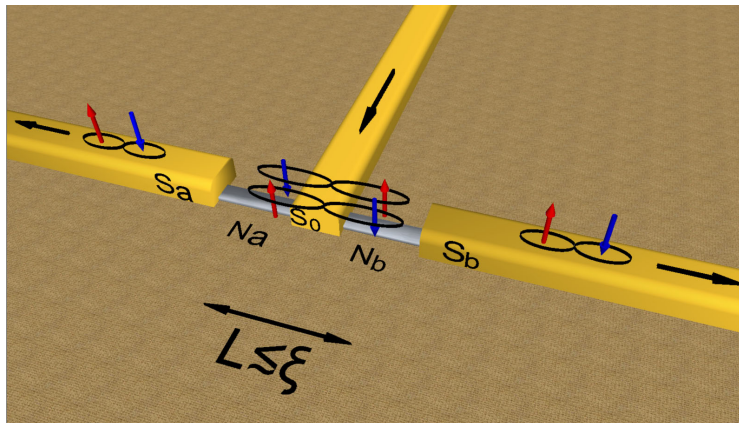


# MULTI-CONDENSATE SUPERFLUID MOTION IN JOSEPHSON JUNCTIONS



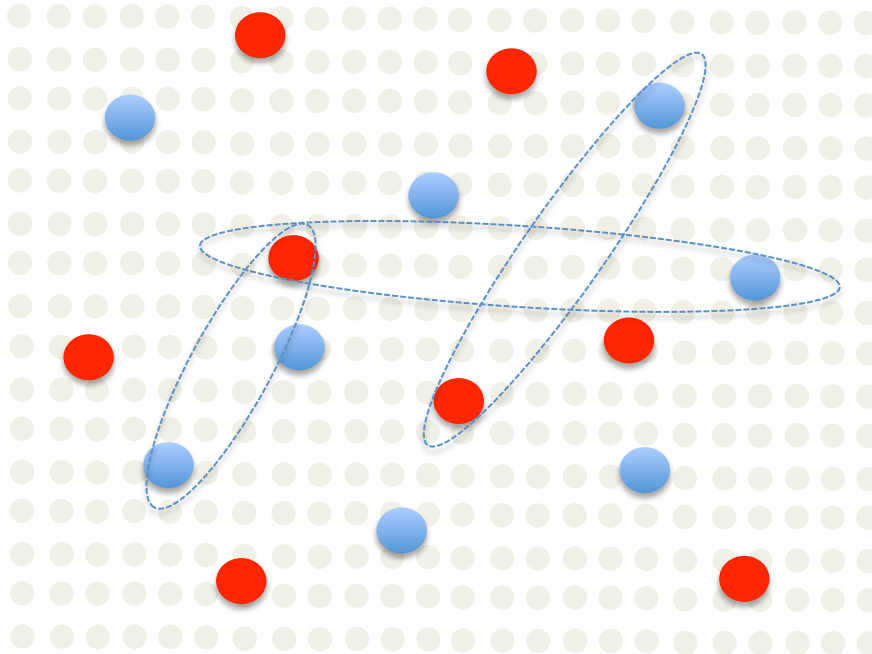
*Denis Feinberg, Régis Mélin et al.*

Institut NEEL  
CNRS and Grenoble University

## ***OUTLOOK***

1. Basics of SIS and SNS Josephson junctions
2. Multiterminal junctions
  - Equilibrium
  - Commensurate biases : quartets and more
3. Towards BEC ?

# BCS superconductors : large overlapping Cooper pairs

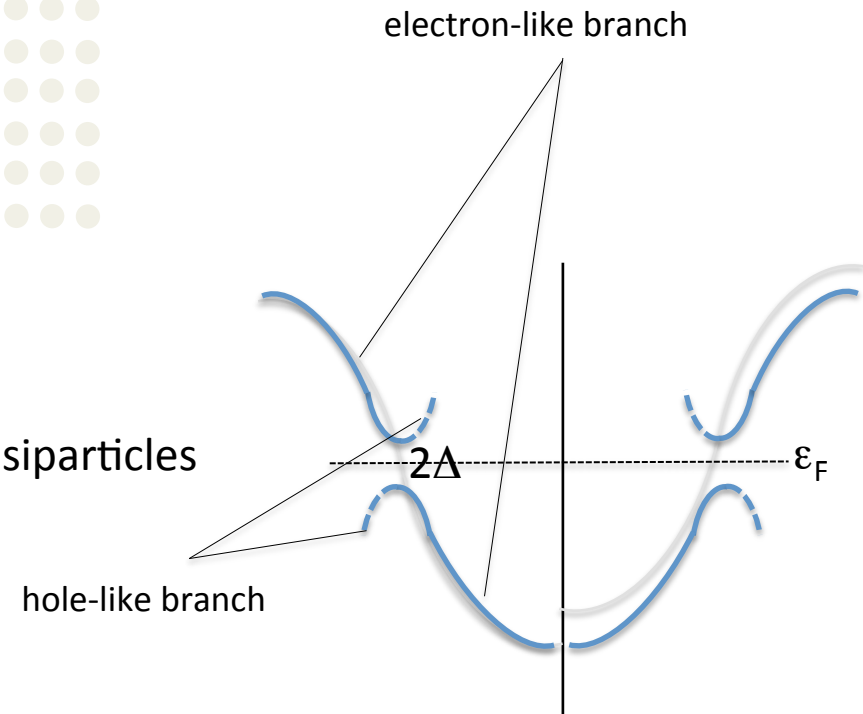


$$\xi / a_{\text{latt}} \gg 1$$

Remaining interactions negligible (Fermi liquid)

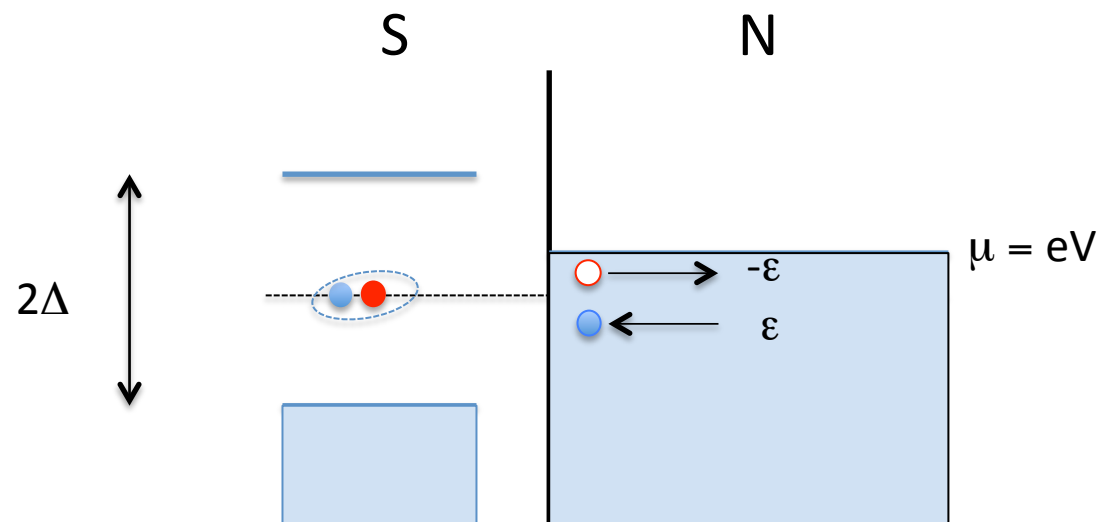
Excitation spectrum : Fermionic Bogoliubov quasiparticles

$$\gamma_{ks} = u_k c_{ks} + (-1)^s v_k c_{ks}^+$$



## Transport at a Superconductor- Normal metal contact

Andreev reflection of spin-up electrons into spin-down holes



## Josephson junction : tunnel barrier or metallic

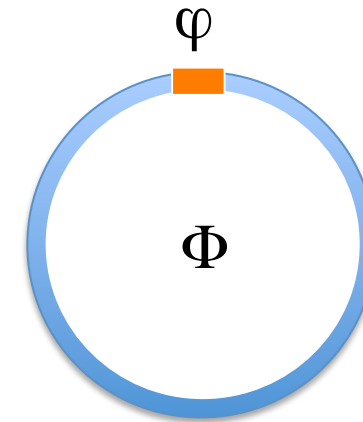
### DC Josephson effect at equilibrium

$$E = E_J (\varphi_1 - \varphi_2), \text{ period } 2\pi$$

$$I = (2e/\hbar) (dE/d\varphi), \quad \varphi = \varphi_1 - \varphi_2$$

if  $\varphi$  forced by a magnetic flux

*Equilibrium state*



$$[N, \varphi] = i$$

### Voltage biased junction

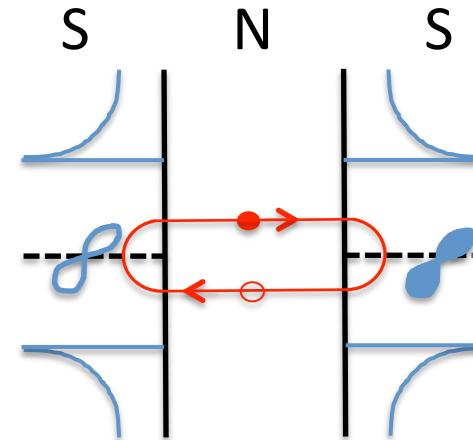
$$d\varphi/dt = 2eV / \hbar \rightarrow \varphi(t) = \varphi_0 + \omega_J t$$

$$\text{adiabatic} \rightarrow I = I_c \sin(\varphi_0 + \omega_J t)$$

## S-N-S contact

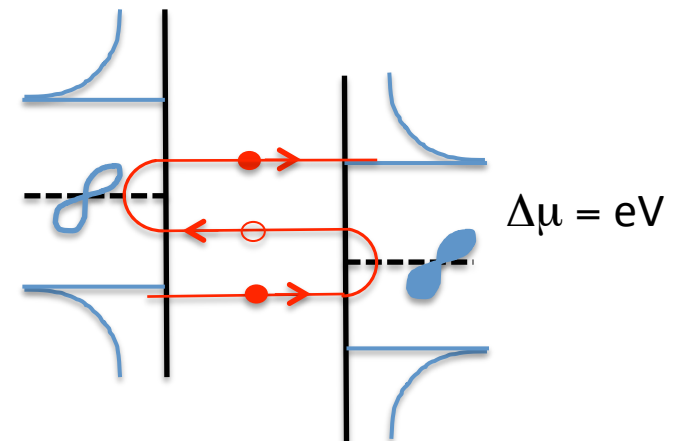
### *At equilibrium*

Andreev bound states in the gap  
(trapped Bogoliubov quasiparticles)

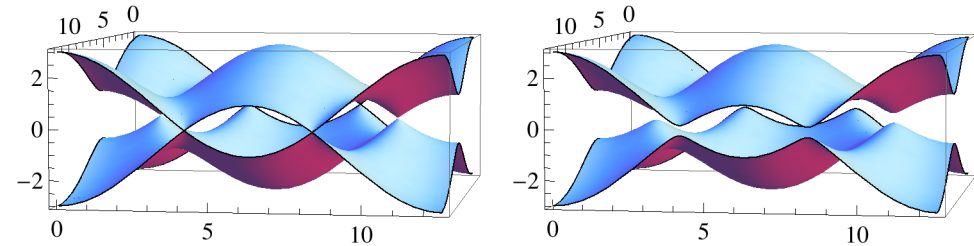
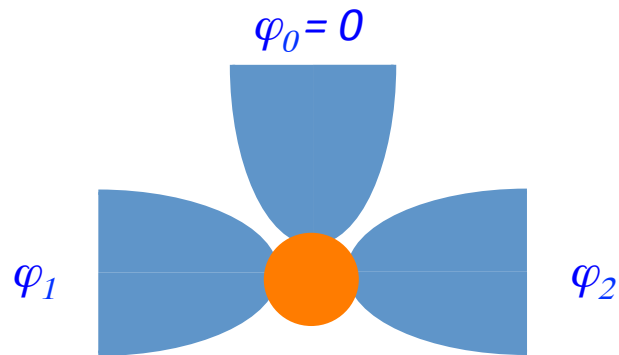


### *Voltage-biased junction*

Multiple Andreev reflections :  
subgap transport



## N – terminal junctions : equilibrium



Andreev bound state dispersion  
in two (phase) dimensions

one level in junction : gapped vs ungapped

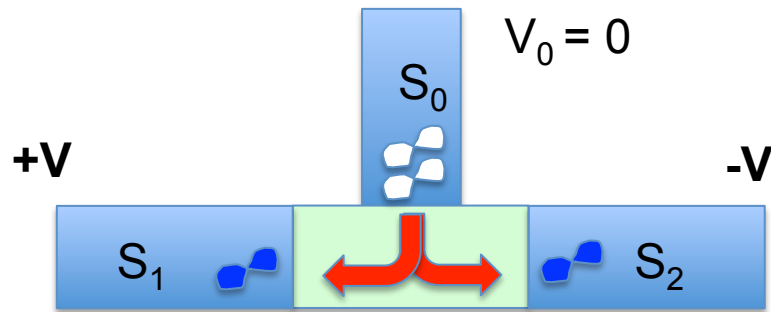
(Akhmerov et al., Padurariu et al.)

$N > 3$  terminals  $\rightarrow$  Topological properties (Riwar, Meyer, Houzet, Nazarov 2016)

A platform to simulate topological materials built in the Cooper pair number space  $|N_i\rangle$

$$[N_i, \varphi_i] = i$$

## Biased 3-terminal junctions : quartets and multipair transport



$$\varphi_1(\mathbf{t}) = \varphi_{10} + \omega_J \mathbf{t}$$

$$\varphi_2(\mathbf{t}) = \varphi_{20} - \omega_J \mathbf{t}$$

$$\omega_J = 2eV/\hbar$$

$$\varphi_1(\mathbf{t}) + \varphi_2(\mathbf{t}) = \varphi_{10} + \varphi_{20} - 2\varphi_{00} = \varphi_Q$$

**3 – body phase combination**

*Lowest harmonic, dc current*       $I_1 = I_2 = I_q \sin [\varphi_1(\mathbf{t}) + \varphi_2(\mathbf{t})] = I_q \sin \varphi_Q$

*Quartet current made of 2 correlated pairs : energy-conserving process*

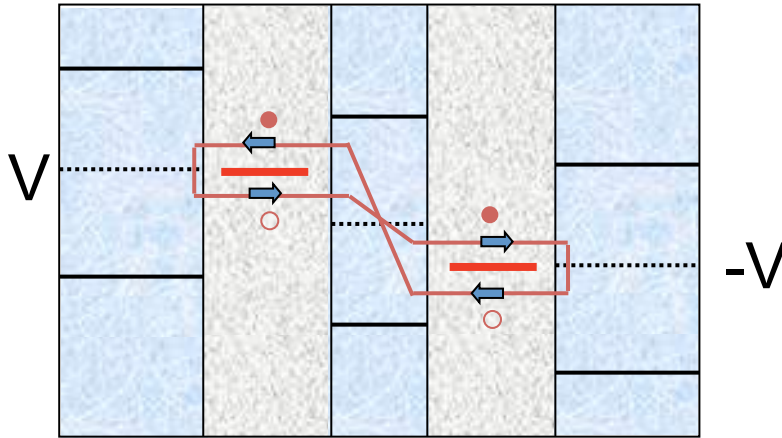
(Generalizes to higher order :       $n V_1 + m V_2 = 0 \rightarrow n+m$  pairs altogether)

*Tripartite number entanglement :*

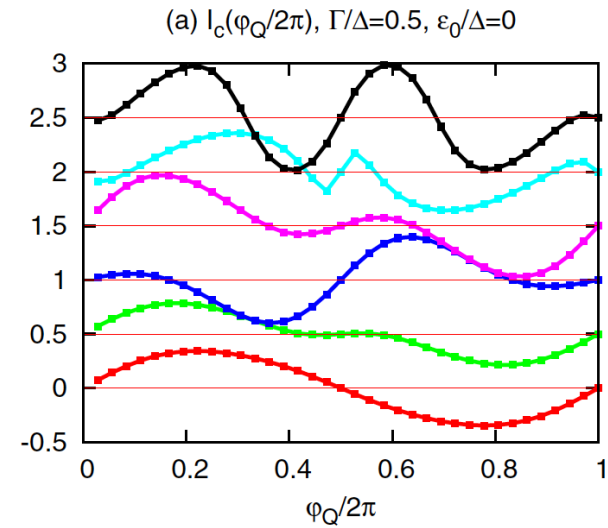
$$|\Psi\rangle = \sum A_n e^{in\varphi_Q} |N_0 - 2n\rangle_0 |N_1 + n\rangle_1 |N_2 + n\rangle_2$$



### Quartet current

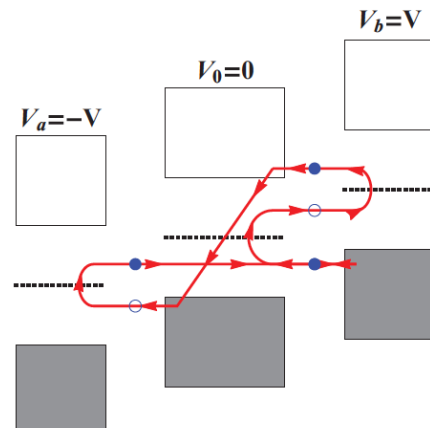


Four coherent Andreev reflections,  $0-\pi$  transition with the voltage  $V$

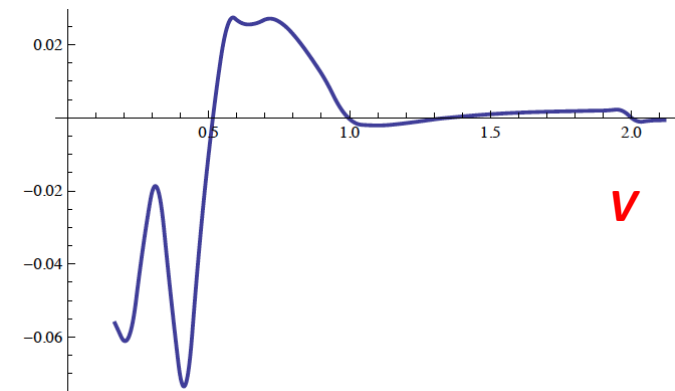


$eV/\Delta=0.60$  (red line)     $eV/\Delta=0.30$  (magenta line)  
 $eV/\Delta=0.50$  (green line)     $eV/\Delta=0.20$  (cyan line)  
 $eV/\Delta=0.40$  (blue line)     $eV/\Delta=0.10$  (black line)

### Phase-dependent multiple Andreev reflections (quasiparticle current)



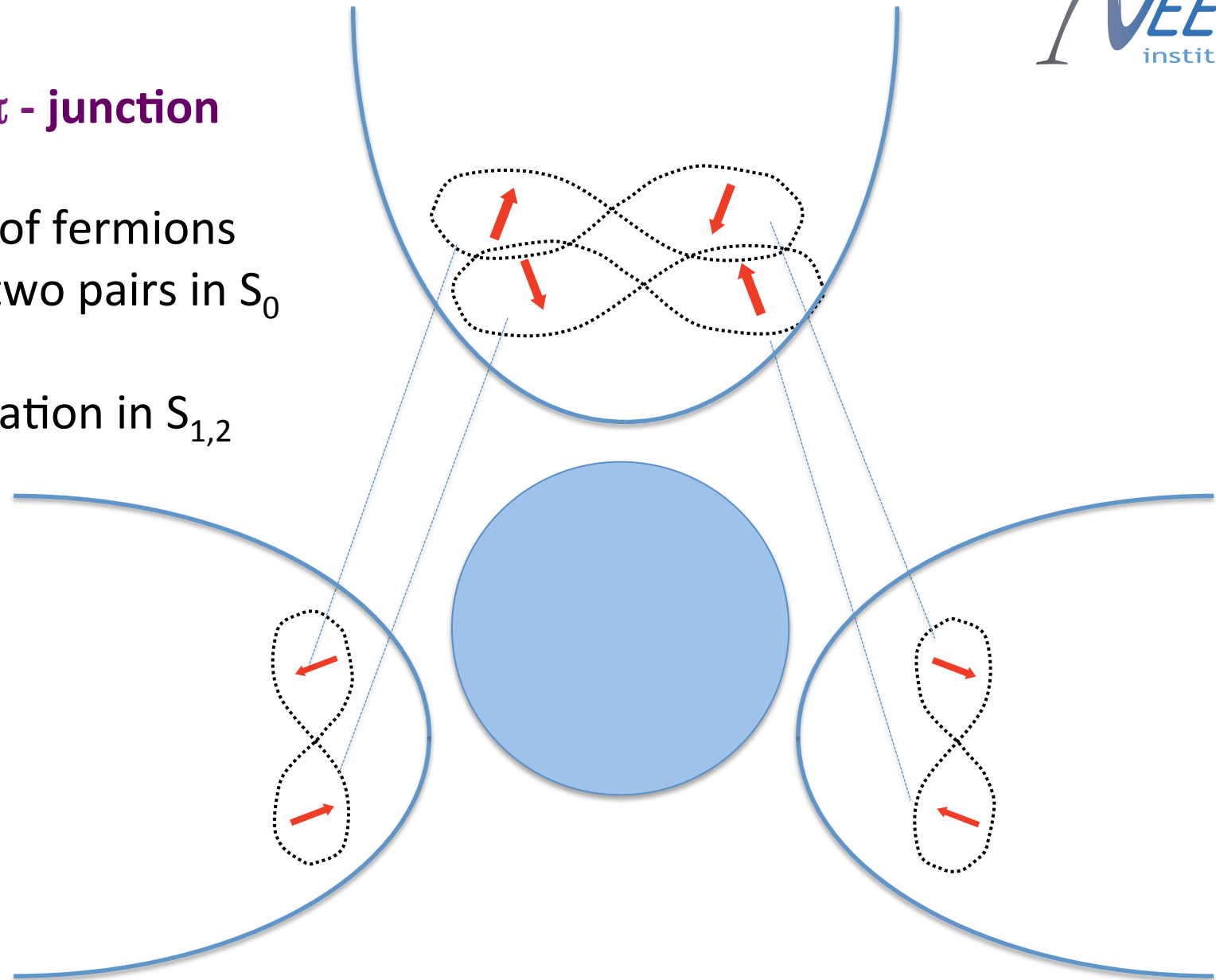
### Quartet current (fixed $\varphi_Q$ )



## Quartet $\pi$ - junction

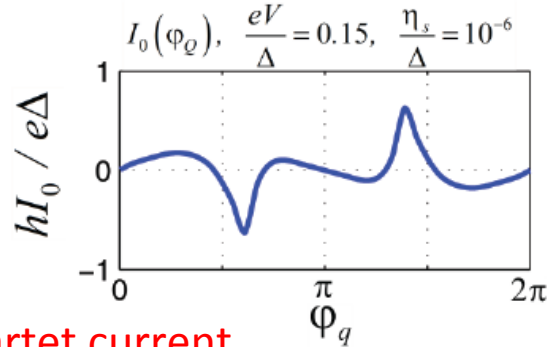
Exchange of fermions  
between two pairs in  $S_0$

Recombination in  $S_{1,2}$

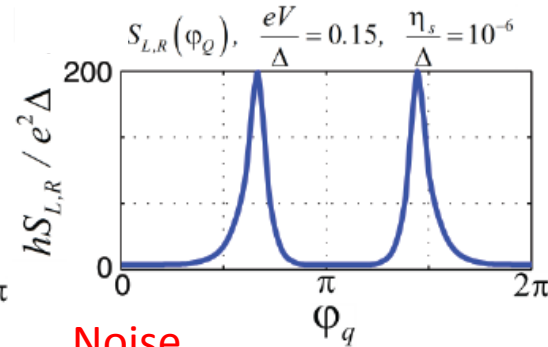


# More theory : a double dot model (R. Mélin)

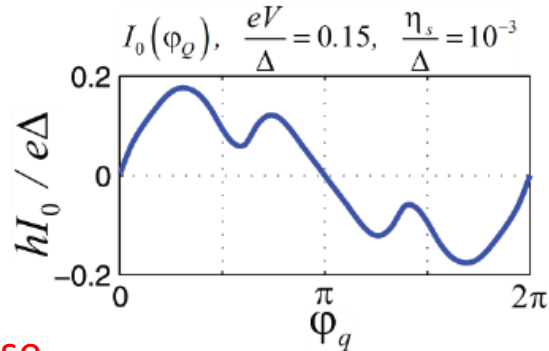
## Quartet current



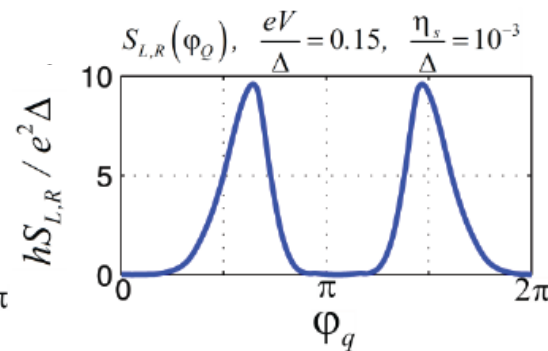
## Noise



## Quartet current

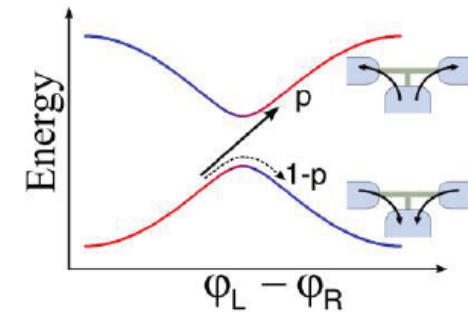
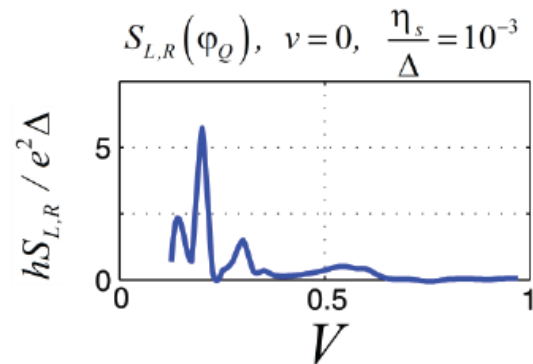


## Noise



Strong dependence on the Dynes parameter !

## Noise

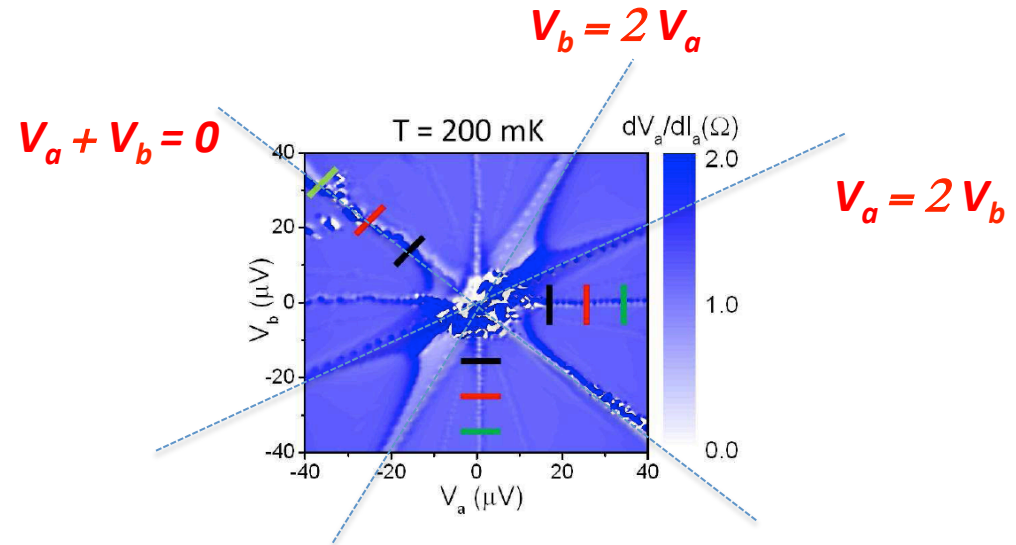
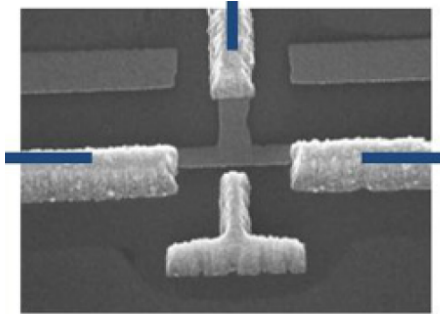


Nonadiabatic transitions between Andreev quasi-bound states

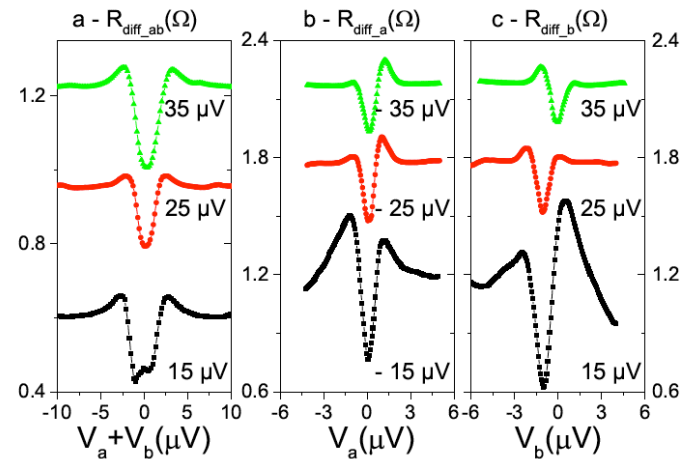
**EXPERIMENT 1** : Pfeffer, Lefloch, Duvauchelle, Courtois, PRB 2014 (Grenoble)

*Al-Cu diffusive long junctions*

$$E_{Th} < eV \ll \Delta$$



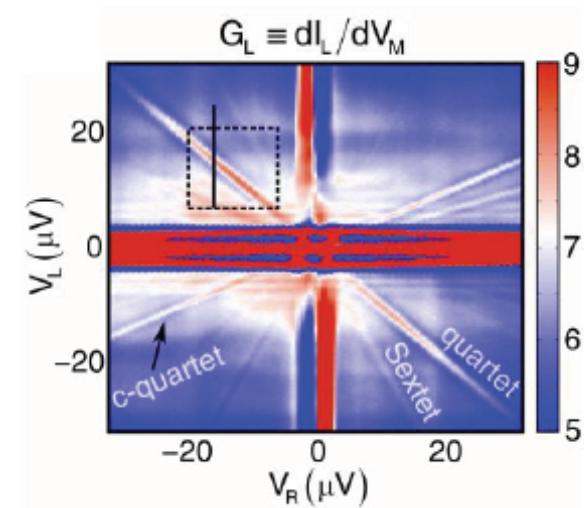
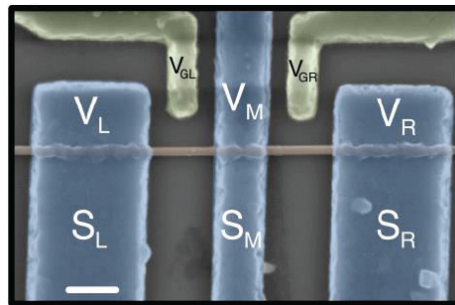
*Quartet dc signatures*



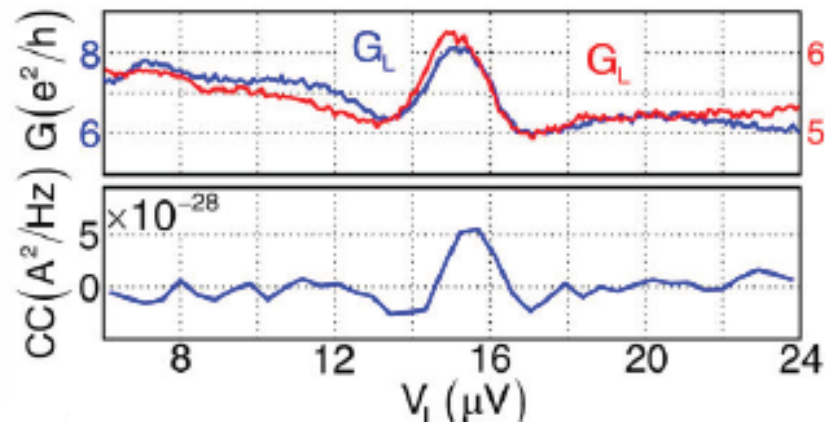
## EXPERIMENT 2 : Cohen, Ronen, Heiblum, Shtrikman 2016 (Weizmann)

Al - InSb nanowires

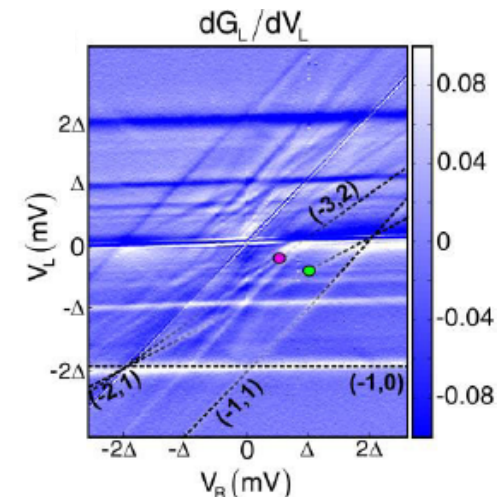
$$eV \ll \Delta < E_{th}$$



Large and positive cross-correlation noise



Nonlocal MAR



## Towards BEC ... (prospective part)

### An infinite gap model :

Single dot level, non-interacting  
 Equivalent to hard-core bosons  
 states  $|0\rangle$ ,  $|\uparrow\downarrow\rangle$

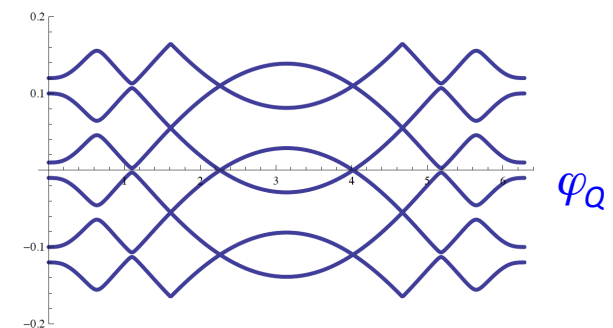
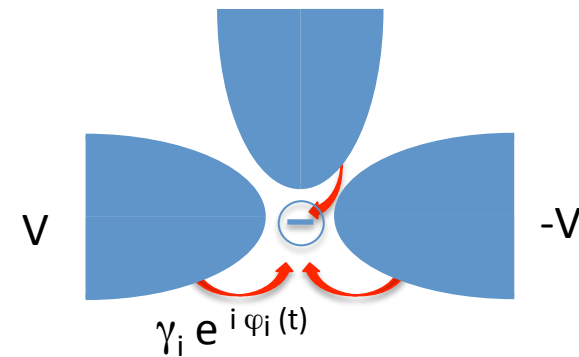
mean field, no interactions in the reservoirs

$$H = \varepsilon_0 b^\dagger b + \sum_i \gamma_i e^{i\varphi_i(t)} + \text{h. c.}$$

Floquet solution  $E_{Fl, n\pm}(\varphi_Q)$

Current in state  $|0, -\rangle$  : generalization of  
 the Josephson relation

$$\langle I \rangle = (2e/\hbar) d E_{Fl} / d \varphi_Q$$



## *Multi-BEC condensates : some directions*

### - Multi-boson superfluidity in tri-condensates

Fermionic BEC : analogous to BCS superconductors ?

Bosonic BEC : what ingredient to correlate 2 bosonic flows ?

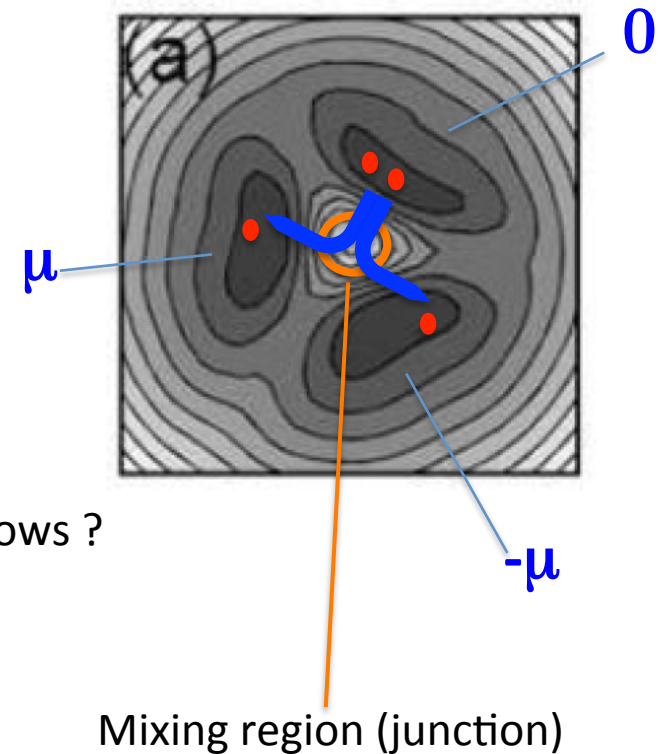
Role of interactions (in the reservoirs, in the junction..) ?

Detection of 3-body phase coherence

### - Simulation of topological lattices by multi-condensates

Realization of edge states ?

Detection of topological transitions



R. Mélin (Grenoble, NEEL)  
A. Freyn (ex Grenoble, NEEL)  
C. Padurariu (ex Grenoble, NEEL)  
M. Sotto (ex Grenoble, NEEL)

B. Douçot (Paris, LPTHE)

T. Jonckheere (Marseille, CPT)  
J. Rech (Marseille, CPT)  
T. Martin (Marseille, CPT)

J.G. Caputo (Rouen)

F. Lefloch (Grenoble, INAC)  
A. Pfeffer (ex Grenoble, INAC)  
J. E. Duvauchelle (ex Grenoble, INAC)

H. Courtois (Grenoble, NEEL)

Yu. Ronen (Rehovot, Weizmann)  
Y. Cohen (Rehovot, Weizmann)  
M. Heiblum (Rehovot, Weizmann)  
H. Shtrikman (Rehovot, Weizmann)  
J-H. Kang (Rehovot, Weizmann)

*Freyn et al. PRL 2011*

*Jonckheere et al. PRB 2013*

*Mélin et al. PRB 2016*