Long-standing Challenging Problem: Onset of Condensation?

NON-EQUILIBRIUM TECHNIQUES FOR ATOMTRONIC MODELLING

NICK PROUKAKIS

EPSRC
Engineering and Physical Sciences Research Council
NON-EQUILIBRIUM TRAPPED

THE STOCHASTIC GROSS-PITAEVSKII EQUATION (SGPE)

Nick Proukakis

(MY VERY BIASED) “ATOMTRONICS CARTOON”

ATOMTRONICS (!)

INTERFEROMETRIC MEASUREMENT

JOSEPHSON JUNCTION

RING TRAP

MIXTURE

NEAR SURFACE

ATOMTRONICS CARTOON
Some Desired Features?

- $T = 0$ (max coherence)
- 1D-ish & Close to Surface (optimal control)
- Rapid Onset (switches?)

...... Reproducibility, etc... .........

(Some) Fluctuations always present
Particularly in 1D & upon rapid quenching

→ Need to Find Optimum Balance: Theory can (hopefully) help?
NON-EQUILIBRIUM TRAPPED THE STOCHASTIC GROSS-PITAEVSKII EQUATION (SGPE)

Nick Proukakis

(MY VERY BIASED) “ATOMTRONICS CARTOON”

(Selected)
ATOMTRONICS MODELLING TECHNIQUES

Transport:
(Self-Consistent) Kinetic Theories

Rapid Dynamics / Fluctuations:
Stochastic Approaches

BEC

Thermal

This Talk:
Present Methods & Selected Applications
Transport: (Self-Consistent) Kinetic Theories

Rapid Dynamics / Fluctuations: Stochastic Approaches

Applications Considered:
* Surface Evaporative Cooling (Tubingen)
* Josephson Junction Dynamics (Florence)
* 2-Component BECs (Aarhus)
* Induced Flow in a Ring (JQI Geometry)

Applications Considered:
* Kibble-Zurek (Trento / Paris)
* Ring Traps (JQI Geometry)
* Double-Rings

*** Future Prospects ??? ***
\[ \frac{i\hbar}{\partial t} \phi = \left( -\frac{\hbar^2 \nabla^2}{2m} + V_{\text{TRAP}} + g(n_C) \right) \phi \]

\[ n_C = |\phi|^2 \]
**“BEST” KINETIC THEORY (“ZNG”)**

\[ \frac{i\hbar}{\partial t} \phi = \left( -\frac{\hbar^2 \nabla^2}{2m} + V_{\text{TRAP}} + g(n_c + 2n') - iR \right) \phi \]

\[ \frac{\partial f}{\partial t} + \frac{p}{m} \cdot \nabla f - \nabla U \cdot \nabla_p f = C_{12} + C_{22} \]

**BOSE-EINSTEIN CONDENSATE**

\[ n_c = |\phi|^2 \]

**THERMAL CLOUD**

\[ n' = \int \frac{d^3 p}{(2\pi\hbar)^3} f \]

**DISSIPATIVE GROSS-PITAEVSKII**

**MEAN FIELD COUPLING**

\[ U(r) = V(r) + 2g(n_c + n') \]

**QUANTUM BOLTZMANN**

**EXCHANGE OF ATOMS BETWEEN SUB-SYSTEMS**

\[ R(r,t) = (\frac{\hbar}{n_c}) \int dp C_{12}[f] \]

**THERMAL COLLISIONS**

Zaremba, Nikuni & Griffin

*J Low Temp Phys* 116, 277 (1999)
Study Surface Evaporative Cooling on a Room-Temperature Surface

by moving thermal cloud (~100’s nK) onto room-temperature surface (300 K) from initial distance ~100μm at constant speed & observing atom loss rate vs. time at different distances from surface

J Maerkle, AJ Allen et al., PRA 90, 023614 (2014)
HEAT BATH

SURFACE EVAPORATIVE COOLING

Measure atom loss rates vs. time for cloud moved towards surface

Theory-Experiment Comparison
(different “hold” positions)

J Maerkle, AJ Allen et al., PRA 90, 023614 (2014)
Measure atom loss rates vs. time for cloud moved towards surface

J Maerkle, AJ Allen et al., PRA 90, 023614 (2014)
In trying to optimize surface cooling efficiency, we find:

Condensate Fraction is maximised very close to surface & saturates at very low velocities

Condensate Number is however maximised further away from the surface

... suggesting the existence of an optimum distance from the surface for large pure condensates

J Maerkle, AJ Allen et al., PRA 90, 023614 (2014)
**JOSEPHSON DYNAMICS MODELLING**

**T = 0**

**HEAT BATH**

**PHASE DIAGRAM**

**DISSIPATIVE REGIME**

See also Piazza-Smerzi-Guilleumas et al. (2011+)
Dissipative Regime (Vortex Phase Slips)

… but …
why do experiments see vortices after a much longer wait time?
Vortex Ring Dynamics with Barrier Removal

E DIAGRAM

Dissipative Regime
(Vortex Phase Slips)

... but ...
why do experiments see vortices after a much longer wait time?

⇒ Barrier Removal (for Imaging) actually enhances vortex ring lifetime!
Only a minor shift to *Phase Diagram* when comparing cases of equal *condensate* number! … but … *Dynamics* within each region significantly affected!
Heat Bath

Josephson Dynamics Modelling

Talk by Alessia Burchianti (LENS)

Phase Diagram

Josephson Regime

Dissipative Regime

$T = 0 \quad T > 0$
JOSEPHSON DYNAMICS MODELLING

PHASE DIAGRAM

- Dissipative Regime

- \( T = 0 \)
- \( 0 < V_0, \mu < T \)
- \( 0 < T < V_0 \)

Talk by Alessia Burchianti (LENS)
Temperature can actually *suppress* “unwanted” BEC excitations – can this be utilised?
HEAT BATH

T > 0 TWO-COMPONENT CONDENSATE THEORY

Single-Component Self-Consistent Kinetic Equation ("ZNG")
Identify & Characterise “Novel” Collisional Processes

Lee, Edmonds & NPP

PRA 91, 011602(R) (2015)
PRA 92, 063607 (2015)
J Phys B 49, 214003 (2016)
PRA 94, 013602 (2016)

and a new (improved) criterion for phase mixing/separation

[jointly with Aarhus experiments (Arlt)]

CAUTION: Expansion Imaging does not always reveal correct in-situ picture!
Miscibility / Immiscibility can also appear *solely* during Expansion!

Lee, Jorgensen, Wacker, Skou, Skalmstang, Arlt & NPP, Preprint
EMERGENCE OF IMMISCIBILITY IN TOF

In Trap | Expanded | Experiments

(a) K | (b) K | Immiscible

0 ms | 14 ms

Rb | Rb

Δ = -0.93

(c) K | (d) K | Miscible

Rb | Rb

Δ = 1.2
Different, yet complementary, approaches to partially condensed \((T > 0)\) Systems

**Kinetic Approaches**
- (explicit BEC separation)
- **BEC + Dynamical Thermal Cloud**
  - with full self-consistent coupling

**Stochastic Approaches**
- (no explicit BEC separation)
- **M**odes up to a cut-off described in a unified manner (classical field) coupled to a Heat Bath

**Ideally suited for:**
- Collective Modes / Transport
- Full BEC – Thermal Coupling (far from critical region)

Random (shot-to-shot) Fluctuations
Quenches / Low-D & Universality
(high-lying modes “unaffected”)
SGPE describes the entire multi-mode system describing the low-lying modes

\[ i\hbar \frac{\partial \Phi(x,t)}{\partial t} = \left(1 - i\gamma \right) \left[-\frac{\hbar^2 \nabla^2}{2m} + V_{\text{TRAP}} - \mu + g|\Phi(x,t)|^2 \right] \Phi(x,t) + \eta(x,t) \]
STOCHASTIC GROSS-PITAEVSKII (SGPE) MODEL

SGPE describes the entire multi-mode system describing the low-lying modes

\[ i\hbar \frac{\partial \Phi(x,t)}{\partial t} = \left(1 - i\gamma \right) \left[-\nabla^2 + V_{\text{TRAP}} - \mu + g|\Phi(x,t)|^2 \right] \Phi(x,t) + \eta(x,t) \]

→ Results obtained by averaging over noise realizations \( \eta(x,t) \)

\[ \langle \eta^* (x,t) \eta(x',t') \rangle = 2\hbar \gamma k_B T \delta(x - x') \delta(t - t') \]

so supposed to be interpreted after suitable ‘trajectory’ averaging

SGPE describes the entire multi-mode system describing the low-lying modes

\[ i\hbar \frac{\partial \Phi(x,t)}{\partial t} = \left( 1 - i\gamma \right) \left[ -\frac{\hbar^2 \nabla^2}{2m} + V_{\text{TRAP}} - \mu + g|\Phi(x,t)|^2 \right] \Phi(x,t) + \eta(x,t) \]

→ Results obtained by averaging over noise realizations \( \eta(x,t) \)

\[ \left\langle \eta^*(x,t)\eta(x',t') \right\rangle = 2\hbar \gamma k_B T \delta(x-x')\delta(t-t') \]

so supposed to be interpreted after suitable ‘trajectory’ averaging


Single Run

vs.

Averaged Profiles

* Contain element of stochasticity
* **Qualitatively** reproduces single experimental realisations

* Wash out density fluctuations to produce smooth profiles
* Suitable for extracting global features (densities, correlation functions, etc.)
**LOW-D EQUILIBRIUM PROPERTIES**

Properties Characterised by Densities & Lowest Order Correlation Functions

**Quasi-1D:**
Ab Initio Prediction of densities & coherences

**Quasi-2D:**
Scale-invariance & Universality

---

**Density Profiles**

**Density Fluctuations**

**Phase Fluctuations**

---

**Experiments:**
- Paris & Amsterdam
  - PRL 97, 250403 (2006)
  - PRL 100, 090402 (2008)
  - PRL 105, 230402 (2010)
  - PRL 91, 010405 (2003)
  - EPJD 35, 155 (2005)

**Ab Initio SGPE Modelling:**
- NPP et al., PRA 84, 023613 (2011)
- PRA 86, 013627 (2012)

**Experiment:**
- Chicago
  - Nature 470, 236 (2011)

**Ab Initio SGPE Modelling:**
- Cockburn & NPP
  - PRA 86, 033610 (2012)

---

**Detailed Theoretical Benchmarking:**
- Cockburn et al. PRA 83, 043619 (2011)
Consider rapidly quenching a system through the BEC phase transition

As system approaches phase transition, it cannot follow external drive (e.g. cooling ramp)

Dynamics “freezes out” with coherence forming in “local patches”

**Kibble-Zurek Model:**

Number of Defects: \( N \propto (\tau_Q)^{-\alpha} \)

**1D:** Soliton Formation

**Quasi-1D:** Solitonic Vortices

**Quasi-2D / 3D:** Vortex Formation

**Ring Trap:** Persistent Current

**Experimentally also Characterised in a 3D/2D Box-like Trap**

TRENTO EXPERIMENT SUMMARY

Quench Protocol
(Forced Evaporative Cooling)


Observations (Quasi-1D System; TOF Imaging)

(a) BEC
Radial view
Axial view

(b) Counts
Number of vortices

"Plateau"
"Kibble-Zurek"
$^{23}\text{Na}$

$\omega_z (\omega_\perp) = 2\pi \cdot 13(131) \text{Hz}$

$T = (790 \rightarrow 210) nK$

$N = (20 + 0.4) \times 10^6 \rightarrow N = (6.6 + 4.9) \times 10^6$

$\tau_{\text{evap}} = 18 ms$
Growth with Persistent Current

Growth without Persistent Current

See also Das, Sabbatini & Zurek, Scientific Reports (2012) & Jerome Beugnon’s Talk
Next, Density Engineer a Dark-Soliton Pair of desired speed and study its dynamical evolution.

Focus on JQI Ring Trap Parameters
[ see also Murray et al., PRA 88, 053615 (2013) for parameter details ]

Planar Geometry

\[ V(r) = V_G \left( 1 - e^{-2(r-r_0)^2/w^2} \right) \]

- \( l_r = 2 \mu m \)
- \( r_0 = 18.5 \mu m \)
- \( w = 9.45 \mu m \)
- \( \omega_\perp = 2\pi \times 600 Hz \)
OPTIMIZING THE EXCITATION SCHEME

Laser Potential of form

\[ V_L(t) = V_0 f(t) \]

Gradual Turn-on Minimises Linear (sound) excitations

See also Shomroni et al., Nat. Phys. 5, 193 (2009)

Narrow Width of Excitation Laser

\[ V = ... + V_L(t) e^{-y^2/2\sigma^2} \]

Ensures only single Counter-propagating soliton pair is generated

Relevant ‘control’ parameters thus reduce only to:

\[ V_0, \, \xi \]

To make soliton engineering findings universal, we hence consider the following dimensionless ratios:

\[ \left( \frac{L_r}{\xi} \right) \& \left( \frac{V_0}{\mu} \right) \]
LONG-LIVED DARK SOLITONS IN RING TRAPS

$t = 36 \text{ ms}$  $t = 43 \text{ ms}$  $t = 72 \text{ ms}$  $t = 119 \text{ ms}$  $t = 750 \text{ ms}$

Renormalized Density

Phase

$(l_0 = 10 \mu m)$  $\sigma/\xi = 0.7$  $l_r/\xi = 1.3$  $V_0/\mu = 2$  $\xi = 1.5 \mu m$

Gallucci & NPP, New J Phys 18, 025004 (2016)
Identify $T = 0$ Phase Diagram Revealing Distinct Dynamics

Gallucci & NPP
New J Phys
18, 025004 (2016)
2D Solitonic Regime
Identify T = 0 Phase Diagram Revealing Distinct Dynamics

Gallucci & NPP
New J Phys
18, 025004 (2016)
Identify $T = 0$ Phase Diagram Revealing Distinct Dynamics

Gallucci & NPP
New J Phys
18, 025004 (2016)

Dynamical Instability
What about Thermodynamic Instabilities?

Add realistic “noisy” background density modulation & Monitor Evolution

Initial Condition \((t=0)\)  

Post Density-Engineering

Observe Counter-Propagating Dark “Solitons” with \(\Delta \phi \approx 0.4 \pi\) persisting for multiple revolutions / collisional cycles
What about Thermodynamic Instabilities?

Add realistic “noisy” background density modulation & Monitor Evolution

Observe Counter-Propagating Dark “Solitons” with \( \Delta \phi \approx 0.4 \pi \)
persisting for multiple revolutions / collisional cycles

Optimal Parameters:

* Tight Transverse Confinement
* Small Atom Number
* Not too high Temperature

\[ \mu, k_B T \leq \hbar \omega \]

* Narrow Ring Trap Radius

Gallucci & NPP
New J Phys 18, 025004 (2016)
Can flow patterns in complex ring geometries be useful for measurements?

Single Ring

Double Rings

or even more complex Structures?
Have done basic “brute-force” investigation of emerging phase patterns (by stochastic condensation into such geometry)
Example of (an unlikely?) atomtronic scheme [work in progress]

Controllable Barrier

Conceivably use a 3\textsuperscript{rd} ring as reference frame?

or Move Rings Apart & bring together again?
Example of (an unlikely?) atomtronic scheme [work in progress]

Middle ring acts as gate controlling flow / phase???

... Concrete Ideas Wanted !!! ...
ANNOUNCEMENTS

NJP Interdisciplinary Spotlight Collection on MultiComponent Quantum Matter


Your Invitation to Submit a Contribution within 2017
(usual NJP application/acceptance criteria apply)

Managed by:
Frédéric Chevy (ENS)
Milorad Miloševic (Antwerp)
Nick Proukakis (Newcastle/JQC)

New Journal of Physics
The open access journal at the forefront of physics
www.njp.org

Announcement and your invitation to contribute

New Journal of Physics (NJP) is pleased to announce the publication of a collection of research articles on multicomponent quantum matter.

Universal Themes of Bose-Einstein Condensation
(Cambridge University Press, PUBLISHED, April 2017)

ISBN-9781316084366

Edited by NP Proukakis, DW Snoke & PB Littlewood
The Onassis Foundation

2017 Lectures in Physics

Quantum Physics Frontiers Explored with Cold Atoms, Molecules & Photons

Hosted by Serge Haroche

Other Lecturers: Aspect – Bloch – Dalibard

Davidovich – Proukakis - Ye

24-28 July 2017, Crete

Full Financial Support for 15 International UG/PG Students, or RAs
ACKNOWLEDGEMENTS

Kinetic Modelling:

* Kean Loon Lee
  * Klejdja Xhani

Stochastic Modelling:

  * Gary Liu
  * Donatello Gallucci
  * Quentin Marolleau

Matthew Edmonds

EPSRC

Gabriele Ferrari
Giacomo Lamporesi
Franco Dalfovo
Iacopo Carusotto

Alessia Burchianti
Giacomo Roati

Nils Jorgensen
Jan Arlt

Eugene Zaremba

Carsten Henkel

Nir Navon
Zoran Hadzibabic

UCL

Simon Gardiner & Simon Cornish

Nick Parker, Carlo Barenghi, Tom Billam, Luca Galantucci

Shih-Chuan Gou
(Changhua Uni, Taiwan)
**SUMMARY / FOR DISCUSSION**

**Kinetic Approaches**  
(explicit BEC separation)

**Stochastic Approaches**  
(no explicit BEC separation)

- *Fully Dynamical*
- *Static (Heat Bath)*

- *NON-BEC*
- *BEC*

---

**Surface Cooling**

**TOF Analysis**

**Josephson Junction**

**Ring Dynamics**

**Quenched Growth**

- *Dark Solitons*
- *Double Ring Dynamics*
**T > 0 TWO-COMPONENT CONDENSATE THEORY**

→ Identify & Characterise “Novel” Collisional Processes

\[
\begin{align*}
  i\hbar \frac{\partial \phi_j}{\partial t} &= \left[-\frac{\hbar^2}{2m_j} \nabla^2 + V_j(r) + g_{jj}(n_{c,j} + 2\tilde{n}_j) + g_{kj}(n_{c,k} + \tilde{n}_k)\right] \\
  \frac{\partial}{\partial t}f^j + \frac{1}{m_j} \mathbf{p} \cdot \nabla f^j - \nabla \cdot (\mathbf{p} f^j) - \nabla U^j_{\text{eff}} &= 0
\end{align*}
\]
Identify & Characterise “Novel” Collisional Processes

\[ i\hbar \frac{\partial \phi_j}{\partial t} = \left[ -\frac{\hbar^2}{2m_j} \nabla^2 + V_j(\mathbf{r}) + g_{jj}(n_{c,j} + 2\tilde{n}_j) + g_{kj}(n_{c,k} + \tilde{n}_k) \right] \phi_j \]

\[ \frac{\partial}{\partial t} f^j + \frac{1}{m_j} \mathbf{p} \cdot \nabla f^j - \nabla_{\mathbf{r}} f^j \cdot \nabla_{\mathbf{r}} U_{\text{eff}}^j = \left( C_{12}^{jj} + C_{12}^{kj} \right) + C_{12}^{kj} + \left( C_{22}^{jj} + C_{22}^{kj} \right) \]
Phase Profiles Controlled by Inter- / Intra- Atomic Interactions

**Phase Mixing** (overlapping components)

Consider here repulsive interactions

**Phase Separation** (non-overlapping components)

**Usual Criterion** (Homogeneous)

$\Delta = \left( \frac{g_{11}g_{22}}{2g_{12}} - 1 \right)$

Interspecies interactions dominate

Intraspecies interactions dominate

$g_{11} (g_{22})$: Interactions *within* Species 1 (or 2)

$g_{12}$: Interactions *between* Species 1 & 2

$\Delta > 0$

$\Delta < 0$
**PHASE PROFILES OF 2-COMPONENT BECs**

**Phase Profiles Controlled by Inter- / Intra- Atomic Interactions**

**Phase Mixing** (overlapping components) *repulsive* interactions

\[ \Delta > 0 \]

Interspecies interactions dominate

\[ \Delta = \left( \frac{g_{11} g_{22}}{g_{12}} \right) - 1 \]

**Phase Separation** (non-overlapping components)

\[ \Delta < 0 \]

Intraspecies interactions dominate

**Consider here**

**Usual Criterion** (Homogeneous)

**Define New (Im)Miscibility Parameter for Symmetric Trapped BEC Mixtures:**

\[ \Delta n_{\text{norm}} = \frac{n_{c,1}(0)}{\max n_{c,1}(r)} - \frac{n_{c,2}(0)}{\max n_{c,2}(r)} \]

New Relevant (Im)Miscibility Parameter for Symmetric Trapped BEC Mixtures:

\[
\Delta n_{\text{norm}} = \frac{n_{c,1}(0)}{\max n_{c,1}(r)} - \frac{n_{c,2}(0)}{\max n_{c,2}(r)}
\]

Phase separation Boundary: Green $\rightarrow$ Blue

Criterion in Trap can Deviate Significantly from Homogeneous Condition!
This depends critically also on atom numbers

T = 0 Analysis (Gross-Pitaevskii)
New Criterion fully captures “crossover” between separation & overlap

Probing Blue $\rightarrow$ Green Phase Separation Boundary

Magnetic field [G]

124 130 136 142 148

Criterion fully valid at $T > 0$!

(same BEC numbers)

In Situ Density Profiles

How to Characterize / Measure Experimentally?
Induce Collective Modes & Study their Damping

\[ T = 0 \]

Undamped Surface Shape Oscillations

Large Spatial Overlap Enhances Counterflow Instability
New Criterion fully captures “crossover” between separation & overlap

Our Picture is Confirmed by a Simultaneous Shift in Frequency & Damping of Collective Oscillations
TWO-COMPONENT CONDENSATE THEORY

Component A:
- BEC
- Thermal

Component B:
- BEC
- Thermal

Condensate -- Thermal
- $C_{12}^{aa} = \frac{p_4}{p_3} \times \frac{p_2}{p} \times \frac{p_2}{p} \times \frac{p_4}{p_3}$

Thermal Only
- $C_{22}^{aa} = \frac{1}{p_3}$
TWO-COMPONENT CONDENSATE THEORY

Component A: BEC
Component B: BEC

Condensate -- Thermal

Single Component Terms

$C_{12}^{aa}$
$p_4 \times \times \times \times p_4$
$p_3 \times p \times p \times p_3$

Corresponding 2-Component Terms

$C_{12}^{ab}$
$+ C_{12}^{ba}$

Thermal Only

$C_{22}^{aa}$
$- \times \times \times \times -$

$C_{22}^{ab}$
$- \times \times \times \times -$
TWO-COMPONENT CONDENSATE THEORY

**Component A:**
- BEC
- Thermal

**Component B:**
- BEC
- Thermal

**Conducte -- Thermal**

- $p_4$
- $p_2$
- $p_2$
- $p_4$
- $C_{12}^{aa}$
- $p_3$
- $p$
- $p$
- $p_3$
- $C_{22}^{aa}$

**Thermal Only**

- $C_{12}^{ab}$
- $C_{12}^{ba}$

"Novel Cross-Coupling Processes" Important when both components are partly condensed
We now examine the Temperature-dependence of this collisional process


Remarkably this “cross-condensate” scattering process is found to dominate near equilibrium even at rather high temperatures!

We thus anticipate it to play a dominant role in sympathetic cooling, from its early stages, completely dominating over collisions at lower temperatures.