From Hessian Geometry to Supergravity Solutions

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Outline

- Hessian geometry and its relation to the special geometry of vector multiplets.
- Dimensional reduction to three (spacelike) dimensions.
- Example: construction of black brane solutions with vanishing entropy density at zero temperature ('Nernst branes').

(Mostly) based on:

- TM and O. Vaughan, *The Hesse potential, the c-map and black hole solutions*, arXiv:1112.2876, JHEP 1207 (2012) 163.
- D. Errington, TM and O. Vaughan, *Non-extremal black hole solutions from the c-map*, arXiv:1408.0923, JHEP 1505 (2015) 052.
- P. Dempster, D. Errington and TM, *Nernst branes from special geometry*, arXiv:1501.07863, JHEP1505 (2015) 079.
- V. Cortés, P. Dempster, T.M. and O. Vaughan, *Special Geometry of Euclidean Supersymmetry IV: the local c-map*, arXiv:1507.04620, JHEP1510 (2015) 066 → Talk by Vicente.
- Work to appear, with P. Dempster, D. Errington and J. Gutowski, on five-dimensional Nernst branes.

Part I: Hessian Geometry and its relation to Special Geometry

Hessian manifolds

(M, g, ∇) such that

- (M, g) (semi-)Riemannian
- ∇ flat, torsion-free connection, such that ∇g totally symmetric.

When using affine coordinates q^a, (that is, $\nabla dq^a = 0$),

$$g = \partial^2 H \; ,$$

where H is the Hesse potential.

Conical Hessian manifolds

(M, g, ∇, ξ) such that

- (M, g, ∇) Hessian
- ξ vector field satisfying

$$\frac{2}{n} D\xi = \nabla\xi = \mathrm{Id}_{TM}$$

where $D =$ Levi-civita connection, and $n \neq 0$.

$D\xi = \frac{n}{2}\mathrm{Id}_{TM}$ implies that ξ is a homothetic Killing vector field, which is hypersurface orthgonal:

$$L_\xi g = ng \;, \quad d(g\xi) = 0 \;.$$

I.p., for $n = 2$, (M, g, ξ) is locally a metric cone.

$\nabla\xi = \mathrm{Id}_{TM}$ implies that ξ is the Euler field with respect to affine coordinates,

$$\xi = q^a \frac{\partial}{\partial q^a} \ .$$

(Restrict q^a such that ξ acts by $q^a \to \lambda q^a$.)

Then there exists a Hesse potential (unique up to linear transformations), which is homogeneous of degree n in q^a:

$$H(\lambda q^a) = \lambda^n H(q^a) \ .$$

Dual coordinates and dual Hessian structures

Dual Hessian structure obtained by Legendre transformation:

$$\begin{aligned} \nabla' &= 2D - \nabla \\ q'_a &= \frac{\partial H}{\partial q^q} \\ H' &= q^a H_a - H \\ ds^2 &= H_{ab} dq^a dq^b = H^{ab} dq'_a dq'_b \end{aligned}$$

For n-conical Hessian structures:

$$H' = (n-1)H$$

'Log-ing' a homogeneous Hesse potential

If H is homogeneous of degree n:

- $\tilde{H} = \log H$ is not (quite) a homogeneous function, but
- $\partial^k_{a_1\ldots}\tilde{H}$ is homogeneous of degree $-k$.

Then

$$\tilde{H}_{ab} = \frac{\partial^2 \tilde{H}}{\partial q^a \partial q^b}$$

defines a Hessian metric, which is 'conical with $n = 0$', i.e. ξ acts isometrically.

Dual coordinates

$$\tilde{q}_a := \tilde{H}_a = -\tilde{H}_{ab} q^b$$

Dual Hesse potential $\tilde{H}' = -\tilde{H}$:

$$\tilde{H}^{ab} = \frac{\partial^2 \tilde{H}'}{\partial \tilde{q}_a \partial \tilde{q}_b} .$$

Special real geometry

$$
\begin{aligned}
e_5^{-1}\mathcal{L}_5 &= -\frac{1}{2}R_5 - \frac{1}{2}g_{xy}(\phi)\partial_\mu\phi^x\partial^\mu\phi^y - \frac{1}{4}a_{ij}(\phi)\mathcal{F}^i_{\mu\nu}\mathcal{F}^{j|\mu\nu} \\
&\quad + \frac{1}{6\sqrt{6}}e_5^{-1}c_{ijk}\epsilon^{\mu\nu\rho\sigma\tau}\mathcal{F}^i_{\mu\nu}\mathcal{F}^j_{\rho\sigma}\mathcal{A}^k_\tau - V(\phi)
\end{aligned}
$$

Hesse potential $H = c_{ijk}h^i h^j h^k$ must be a homogeneous cubic polynomial.
Physical scalars ϕ^x parametrize a projective special real manifold

$$
\bar{M} = \{h^i \in \mathbb{R}^n | H = c_{ijk}h^i h^j h^k = 1\} \subset M ,
$$

where M is 3-conical Hessian (with a polynomial Hesse potential).

$\iota : \bar{M} = \{H = 1\} \to M.$

- Scalar field metric (on $\bar{M}$):

$$g_{PSR} \simeq \iota^* \partial^2 H \simeq \iota^* \partial^2 \log H$$

- Vector field metric (section of $\mathrm{Sym}^2(TM)_{|\bar{M}}$):

$$a_{ij} \simeq \partial^2_{ij} \log H \; .$$

Special Kähler geometry

$$
\begin{aligned}
e_4^{-1}\mathcal{L}_4 &= -\frac{1}{2}R_{(4)} - g_{A\bar{B}}\partial z^A \partial \bar{z}^B + \frac{1}{4}\mathcal{I}_{IJ}F^I_{\hat{\mu}\hat{\nu}}F^{J|\hat{\mu}\hat{\nu}} + \frac{1}{4}\mathcal{R}_{IJ}F^I_{\hat{\mu}\hat{\nu}}\tilde{F}^{J|\hat{\mu}\hat{\nu}} \\
&\quad - V\left(X, \bar{X}\right) \; .
\end{aligned}
$$

Couplings $g_{A\bar{B}}$, $\mathcal{I}_{IJ}$, $\mathcal{R}_{IJ}$ are determined by prepotential $F(X^I)$, $I = 0, 1, \ldots n$, which is holomorphic and homogeneous of degree two in scalars X^I subject to complex rescalings $X^I \rightarrow \lambda X^I$, $\lambda \in \mathbb{C}^*$. Physical scalars:

$$
z^A = \frac{X^A}{X^0} \; , \quad A = 1, \ldots, n \; .
$$

Scalar manifold $\bar{N}$ is a projective special Kähler (PSK) manifold,

$$
\bar{N} = N/\mathbb{C}^* = N//U(1) \; ,
$$

where N is a conical affine special Kähler (CASK) manifold.

Affine special Kähler manifolds

(N, g, J, ∇), where

- (N, g, J) Kähler with Kähler form $\omega = gJ$.
- ∇ is a flat, torsion-free, symplectic connection satisfying

$$d^{\nabla} J = 0$$

equivalently:

$$\nabla g \;\; \text{totally symmetric} \; .$$

Thus Kähler and Hessian.

Kähler potential has a holomorphic prepotential:

$$K = -i(X^I \bar{F}_I - \bar{X}^I F_I) \; .$$

Hesse potential $H(q^a)$ and holomorphic prepotential $F(X^I)$ are related by a Legendre transformation ($F_I = \partial F / \partial X$, etc)

$$\begin{array}{rcl} X^I & = & x^I + i u^I(x,y) \\ F_I & = & y_I + i v_I(x,y) \end{array}$$

Special real coordinates = ∇-affine coordinates which are ω-Darboux coordinates: $(q^a) = (x^I, y_I)$.

$$H(x,y) = 2\left(\mathrm{Im}(F(x, u(x,y))) - y_I u^I(x,y)\right)$$

Conical affine special Kähler manifolds

(N, g, J, ∇, ξ) such that

- (N, g, J, ∇) is ASK.
- ξ is a vector field such that

$$D\xi = \nabla\xi = \mathrm{Id}_{TN}$$

Vector fields

$$\xi = q^a \frac{\partial}{\partial q^a} = X^I \frac{\partial}{\partial X^I} + \text{c.c.} \quad \text{and} \quad J\xi = \frac{1}{2} H_a \Omega^{ab} \frac{\partial}{\partial q^b} = iX^I \frac{\partial}{\partial X^I} + \text{c.c.}$$

generate a homothetic, holomorphic $\mathbb{C}^*$ action.

Assuming group action can take Kähler quotient to define $\bar{N} = N/\mathbb{C}^* = N//U(1)$.
Level surface of $U(1)$ moment map:

$$S = \{g(\xi, \xi) = 1\} = \{-2H = 1\} = \{-i(X^I \bar{F}_I - F_I \bar{X}^I) = 1\}$$

$F(X^I)$ and $H(q^a)$ are homogeneous of degree two in (conical) special (holomorphic/real) coordinates X^I/q^a, and $H(q^a)$ is $U(1)$ invariant.

Scalar potential

Potential:

$$V(X,\bar{X}) = N^{IJ}\partial_I W \partial_J \bar{W} - 2\kappa^2|W|^2, \quad (N^{IJ}) = (2\mathrm{Im}F_{IJ})^{-1} \ ,$$

Superpotential:

$$W = 2\left(g^I F_I - g_I X^I\right) \ .$$

(g^I, g_I) parameters of magnetic/electric FI gauging.

Potential (real coordinates):

$$V = g^a g^b \left[H_{ab} + \frac{H_a H_b + 4\,(\Omega q)_a\,(\Omega q)_b}{H}\right] \ , \quad -2H \stackrel{D}{=} \kappa^{-2} \ .$$

Superpotential (real coordinates)

$$W = W(q^a) = ig^a\,(H_{ab} - 2i\Omega_{ab})\,q^b, \quad (\Omega_{ab}) = \begin{pmatrix} 0 & \mathbb{1} \\ -\mathbb{1} & 0 \end{pmatrix} \ ,$$

where $(g^a) := (g^I, g_I)$.

ε-complex structures

Almost complex structure:

$$J \in \Gamma(\mathrm{End}(TM)) \, , \quad J^2 = -\mathrm{Id}_{TM} \, .$$

Almost para-complex structure:

$$J \in \Gamma(\mathrm{End}(TM)) \, , \quad J^2 = \mathrm{Id}_{TM}$$

with the eigendistributions having equal dimension.
Unified notation: ε-complex structure:

$$J \in \Gamma(\mathrm{End}(TM)) \, , \quad J^2 = \varepsilon \mathrm{Id}_{TM} \, , \quad \varepsilon = \pm 1 \, .$$

Various concepts of complex geometry (Hermitian, Kähler, hyper-Kähler, quaternionic-Kähler, affine and projective special Kähler) can be adapted to para-complex geometry.

ε-quaternionic structures

$J_1, J_2, J_3 \in \mathrm{End}(V)$, pairwise anti-commuting, $J_1 J_2 = J_3$.

- Quaternionic structure:

$$J_1^2 = J_2^2 = J_3^2 = -\mathrm{Id}\ .$$

- Para-quaternionic structure:

$$J_1^2 = J_2^2 = -J_3^2 = \mathrm{Id}\ .$$

- Unified notation: ε-quaternionic structure:

$$J_1^2 = J_2^2 = -\varepsilon J_3^2 = \varepsilon \mathrm{Id}\ .$$

Part II: Dimensional reduction to three dimensions

Dimensional reduction to three dimensions

Metric	$g_{\hat{\mu}\hat{\nu}}$	Metric KK vector KK scalar	$g_{\mu\nu}$ $A_\mu \sim \tilde{\phi}$ ϕ
$n+1$ Vector fields	$A^I_{\hat{\mu}}$	$n+1$ Vector fields $n+1$ scalars	$A^I_\mu \sim \tilde{\zeta}_I$ $A^I_\star = \zeta^I$
n complex scalars	z^A	n complex scalars	z^A

$4n+4$ independent real scalar fields: $z^a, \zeta^I, \tilde{\zeta}_I, \phi, \tilde{\phi}$.

Re-packaging: use homogeneous variables X^I or q^a to encode scalars z^A, and absorbe the KK-scalar ϕ by a field redefinition:

$$Y^I = e^{\phi/2} X^I \ , \quad q^a_{\rm new} = e^{\phi/2} q^a_{\rm old}$$

$4n+5$ real scalar fields $q^a, \hat{q}^a, \tilde{\phi}$, subject to $U(1)$ transformations $= 4n+4$ independent fields.

3d Lagrangian

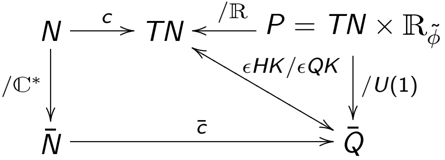
$$
\begin{aligned}
e_3^{-1}\mathcal{L}_3 &= -\frac{1}{2}R_{(3)} - \tilde{H}_{ab}\left(\partial_\mu q^a \partial^\mu q^b - \epsilon \partial_\mu \hat{q}^a \partial^\mu \hat{q}^b\right) + \frac{1}{2H}V \\
&\quad -\frac{1}{H^2}(q^a \Omega_{ab} \partial_\mu q^b)^2 + \epsilon \frac{2}{H^2}(q^a \Omega_{ab} \partial_\mu \hat{q}^b)^2 \\
&\quad -\frac{1}{4H^2}(\partial_\mu \tilde{\phi} + 2\hat{q}^a \Omega_{ab} \partial_\mu \hat{q}^b)^2 .
\end{aligned}
$$

where

$$
\Omega_{ab} = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix} , \quad \tilde{H}_{ab} = \partial^2_{a,b}\tilde{H} , \quad \tilde{H} = \log(-2H)
$$

where H = Hesse potential of the CASK manifold, and $\epsilon = -1$ ($\epsilon = 1$) for space-like (time-like)reduction.
Note: $\mathcal{L}_3$ is locally $U(1)$-invariant, only $4n + 4$ propagating scalar fields.

Geometrically: $\mathcal{L}_3$ defines a projectable ($U(1)$ invariant and $U(1)$ transverse, hence degenerate) symmetric tensor field on the total space of a $U(1)$ principal bundle $P \to \bar{Q}$ over the physical scalar manifold $\bar{Q}$, which is (para-)quaternionic Kähler.



HK/QK correspondence and its relation to the c-map: A. Haydys, J. Geom. Phys. 58 (2008) 293, arXiv:0706.4473; S. Alexandrov, D. Persson and B. Pioline, JHEP 1112 (2011) 027, arXiv:1110.0466; D. Alekseevsky, V. Cortés, T.M., Commun. Maths. Phys. 324 (2013) 637, arXiv:1205.2964; D. Alekseevsky, V. Cortés, M. Dyckmanns T.M., J. Geom. Phys. 92 (2015) 271, arXiv:1305.3549; V. Cortés, P. Dempster, T. Mohaupt and O. Vaughan, JHEP 1510 (2015) 066, arXiv:1507.04620; M. Dyckmanns and O. Vaughan, arXiv:1601.05001.

PI field configurations

For a certain class of field configurations, interesting solutions can be found by integrating the field equations elementarily.

Impose the following conditions:

- 4d field configuration is static.
- Impose that 4d scalars are ‘purely imaginary’ (‘axion-free’).
- Impose analogous conditions on gauge fields (and, in presence of a potential, gauging parameters).

This sets half of the three-dimensional scalars constant, while the remaining scalars parametrize a para-Kähler submanifold.

$$
\begin{aligned}
(q^a)|_{\mathrm{PI}} &= (x^0, 0, \ldots, 0; 0, y_1, \ldots, y_n), \\
(\partial_\mu \hat{q}^a)|_{\mathrm{PI}} &= \frac{1}{2}(\partial_\mu \zeta^0, 0, \ldots, 0; 0, \partial_\mu \tilde{\zeta}_1, \ldots, \partial_\mu \tilde{\zeta}_n), \\
(g^a)|_{\mathrm{PI}} &= (g^0, 0, \ldots, 0; 0, g_1, \ldots, g_n)\ .
\end{aligned}
$$

Additional assumption: prepotential is of 'very special type' $\Leftrightarrow$ can lift to five dimensions:

$$F = \frac{f(Y^1, \ldots, Y^n)}{Y^0} \;, \quad f \quad \text{homogeneous of degree 3.}$$

(This can be relaxed, essential point is to have some factorization of variables and some homogeneity property.)

Then one can obtain an explicit formula for Hesse potential

$$H = -\frac{1}{4} \left(-q_0 f(q_1, \ldots, q_n)\right)^{-\frac{1}{2}} \;, \quad \text{dual scalars} \quad q_a := \tilde{H}_a \;.$$

(Have shifted indices $a = 0, n+2, n+3, \ldots, 2n+1 \rightarrow$
$n = 0, 1, \ldots n$.)

Integrating the equations of motion

- $\hat{q}_a$ equations are trivial to integrate.
- Einstein equation can be solved in terms of q_a (they impose constraints on the integration constants).
- Block decomposition of $\tilde{H}_{ab}$ leads to decoupling of the q_0 equation of motion.
- Homogeneity allows to solve q_a equations of motion by taking fields q_a to be proportional to one another. (Constants of proportionality fixed by equations of motion.)

Part III: Nernst branes

Looking for Nernst branes

We wanted to find non-extremal versions of the 'Nernst branes' of Barisch et al.

S. Barisch, G. Lopes Cardoso, M. Haack, S. Naampuri and N.A. Obers, JHEP 1111 (2011) 090, [arXiv: 1108.02960].

Brane-like ansatz for 3d metric:

$$ds_3^2 = e^{4\psi} d\tau^2 + e^{2\psi}(dx^2 + dy^2),$$

where $\psi = \psi(\tau)$, $\tau =$ 'affine' radial coordinate. (Horizon at $\tau = \infty$, asymptotic regime at $\tau = 0$.) Static 4d metric of the form:

$$ds_4^2 = -e^{\phi} dt^2 + e^{-\phi+4\psi} d\tau^2 + e^{-\phi+2\psi}(dx^2 + dy^2).$$

Allow one electric charge Q_0, take gauging purely electrical: $g^0 = 0$, keep $g_1, \ldots, g_n \neq 0$.

3d solution

Can integrate:

$$
\begin{aligned}
\dot{\hat{q}}_0 &= -Q_0, \\
e^{-4\psi} &= \frac{1}{B_0^3}\sinh^3(B_0\tau)e^{B_0\tau}, \\
e^{\phi} &= \frac{1}{2}(-q_0)^{-\frac{1}{2}}(f(q_1,\ldots,q_n))^{-\frac{1}{2}}\ , \\
q_0 &= \pm - \frac{Q_0}{B_0}\sinh\left(B_0\tau + B_0\frac{h_0}{Q_0}\right), \\
q_A &= \pm\frac{1}{8g_A}B_0^{-\frac{1}{2}}e^{\frac{1}{2}B_0\tau}\left(\sinh(B_0\tau)\right)^{\frac{1}{2}} \quad \text{for} \quad A = 1,\ldots,n\ .
\end{aligned}
$$

Have already imposed that 4d solution has a regular event horizon $\Rightarrow$ Only 3 integration constants remain: Q_0 (electric charge), $B_0 \geq 0$, extremality parameter ($\sim$ temperature), h_0 ($\sim$ chemical potential).
Consistent with '1st order rewriting' $\rightarrow$ Dietmar Klemm's talk.

4d solution

New radial coordinate:

$$e^{-2B_0\tau} = 1 - \frac{2B_0}{\rho} =: W(\rho)$$

Asymptotic region: $\rho \to \infty$, horizon: $\rho = 2B_0$.

4d metric:

$$ds_4^2 = -\mathcal{H}^{-\frac{1}{2}} W \rho^{\frac{3}{4}} dt^2 + \mathcal{H}^{\frac{1}{2}} \rho^{-\frac{7}{4}} \frac{d\rho^2}{W} + \mathcal{H}^{\frac{1}{2}} \rho^{\frac{3}{4}} (dx^2 + dy^2),$$

where

$$\mathcal{H}(\rho) \equiv \pm 4 \left(\frac{1}{8}\right)^3 f\left(\frac{1}{g_1}, \ldots, \frac{1}{g_n}\right) \mathcal{H}_0(\rho) \,, \quad \mathcal{H}_0(\rho) = -\left[\frac{Q_0}{B_0} \sinh\left(\frac{B_0 h_0}{Q_0}\right) + \frac{Q_0 e^{-\frac{B_0 h_0}{Q_0}}}{\rho}\right].$$

Scalars:

$$z^A = -i\left(\pm\frac{8}{g_A^2}f\left(\frac{1}{g_1},\ldots,\frac{1}{g_n}\right)^{-1}\rho^{1/2}\mathcal{H}_0\right)^{\frac{1}{2}} .$$

Asymptotics for $\rho \to \infty$:

$$z^A \sim \begin{cases} \rho^{1/4} & \text{for } \; h_0 \neq 0 \; , \\ \rho^{-1/4} & \text{for } \; h_0 = 0 \; . \end{cases}$$

Asymptotics for $\rho \to 2B_0$:

$$z^A \sim \begin{cases} \text{finite} & \text{for } \; B_0 \neq 0 \; , \\ \rho^{-1/4} & \text{for } \; B_0 = 0 \; . \end{cases}$$

Black brane thermodynamics

Hawking temperature:

$$4\pi T = Z^{-1/2}(2B_0)^{3/4} e^{-\frac{B_0 h_0}{2Q_0}}.$$

$Z =$ combination of constants.

Chemical potential:

$$\mu \equiv A_t(\tau = 0) = \frac{1}{2}\left(\frac{B_0}{Q_0}\right)\left[\coth\left(\frac{B_0 h_0}{Q_0}\right) - 1\right],$$

diverges for $h_0 \to 0$.

Entropy density:

$$s = Z^{1/2}(2B_0)^{1/4} e^{\frac{B_0 h_0}{2Q_0}}$$

Note limits: $T = 0 \Leftrightarrow B_0 = 0$ and $\mu = \infty \Leftrightarrow h_0 = 0$.

Can eliminate B_0:

$$B_0 = 2\pi s T.$$

Equation of state:

$$s^3 = 4\pi Z^2 T \left(1 + \frac{2\pi s T}{Q_0 \mu}\right).$$

Nernst law:

$$s \xrightarrow[T \to 0]{} 0 \,, \quad \mu, Q_0, Z \text{ fixed}$$

Scaling regimes:

$$\begin{aligned} s \sim T^{1/3} \quad &\text{for} \quad T/\mu \ll 1 \\ s \sim T \quad &\text{for} \quad T/\mu \gg 1 \end{aligned}$$

hvLif geometries

Hyperscaling violating Lifshitz geometries with d transverse spatial dimensions:

$$ds^2_{d+2} = r^{-\frac{2(d-\theta)}{d}} \left(-r^{-2(z-1)} dt^2 + dr^2 + dx_i^2\right) ,$$

Scaling behaviour:

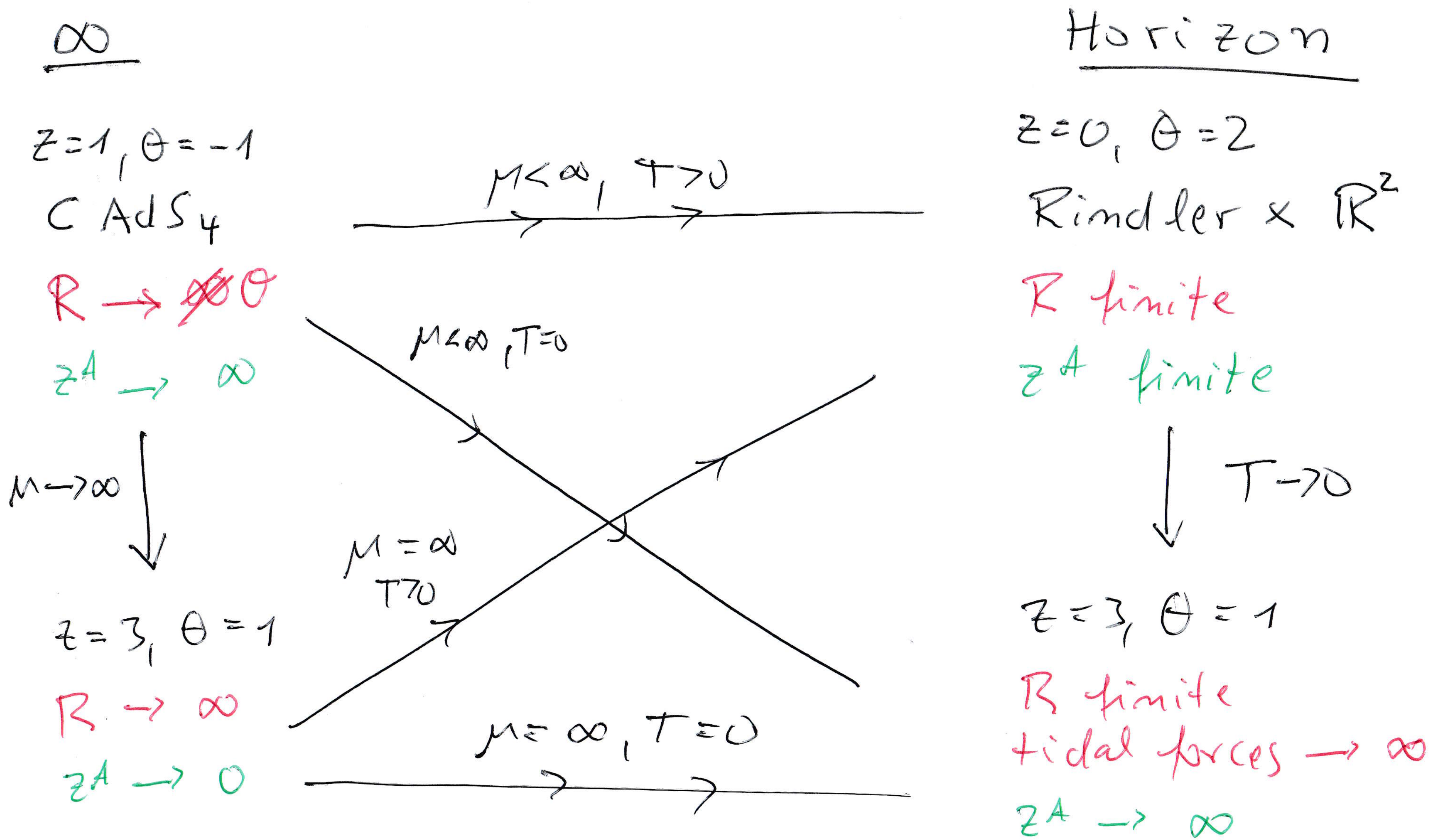
$$(r, x_i) \mapsto \lambda(r, x_i) \ , \quad t \mapsto \lambda^z t \ , \quad ds^2_{d+2} \mapsto \lambda^{2\theta/d} ds^2_{d+2} \ .$$

$z =$ Lifshitz exponent, measures deviations from relativistic symmetry ($\lambda \neq 1$).
$\theta =$ hyperscaling violating exponent, measures deviation from scale invariance ($\theta \neq 0$).

Thought to be dual to $\mathrm{QFT}_{1,d}$, with above scaling behaviour, i.p.

$$s \sim T^{(d-\theta)/z} \ .$$

4d Nernst branes



μ = chemical potential, T = temperature

R = Ricci Scalar, z^A = scalar fields

5d solution

Lifting 3d solution to 5d gives:

$$ds^2_{(5)} = \frac{f(\rho)\rho^{1/2}}{6\tilde{\lambda}}(dx^0 + \mathcal{A}^0_4 dx^4)^2 - \frac{\rho^{1/2}W(\rho)}{\tilde{\lambda}f(\rho)}(dx^4)^2 + \frac{6\tilde{\lambda}^2 d\rho^2}{\rho^2 W(\rho)} + 6\tilde{\lambda}^2\rho^{1/2}(dx^2 + dy^2)$$

where

$$f = A + \frac{\Delta}{\rho}\ , \quad W = 1 - \frac{2B_0}{\rho}\ , \quad \mathcal{A}^0_4 = -\sqrt{\frac{6\Delta}{\Delta + 2B_0 A}\,\frac{W}{f}}\ ,$$

$$\tilde{\lambda} = \left(\frac{1}{6}c_{ijk}\lambda^i\lambda^j\lambda^k\right)^{1/3}\ , \quad \lambda^i \sim \frac{1}{g_i}\ , \quad h^i = \frac{\lambda^i}{(c_{ijk}\lambda^i\lambda^j\lambda^k)^{1/3}}$$

Properties of 5d solution

- 5d solution is stationary, but not static, and carries no electric charge.
- 5d scalars are constant, with values determined by gauging parameters, acting as an effective cosmological constant. Can consistently truncate to pure gauged supergravity.
- Solution is asymptotically AdS_5.
- Entropy density shows expected behaviour: $s \sim T^3$ at infinity/UV, $s \sim T^{1/3}$ at horizon/IR.
- Solution is a boosted AdS-Schwarzschild black brane. $\Delta =$ boost parameter. Matches with the reductions of certain ten-dimensional solutions, which describe limits of a stack of boosted black D3 branes in IIB.

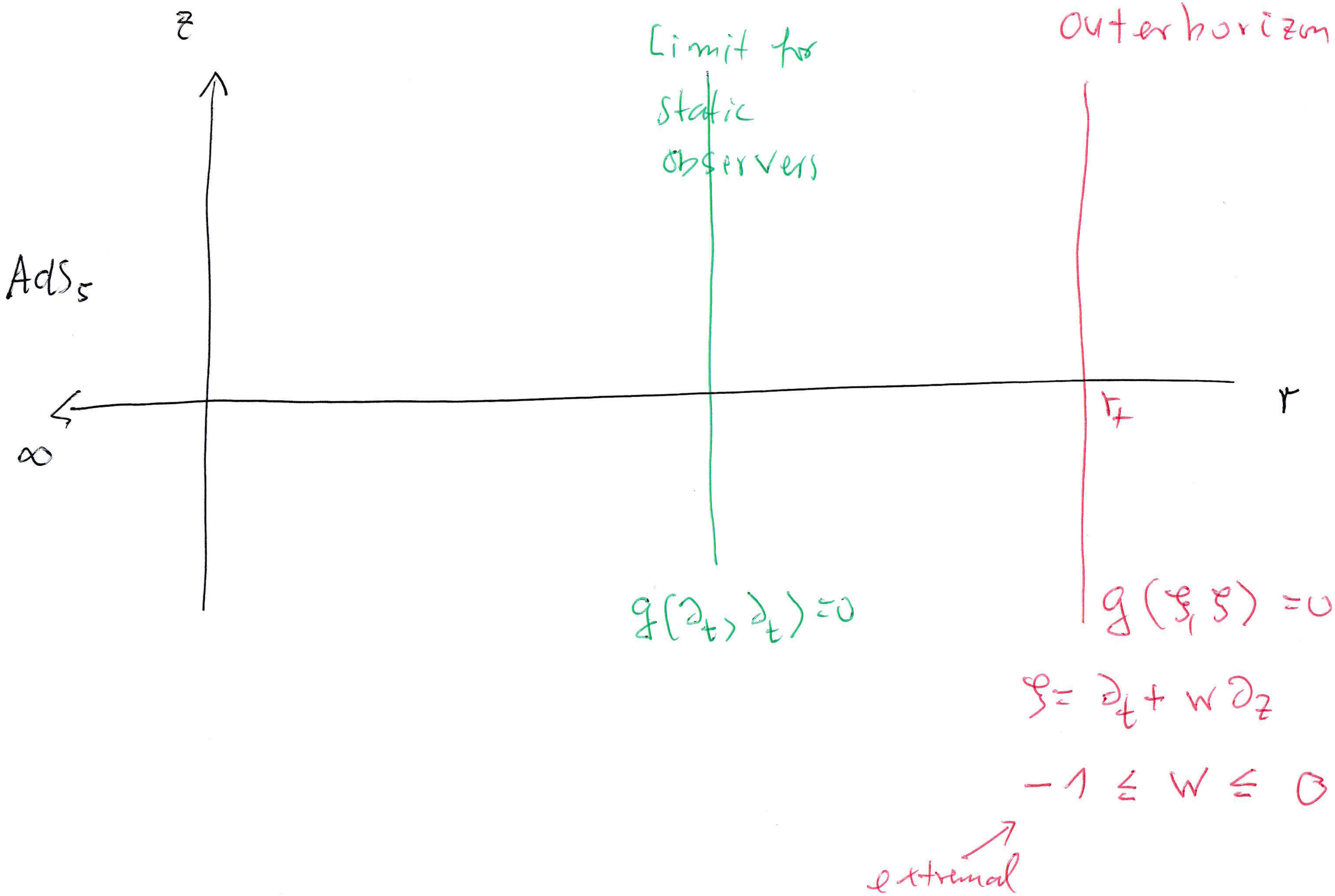
  H. Singh, JHEP 1012 (2010) 061, K. Narayan, Phys. Rev. D 85:106006 (2012), H. Singh, JHEP 1308 (2013) 097.

- Extremal limit: zero temperature, infinite boost. (Is BPS).

  Extremal 5d Nernst branes for STU model: Barisch, Cardoso, Haack, and Nampuri JHEP 1302 (2013) 103.

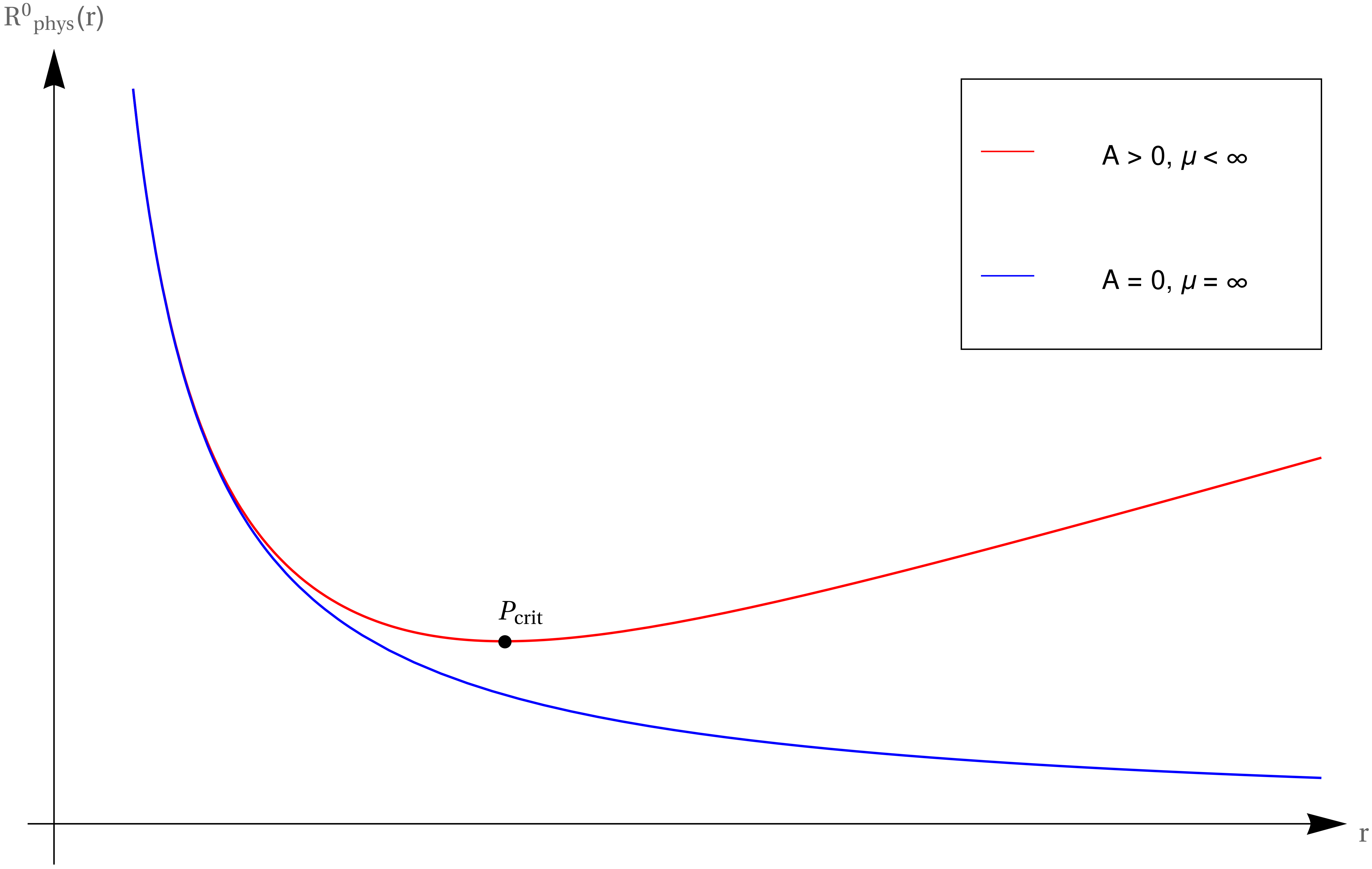
  'Heated up version': Goldstein, Nampuri, and Véliz-Osorio, JHEP 1408 (2014) 151.

AdS boosted Schwarzschild Black Brane



Parameter puzzle

1. $B_0 \leftrightarrow$ Temperature.
2. $\Delta \leftrightarrow$ 5d momentum should correspond to 4d electric charge Q_0 (hence become discrete upon reduction to 4d).
3. Parameter A is 'quasi-discrete': $A = 0$ or $A > 0$, but all values $A > 0$ are equivalent. How to account for the 4d chemical potential?


$R^0{}_{phys}(r)$
$A > 0, \mu < \infty$
$A = 0, \mu = \infty$
P_{crit}
r

5d vs 4d Nernst branes

∞

AdS_5

dynamical decompactification

$\mu < \infty$	$\mu = 0$
Conf AdS_4	$z = 3, \theta = 1$
$R \to 0$	$R \to \infty$
$z^A \to \infty$	$z^A \to 0$

$R_0 \to \infty, \quad \sigma$

Horizon

$T > 0$	$T = 0$
	$z = 3, \theta = 0$
	$z' = -1$
finite	tidal forces $\to \infty$
	$z = 3, \theta = 1$
finite	tidal forces $\to \infty$
	$z^A \to \infty$
$R_0 < \infty$	$R_0 \to \infty$

Outlook

- More charges, more gauging parameters
- Hypermultiplets
- Relation between regularity of solutions to second order equations and first oder rewritings
- Methods for solving equations, e.g. integrability
- Relation to higher-dimensional geometries
- Higher curvature terms (and the fate of the third law)
- Geometry of submanifolds of 3d scalar manifold
- Completeness of classes of geodesics in semi-Riemannian setting