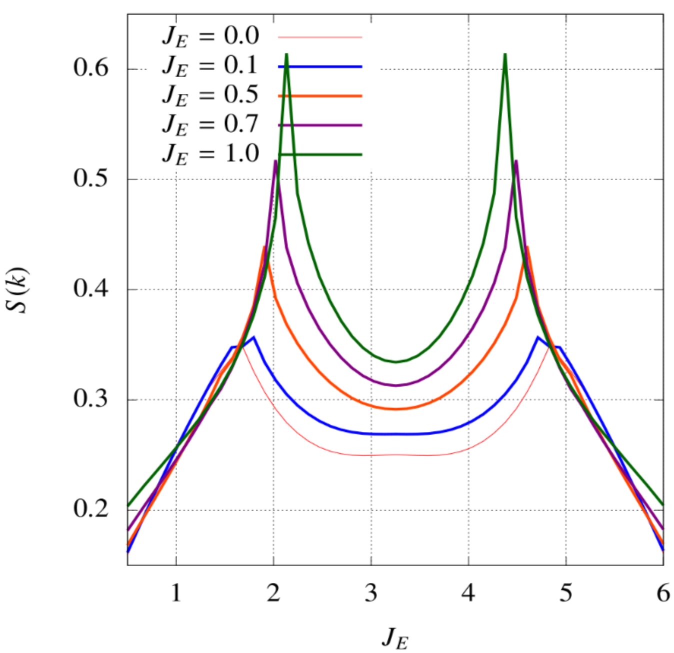




0 → J_E → J_O

Melted vortex → Meissner charge density wave → Meissner

Similar to attractive interleg interaction, [Orignac et al, arXiv:1703.07742]

Working to see evidence of commensurate-incommensurate transition

Further steps

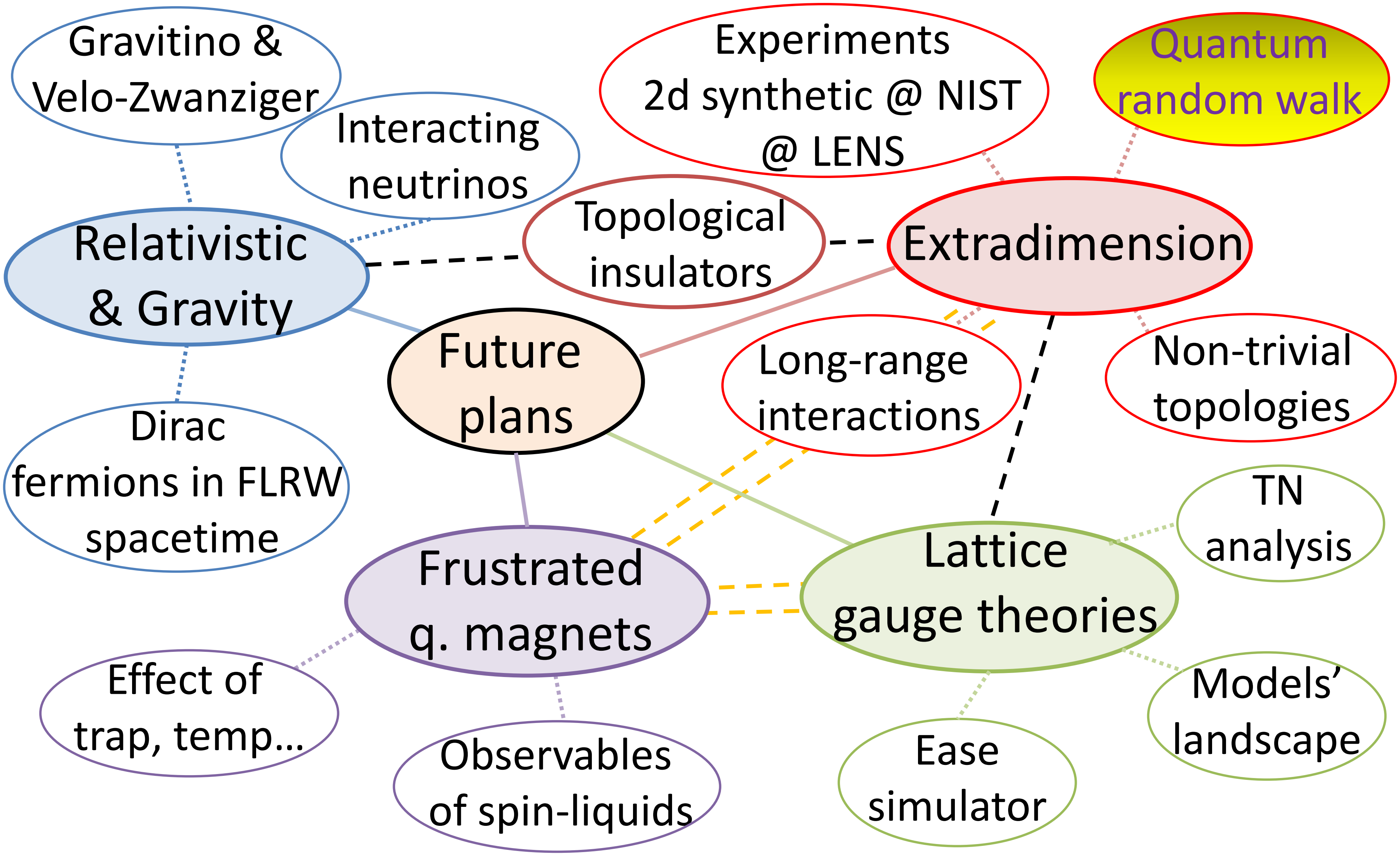
- No hard boson limit: bosons different fermions

- Study the accessible experimental parameters

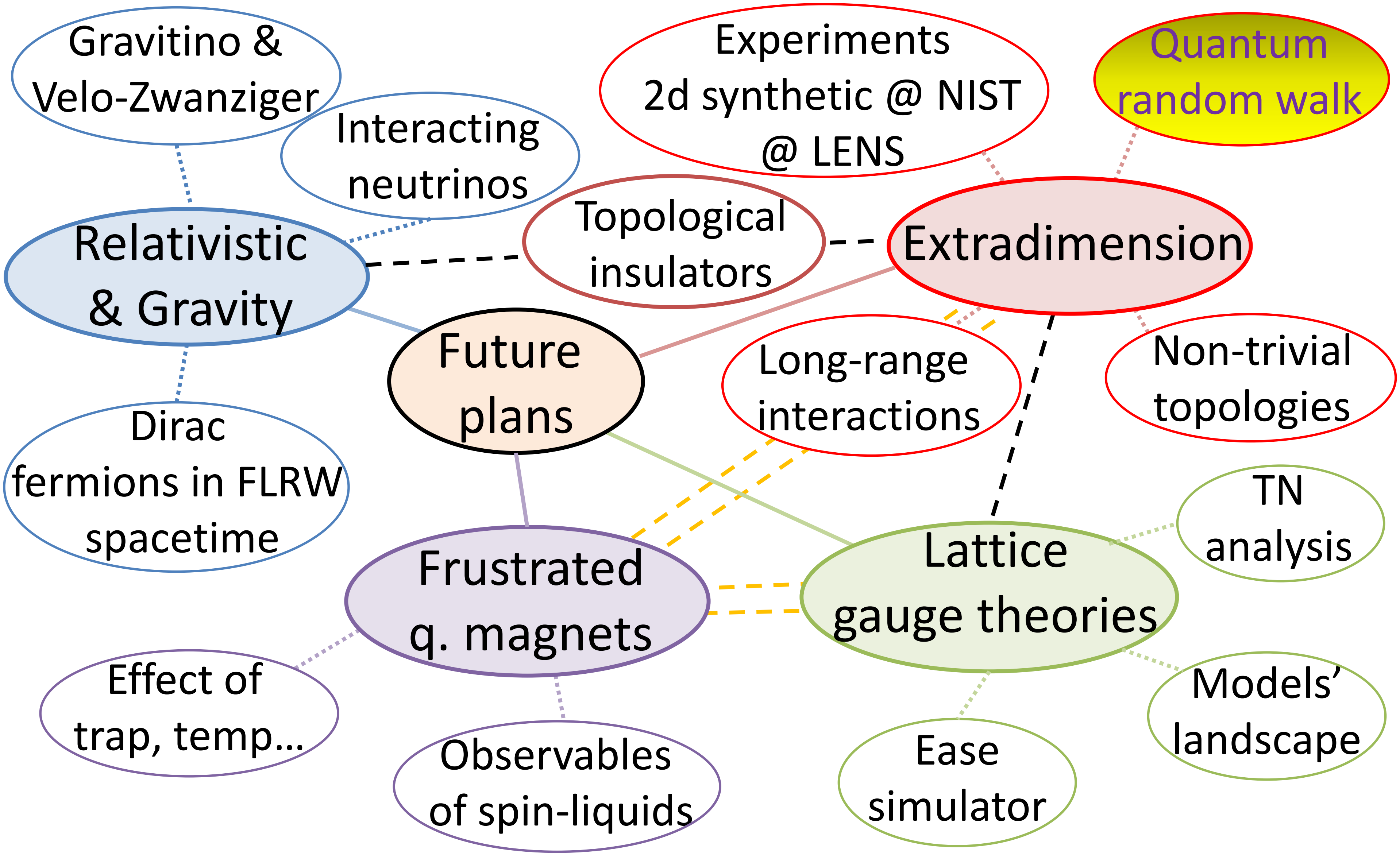
- Search for “visible” Laughling-like states in such regimes

….. *Hopefully many more*

Exciting time for Quantum Simulation



New opportunities for Atomtronics



“Extradimensional” collaborators

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O. Boada

M. Lewenstein

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