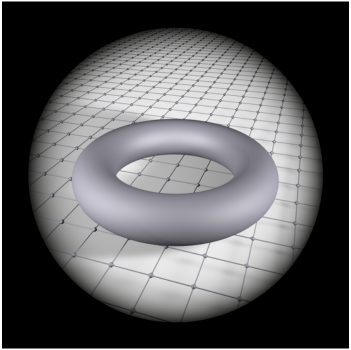
Probing topology by "heating"

Nathan Goldman



Atomtronics, Benasque, May 9th 2017

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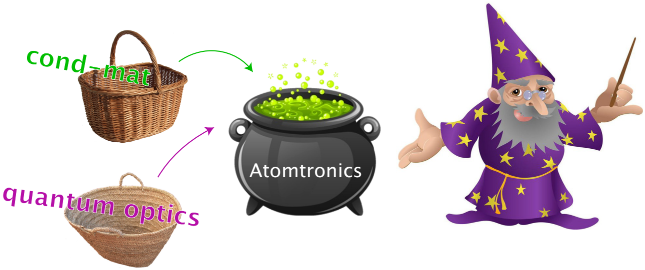

"See cond-mat physics from a different perspective"

"Probe new effects not accessible in solid-state"

In this talk: topological states of matter with cold atoms

"*See* **topological** *physics from a different perspective*"

"*Probe* **topological** *effects not accessible in solid-state*"

In this talk: topological states of matter with cold atoms

"*See* **topological** *physics from a different perspective*"

"*Probe* **topological** *effects not accessible in solid-state*"

For a recent review on the topic:

nature physics

FOCUS | PROGRESS ARTICLE

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Topological quantum matter with ultracold gases in optical lattices

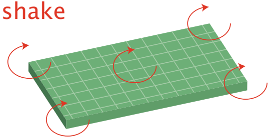
N. Goldman[1*], J. C. Budich[2,3] and P. Zoller[2,3]

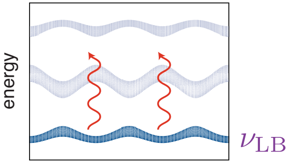
Since the discovery of topological insulators, many topological phases have been predicted and realized in a range of different systems, providing both fascinating physics and exciting opportunities for devices. And although new materials are being developed and explored all the time, the prospects for probing exotic topological phases would be greatly enhanced if they could be realized in systems that were easily tuned. The flexibility offered by ultracold atoms could provide such a platform. Here, we review the tools available for creating topological states using ultracold atoms in optical lattices, give an overview of the theoretical and experimental advances and provide an outlook towards realizing strongly correlated topological phases.

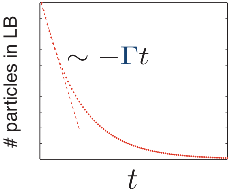
Since the discovery of the quantum Hall (QH) effect[1] and topological insulators[2,3], intense effort has been devoted to the exploration of novel topological phases of matter. This quest is driven by the prediction of fundamentally new physical phenomena, some of which envisage potential applications[2–5]. Topological phases of matter elude the conventional Landau

The main interface to control atoms through light–matter interaction is the optical dipole potential[8,9], which can be generated by subjecting atoms to laser fields. It can be expressed as $V(\boldsymbol{x}) = \alpha|\boldsymbol{E}(\boldsymbol{x})|^2$, where $\boldsymbol{E}(\boldsymbol{x})$ denotes the electric field associated with the lasers, α is the polarizability, which typically depends on the laser frequency and $\boldsymbol{x}$ denotes the position vector. By

This talk: Probing topology by "heating" ?







Signature of topology in the **Depletion rate** of a Bloch band ?

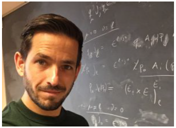
Probing topology by "heating"

In collaboration with

Duc Thanh Tran (ULB)

Alexandre Dauphin (ICFO)

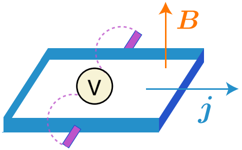
Adolfo Grushin (Berkeley)



Peter Zoller (IQOQI)

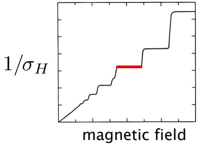
“*back to the basics...*”

The quantum Hall effects (1985 / 1998)



quantized Hall conductance

$$\sigma_H = \nu \times (e^2/h)$$



von Klitzing et al. **PRL 1980**

The Nobel Prize in Physics 2016

© Trinity Hall, Cambridge University. Photo: Kiloran Howard

David J. Thouless

Prize share: 1/2

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F. Duncan M. Haldane

Prize share: 1/4

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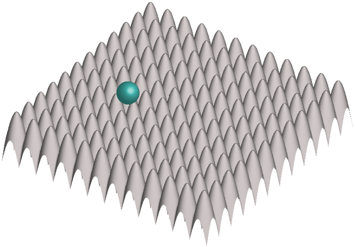
J. Michael Kosterlitz

Prize share: 1/4

ν is a **topological invariant** : a **Chern number**

Thouless et al. (TKNN) **PRL 1982**

- Consider a particle moving on a two-dimensional lattice:

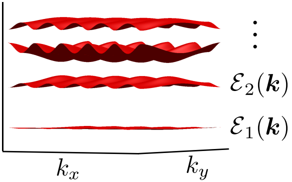


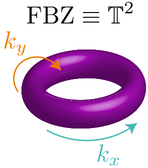
- The eigenfunctions are Bloch waves

$$\psi_{n,\boldsymbol{k}}(\boldsymbol{r}) = e^{i\boldsymbol{r}\cdot\boldsymbol{k}} u_{n,\boldsymbol{k}}(\boldsymbol{r})$$

- The eigenenergies are Bloch bands

$$\hat{H}_{\boldsymbol{k}}\, u_{n,\boldsymbol{k}} = \mathcal{E}_n(\boldsymbol{k})\, u_{n,\boldsymbol{k}}$$



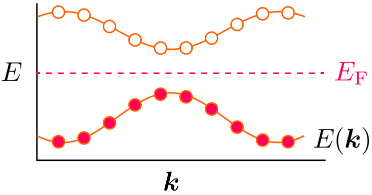


- The **Berry curvature** of the n**th band**: $\Omega_n = \Omega_n^{xy} \mathrm{d}k_x \wedge \mathrm{d}k_y$

$$\Omega_n^{xy} = i\left[\langle \partial_{k_x} u_n | \partial_{k_y} u_n \rangle - \langle \partial_{k_y} u_n | \partial_{k_x} u_n \rangle\right]$$

Topology of the nth Bloch band: $\boxed{\frac{1}{2\pi}\int_{\mathbb{T}^2} \Omega_n = \nu_n \in \mathbb{Z}}$ **: Chern number of the band**

- **Filled Bloch band + electric field** E_y



Equations of motion (semiclassics)

$$v_n^x(\boldsymbol{k}) = \frac{\partial E(\boldsymbol{k})}{\hbar \partial k_x} - \frac{eE_y}{\hbar}\Omega^{xy}(\boldsymbol{k})$$

$$v_n^y(\boldsymbol{k}) = \frac{\partial E(\boldsymbol{k})}{\hbar \partial k_y}$$

Karplus & Luttinger 1954

$$j_x = \sigma_{xy} E_y \longrightarrow \boxed{\sigma_\mathrm{H} = \frac{e^2}{h}\,\nu_1}$$

TKNN formula 

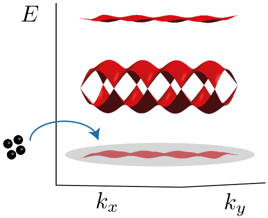
Topological responses are universal : They appear in band systems with **non-trivial topology**

$\longrightarrow$ **topological physics in a wide range of physical settings (cold atoms, photonics , ...)**

Reviews: *Focus on Topological matter* in **Nature Physics '16**

Atomtronics: solid-state versus cold-atom measurements

- **Solid-state physics**: filling a band with electrons



electric field

$$+\boldsymbol{E} = E_y \mathbf{1}_y$$

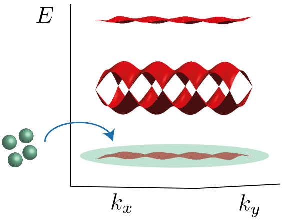
Transport equation for the **current density**

$$\boxed{j^x = \frac{e^2}{h} \nu \; E_y}$$

(**TKNN** formula)

$\longrightarrow$ quantized Hall conductivity (IQHE)

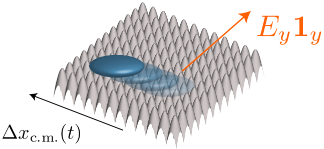
- **Ultracold atoms in optical lattices**: an analogy



linear gradient

$$+\boldsymbol{E} = E_y \mathbf{1}_y$$

Center-of-mass imaged in-situ



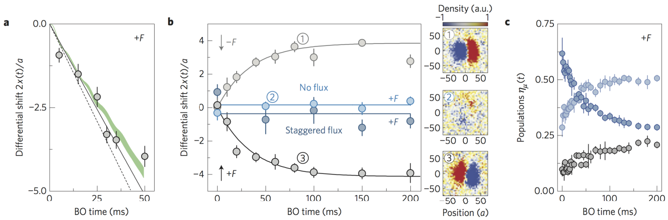
$$\boxed{\boldsymbol{v}_{\mathrm{c.m.}} = \frac{\boldsymbol{j}(\nu)}{n} = \boldsymbol{j}(\nu) V_{\mathrm{cell}}}$$

Ref: Dauphin and Goldman **PRL '13**

LETTERS
PUBLISHED ONLINE: 8 DECEMBER 2014 | DOI: 10.1038/NPHYS3171
nature physics

Measuring the Chern number of Hofstadter bands with ultracold bosonic atoms

M. Aidelsburger[1,2*], M. Lohse[1,2], C. Schweizer[1,2], M. Atala[1,2], J. T. Barreiro[1,2†], S. Nascimbène[3], N. R. Cooper[4], I. Bloch[1,2] and N. Goldman[3,5]



By fitting this equation to the experimental data $x(t)$, with the Chern number being the only fit parameter, we obtain an experimental value for the Chern number of the lowest band

$$\boxed{\nu_{\rm exp} = 0.99(5)}$$

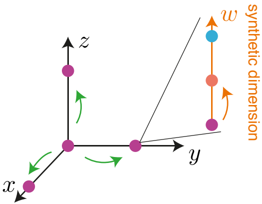
A small gallery of recent (related) works from Brussels

Measurement of Chern numbers through center-of-mass responses
H. M. Price, O. Zilberberg, T. Ozawa, I. Carusotto and N. Goldman,
Phys. Rev. B 93, 245113 (2016)

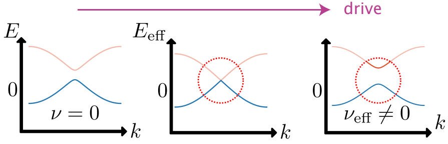
$$\boldsymbol{v}_{\text{c.m.}} = \frac{\boldsymbol{j}(\nu)}{n(\nu)} \xrightarrow{B \neq 0} v^x_{\text{c.m.}} \approx \frac{2\pi}{A_{\text{MBZ}}} E_y \nu - \left(\frac{2\pi}{A_{\text{MBZ}}}\right)^2 E_y B\,(\nu)^2$$

Four-Dimensional Quantum Hall Effect with Ultracold Atoms
H. M. Price, O. Zilberberg, T. Ozawa, I. Carusotto and N. Goldman,
Phys. Rev. Lett. 115, 195303 (2015)

$$v^\mu_{\text{c.m.}} \sim \frac{\nu_2}{4\pi^2} \varepsilon^{\mu\alpha\beta\nu} E_\nu B_{\alpha\beta}, \quad \nu_2 = \frac{1}{4\pi^2} \int_{\mathbb{T}^4} \Omega^{zx} \Omega^{yw} \mathrm{d}^4 k$$



Loading Ultracold Gases in Topological Floquet Bands: The Fate of Current and Center-of-Mass Responses
A. Dauphin, D. T. Tran, M. Lewenstein and N. Goldman,
2D Materials (accepted manuscript 2017)



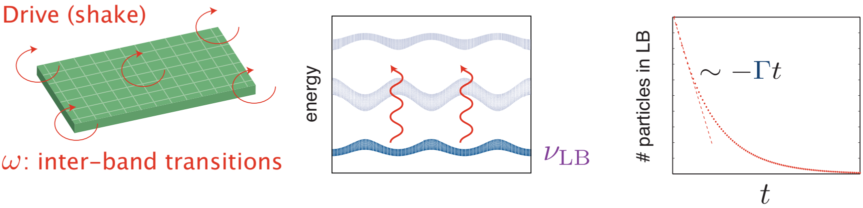
see also Hu, Zoller, Budich PRL '16

Current response is affected by:
(1) inter-band interferences, (2) micro-motion (drive)

$$\Omega_{\text{drive}} \gg \omega_{\text{inter-band}} \gg \omega_{\text{Bloch}}$$

COM displacement: more robust (time-integration)

This talk: Probing topology by “heating” ?



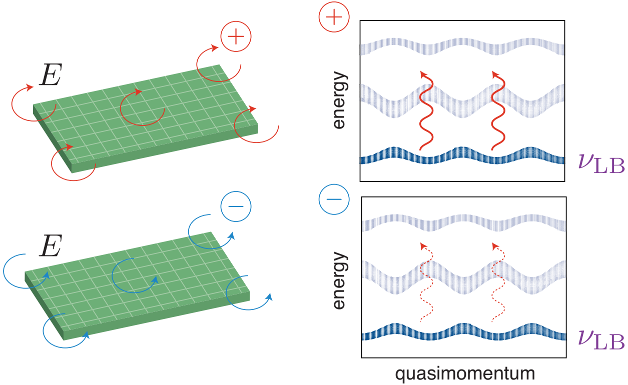
Depletion rate of a Bloch band: topological signature?

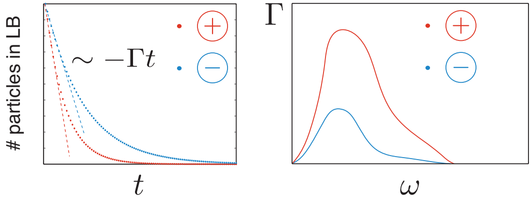
Fundamentally distinct from TKNN paradigm:

Depletion rate reflects the **non-linear inter-band response** to a **time-dependent perturbation**

(TKNN: linear response to constant field, single-band effect)

Summary: Depletion rate, chirality and topology



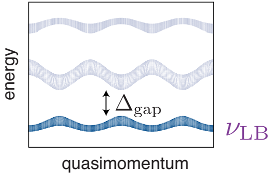


Differential integrated rate (DIR)

$$\boxed{\int d\omega \, (\Gamma_+ - \Gamma_-) \sim \nu_{\mathrm{LB}} E^2}$$

A time-modulated 2D Chern insulator

- Consider a filled Bloch band of Chern number ν_{LB}, separated by a gap Δ_{gap}



- Subjecting the system to a constant "electric" field $\boldsymbol{E} = E_y \mathbf{1}_y \rightarrow \sigma_{\mathrm{H}} \sim \nu_{\mathrm{LB}}$ (TKNN)

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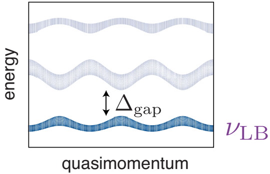

- Subjecting the system to a constant "electric" field $\boldsymbol{E} = E_y \mathbf{1}_y \rightarrow \sigma_{\mathrm{H}} \sim \nu_{\mathrm{LB}}$ (TKNN)

- **Here** : We study the response against a **circular time-dependent perturbation**

$\hat{H}_{\pm}(t) = \hat{H}_0 + 2E\,[\cos(\omega t)\hat{x} \pm \sin(\omega t)\hat{y}]\,,\quad \pm :$ drive orientation/chirality, ω : inter-band

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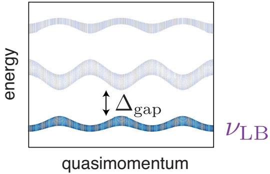

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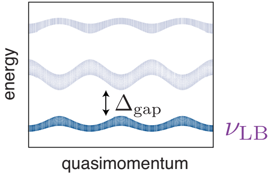

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- The **depletion rate** $\Gamma_{\pm}$ can be evaluated through Fermi's Golden Rule (FGR) :

  $$\Gamma_{\pm}(\omega) = \frac{2\pi}{\hbar} E^2 \sum_{e \notin \text{LB}} \sum_{g \in \text{LB}} |\langle e|\hat{x} \pm i\hat{y}|g\rangle|^2 \delta^{(t)}(\varepsilon_e - \varepsilon_g - \hbar\omega), \qquad E: \text{ perturbation}$$

  g : initially occupied states (LB), $\quad e$: initial. unoccupied states (higher bands)

  $\delta^{(t)}(\omega) = (2/\pi t)\sin^2(\omega t/2)/\omega^2 \longrightarrow \delta(\omega)$: "long-time" limit

Simple derivation of main result

- Starting point : the total time-dependent Hamiltonian

$$\hat{H}_{\pm}(t) = \hat{H}_0 + 2E\ [\cos(\omega t)\hat{x} \pm \sin(\omega t)\hat{y}]$$

- We perform a frame/gauge transformation (to recover translational symmetry)

$$\hat{R}_{\pm} = \exp\left\{ i\frac{2E}{\hbar\omega}\left[\sin(\omega t)\hat{x} \mp \cos(\omega t)\hat{y}\right]\right\}, \quad \hat{\mathcal{H}}(t) = \hat{R}\hat{H}(t)\hat{R}^{\dagger} - i\hbar\hat{R}\partial_t\hat{R}^{\dagger}$$

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- To lowest order in E, the momentum-representation Hamiltonian reads :

$$\hat{\mathcal{H}}_{\pm}(\boldsymbol{k},t) \approx \hat{H}_0(\boldsymbol{k}) + \frac{2E}{\hbar\omega}\left\{ \sin(\omega t)\frac{\partial \hat{H}_0(\boldsymbol{k})}{\partial k_x} \mp \cos(\omega t)\frac{\partial \hat{H}_0(\boldsymbol{k})}{\partial k_y} \right\}$$

- Writing the FGR in this frame :

$$\Gamma_{\pm}(\omega) = \sum_{\boldsymbol{k}} \Gamma_{\pm}(\boldsymbol{k};\omega),$$

$$\Gamma_{\pm}(\boldsymbol{k};\omega) = \frac{2\pi E^2}{\hbar^3\omega^2} \sum_{n>0} |\langle n| \frac{1}{i}\frac{\partial \hat{H}_0}{\partial k_x} \mp \frac{\partial \hat{H}_0}{\partial k_y} |0\rangle|^2 \delta^{(t)}(\varepsilon_n(\boldsymbol{k}) - \varepsilon_0(\boldsymbol{k}) - \hbar\omega).$$

where $|g\rangle \equiv |0(\boldsymbol{k})\rangle$ and $|e\rangle \equiv |n(\boldsymbol{k})\rangle$ (n : band index) ;

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where $|g\rangle \equiv |0(\boldsymbol{k})\rangle$ and $|e\rangle \equiv |n(\boldsymbol{k})\rangle$ (n : band index) ;

- Let us introduce the **differential rate** $\Delta\Gamma = (\Gamma_+ - \Gamma_-)/2 = \sum_{\boldsymbol{k}} \Delta\Gamma(\boldsymbol{k})$

$$\Delta\Gamma(\boldsymbol{k}) = \frac{\Gamma_+(\boldsymbol{k};\omega) - \Gamma_-(\boldsymbol{k};\omega)}{2} = \frac{4\pi E^2}{\hbar^3} \sum_{n>0} \text{Im}\, \frac{\langle 0|\partial_{k_x}\hat{H}_0|n\rangle\langle n|\partial_{k_y}\hat{H}_0|0\rangle}{(\varepsilon_0 - \varepsilon_n)^2} \delta(\varepsilon_n(\boldsymbol{k}) - \varepsilon_0(\boldsymbol{k}) - \hbar\omega)$$

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- Compare with the Berry curvature :

$$\Omega_{xy}(\boldsymbol{k}) = 2 \sum_{n>0} \mathrm{Im} \frac{\langle 0|\partial_{k_x} \hat{H}_0|n\rangle\langle n|\partial_{k_y} \hat{H}_0|0\rangle}{(\varepsilon_0 - \varepsilon_n)^2}$$

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- For a 2-band model $(n=1)$: the local differential rate probes the LB's geometry

  $\Delta\Gamma(\boldsymbol{k}) \sim \Omega_{xy}(\boldsymbol{k})\delta^{(t)}(\varepsilon_1(\boldsymbol{k}) - \varepsilon_0(\boldsymbol{k}) - \hbar\omega) \rightarrow$ measurement from wave-packets

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- More generally (N-band models) : Integrating over drive frequency (*activate all inter-band transitions*) $\rightarrow$ "**differential integrated rate (DIR)**"

$$\Delta\Gamma^{\text{int}} = \frac{4\pi E^2}{\hbar^2} \sum_{\boldsymbol{k}} \sum_{n>0} \text{Im} \frac{\langle 0|\partial_{k_x} \hat{H}_0|n\rangle\langle n|\partial_{k_y} \hat{H}_0|0\rangle}{(\varepsilon_0 - \varepsilon_n)^2}$$

- Compare with the Chern number

$$\nu_{\text{LB}} = \frac{2\pi}{A_{\text{syst}}} \sum_{\boldsymbol{k}} \Omega_{xy}(\boldsymbol{k}) = \frac{4\pi}{A_{\text{syst}}} \text{Im} \sum_{\boldsymbol{k}} \sum_{n>0} \frac{\langle 0|\partial_{k_x} \hat{H}_0|n\rangle\langle n|\partial_{k_y} \hat{H}_0|0\rangle}{(\varepsilon_0 - \varepsilon_n)^2}$$

The main result

- One obtains a simple quantization law for the **differential integrated rate (DIR)**

$$\Delta\Gamma^{\text{int}}/A_{\text{syst}} = \eta_0 E^2, \qquad \eta_0 = (1/\hbar^2)\,\nu_{\text{LB}} \tag{1}$$

$\longrightarrow$ *Activating all transitions reveals the LB's topology !*

- Compare with TKNN (quantization of the Hall conductivity)

$$J^x/A_{\text{syst}} = \sigma_{\text{H}} E, \qquad \sigma_{\text{H}} = (e^2/h)\,\nu_{\text{LB}}$$

- The quantized response associated with the DIR : the main differences
  - The response (1) is **non-linear** with respect to the driving field E
  - The response (1) involves **inter-band** (dissip.) processes ($\neq$ single-band semiclassics)
  - The response (1) **probes the chirality** (through differential measurement)

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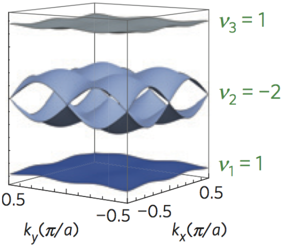
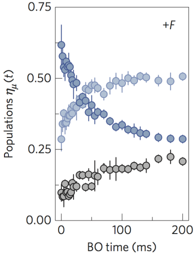
- Measuring the DIR in practice : Sample the frequency $\omega \to \omega_l \in [\Delta_{\text{gap}}, W_{\text{bandwidth}}]$

$\Delta\Gamma^{\text{int}} = \sum_l \left[\Gamma_+(\omega_l) - \Gamma_-(\omega_l)\right] \Delta_\omega/2 \to$ validated numerically (Haldane model) quantum optics

"*Measuring band populations*
is possible using band mapping"





depletion rate $\Gamma(\omega)$

Aidelsburger et al. Nature Phys '15

The crucial role of boundaries

- All previous calculations assumed **periodic boundary conditions** (no edges)
- Edges can be treated within a real-space approach

$$\Gamma_\pm(\omega) = (2\pi/\hbar)E^2 \sum_{e\notin \mathsf{LB}} \sum_{g\in \mathsf{LB}} |\langle e|\hat{x} \pm i\hat{y}|g\rangle|^2 \delta^{(t)}(\varepsilon_e - \varepsilon_g - \hbar\omega)$$

$g:$ initially occupied states (LB), $e:$ initial. unoccupied states (higher bands/edge)

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where $\hat{P} = \sum_g |g\rangle\langle g| = 1 - \hat{Q}$.

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- For open BC : Tr $\hat{\mathfrak{C}} = 0$ ... (technical : trivial fibre bundle ; see Resta, Vanderbilt)

The crucial role of boundaries

- Summary : $\Delta\Gamma^{\text{int}} = (E/\hbar)^2 \,\text{Tr}\,\hat{\mathfrak{C}}$ and $\text{Tr}\,\hat{\mathfrak{C}} = 0$ (OBC)
- Insight : we can perform the trace over the position-basis $\{|\boldsymbol{r}_j\rangle\}$ and write

$$\Delta\Gamma^{\text{int}}_{\text{OBC}} = (E/\hbar)^2 \left\{ \sum_{\boldsymbol{r}_j \in \text{bulk}} C(\boldsymbol{r}_j) + \sum_{\boldsymbol{r}_j \in \text{edge}} C(\boldsymbol{r}_j) \right\}, \quad C(\boldsymbol{r}_j) = \langle \boldsymbol{r}_j | \hat{\mathfrak{C}} | \boldsymbol{r}_j \rangle$$

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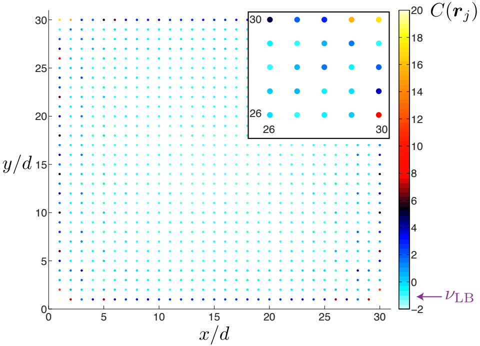
- Illustration on the Haldane model :



- In the bulk : $C(\boldsymbol{r}_j) \approx \nu_{\text{LB}} \longrightarrow (1/A_{\text{syst}}) \sum_{\boldsymbol{r}_j \in \text{bulk}} C(\boldsymbol{r}_j) = \nu_{\text{LB}}$
- The edge exactly compensates the bulk contribution (even for large systems !).

Methods to get rid of the edge-state contribution

- Local measurement in the bulk (R) ? Does **not lead** to a quantized response !

$$\nu_{\mathsf{R}} = (4\pi/A_R)\mathrm{Tr}\ \mathrm{Im}\left(\hat{R}\hat{P}\hat{x}\hat{Q}\hat{y}\hat{P}\right) \approx \nu_{\mathsf{LB}} \quad \text{but} \quad \nu_{\mathsf{R}} \neq (4\pi/A_R)\mathrm{Tr}\ \mathrm{Im}\left(\hat{P}\hat{R}\hat{x}\hat{Q}\hat{R}\hat{y}\hat{P}\right)$$

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  Validated numerically (OK on large systems) ; but **requires band-structure knowledge** !

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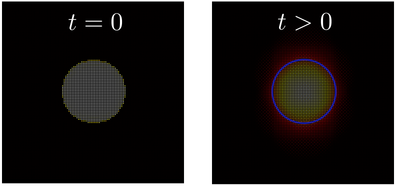
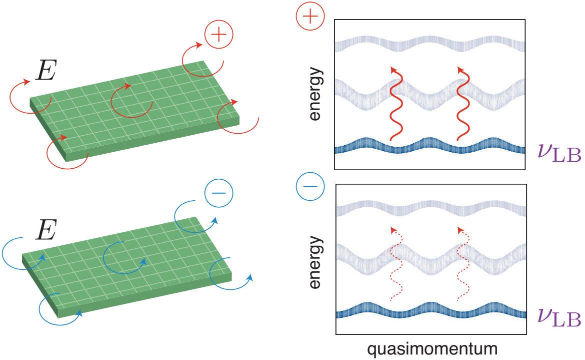
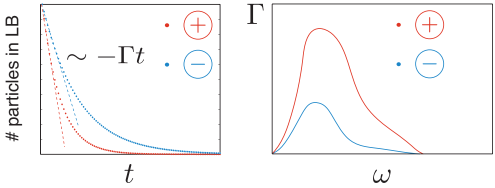
- Remove trap before heating protocol :
  $\rightarrow$ projection onto bulk states ($\approx$ LB for large clouds) only !



$\rightarrow$ **efficient and model-independent** method (validated numerically) !

Summary: Topology through depletion-rates measurements





Differential integrated rate (DIR)

$$\boxed{\Delta\Gamma^{\mathrm{int}}/A_{\mathrm{syst}} = \eta_0 E^2, \quad \eta_0 = (1/\hbar^2)\,\nu_{\mathrm{LB}}}$$

Concluding remarks

- Topological order can be detected by **heating** a system (in a proper way)
- **Atomtronics** : This measurement can be performed in **cold atoms** (shake/band-mapping/remove edge contributions) !
- Results presented for 2D Chern insulators, but can be generalized (e.g. 3D)
- The result $\Delta\Gamma^{\text{int}} = (E/\hbar)^2 \,\text{Tr}\, \hat{\mathfrak{C}}, \qquad \hat{\mathfrak{C}} = 4\pi \,\text{Im} \hat{P}\hat{x}\hat{Q}\hat{y}\hat{P}$

  $\rightarrow$ suggests possible generalizations to interacting topological phases

Probing topology by "heating"
D. T. Tran, A. Dauphin, A. G. Grushin, P. Zoller, and N. Goldman
arXiv :1704.01990

Thank you for your attention !