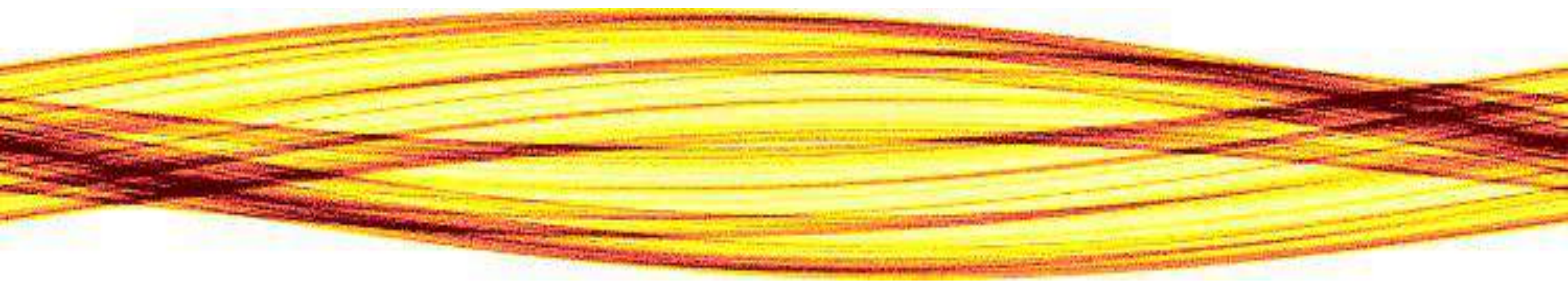
Shortcuts to adiabaticity for matter-waves

Adolfo del Campo

Department of Physics
University of Massachusetts, Boston



Benasque, Spain
Atomtronics, May 7-20th 2017

Contents

Adiabatic dynamics

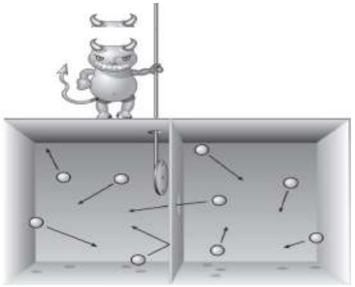
Slow driving of a system

Provides good control

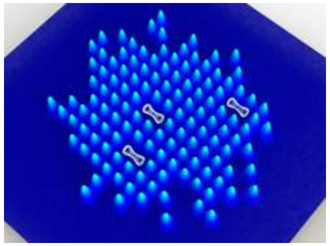
No excitations

So, why shortcuts?

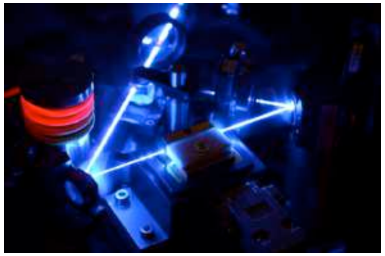
Well …



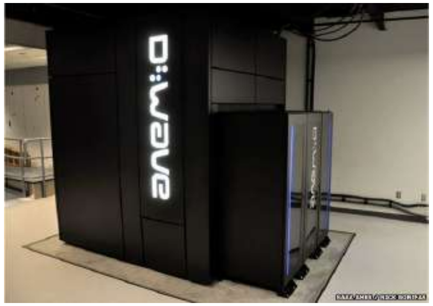
Quantum thermodynamics
energy conversion
ground state cooling



Quantum Simulation & Condensed Matter
defect suppression



Quantum Information Quantum Optics
decoherence, noise



Adiabatic Quantum Computation

Shortcuts to adiabaticity

Fast non-adiabatic process that mimics adiabatic dynamics
e.g. to prepare a state

[Review: Adv. At. Mol. Opt. Phys. **62**, 117 (2013)]

Processes: Expansion, transport, splitting, adiabatic passage, phase transitions, …

Systems: ultracold atoms, ions chains, quantum dots, spin systems, NVC, …

Experiments: Nice, NIST, Mainz, PTB, MPQ, Florence, Trento, Tsukuba, …

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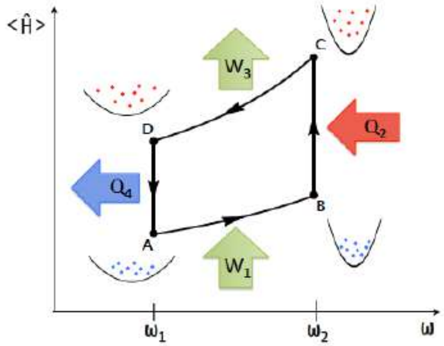
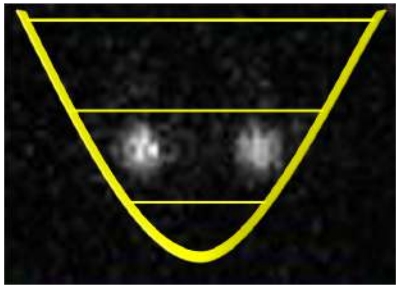
Adolfo del Campo: adolfo.delcampo@umb.edu

Shortcuts to adiabaticity



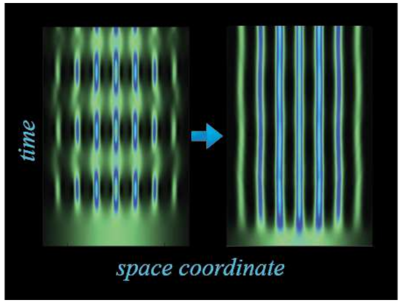
Quantum thermodynamics

Chen et al, PRL 104, 063002 (2010)
AdC & Boshier, Sci. Rep. 2, 648 (2012)
AdC, Goold, Paternostro Sci. Rep. 4, 6208 (2014)
Jaramillo, Beau, AdC, arXiv:1510.04633 (2016)
Beau, Jaramillo, AdC, Entropy 18, 168 (2016)



Transport

Deffner, Jarzynski, AdC PRX 4, 021013 (2014)
An, Lv, AdC, Kihwan Kim, arXiv:1601.05551

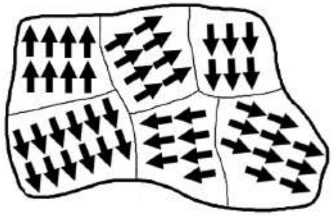


Loading optical lattice

Masuda, Nakamura, AdC PRL 113, 063003 (2014)

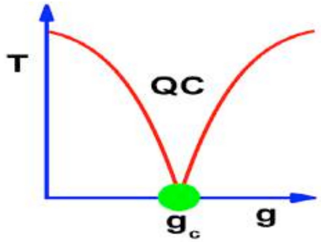
Quantum microscopy

AdC, EPL 96, 60005 (2011)
AdC, PRA 84, 031606(R) (2011)
AdC, PRL 111, 100502 (2013)



Topological Defect suppression

AdC et al. PRL105, 075701 (2010)
AdC et al. NJP 13, 083022 (2011)
Pyka et al. Nat. Commun. 4, 2291 (2013)
AdC, Kibble, Zurek, JPCM 25, 404210 (2013)
AdC & Zurek Int. J. Mod. Phys. A 29, 1430018 (2014)



Adiabatic crossing of quantum phase transition

AdC, Rams, Zurek PRL 109, 115703 (2012)
Saberi, Opatrný, Mølmer, AdC, PRA 90, 060301(R)
AdC & Sengupta, EPJ ST 224, 189 (2015)
Rams, Mohseni, AdC, TBS (2015)

And many other applications
(chemical rate processes, quantum logic gates, soliton dynamics, atom interferometry, …)

Universal approach: Counterdiabatic driving

Consider driving a system Hamiltonian

$$\hat{H}_0(t)|n(t)\rangle = E_n(t)|n(t)\rangle$$

Write the adiabatic approximation

$$|\psi_n(t)\rangle = \exp\left[-\frac{i}{\hbar}\int_0^t E_n(s)ds - \int_0^t \langle n(s)|\partial_s n(s)\rangle ds\right]|n(t)\rangle$$

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Is there a Hamiltonian for which the adiabatic approximation is exact?

$$i\hbar\partial_t|\psi_n(t)\rangle = \hat{H}(t)|\psi_n(t)\rangle$$

Universal approach: Counterdiabatic driving

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Is there a Hamiltonian for which the adiabatic approximation is exact?

$$i\hbar\partial_t|\psi_n(t)\rangle = \hat{H}(t)|\psi_n(t)\rangle$$

Yes, indeed!

$$\hat{H}(t) \equiv \hat{H}_0(t) + \hat{H}_1(t)$$

$$\hat{H}_1(t) = i\hbar\sum_n \left(|\partial_t n\rangle\langle n| - \langle n|\partial_t n\rangle|n\rangle\langle n|\right)$$

Counterdiabatic driving

\

Is there a Hamiltonian for which the adiabatic approximation is exact?

$$i\hbar\partial_t|\psi_n(t)\rangle = \hat{H}(t)|\psi_n(t)\rangle$$

Yes, indeed!

$$\hat{H}(t) \equiv \hat{H}_0(t) + \hat{H}_1(t)$$

$$\hat{H}_1(t) = i\hbar \sum_{n\neq m}\sum_{m} \frac{|m\rangle\langle m|\partial_t \hat{H}_0|n\rangle\langle n|}{E_n(t) - E_m(t)}$$

Theory: Demirplak & Rice 2003; = M. V. Berry 2009 “Transitionless quantum driving”
CD inspired experiment for TLS: Morsch’s group Nature Phys. 2012; NVC: Suter’s group PRL 2013

Counterdiabatic driving: Experiments

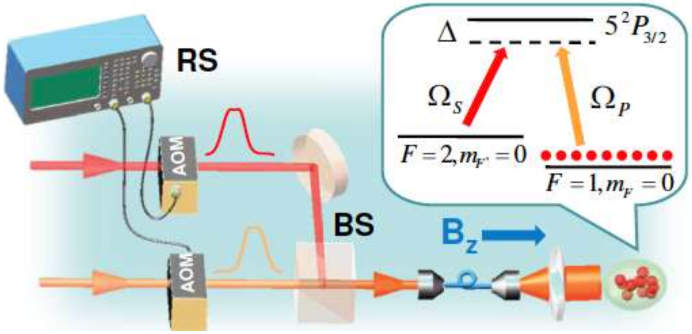
nature COMMUNICATIONS

ARTICLE

Received 28 Jan 2016 | Accepted 6 Jul 2016 | Published 11 Aug 2016 DOI: 10.1038/ncomms12479 OPEN

Experimental realization of stimulated Raman shortcut-to-adiabatic passage with cold atoms

Yan-Xiong Du[1], Zhen-Tao Liang[1], Yi-Chao Li[2], Xian-Xian Yue[1], Qing-Xian Lv[1], Wei Huang[1], Xi Chen[2], Hui Yan[1] & Shi-Liang Zhu[1,3,4]



CD for 2 & 3 Level systems

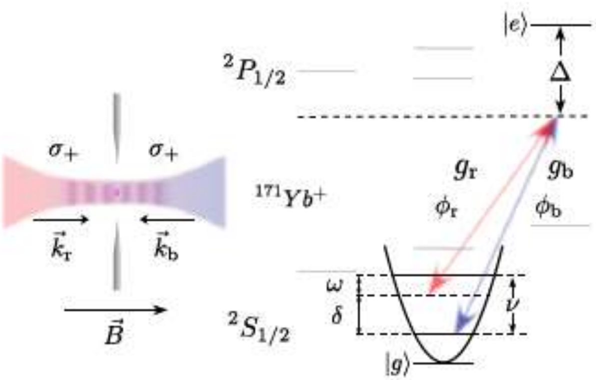
nature COMMUNICATIONS

ARTICLE

Received 29 Feb 2016 | Accepted 19 Aug 2016 | Published 27 Sep 2016 DOI: 10.1038/ncomms12999 OPEN

Shortcuts to adiabaticity by counterdiabatic driving for trapped-ion displacement in phase space

Shuoming An[1], Dingshun Lv[1], Adolfo del Campo[2] & Kihwan Kim[1]



CD for systems with Continuous Variables

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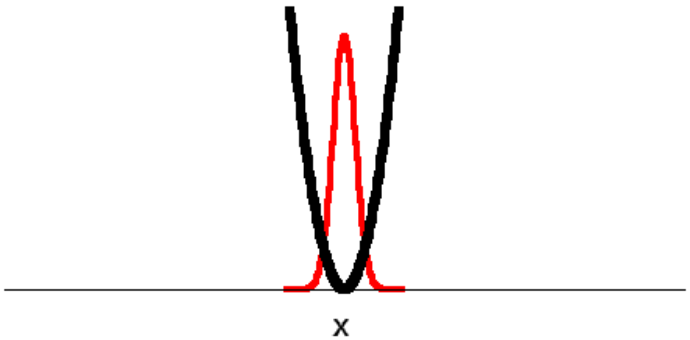
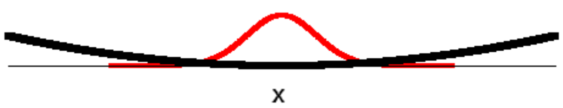
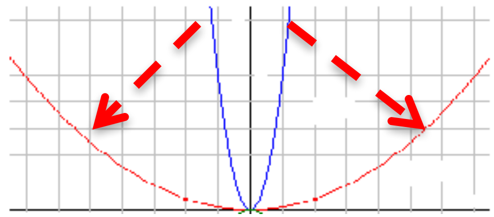
Shortcuts to adiabaticity: superadiabatic expansion?





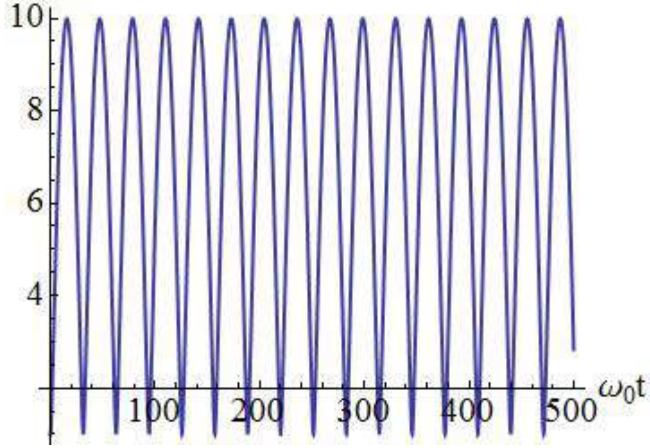
Standard expansion

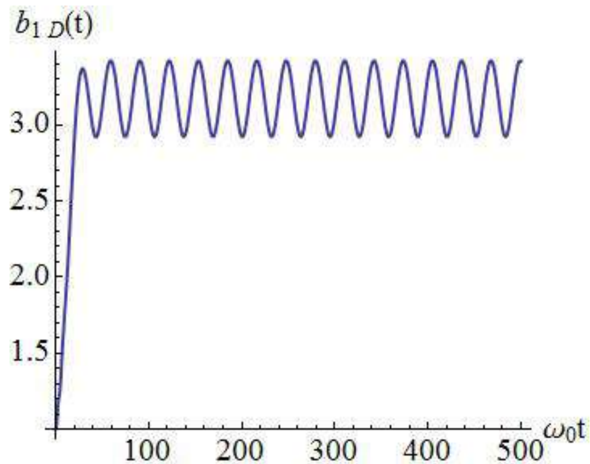
Opening the trap

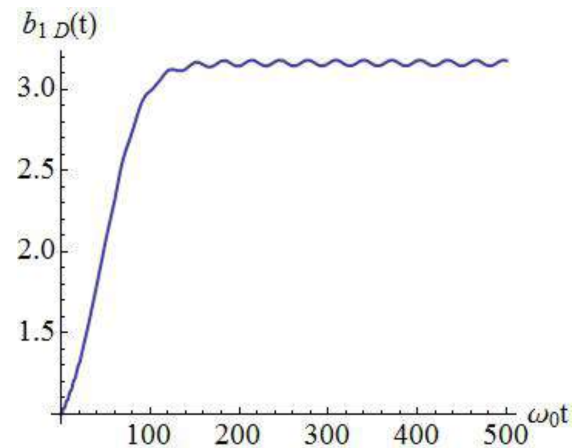


from sudden to adiabatic

$b_{1D}(t)$: width of the cloud







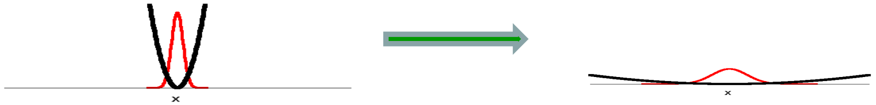
Excitation of the breathing mode of the cloud

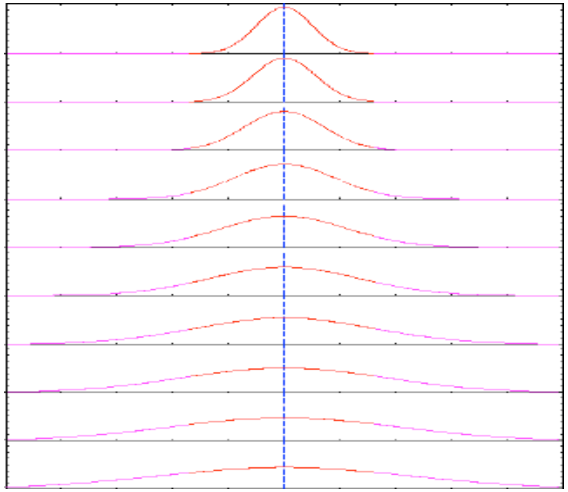
Quantum gases

Interacting quantum fluids

$$\hat{H} = \sum_{i=1}^{N}\left[-\frac{\hbar^2}{2m}\nabla^2 + \frac{1}{2}m\omega(t)^2\mathbf{r}_i^2\right] + \sum_{i<j} V(\mathbf{r}_i - \mathbf{r}_j)$$

Scaling-invariant dynamics when $V(\gamma\mathbf{r}) = \gamma^{-2}V(\mathbf{r})$





A. del Campo, PRL 111, 100502 (2013)

Quantum gases

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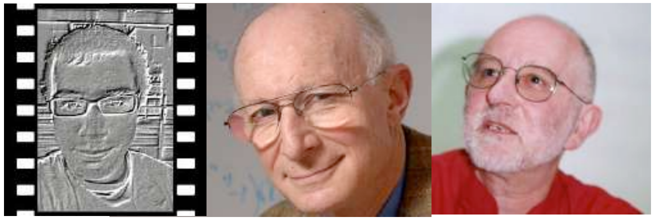
Counterdiabatic Driving?

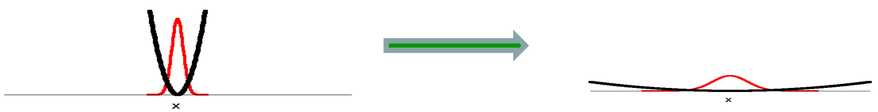


Spectral properties unavailable, even by numerical methods

A. del Campo, PRL 111, 100502 (2013)

Quantum gases

More general case

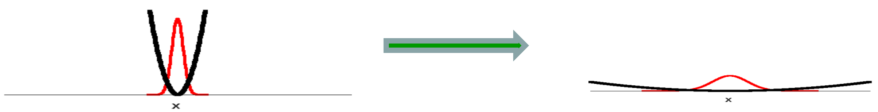


$$\hat{H}_0(t) = \sum_{i=1}^{N}\left[-\frac{\hbar^2}{2m}\Delta_{\mathbf{q}_i} + \frac{1}{2}m\omega^2(t)\mathbf{q}_i^2 + U(\mathbf{q}_i,t)\right] + \epsilon(t)\sum_{i<j}V(\mathbf{q}_i - \mathbf{q}_j)$$

$$\gamma(t) = \left[\frac{\omega(0)}{\omega(t)}\right]^{1/2} \qquad U(\mathbf{q},t) = \frac{1}{\gamma^2(t)}U\left(\frac{\mathbf{q}}{\gamma(t)},0\right), \qquad \mathrm{V}(\lambda\mathbf{q}) = \lambda^{-\alpha}\mathrm{V}(\mathbf{q})$$

A. del Campo, PRL 111, 100502 (2013)

Quantum gases



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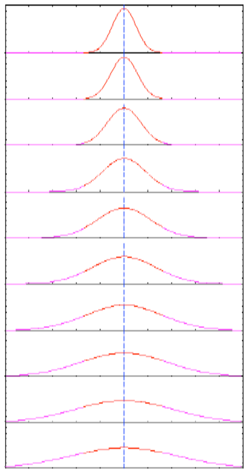

Scaling ansatz

$$\Phi(t) = \gamma^{-\frac{ND}{2}}e^{-i\mu\tau(t)/\hbar}\Phi\left[\frac{\mathbf{q}_1}{\gamma(t)},\dots,\frac{\mathbf{q}_N}{\gamma(t)};0\right]$$

Nonlocal auxiliary Hamiltonian

$$\boxed{\hat{H}_1 = -i\frac{\hbar\dot{\gamma}}{2\gamma}\sum_{i=1}^{N}(\mathbf{q}_i\partial_{\mathbf{q}_i} + \partial_{\mathbf{q}_i}\mathbf{q}_i)}$$

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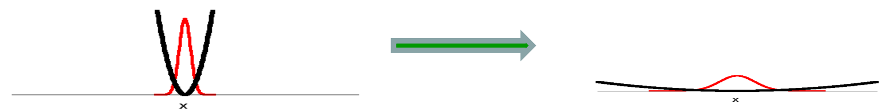
A. del Campo, PRL 111, 100502 (2013)

Quantum gases

More general case



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Unitary transformation

$$\mathcal{U} = \prod_{i=1}^{N}\exp\left(\frac{im\dot{\gamma}}{2\hbar\gamma}\mathbf{q}_i^2\right), \Phi(t)\to\Psi(t) = \mathcal{U}\Phi(t)$$

LOCAL auxiliary Hamiltonian

$$\boxed{\hat{\mathcal{H}}_1 = -\frac{1}{2}m\frac{\ddot{\gamma}}{\gamma}\sum_{i=1}^{N}\mathbf{q}_i^2}$$

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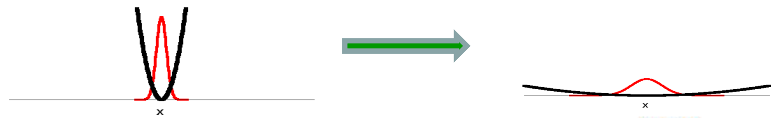
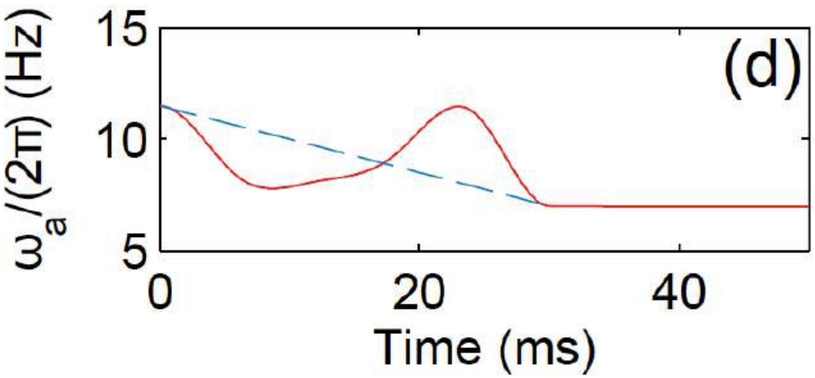
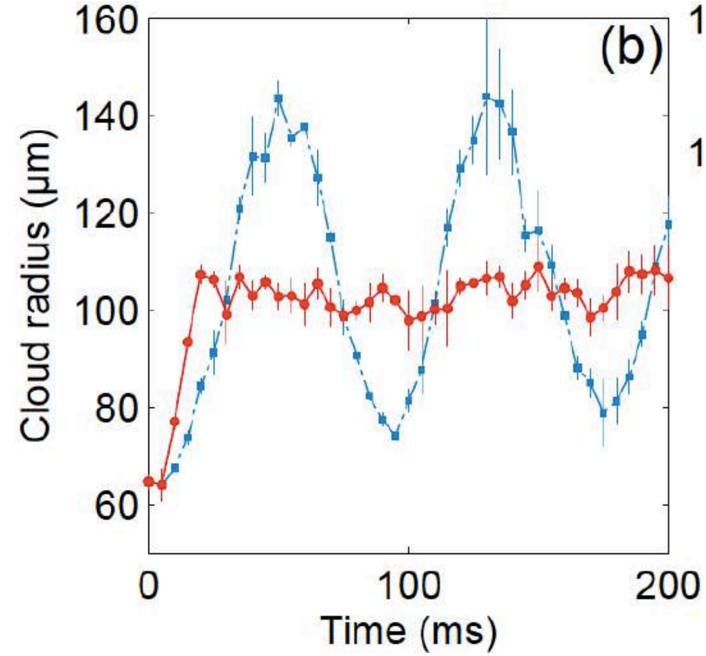
A. del Campo, PRL 111, 100502 (2013)

Experiments: Thermal cloud, BEC and 1D Bose gas



Shortcut vs standard expansion





Experiments: 1D Bose gas
Rohringer et al. Sci. Rep. **5**, 9820 (2015)
Experiments: mean-field BEC
J.-F. Schaff et al. EPL **93**, 23001 (2011)
Experiments: single-particle
J.-F. Schaff et al. Phys. Rev. A **82**, 033430 (2010)

Theory (quantum fluids)
Chen et al. PRL **104**, 063002 (2010)
AdC PRA **84**, 031606(R) (2011)
AdC PRL **111**, 100502 (2013)

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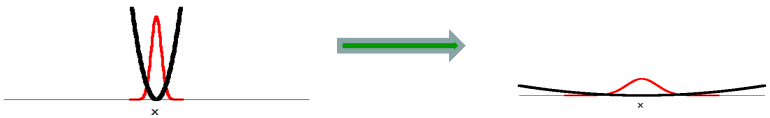
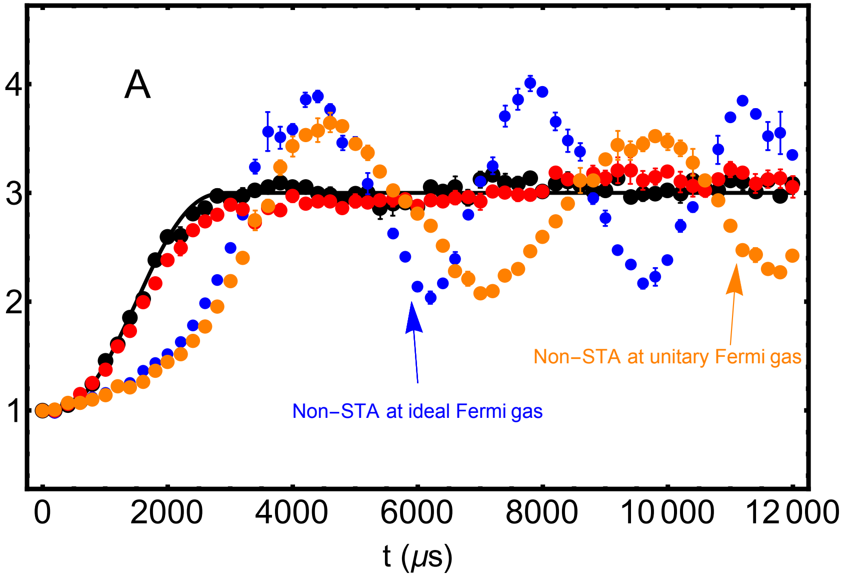
Experiments: Strongly-coupled quantum fluids

3D anisotropic Fermi gas:



- Non-interacting
- At unitarity: emergent scale invariance (broken at finite coupling)



Theory

Quantum fluids

AdC PRA **84**, 031606(R) (2011)

AdC PRL **111**, 100502 (2013)

Papoular & Stringari

PRL **115**, 025302 (2015)

Experiment

Self-similar dynamics

Deng et al Science **353**, 371 (2016)

STA in anisotropic unitary Fermi gas

Deng et al arXiv:1610.09777

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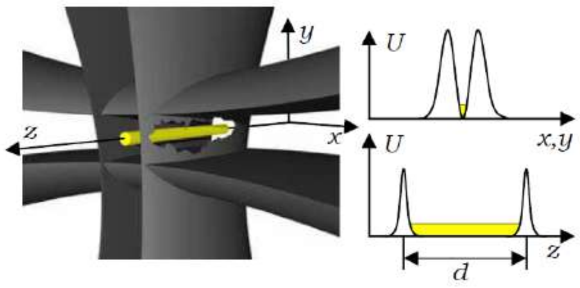
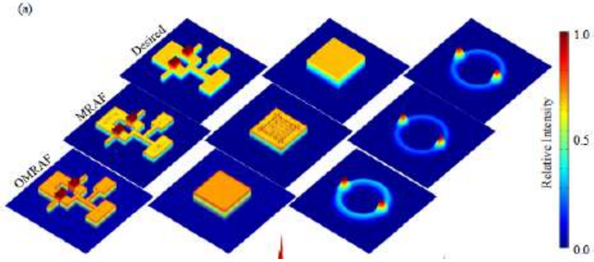
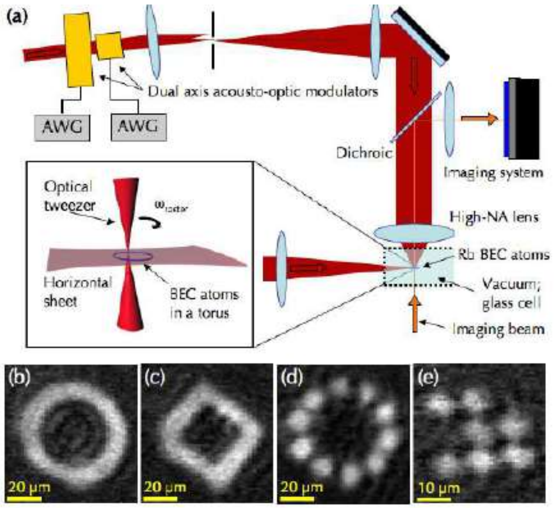
Nonharmonic traps? Boxes?



UT Austin all optical box
at Raizen's Lab
PRA, 71, 041604(R) (2005).

Cambridge's boxes
A. L. Gaunt, Z. Hadzibabic,
Sci. Rep. 2, 721 (2012)

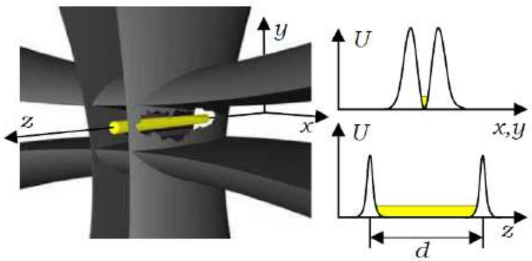




Boshier's group at LANL
New J. Phys. 11, 043030 (2009)

+ implementations in atom chips

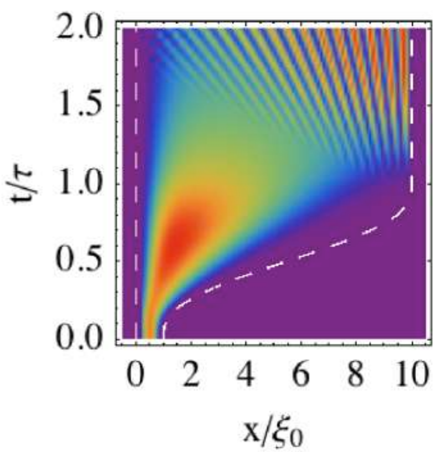
Quantum piston



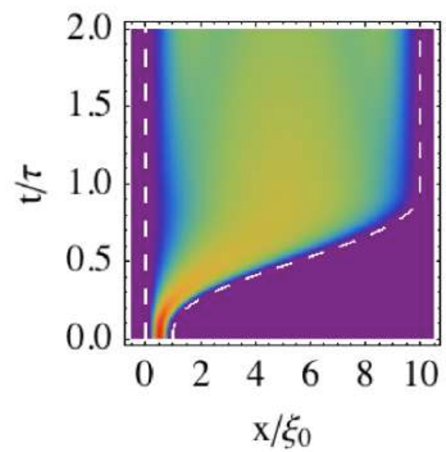
AdC & Boshier, Sci. Rep **2**, 648 (2012)
AdC, PRL **111**, 100502 (2013)

Quantum Piston

normal expansion



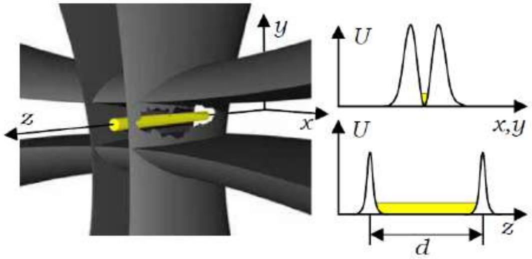
shortcut to adiabaticity



$$\Omega^2(t) = 0$$

$$\Omega^2(t) = -\frac{\ddot{\xi}(t)}{\xi(t)} \sim \frac{1}{\tau}$$

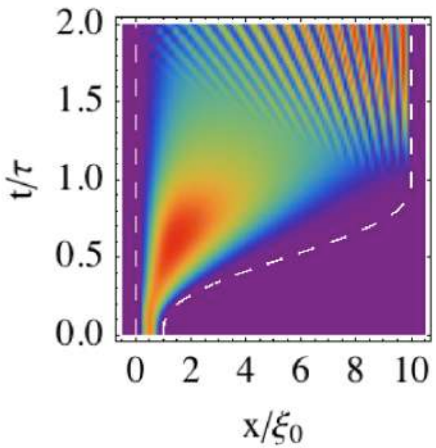
Quantum piston



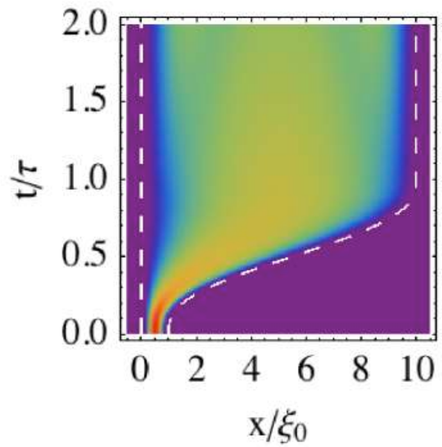
AdC & Boshier, Sci. Rep **2**, 648 (2012)
AdC, PRL **111**, 100502 (2013)

It already implies transport!

normal expansion



shortcut to adiabaticity



$$\Omega^2(t) = 0$$

$$\Omega^2(t) = -\frac{\ddot{\xi}(t)}{\xi(t)} \sim \frac{1}{\tau}$$

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Fast-forward technique

Scale invariance is kind of classical

Really needed?

Theory: Masuda & Nakamura 2008, 2010, 2011
Experiments: ???

Fast-forward technique

Consider the dynamics (mean-field)

$$i\hbar\partial_t\Psi = -\frac{\hbar^2}{2m}\nabla^2\Psi + (\mathcal{V}+\mathcal{V}_{\rm au})\Psi + g|\Psi|^2\Psi,$$

Ansatz for the evolution

$$\Psi(\mathbf{q},t) = \psi[\mathbf{q},R(t)]e^{i\phi(\mathbf{q},t)}e^{-\frac{i}{\hbar}\int_0^t \mu[R(t')]dt'}$$

where

$$-\frac{\hbar^2}{2m}\nabla^2\psi + \mathcal{V}\psi + g|\psi|^2\psi = \mu\psi.$$

Theory: Masuda & Nakamura 2008, 2010, 2011
Experiments: ???

Fast-forward technique

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$$i\hbar\partial_t\Psi = -\frac{\hbar^2}{2m}\nabla^2\Psi + (\mathcal{V}+\mathcal{V}_{\rm au})\Psi + g|\Psi|^2\Psi,$$

Ansatz for the evolution

$$\Psi(\mathbf{q},t) = \psi[\mathbf{q},R(t)]e^{i\phi(\mathbf{q},t)}e^{-\frac{i}{\hbar}\int_0^t \mu[R(t')]dt'}$$

where

$$-\frac{\hbar^2}{2m}\nabla^2\psi + \mathcal{V}\psi + g|\psi|^2\psi = \mu\psi.$$

Substituting ansatz, separating real and imaginary parts

$$\mathcal{V}_{\rm au}({\rm q},t) = -\frac{\hbar^2}{2m}(\nabla\phi)^2 - \hbar\partial_t\phi$$

$$\nabla^2\phi + 2\nabla\ln\psi\cdot\nabla\phi + \frac{2m}{\hbar}\dot{R}\partial_R\ln\psi = 0$$

determine the auxiliary driving potential

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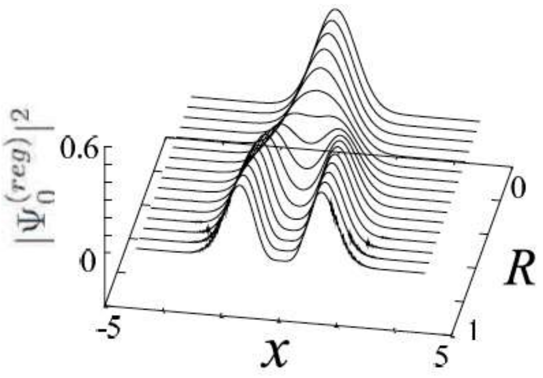
Theory: Masuda & Nakamura 2008, 2010, 2011
Experiments: ???

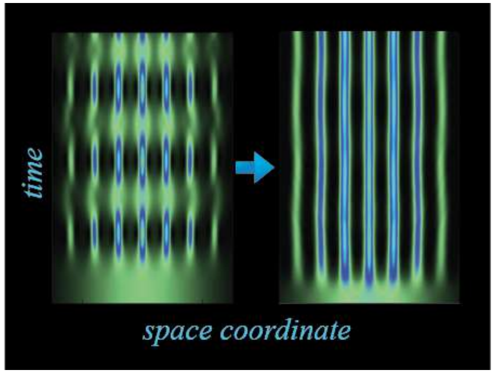
Tapas selection

Matter wave splitting



Masuda & Nakamura,
PRSA **466**, 1135 (2010)
Torrontegui et al
PRA **87**, 033630 (2013)

Ground-state optical lattice loading

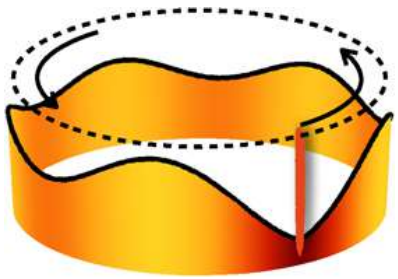


Masuda, Nakamura, AdC
PRL **113**, 063003 (2014)

Auxliary potential
≈ bichromatic lattice

$$\mathcal{V}_{\rm app}(q,t) = U_1(t)\sin^2(k_L q) + U_2(t)\sin^2(2k_L q)$$

Generation of NOON states



Schloss et al.
NJP **18**, 035012 (2016)

Require shaping potential: Ideally suited for painted-potential techniques

Fast-forward technique

Scale invariance is kind of classical

Really needed?

Protocols become energy/state dependent

Critical systems:
AdC, Rams, Zurek PRL **109**, 115703 (2012)
Saberi, Opatrný, Mølmer, AdC PRA **90**, 060301(R) (2014)
Optical lattices:
Masuda, Nakamura, AdC PRL **113**, 063003 (2014)

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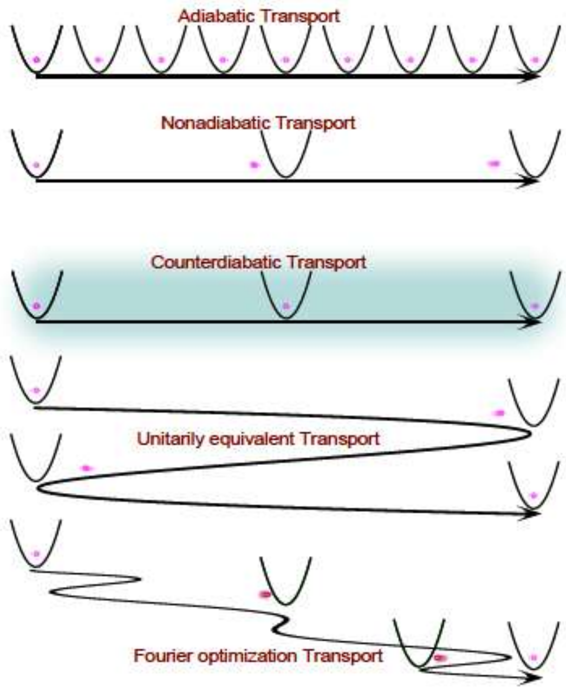
Contents

Part II
Shortcuts to adiabatic transport



Shuoming An, Dingshun Lv, AdC, Kihwan Kim, Nat. Commun 7, 12999 (2016)

Counterdiabatic transport

Transport of quantum fluids

$$\hat{\mathcal{H}} = \sum_{i=1}^{N}\Big[-\frac{\hbar^2}{2m}\Delta^2 + U[\mathbf{r}_i - \mathbf{f}(t)]\Big] + \sum_{i<j} V(\mathbf{r}_i - \mathbf{r}_j),$$

Scale-invariant evolution of an initial stationary state

$$\Phi(\mathbf{r}_1, \dots, \mathbf{r}_N; t) = e^{-i\mu t/\hbar}\Phi\left[\mathbf{r}_1 - \mathbf{f}(t), \dots, \mathbf{r}_N - \mathbf{f}(t); 0\right]$$

NONLOCAL counterdiabatic term

$$\boxed{\hat{\mathcal{H}}_1 = -i\hbar\sum_{i=1}^{N}\dot{\mathbf{f}}\partial_{\mathbf{r}_i} = \sum_{i=1}^{N}\dot{\mathbf{f}}\cdot\mathbf{p}_i.}$$

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S. Deffner, C. Jarzynski, A. del Campo, PRX 4, 021013 (2014)

Counterdiabatic transport

Transport of quantum fluids

$$\hat{\mathcal{H}} = \sum_{i=1}^{N}\Big[-\frac{\hbar^2}{2m}\Delta^2 + U[\mathbf{r}_i - \mathbf{f}(t)]\Big] + \sum_{i<j} V(\mathbf{r}_i - \mathbf{r}_j),$$

Scale-invariant evolution of an initial stationary state

$$\Phi(\mathbf{r}_1,\dots,\mathbf{r}_N;t) = e^{-i\mu t/\hbar}\Phi\left[\mathbf{r}_1 - \mathbf{f}(t),\dots,\mathbf{r}_N - \mathbf{f}(t);0\right]$$

LOCAL CD term
via unitary transformation

$$\mathcal{U} = \exp\left\{\frac{im}{\hbar}\sum_{i=1}^{N}\dot{\mathbf{f}}\cdot\mathbf{r}_i - i\frac{m}{2}\int_0^t \dot{\mathbf{f}}(t')^2 dt'\right\}$$

$$\hat{\mathcal{H}}_{\rm CD} = \hat{\mathcal{H}} - \sum_{i=1}^{N} m\ddot{\mathbf{f}}\cdot\mathbf{r}_i$$

S. Deffner, C. Jarzynski, A. del Campo, PRX 4, 021013 (2014)

Counterdiabatic transport

Transport of quantum fluids

$$\hat{\mathcal{H}} = \sum_{i=1}^{N}\left[-\frac{\hbar^2}{2m}\Delta^2 + U[\mathbf{r}_i - \mathbf{f}(t)]\right] + \sum_{i<j} V(\mathbf{r}_i - \mathbf{r}_j),$$

Scale-invariant evolution of an initial stationary state

$\Phi(\mathbf{r}_1, \ldots$

Single-particle:
S. Masuda, K. Nakamura, Proc. R. Soc. A 466, 1135 (2010)
E. Torrontegui et al, Phys. Rev. A83,013415 (2011).
Many-particle:
S. Masuda, Phys. Rev. A 86, 063624 (2012)

LOCAL CD
via unitary tra

$i=1$

$$\hat{\mathcal{H}}_{\mathrm{CD}} = \hat{\mathcal{H}} - \sum_{i=1}^{N} m\ddot{\mathbf{f}} \cdot \mathbf{r}_i$$

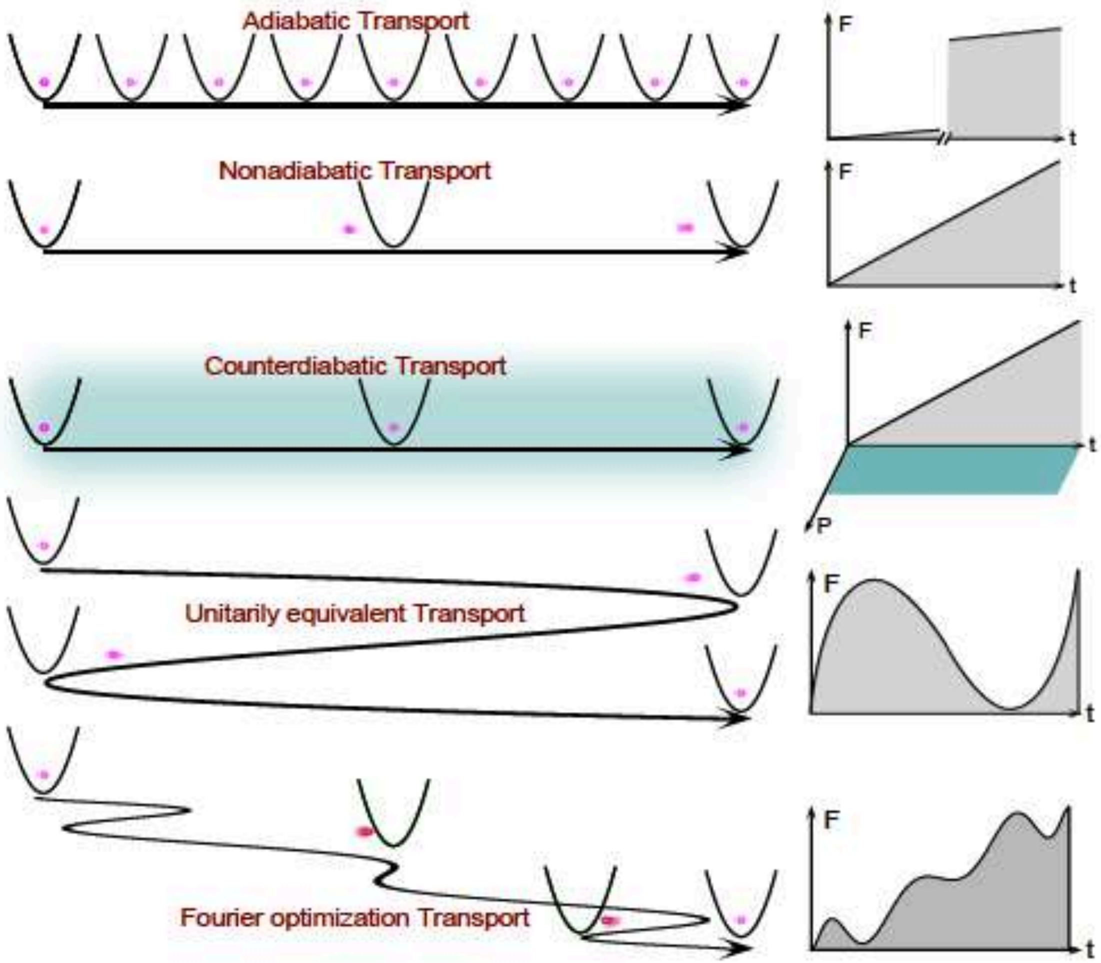
S. Deffner, C. Jarzynski, A. del Campo, PRX 4, 021013 (2014)

Shortcuts to Adiabaticity by Counterdiabatic Driving in Trapped-ion Transport

Shuoming An,[1] Dingshun Lv,[1] Adolfo del Campo,[2] and Kihwan Kim[1]

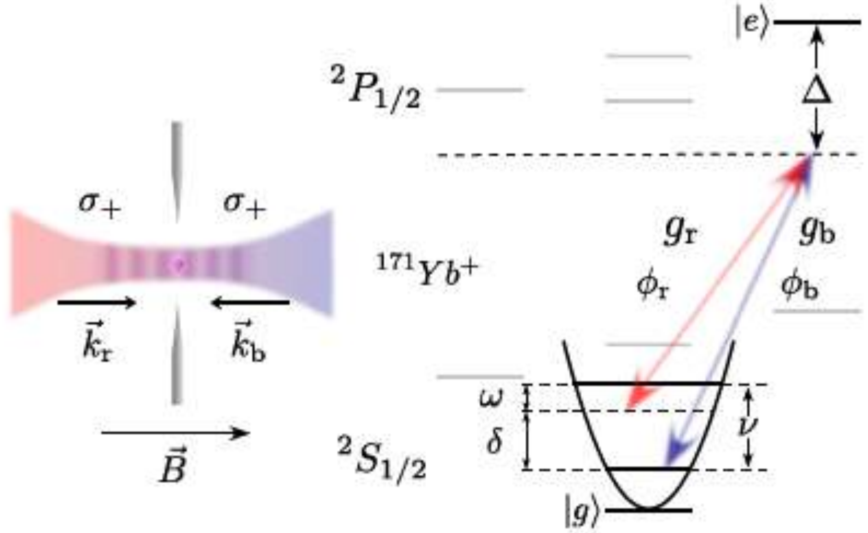
[1]*Center for Quantum Information, Institute for Interdisciplinary Information Sciences, Tsinghua University, Beijing 100084, People's Republic of China*

[2]*Department of Physics, University of Massachusetts, Boston, MA 02125, USA*



Recipe for a dragged harmonic oscillator

- Single trapped $^{171}Yb^{+}$ ion
- Raman beams exert the force
- Dipole approximation
- Rotating Wave approximation (RWA)
- Lamb-Dicke regime

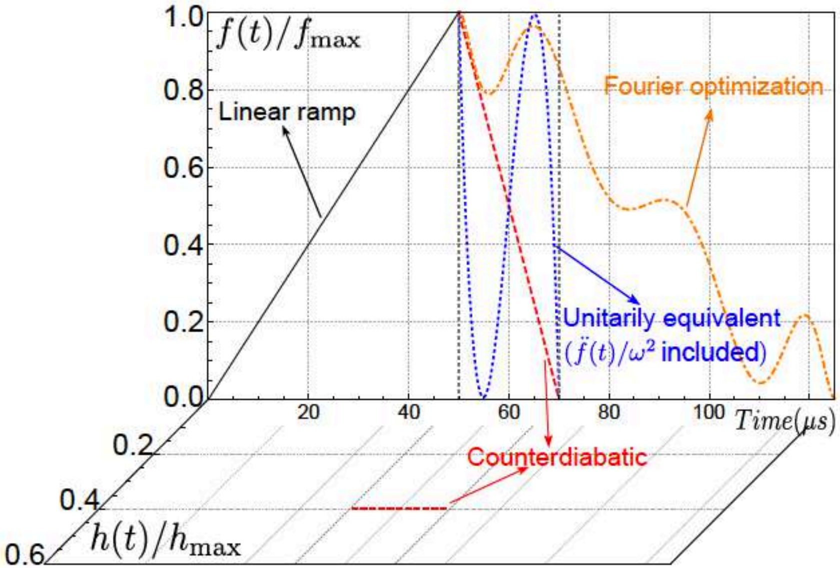


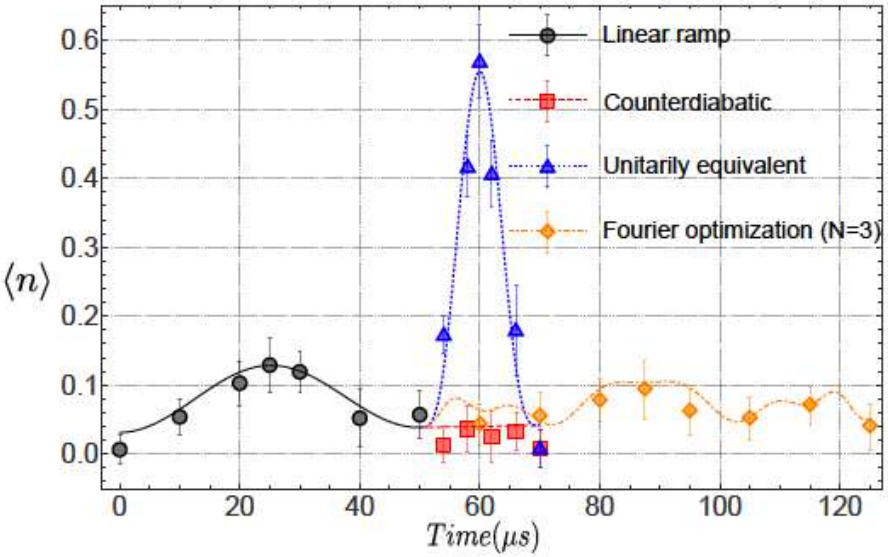
$$\hat{H}_{\rm eff} = \hat{p}^2/2m + m\omega^2\hat{x}^2 + f(t)\hat{x} \qquad \hat{H}_{\rm CD} = -\frac{\dot{f}(t)}{m\omega^2}\hat{p}$$

$$\hat{H}_{\rm eff} = f(t)x_0\left(\hat{a}e^{-i(\omega t+\phi} + \hat{a}^\dagger e^{+i(\omega t+\phi}\right)$$

Adolfo del Campo: adolfo.delcampo@umb.edu

Comparison of transport protocols





Counterdiabatic driving is maximally robust

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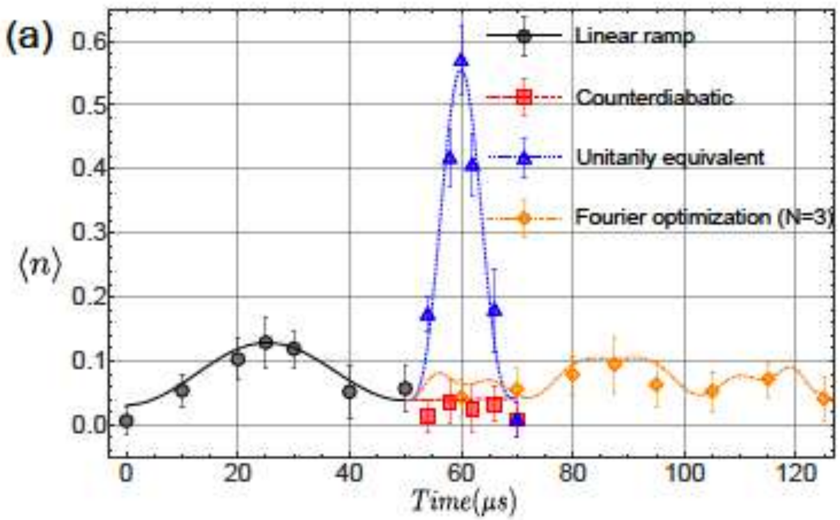
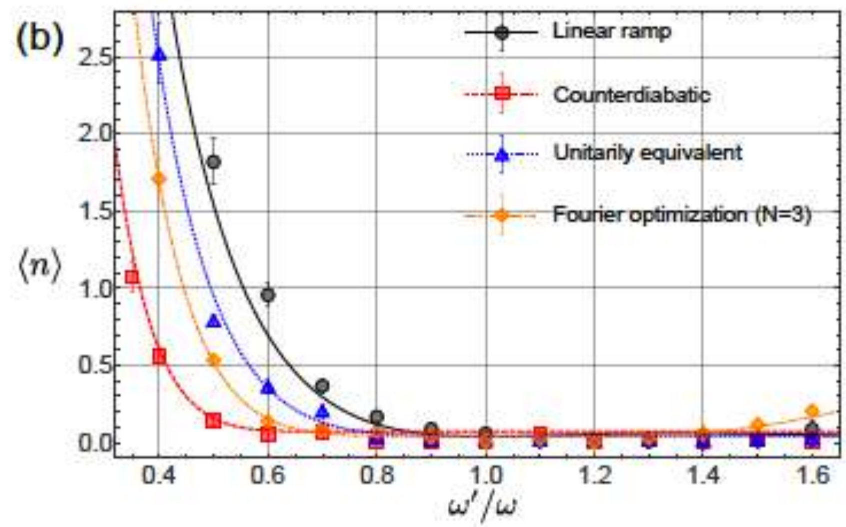
Shuoming An et al. Nat. Commun 7, 12999 (2016)

Robustness against trap frequency errors





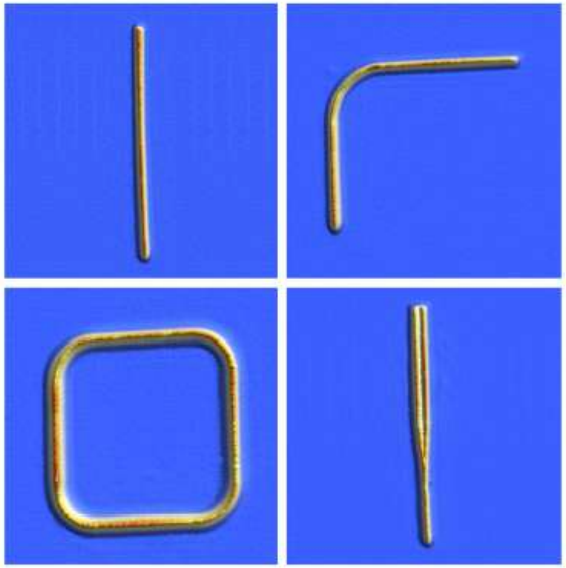
Maximally robust STA via Counterdiabatic Driving

Shuoming An et al. Nat. Commun 7, 12999 (2016)

Contents

Part III

Design of bent waveguides
Tailoring curvature effects



del Campo, Boshier, Saxena, Sci. Rep. **4**, 5274 (2014)
Ryu & Boshier New J. Phys **17**, 092002 (2015)

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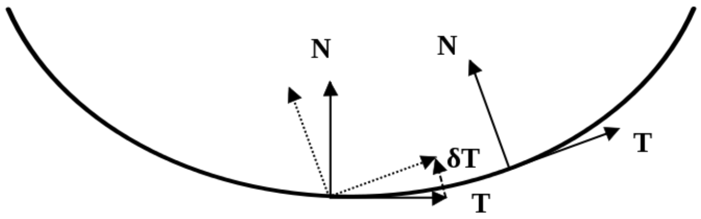
Curvature-induced potential (CIP)

- Waveguide with non-zero curvature
- Dimensional reduction of the Schrödinger equation under tight transverse confinement
- Emergence of quantum-mechanical local attractive potential

$$V_{\rm CIP}(q) = -\frac{\hbar^2}{8m}\kappa(q)^2$$

Curvature: rate of change of unit tangent vector $\kappa(q) = \left\| \frac{d\mathbf{T}}{dq} \right\|$

Switkes, Russel & Skinner, J. Chem. Phys. **67**, 3061(1977)
da Costa, Phys. Rev. A **23**, 1982 (1981)
Exner & Seba, J. Math. Phys. **30**, 2574 (1989)

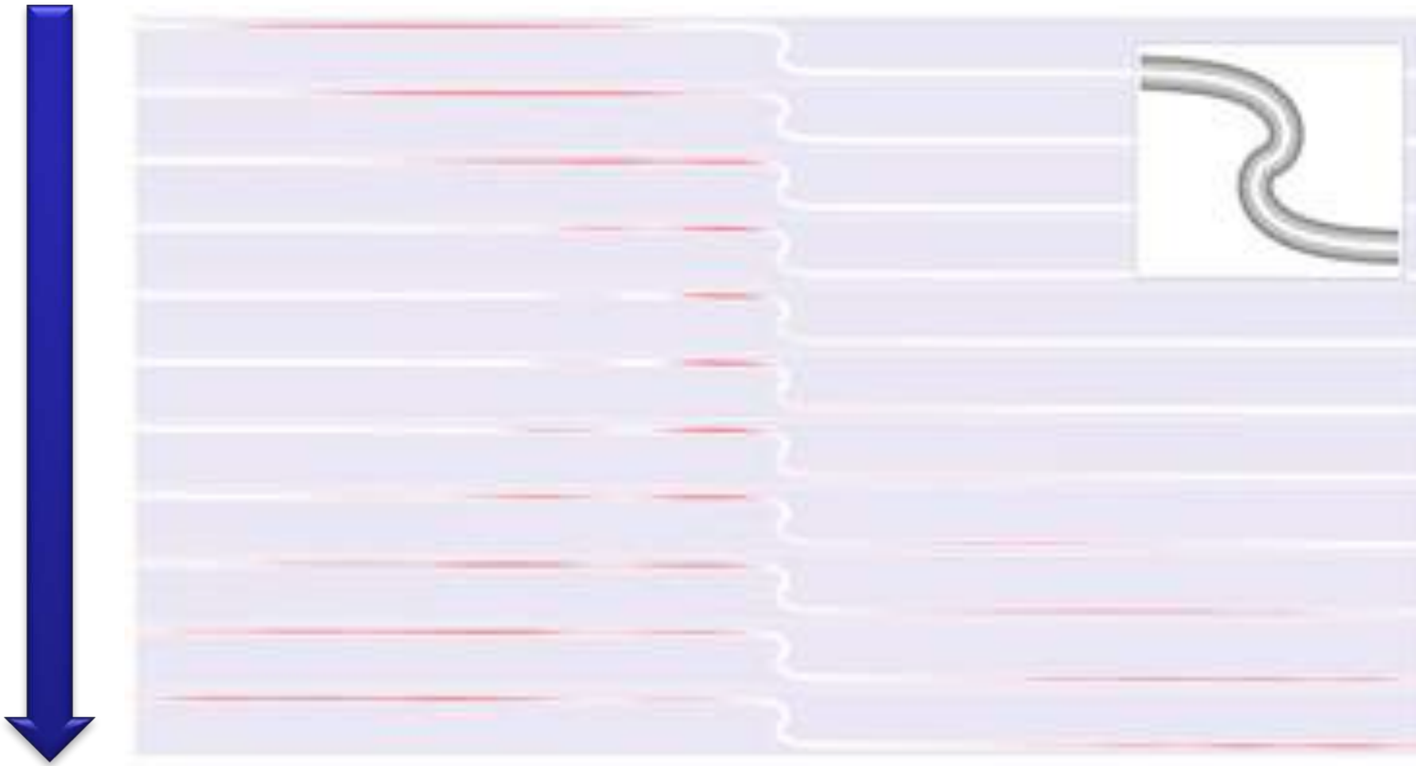


Curvature effects in atomtronics

Curvature affects scattering properties in atom circuits

Example: wavepacket splitting

time



del Campo, Boshier, Saxena, Sci. Rep. **4**, 5274 (2014)

Supersymmetric Quantum Mechanics

- Supersymmetric quantum mechanics identifies families of reflectionless potentials

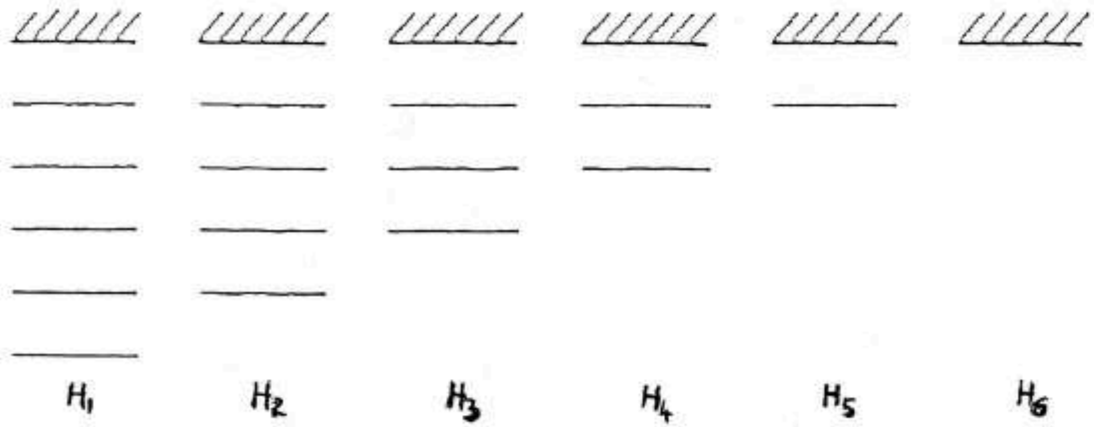


FIGURE 2. Schematic diagram of the eigenvalue spectra of the Hamiltonians in the hierarchy H_n. The number of bound states of H_1 is arbitrarily chosen to be 5.

SUSY partner Hamiltonians share scattering properties

Cooper, Khare, Sukhatme, Phys. Rep. **251**, 267 (1995)

Supersymmetric Quantum Mechanics

SUSY QM identifies families of reflectionless potentials via a superpotential $\Phi(q)$

$$A := \frac{ip}{\sqrt{2m}} + \Phi(q) \qquad A^\dagger := -\frac{ip}{\sqrt{2m}} + \Phi(q)$$

$$H_- := A^\dagger A \geq 0 \qquad H_+ := AA^\dagger \geq 0$$

$$H_\pm = \frac{p^2}{2m} + V_\pm(x) \qquad V_\pm(x) = \mp \frac{\hbar}{\sqrt{2m}} \Phi(q)' + \Phi(q)^2$$

SUSY partner Hamiltonians share scattering properties

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Cooper, Khare, Sukhatme, Phys. Rep. **251**, 267 (1995)

Supersymmetric reflectionless waveguides

- Supersymmetric quantum mechanics identifies families of reflectionless potentials

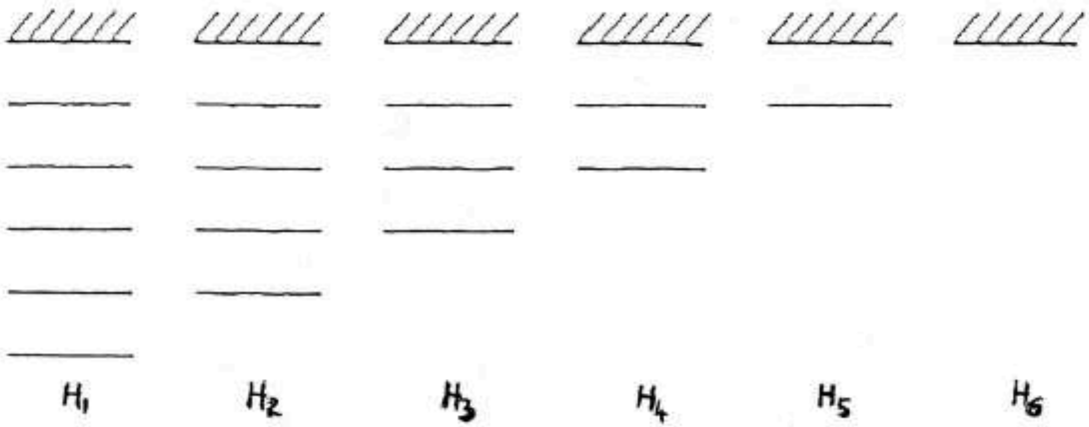

FIGURE 2. Schematic diagram of the eigenvalue spectra of the Hamiltonians in the hierarchy H_n. The number of bound states of H_1 is arbitrarily chosen to be 5.

Goal:
Design reflectionless bent waveguides with unit transmission probability

Idea:
Choose curvature-induced potential that is SUSY partners of V(x)=0 (free dynamics/straight waveguide)

del Campo, Boshier, Saxena, Sci. Rep. **4**, 5274 (2014)

Supersymmetric reflectionless waveguides

- Supersymmetric quantum mechanics identifies families of reflectionless potentials

Unit transmission probability at any energy

- Curvature relation between SUSY waveguides

$$\kappa_+^2(q_1) = \kappa_-^2(q_1) - \frac{8\sqrt{2m}}{\hbar}\Phi'(q_1)$$

- Curvature specifies uniquely the waveguide shape (Frenet-Serret equations)

- Choose curvature to make CIP reflectionless, isospectral to straight waveguide

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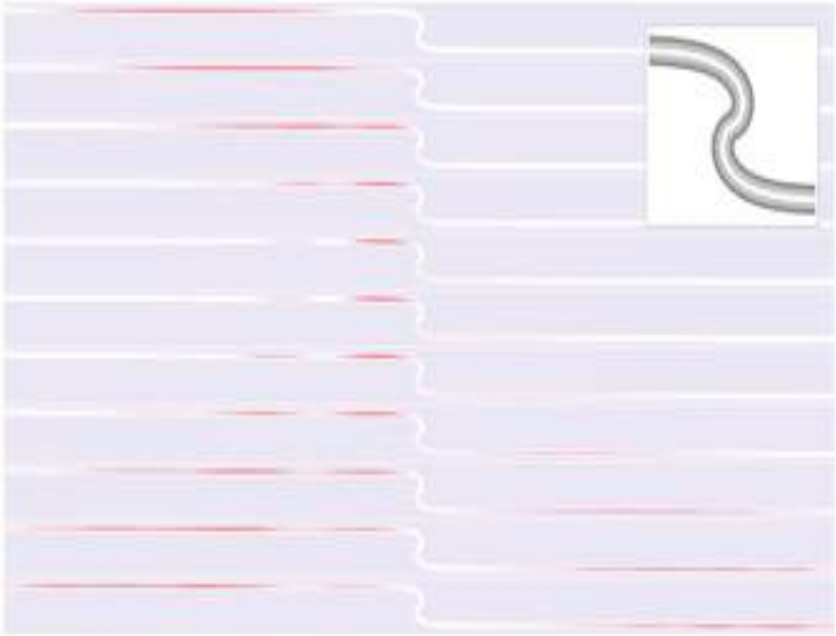
del Campo, Boshier, Saxena, Sci. Rep. **4**, 5274 (2014)

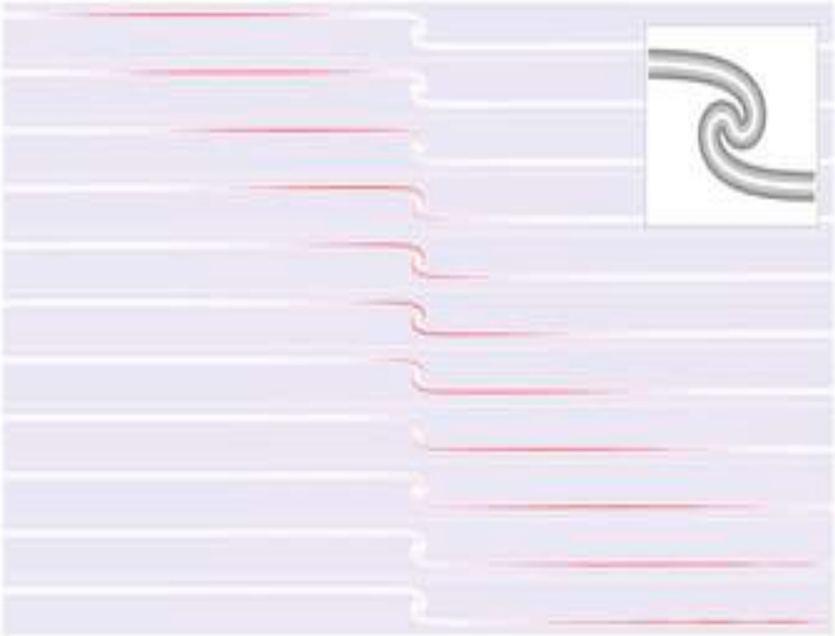
Supersymmetric reflectionless waveguides

- Supersymmetric quantum mechanics identifies families of reflectionless potentials
- Choose curvature to make CIP reflectionless

Curved waveguide



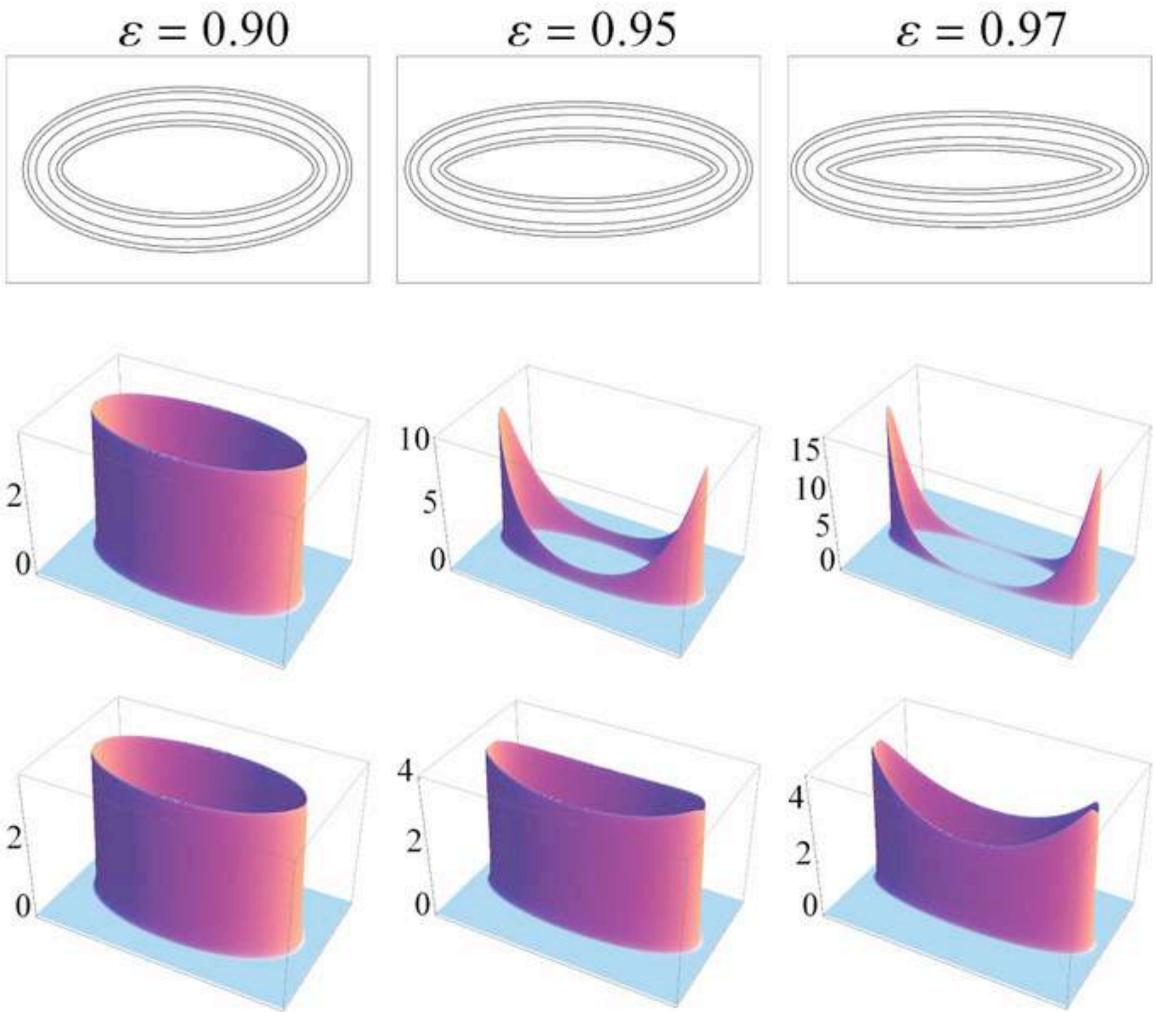
Curved SUSY waveguide



isospectral to straight waveguide

del Campo, Boshier, Saxena, Sci. Rep. **4**, 5274 (2014)

Curvature-induced effects: Elliptical waveguide potentials



del Campo, Boshier, Saxena, Sci. Rep. **4**, 5274 (2014)

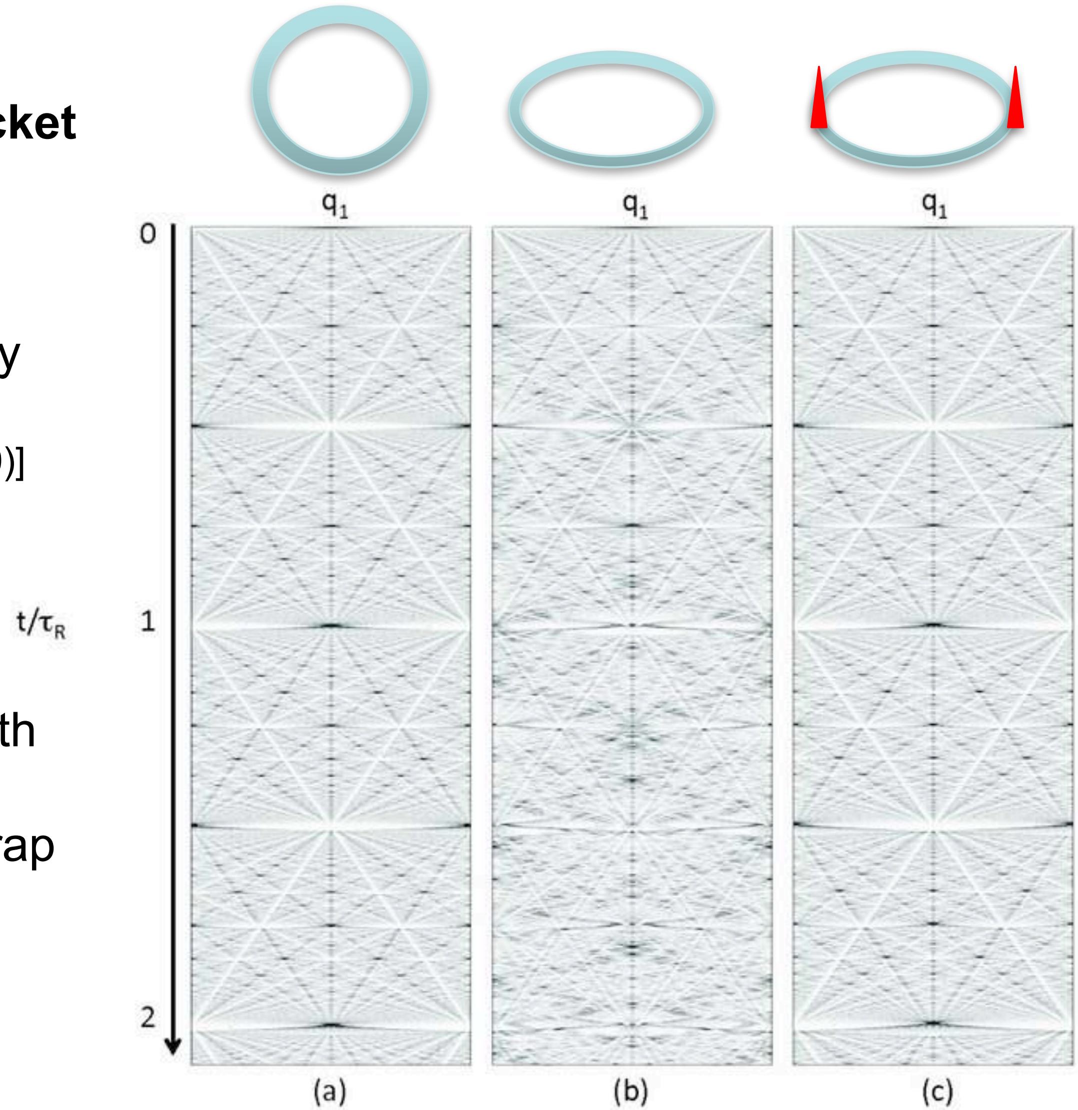
Elliptical waveguide potentials: Quantum carpets

Released localized wavepacket
Talbot oscillations
in the density profile

a) Periodic pattern in the density profile in a ring trap
[see Friesch et al. New J. Phys. **2**, 4 (2000)]

b) Suppressed by curvature in c elliptical trap

c) Recovered in elliptical trap with cancelled curvature-induced potential: isospectral to ring trap



del Campo, Boshier, Saxena, Sci. Rep. **4**, 5274 (2014)

Elliptical waveguide potentials: Quantum carpets

Released localized wavepacket
Talbot oscillations
in the density profile

a) Periodic pattern in the density profile in a ring trap
[see Friesch et al. New J. Phys. **2**, 4 (2000)]

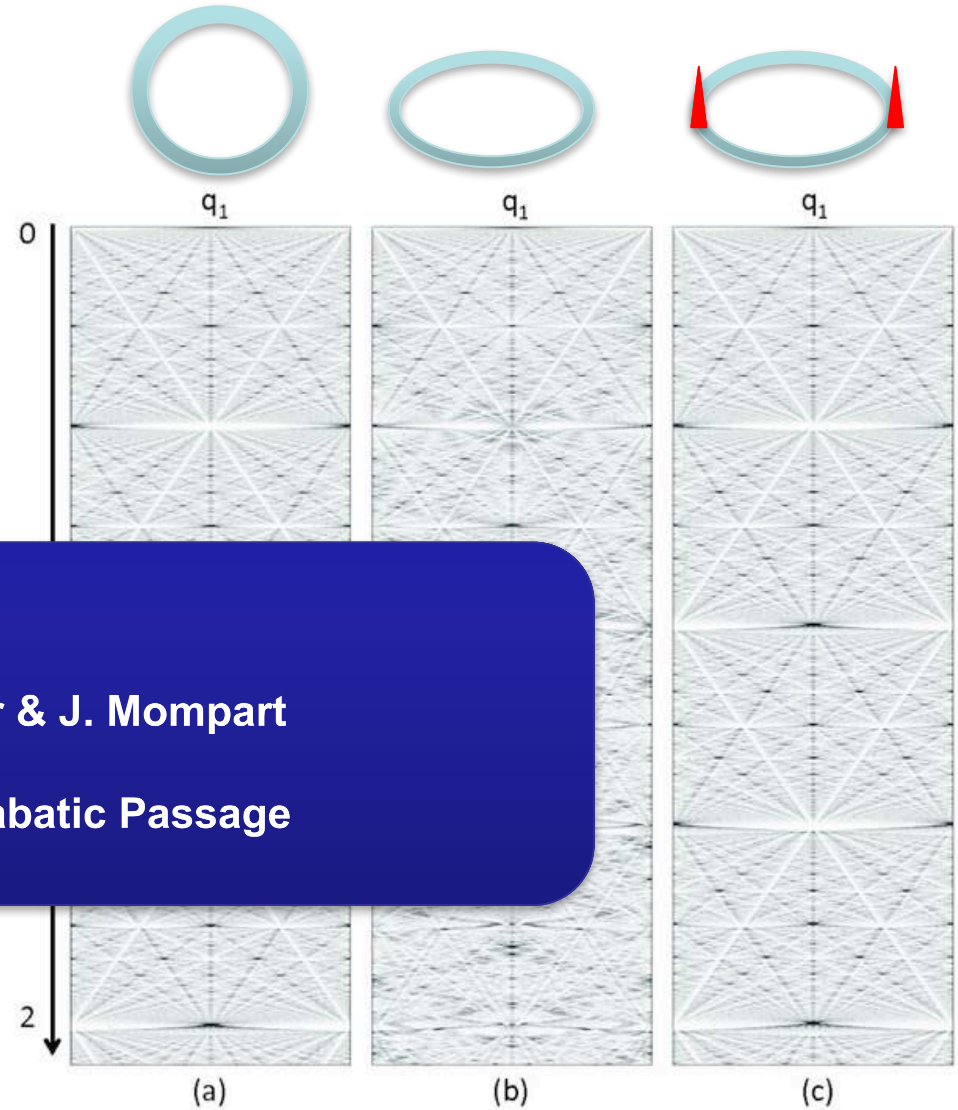
a) Supp
ellipt

b) Rec
canc
pote

Coming developments

UAB group/V. Ahufinger & J. Mompart

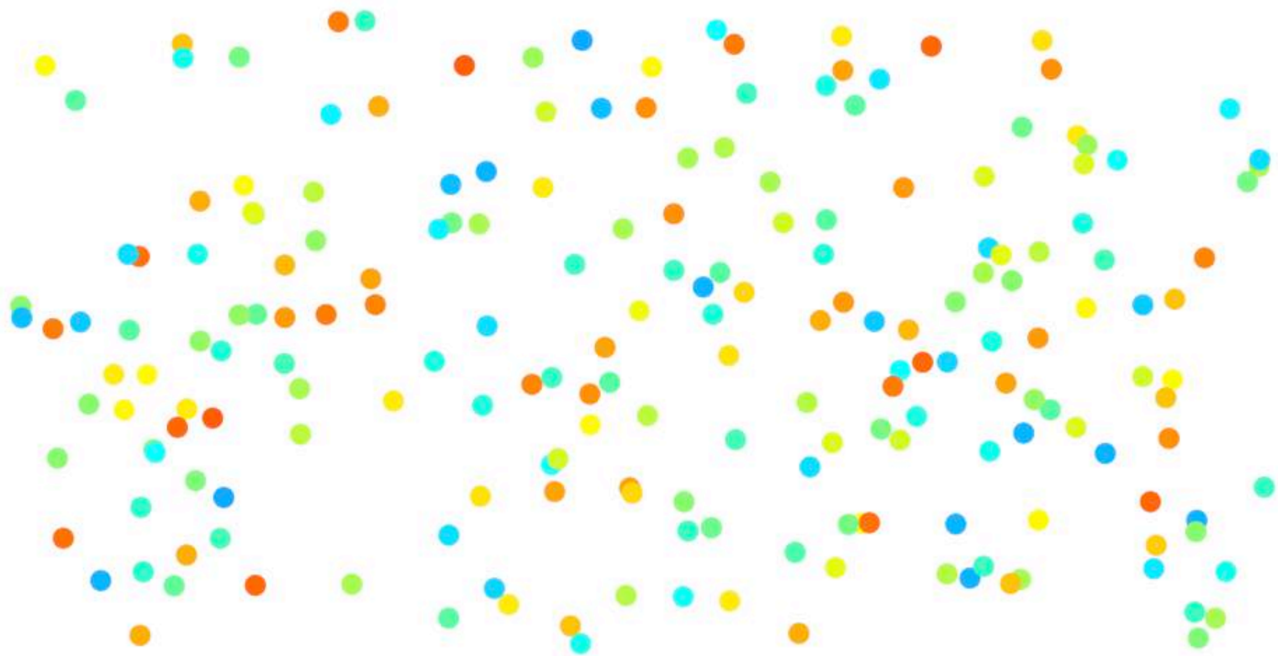
SUSY QM + Spatial Adiabatic Passage



del Campo, Boshier, Saxena, Sci. Rep. **4**, 5274 (2014)

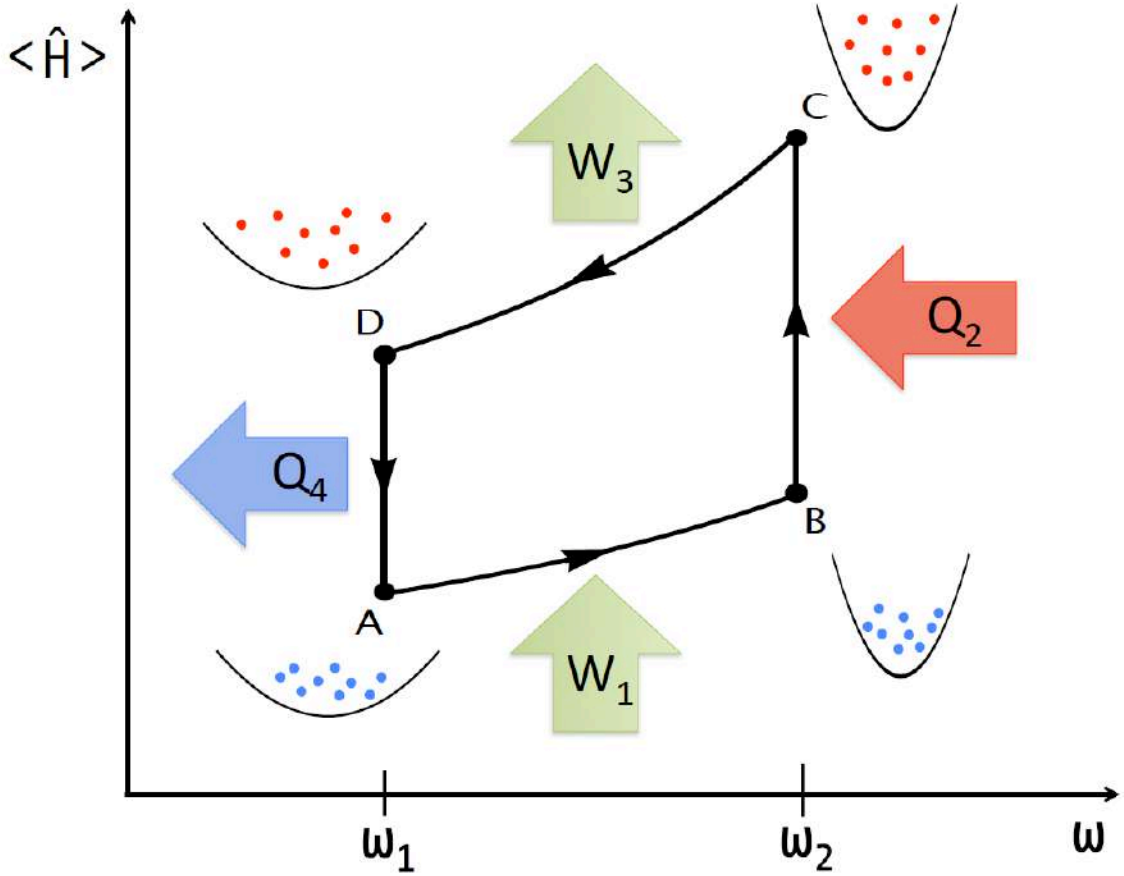
Part IV

Quantum thermal machines

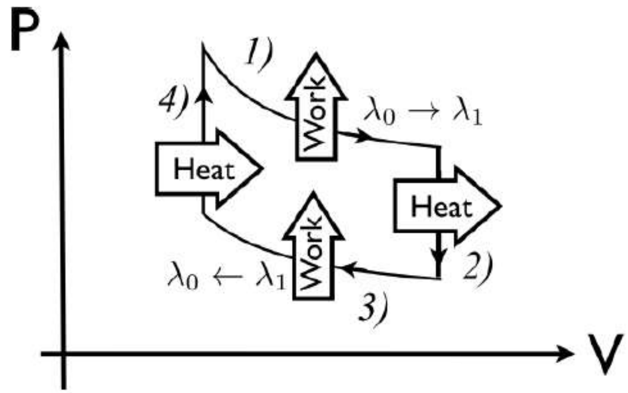


Adolfo del Campo: adolfo.delcampo@umb.edu

Quantum Heat Engines (e.g. Otto Cycle)



Performance of a QHE



$$\langle W \rangle = \langle H_f \rangle - \langle H_i \rangle$$

Efficiency: work done/heat absorbed

$$\eta = -\frac{\langle W_1 \rangle + \langle W_3 \rangle}{\langle Q_2 \rangle}$$

Power: work done per cycle time

$$\mathcal{P} = -\frac{\langle W \rangle_1 + \langle W \rangle_3}{\sum_{j=1}^{4} \tau_j}$$

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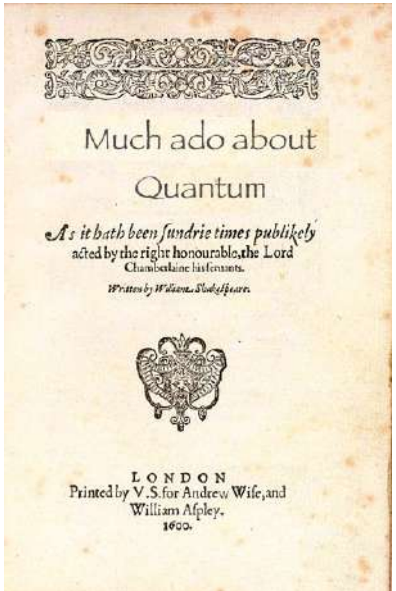
When is quantum thermo really quantum?

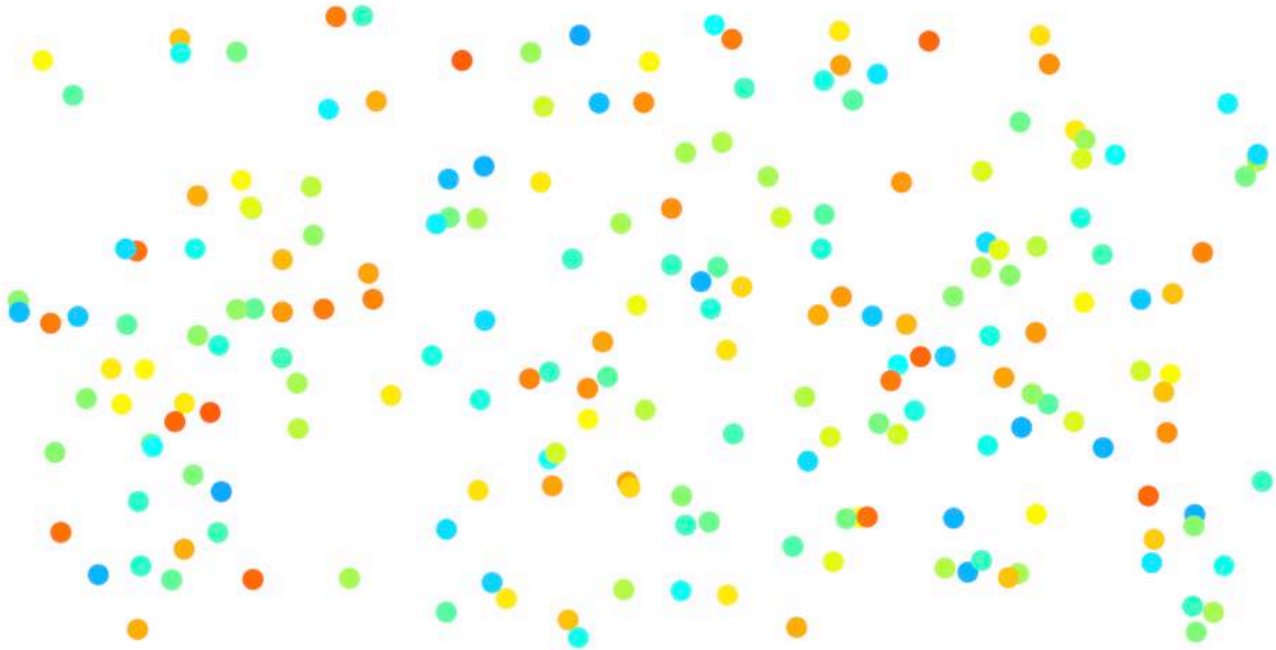

Much ado about
Quantum
As it hath been sundrie times publikely
acted by the right honourable, the Lord
Chamberlaine his seruants.
Written by William Shakespeare.

LONDON
Printed by V.S. for Andrew Wise, and
William Aspley.
1600.

(Phys.org)—A lot of attention has been given to the differences between the quantum and classical worlds. […]
However, when it comes to certain areas of thermodynamics—specifically, thermal engines and refrigerators—quantum and classical systems so far appear to be nearly identical.

Adolfo del Campo: adolfo.delcampo@umb.edu

Many-particle thermal machines



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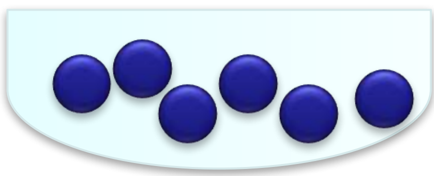
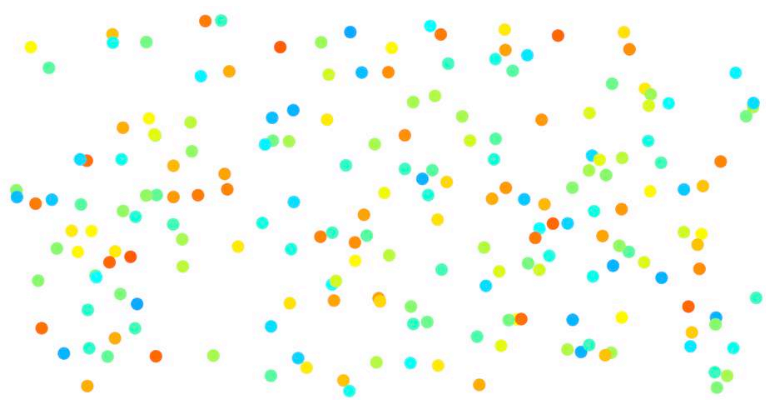
Many-particle QHE

Single N-particle engine vs N single-particle engines?



What substance is optimal as working medium?



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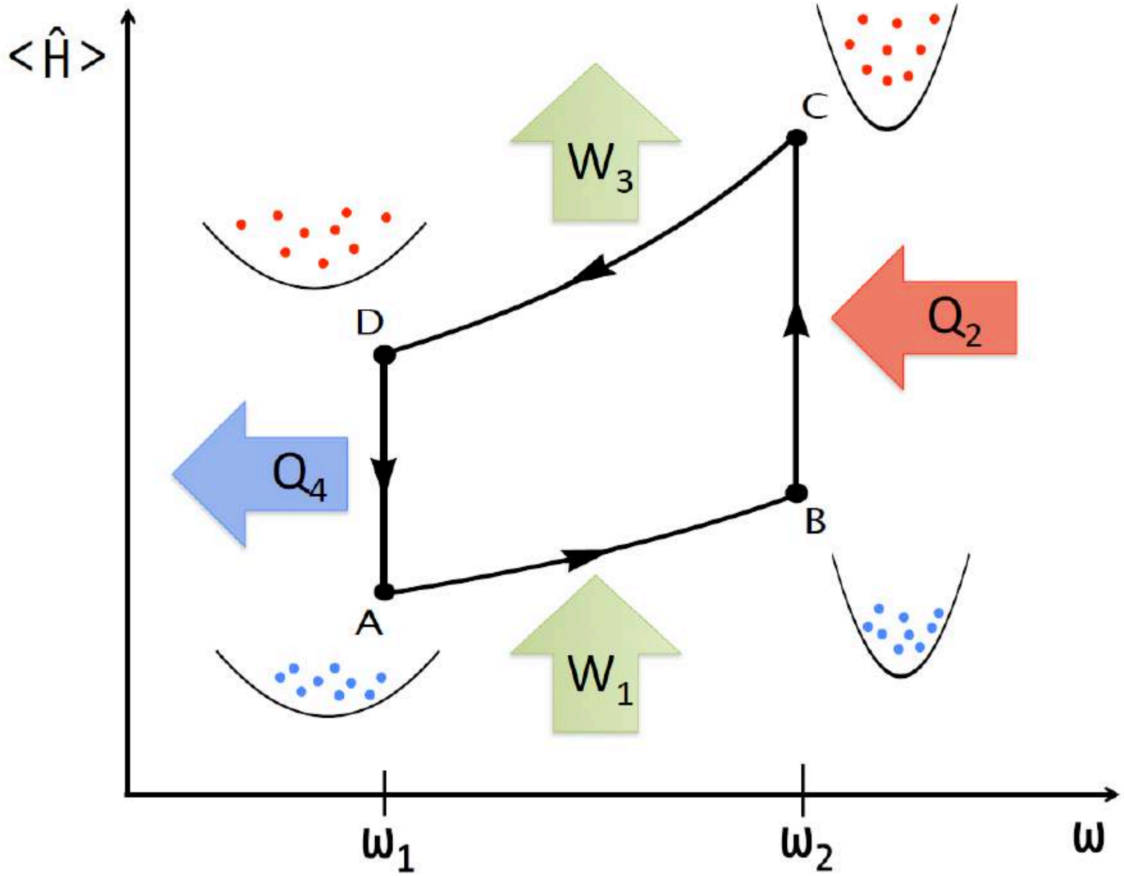
Many-particle QHE (Otto cycle)

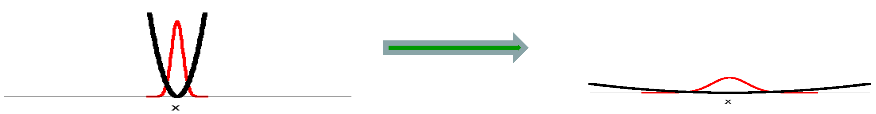


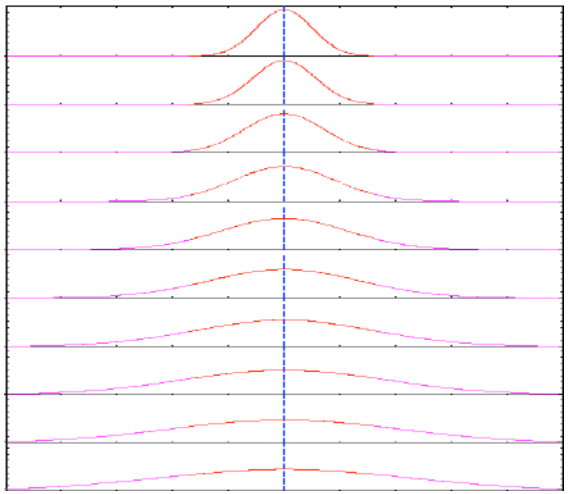
Many-particle working medium

Interacting quantum fluids

$$\hat{H} = \sum_{i=1}^{N}\left[-\frac{\hbar^2}{2m}\nabla^2 + \frac{1}{2}m\omega(t)^2\mathbf{r}_i^2\right] + \sum_{i<j} V(\mathbf{r}_i - \mathbf{r}_j)$$

Scaling-invariant dynamics when $\quad V(\gamma\mathbf{r}) = \gamma^{-2}V(\mathbf{r})$





A. del Campo, PRL **111**, 100502 (2013)

Exact Finite-time thermodynamics

Scale-invariant dynamics with dynamical symmetry [SU(1,1)]

$$\langle H(t)\rangle = \frac{1}{b^2}\langle H(0)\rangle + \sum_{i=1}^{\mathrm{N}} \frac{\dot{b}}{2b}\langle\{z_i, p_i\}(0)\rangle + \sum_{i=1}^{\mathrm{N}} \frac{m}{2}(\dot{b}^2 - b\ddot{b})\langle z_i^2(0)\rangle$$

Scaling factor obeys Ermakov equation

$$\ddot{b} + \omega(t)^2 b = \omega_0^2/b^3$$

Nonadiabatic factor

$$\langle H(t)\rangle = Q^*(t)\langle H(t)\rangle_{\mathrm{adiab}} \qquad Q^*(t) \geq 1$$

$$Q^*(t) = \frac{\omega_0}{\omega(t)}\left(\frac{1}{2b^2} + \frac{\omega(t)^2}{2\omega_0^2}b^2 + \frac{\dot{b}^2}{2\omega_0^2}\right)$$

Jaramillo et al NJP 18, 075019 (2016); Beau et al Entropy 18, 168 (2016)

Example: interacting Bose gas as working medium

Working medium - interacting many-particles with tunable interactions

$$H=\sum_{i=1}^{N}\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_i^2}+\frac{1}{2}m\omega(t)^2x_i^2\right]+\frac{\hbar^2}{m}\sum_{i<j=1}^{N}\frac{\lambda(\lambda-1)}{(x_i-x_j)^2}$$

[1971: Calogero, J. Math. Phys. 12, 419; Sutherland, ibid 12, 246]

Jaramillo et al NJP 18, 075019 (2016); Beau et al Entropy 18, 168 (2016)

Example: interacting Bose gas as working medium

Working medium - interacting many-particles with tunable interactions

$$H=\sum_{i=1}^{N}\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_i^2}+\frac{1}{2}m\omega(t)^2x_i^2\right]+\frac{\hbar^2}{m}\sum_{i<j=1}^{N}\frac{\lambda(\lambda-1)}{(x_i-x_j)^2}$$

[1971: Calogero, J. Math. Phys. 12, 419; Sutherland, ibid 12, 246]

- Includes ideal bosons and hard-core bosons (= fermions) for λ=0,1
- Exact finite-time quantum thermodynamics – no approximations
- Equivalent to ideal gas of particles obeying fractional exclusion [Haldane PRL 67, 937 (1991); Murthy & Shankar PRL 73, 3331 (1994)]
- Universal behavior (Luttinger liquid) of 1D many-body systems
- Tunable zero-point energy + linear spectrum

$$E(\{n_k\})=\frac{\hbar\omega}{2}N[1+\lambda(N-1)]+\sum_{k=1}^{\infty}\hbar\omega k n_k$$

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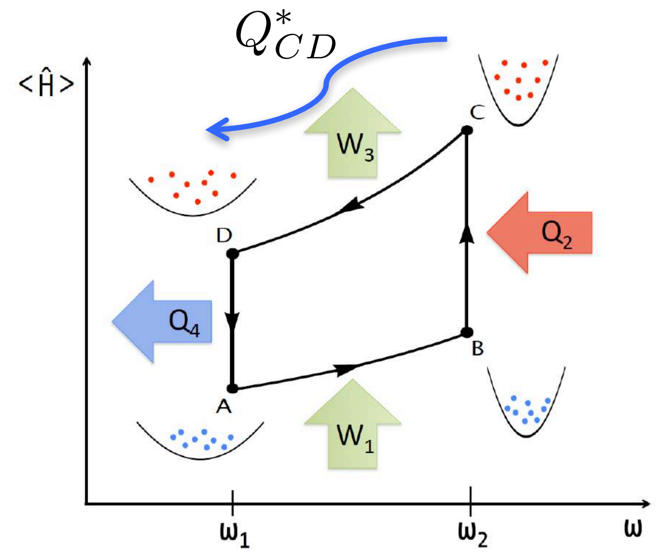
Jaramillo et al NJP 18, 075019 (2016); Beau et al Entropy 18, 168 (2016)

Universal Bound to Quantum Efficiency

Mean work $\langle W \rangle = \langle H_f \rangle - \langle H_i \rangle$

Quantum efficiency of many-particle QHE



$$\eta = -\frac{\langle W_1 \rangle + \langle W_3 \rangle}{\langle \mathrm{Q}_2 \rangle}$$

$$\eta = 1 - \frac{\omega_1}{\omega_2}\left(\frac{Q^*_{CD}\langle H \rangle_C - \frac{\omega_2}{\omega_1}\langle H \rangle_A}{\langle H \rangle_C - Q^*_{AB}\frac{\omega_2}{\omega_1}\langle H \rangle_A}\right)$$

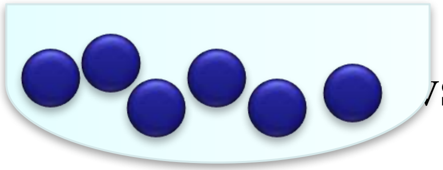
Upper bound to quantum efficiency via nonadiabatic compression factor

$$\boxed{\eta \leq \eta_{\mathrm{nad},O} \equiv 1 - Q^*_{CD}\frac{\omega_1}{\omega_2}}$$

Jaramillo et al NJP 18, 075019 (2016); Beau et al Entropy 18, 168 (2016)

Adolfo del Campo: adolfo.delcampo@umb.edu

Quest for Quantum Supremacy

Comparison  vs 

Worst case: sudden-quench limit (sq)

- Efficiency ratio at maximum power

$$\rho_{\rm sq}^{(N)} := \frac{\eta_{\rm sq}^{(N)}}{\eta_{\rm sq}^{(1)}}$$

- Power ratio

$$r_{\rm sq}^{(N)} := \frac{P_{\rm sq}^{(N)}}{N P_{\rm sq}^{(1)}}$$

Jaramillo et al NJP 18, 075019 (2016)

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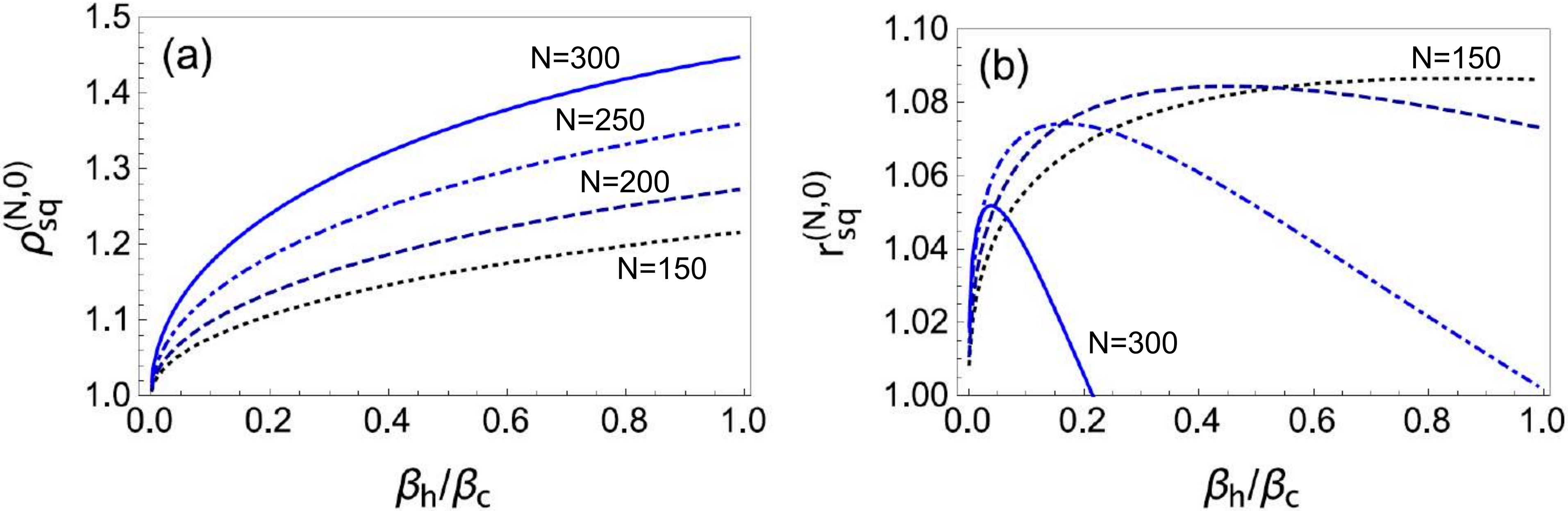
Quantum Supremacy: noninteracting case

Simultaneous enhancement of efficiency and power



Up to 50% efficiency enhancement

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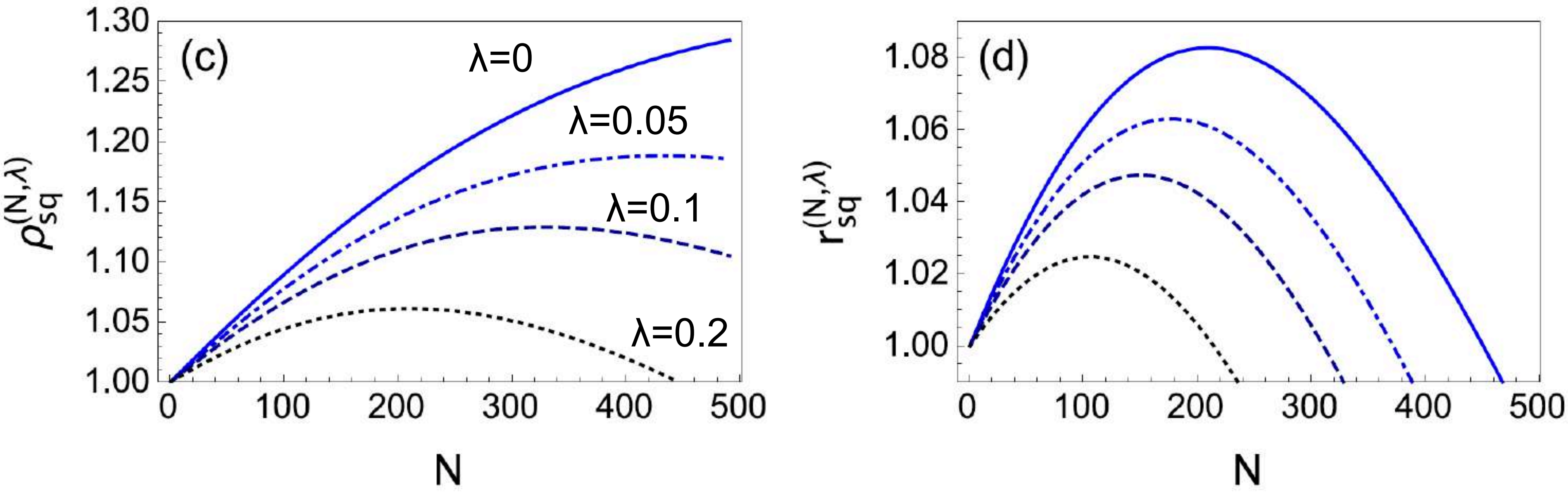
Jaramillo et al NJP 18, 075019 (2016)

Quantum Supremacy: interacting case

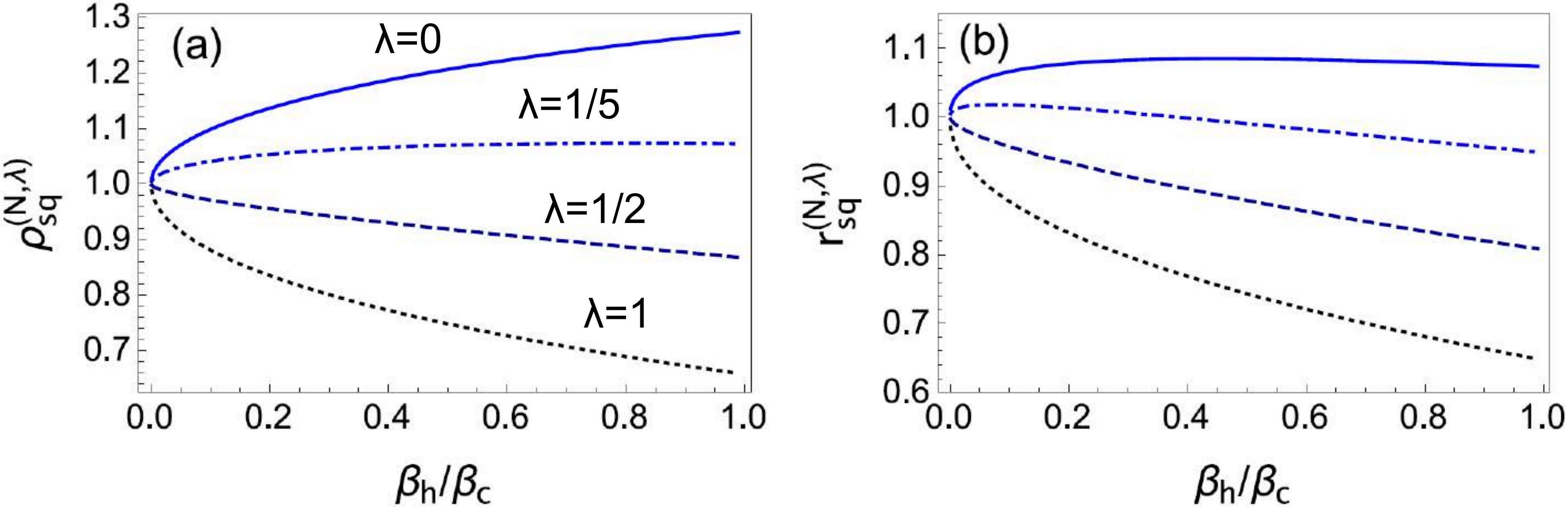


Jaramillo et al NJP 18, 075019 (2016)

Quantum Supremacy

Simultaneous enhancement of efficiency and power (N=200)

QS suppressed by strong interactions



Jaramillo et al NJP 18, 075019 (2016)

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Quantum Supremacy

Simultaneous Enhancement

of Efficiency and Power

in finite-time thermodynamics

[Jaramillo, Beau, AdC, NJP 18, 075019]

$$\left.\begin{array}{l}\blacklozenge\ \text{Quantum / low T}\\ \\ \blacklozenge\ \text{Many-particle}\\ \\ \blacklozenge\ \text{Non-adiabatic}\end{array}\right\}\ \text{effect}$$

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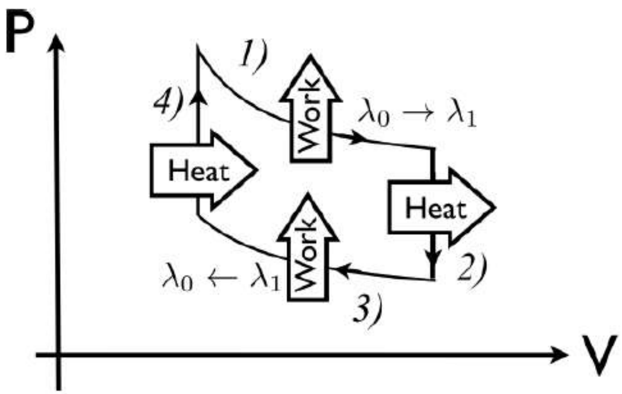
Efficiency vs Power



Quantum efficiency

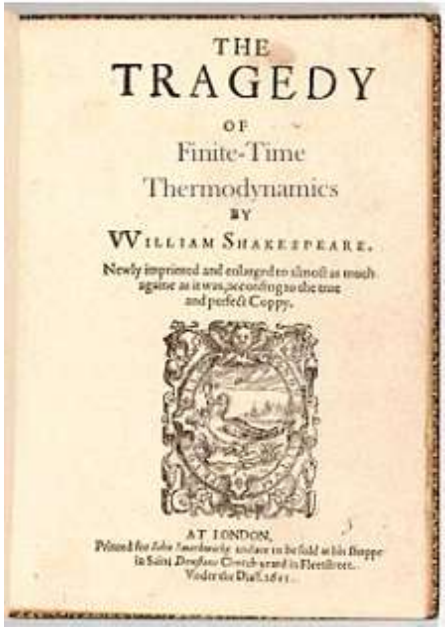
$$\eta = -\frac{\langle W_1 \rangle + \langle W_3 \rangle}{\langle Q_2 \rangle}$$

Nonadiabatic Efficiency e.g. of a single-particle Otto cycle

$$\eta = 1 - \frac{\omega_1}{\omega_2} f[\omega(t)] \leq 1 - \frac{\omega_1}{\omega_2}$$

Essence of finite-time thermodynamics:

Trade-off between efficiency and power



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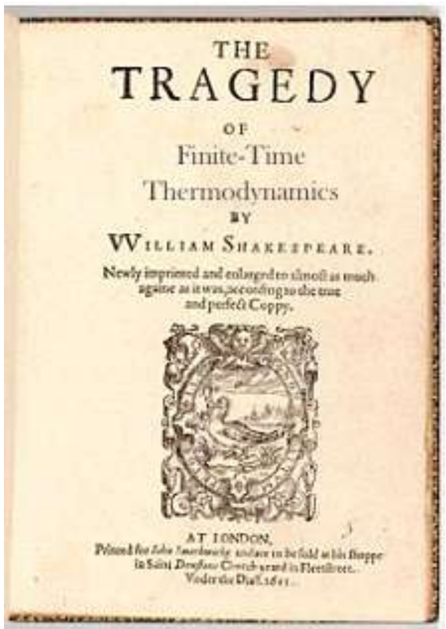
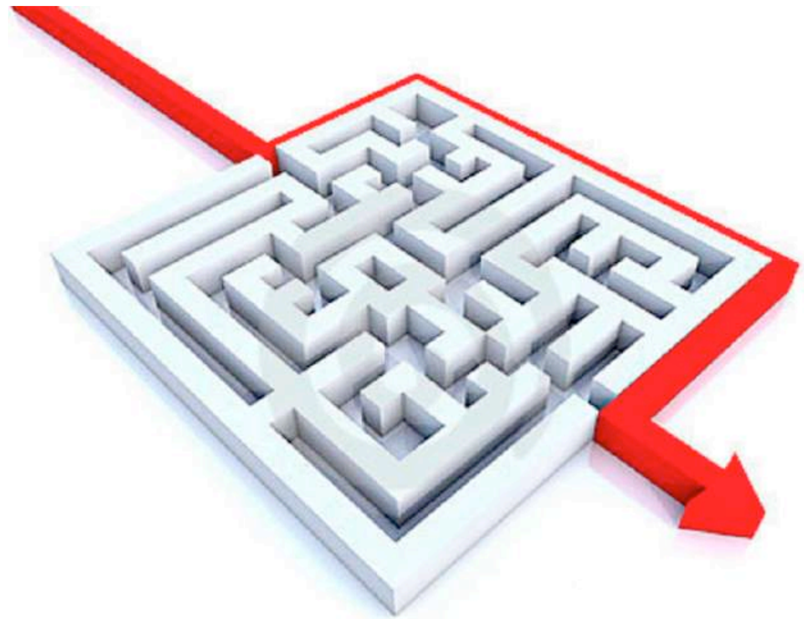
Shortcuts as a way out of the tragedy



Shortcuts to adiabaticity



- AdC, J. Goold, M. Paternostro, Sci. Rep. **4**, 6208 (2014) (single-particle)
- M. Beau, J. Jaramillo, AdC, Entropy 18, 168 (2016) (many-particle)

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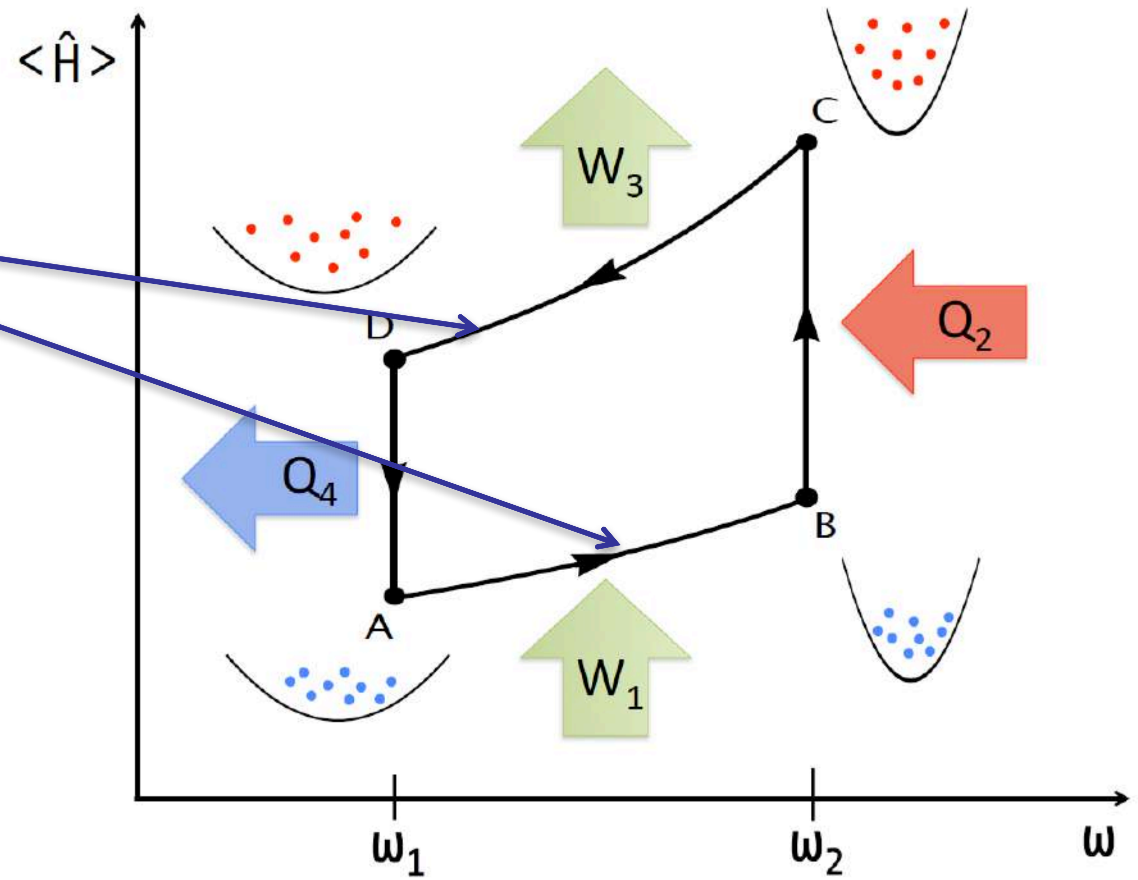
Quantum Heat Engines (e.g. Otto Cycle)

STA to expansion and compression



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Superadiabatic quantum engine

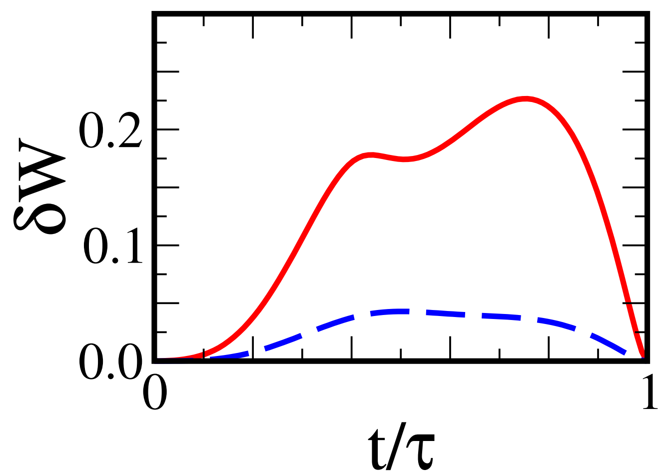
STA in unitary strokes, slower than thermalization time

Cycle at finite power and zero friction i.e., maximum efficiency

$$\mathcal{E}_{\max} = 1 - \frac{\omega(\tau)}{\omega(0)}$$

Initial state thermal

$$\delta W = \langle W \rangle - \langle W_{\rm ad} \rangle = \frac{1}{\beta_t} S(\rho_t || \rho_t^{\rm ad}), \qquad \rho_t^{\rm ad} = \sum_n p_n^0 |n(t)\rangle\langle n(t)|$$



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Scalable QHE assisted by shortcuts to adiabaticity

- **Thermodynamic cycle working at tunable finite power and zero friction**

- **Quantum speed limits impose ultimate performance bounds**


Eco-friendly Lamborghini!

- AdC, J. Goold, M. Paternostro, Sci. Rep. **4**, 6208 (2014) (single-particle)
- M. Beau, J. Jaramillo, AdC, Entropy **18**, 168 (2016) (many-particle)

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Summary

Shortcuts to adiabaticity speed up processes by tailoring excitations

- Superadiabatic expansions/compressions and other processes
- Test of counterdiabatic transport with matter-waves: max. robust STA
- Waveguides: Tailoring curvature via SUSY QM and the painting potential technique
- Quantum Thermal machines: nonadiabatic effects and STA

The UMass Group

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STA in quantum fluids

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David Guery-Odelin (Toulouse)

Chris Jarzynski (UMD)

Kihwan Kim (Tsinghua)

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Masahito Ueda (Tokyo)

Haibin Wu (ECNU)

Wojciech H Zurek (LANL)

Bent waveguides

Malcolm Boshier (LANL)

Avadh Saxena (LANL)

Quantum Thermo

John Goold (ICTP)

Mauro Paternostro (Belfast)

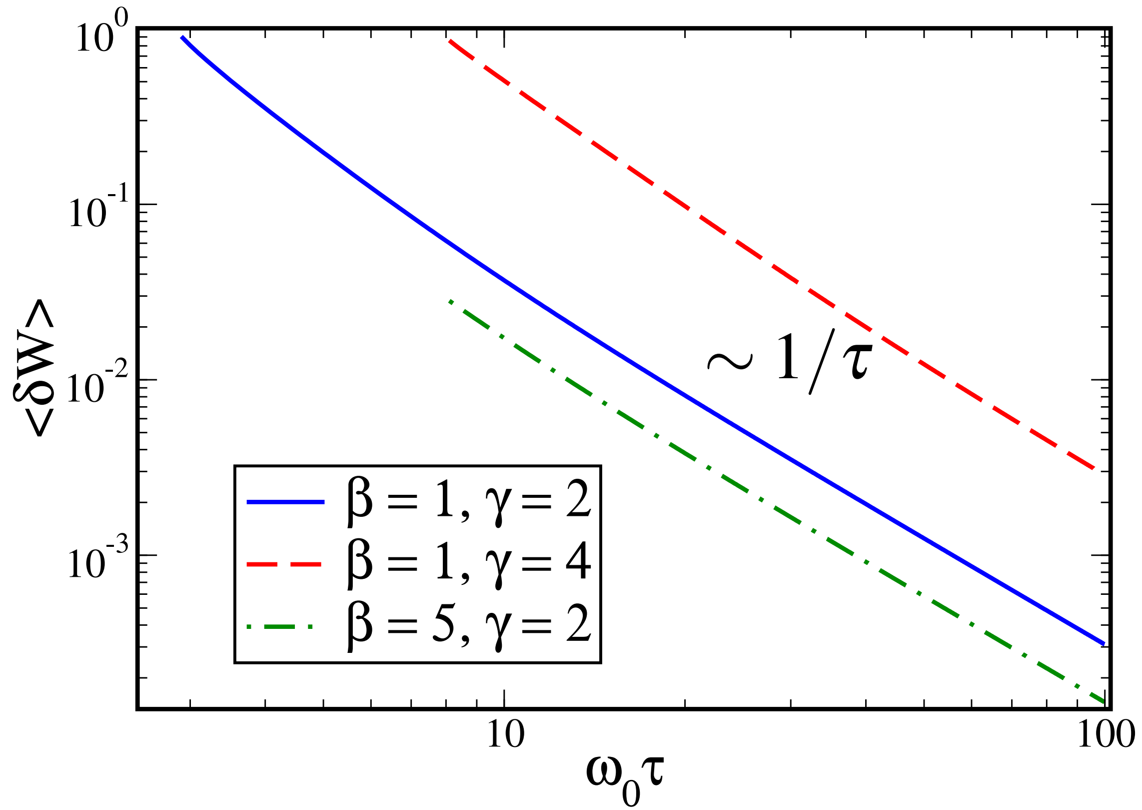
Thanks for your attention!!

Please visit

welcome MSc, PhD students & postdocs!

Energy Cost of Shortcuts to Adiabaticity

$$\delta W = \langle W \rangle - \langle W_{\rm ad} \rangle \qquad \langle \delta W \rangle = \frac{1}{\tau}\int_0^\tau \delta W dt$$



Time-energy uncertainty relation

Beautiful history

Passage time: Minimum time required for a state to reach an orthogonal state

Landau

Krylov

1945 Mandelstam and Tamm "MT"

$$\tau \geq \frac{\pi}{2}\frac{\hbar}{\Delta H}$$

1967 Fleming

1990 Anandan, Aharonov

1992 Vaidman, Ulhman

1993 Uffnik

1998 Margolus & Levitin "ML"

2000 Lloyd

2003 Giovannetti, Lloyd, Maccone: MT & ML unified

2003 Bender: no bounds in PT-symmetric QM

2009 Levitin,Toffoli

2012 2012 Bound for open (as well as unitary) system dynamics!

Time-energy uncertainty relation

2013
Taddei-Escher- Davidovich-de Matos Filho
AdC-Egusquiza-Plenio-Huelga
Deffner–Lutz

Rate of decay of the relative purity $f(t) = \frac{\mathrm{tr}\left[\rho_0 \rho_t\right]}{\mathrm{tr}(\rho_0^2)}$

Master equation $\frac{d\rho_t}{dt} = \mathscr{L}\rho_t$

Example: Markovian dynamics

$$\mathscr{L}\rho = -\frac{i}{\hbar}[H,\rho] + \sum_k \left(F_k \rho F_k^\dagger - \frac{1}{2}\left\{ F_k^\dagger F_k, \rho \right\} \right)$$

2012 MT-like bound for open (as well as unitary) system dynamics

$$f(t) = \cos\theta \qquad \tau_\theta \geq \frac{|\cos\theta - 1|\mathrm{tr}\rho_0^2}{\sqrt{\mathrm{tr}[(\mathscr{L}^\dagger \rho_0)^2]}} \geq \frac{4\theta^2 \mathrm{tr}\rho_0^2}{\pi^2 \sqrt{\mathrm{tr}[(\mathscr{L}^\dagger \rho_0)^2]}}$$

Bound to the velocity of evolution

$$v = \sqrt{\mathrm{tr}[(\mathscr{L}^\dagger \rho_0)^2]}$$

Counterdiabatic driving: applications

Counterdiabatic terms are often **nonlocal**

Search for experimentally-realizable local Unitarily equivalent Hamiltonians
(e.g. Deffner's talk)

$$\hat{H}' = U\hat{H}U^\dagger - i\hbar U\partial_t U^\dagger$$

RAP in Two level system (spin flip) $\hat{H}_1 \propto \sigma_y$ $\hat{H}_1' \propto \sigma_z$

Demirplak & Rice 2003 Bason et al 2012

Time-dependent harmonic oscillator $\hat{H}_1 \propto (xp + px)$ $\hat{H}_1' \propto x^2$

Muga el at 2010, Jarzynski 2013 Ibáñez et al 12, AdC 13

Transport of matter waves $\hat{H}_1 \propto p$ $\hat{H}_1' \propto x$

Deffner-Jarzynski-AdC 14

Theory: Demirplak & Rice 2003; M. V. Berry 2009
Experiment for TLS: Morsch's group Nature Phys. 2012; NVC: Suter's group PRL 2013

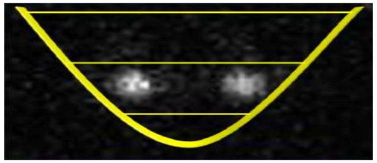
Fast-forward technique: application

For self-similar processes is equivalent to other techniques

Example: transport of ion chains/strongly correlated systems (beyond mean-field)

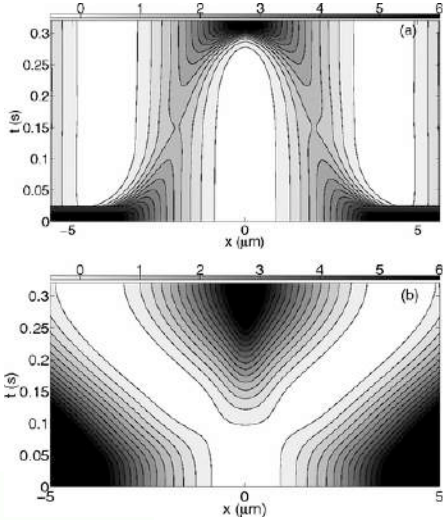
(Masuda PRA 2012)

Auxiliary potential = linear potential



"Favourite" technique for non-self similar driving of matter-waves

Matter wave splitting
(Torrontegui et al PRA 2013)



Loading an optical lattice
(Masuda, Nakamura, AdC PRL 2014)
Auxliary potential ≈ bichromatic lattice