

The exceptional form of massive IIA supergravity

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Superstring solutions, supersymmetry and geometry
Benasque May 4 2016

Take-home message (1/2)

- **Exceptional Field Theory** captures (*locally on a coordinate patch*) the **IId** and **massless Type II supergravities** in an Exc. Generalised Geometry form
- It was unclear how to capture **massive Type IIA supergravity**:
 - There should be a **deformation** of the **massless IIA Ex. Gen. Geom.**
 - On the other hand, in EFT some **non-geometry** seemed necessary
 - There is a puzzle here...
- We solve this puzzle defining **deformed EFT's** (XFT's)
(and show the relation with the expected, but not necessary, non-geometry)

Take-home message (2/2)

EFT is based on a **generalised Lie derivative**
on an extended internal space (analogous to DFT)

$$\mathbb{L}_\Lambda U^M = \Lambda^N \overset{\frac{\partial}{\partial Y^M}}{\partial_N} U^M - U^N \partial_N \Lambda^M + \underset{\substack{\uparrow \\ \text{A (very specific) } E_{n(n)} \text{ invariant tensor}}}{Y^{MN}}{}_{PQ} \partial_N \Lambda^P U^Q + (\lambda_U - \omega) \partial_P \Lambda^P U^M$$

section condition: $Y^{PQ}{}_{MN} \partial_P \otimes \partial_Q = 0$

We define **non-derivative deformations** of the form

$$\tilde{\mathbb{L}}_\Lambda = \mathbb{L}_\Lambda + \Lambda^M X_M$$

with X_M an embedding tensor. Extra **X-constraint:**

$$X_{MN}{}^P \partial_P = 0$$

For a certain X , we reproduce the full massive IIA. No dimensional reduction!!

(few-lines application: what massless IIA sphere truncations are consistent for mIIA)

Outline

- A Gauged Supergravity Appetizer
- Consistent truncations, EFT & EGG
- Massless IIA vs. EFT
- The Romans mass & deformed EFT
- Relation to (non-)geometric EFT
- massive IIA on spheres (blackboard)

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Deformations of Gauged Supergravities

- We will not do gauged supergravity here, but part of the motivation comes from it:
 - Many gauged supergravities, only *some* have **uplift to 11d/10d**
 - **Classification** of gauged supergravities is **hard**.
 - Finding which ones have an **uplift** is **even harder**.
- **Exc. Generalised Geometry & Exc. Field Theory** help us
...but they go beyond this: they can capture the **full 11d/10d theories**

Deformations of Gauged Supergravities

Many gauged (D=4) supergravities come in **families of inequivalent gaugings** (all sharing same gauge group)

- ω -deformation of $SO(8)$ gauged max. sugras [Dall'Agata, GI, Trigiante]
 only $\omega=0 \bmod \pi/4$ lifts to 11d sugra on S^7 [De Wit, Nicolai]

These are related to inequivalent E.m. embeddings of gauge connections

Explicit construction & classification: **“Symplectic Deformations”** [Dall'Agata, GI, Marrani]

$$\mathfrak{G}_{\text{red}}^0 \equiv \mathbb{S}(X^0, \theta^0) \setminus \mathcal{N}_{\text{Sp}(2n_v, \mathbb{R})}(\mathbf{G}_X) / \mathcal{N}_{\mathcal{G}_d}(\mathbf{G}_X)$$

normalizer
gauge group on vectors

↑ embedding tensors
↑ duality symmetries

$$\mathcal{N} \geq 2$$

$$\mathbb{S}(X^0, \theta^0) \equiv \left\{ S \in \mathcal{S}_{\text{Sp}(2n_v, \mathbb{R})}(X^0) \mid S\theta^0 = \theta^0 m, m \in \frac{\mathcal{N}_{\mathcal{G}_m}(\mathbf{H}_m)}{\mathcal{C}_{\mathcal{G}_m}(\mathbf{H}_m)} \right\} \quad \text{[GI]}$$

stabilizer
matter symmetries

↑ gauge group on matter

Symplectic deformations of some N=8 gaugings

$$\begin{aligned} \text{SO}(8), \text{SO}(4,4) : & \quad \mathfrak{G}_{\text{red}} = S^1/D_8, & \text{fundamental domain: } \omega \in [0, \pi/8]. \\ \text{SO}(p, 8-p), p \neq 0,4 : & \quad \mathfrak{G}_{\text{red}} = S^1/D_4, & \text{fundamental domain: } \omega \in [0, \pi/4]. \end{aligned}$$

No (geometric) uplift to IId sugra for the ω -deformation

[De Wit, Nicolai] [Lee, Strickland, Constable, Waldram]

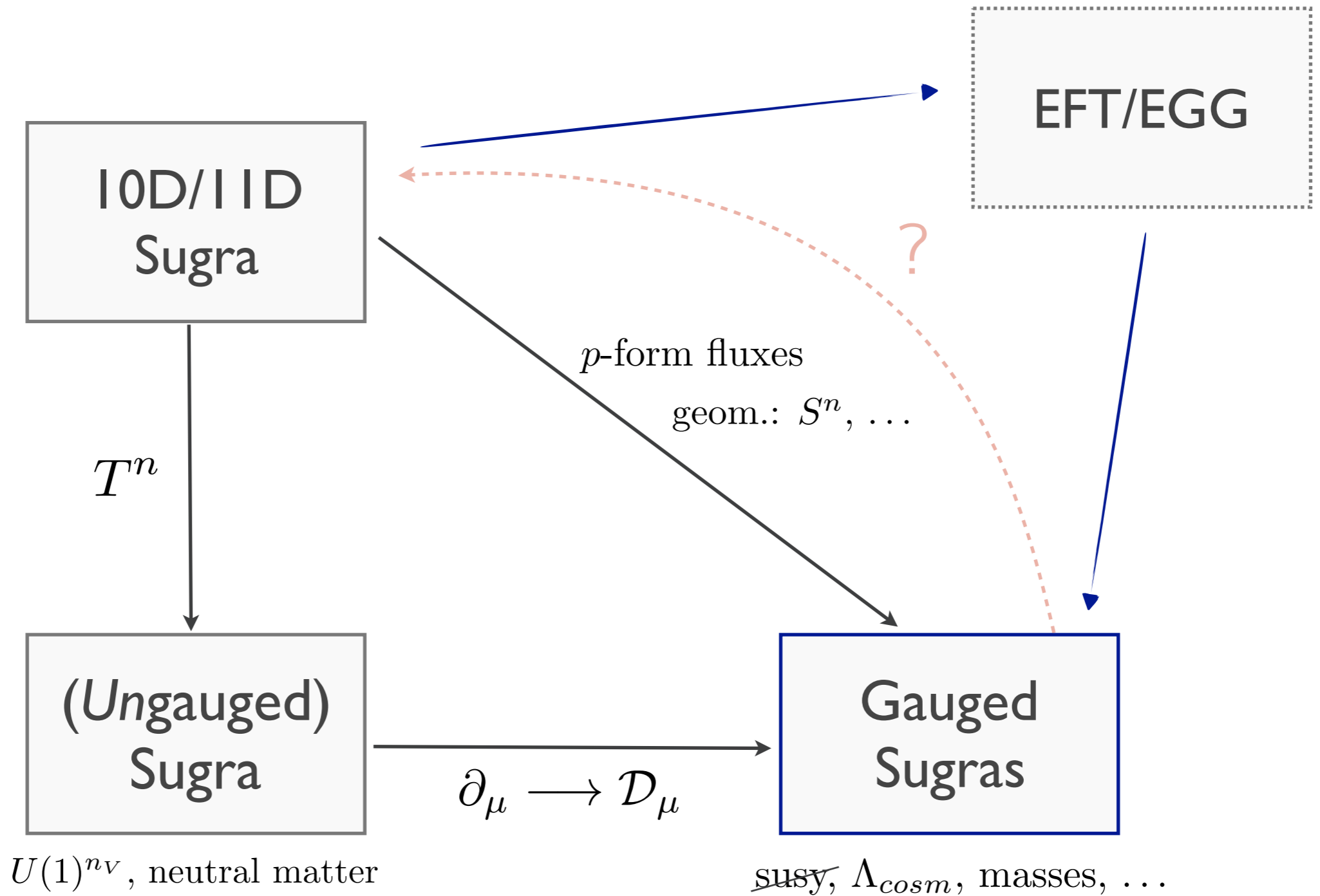
$$\text{ISO}(p, 7-p) : \quad \omega = 0 \quad \text{or} \quad \omega \neq 0 \pmod{\pi/2}.$$

Crucially, in [Dall'Agata, GI, Marrani] this is proven to be an **on/off** deformation.

Lifts to the on/off deformation of IIA on S^6 : **the Romans mass.** [Guarino, Varela]

(deformed ISO(7) is not really “dyonic”, all charges are mutually local)

Consistent Truncations & EFT



(inspired by Samtleben, 0808.4076)

Consistent Truncations & EFT

- **Exc. Field Theory** can be used to construct **consistent truncations** [Hohm, Samtleben]
- **IId** and **massless type II** in single framework
- $E_{n(n)} \times \mathbb{R}^+$ structures are made evident and can be exploited
- **Question: how can we embed massive type IIA in EFT?**
- massive IIA is reduced on S^6 using same Ansatz as massless (doable in EFT). Does this hold for lower-dim. spheres? [Guarino, Varela]
- **Puzzle:**
 - **mIIA** is a sugra theory **in its own right**.
It should be captured by EFT upon solution of its (strong) **section constraint**.
BUT this is not the case! [Berman et al.], [Blair Malek Park], [Bossard Kleinschmidt], [Bandos]
 - On the other hand, **in DFT non-geometry was necessary** [Hohm, Kwak]
(dependence on a winding coord. for some RR potential)

Supergravity, geometry & $E_{n(n)}$

Basic idea of exceptional geometry:

- **Repackage** fields and symmetries of II_d/IO_d SUGRA so that it **looks like** an $(II-n)$ dimensional maximal supergravity.
- However, **no truncation is performed!**
Equivalent form of the full II_d/IO_d SUGRA we began with.
- Fields and symmetry parameters **fill out $E_{n(n)}$ representations**
($K(E_{n(n)})$ for fermions)
- Very **convenient** (if not necessary!) if you want to do **consistent truncation** to an $(II-n)$ dimensional theory.
- But you may as well study **dynamics of full theory** without truncation!

Exceptional Field Theory

There are $E_{n(n)} \times \mathbb{R}^+$ **Exceptional Generalised Geometries** for

- **11d Supergravity** and **type IIA** (massless, so far!)
- **Type IIB**

Exceptional Field Theory is a framework that **captures both** (in coord. patch) by introducing extra internal coordinates

$$\begin{aligned} x^{\hat{\mu}} &= (x^{\mu}, y^m) \longrightarrow (x^{\mu}, Y^M) \\ \mathbf{d=11} &= (\mathbf{11-n}) + \mathbf{n} \longrightarrow (\mathbf{11-n}) + \mathbf{R_{vec}} \\ \text{(or 10)} & \qquad \qquad \qquad \text{(n-1)} \end{aligned}$$

↑
Full $E_{n(n)}$ representation

Similar to Double Field Theory.

Exceptional Field Theory

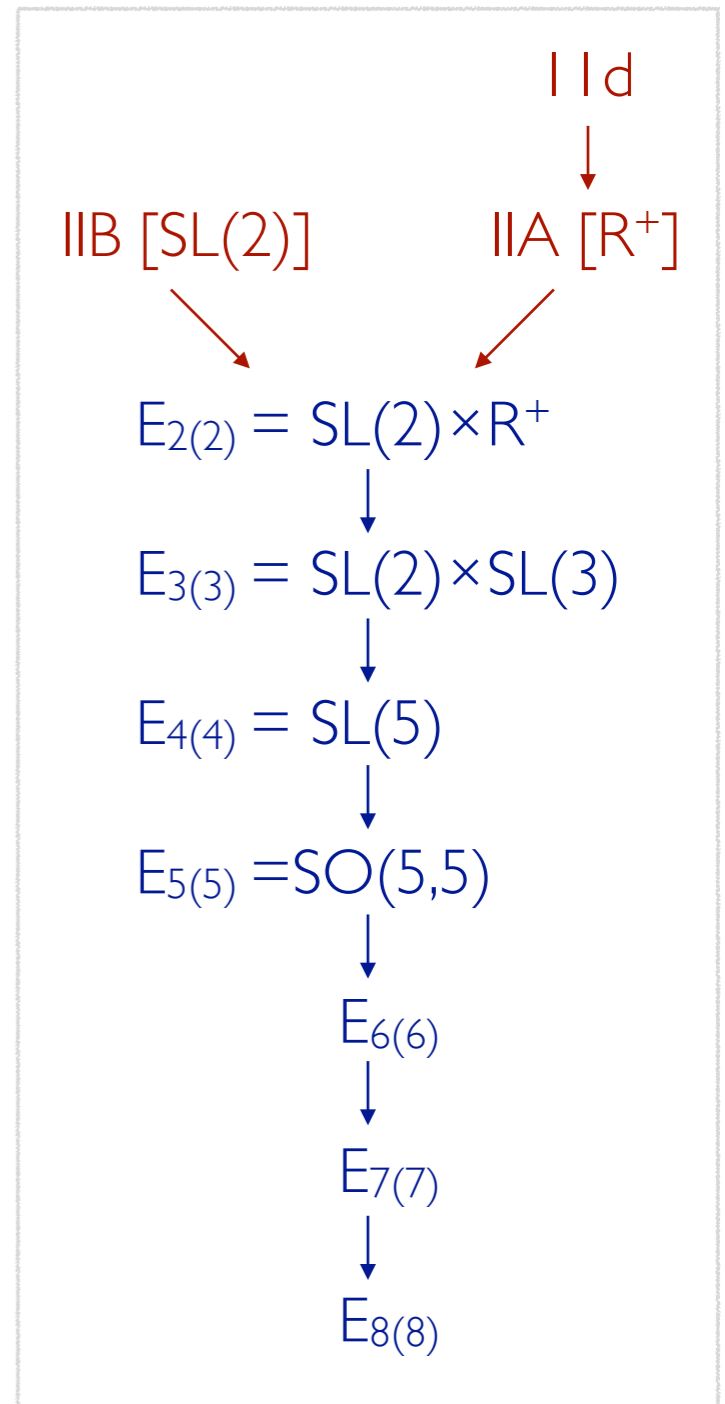
- There are EFT's in $D = 9, \dots, 3$ corresponding to the $E_{n(n)}$ series
- All fields depend on (x^μ, Y^M) but there is a section condition on Y dependence

$$Y^{MN}{}_{PQ} \overset{\frac{\partial}{\partial Y^M}}{\partial}_M \otimes \partial_N = 0$$

A (very specific) $E_{n(n)}$ invariant tensor

imposed on any field, parameter, etc.

- Result: fields only really depend on (x^μ, y^m) ,
- but what y^m ? **Two maximal solutions:**
 - n dimensional: **II d** SUGRA coordinates
 - $(n - 1)$ dimensional: **Type IIB** coordinates



Exceptional Field Theory

- EFTs look like a (D=9,...,3) supergravity theory, but with an infinite set of fields and internal symmetries.
- E.g. in D = 4, bosonic pseudo-action

$$S_{\text{EFT}} = \int d^4x \int d^{56}Y e \left[\overset{\text{Ricci scalar}}{\hat{R}} + \frac{1}{48} g^{\mu\nu} \overset{E_{7(7)}/SU(8) \text{ scalar fields}}{\mathcal{D}_\mu \mathcal{M}^{MN}} \mathcal{D}_\nu \mathcal{M}_{MN} - \frac{1}{8} \mathcal{M}_{MN} \mathcal{F}^{\mu\nu M} \mathcal{F}_{\mu\nu}{}^N + e^{-1} \mathcal{L}_{\text{top}} - V_{\text{EFT}}(\mathcal{M}, g) \right]$$

“scalar potential”

- Invariant under:
 - External diffeomorphisms $\xi^\mu(x, Y)$
 - Internal **Generalised diffeomorphisms** acting as:

$$\mathbb{L}_\Lambda U^M = \Lambda^N \overset{\frac{\partial}{\partial Y^M}}{\partial_N} U^M - U^N \partial_N \Lambda^M + Y^{MN}{}_{PQ} \partial_N \Lambda^P U^Q + (\lambda_U - \omega) \partial_P \Lambda^P U^M$$

A (very specific) $E_{n(n)}$ invariant tensor

IIA vs. EFT

- Section condition imposes:

$$\begin{aligned} (x^\mu, Y^M) &\longrightarrow (x^\mu, y^m) = x^{\hat{\mu}} \\ \mathbf{D} + \mathbf{R}_1 &\longrightarrow \mathbf{D} + \mathbf{k} \leq \mathbf{11} \text{ or } \mathbf{10} \end{aligned}$$

- For a certain choice of $\{y^m\}$ we get **massless IIA**.

..and other p-form trfs in D dimensions

$$\left. \begin{array}{l} \mathcal{L}_{\xi^\mu(x,y)} \\ \mathbb{L}_{\Lambda^M(x,y)} \end{array} \right\} \longrightarrow \left\{ \begin{array}{l} 10\text{d diffeos } (\xi^{\hat{\mu}}) \\ p\text{-form gauge trfs. } (\lambda_{(0)}, \Xi_{(1)}, \theta_{(2)}) \end{array} \right.$$

- For instance one can match the gauge trfs of the D -dimensional vectors

$$\delta B_\mu{}^\alpha = (\partial_\mu - B_\mu{}^\delta \partial_\delta) \xi^\alpha + \xi^\delta \partial_\delta B_\mu{}^\alpha ,$$

$$\delta C_\mu = \xi^\delta \partial_\delta C_\mu + (\partial_\mu - B_\mu{}^\delta \partial_\delta) \lambda ,$$

$$\delta C_{\mu\beta} = \xi^\delta \partial_\delta C_{\mu\beta} + C_{\mu\delta} \partial_\beta \xi^\delta + (\partial_\mu - B_\mu{}^\delta \partial_\delta) \Xi_\beta + B_\mu{}^\delta \partial_\beta \Xi_\delta ,$$

$$\begin{aligned} \delta C_{\mu\beta\gamma} &= \xi^\delta \partial_\delta C_{\mu\beta\gamma} + 2 C_{\mu\delta[\gamma} \partial_{\beta]} \xi^\delta + (\partial_\mu - B_\mu{}^\delta \partial_\delta) \theta_{\beta\gamma} + 2 B_\mu{}^\delta \partial_{[\beta} \theta_{\delta|\gamma]} \\ &\quad + 2 C_\mu \partial_{[\beta} \Xi_{\gamma]} - 2 C_{\mu[\beta} \partial_{\gamma]} \lambda . \end{aligned}$$

mIIA vs. EFT

- **Massive IIA is not embedded in EFT**
- Solutions of the section constraint only yield IId or massless type II
- A clear **obstruction** comes from the p-form gauge variations:

$$\delta B_\mu^\alpha = (\partial_\mu - B_\mu^\delta \partial_\delta) \xi^\alpha + \xi^\delta \partial_\delta B_\mu^\alpha ,$$

$$\delta C_\mu = \xi^\delta \partial_\delta C_\mu + (\partial_\mu - B_\mu^\delta \partial_\delta) \lambda - m_R B_\mu^\delta \Xi_\delta ,$$

$$\delta C_{\mu\beta} = \xi^\delta \partial_\delta C_{\mu\beta} + C_{\mu\delta} \partial_\beta \xi^\delta + (\partial_\mu - B_\mu^\delta \partial_\delta) \Xi_\beta + B_\mu^\delta \partial_\beta \Xi_\delta ,$$

$$\begin{aligned} \delta C_{\mu\beta\gamma} = & \xi^\delta \partial_\delta C_{\mu\beta\gamma} + 2 C_{\mu\delta[\gamma} \partial_{\beta]} \xi^\delta + (\partial_\mu - B_\mu^\delta \partial_\delta) \theta_{\beta\gamma} + 2 B_\mu^\delta \partial_{[\beta} \theta_{\delta|\gamma]} \\ & + 2 C_\mu \partial_{[\beta} \Xi_{\gamma]} - 2 C_{\mu[\beta} \partial_{\gamma]} \lambda - 2 m_R C_{\mu[\beta} \Xi_{\gamma]} . \end{aligned}$$

- **Terms without derivatives,**
but action of ξ^μ and Λ^M in EFT contain derivatives in every term!

IIA vs. EFT

- We could take the **non-geometric route** (violate section condition),
 - **BUT: massive IIA is a fine maximal SUGRA!**
There must be an exceptional generalised geometry for it.
 - Which means: **new EFTs**
with solutions of the section condition **that yield massive IIA**
 - We reverse-engineered these new EFTs by **comparing gauge trfs**
- Trick: it is sufficient to do it on a subset of fields transforming faithfully under $\mathbb{E}_{(1)}$ variations, the rest follows by covariance.

IIA vs. EFT

Massless IIA

$$\left. \begin{array}{l} \mathcal{L}_{\xi^\mu(x,y)} \\ \mathbb{L}_{\Lambda^M(x,y)} \end{array} \right\} \longrightarrow \left\{ \begin{array}{l} 10\text{d diffeos } (\xi^{\hat{\mu}}) \\ p\text{-form gauge trfs. } (\lambda_{(0)}, \Xi_{(1)}, \theta_{(2)}) \end{array} \right.$$

↓ Turn on m_R

$$\left. \begin{array}{l} \mathcal{L}_{\xi^\mu(x,y)} \\ \tilde{\mathbb{L}}_{\Lambda^M(x,y)} \end{array} \right\} \xleftarrow{\text{Deduce}} \begin{array}{l} \delta A_{\hat{\mu}} = \partial_{\hat{\mu}} \lambda + m_R \Xi_{\hat{\mu}} \\ \delta B_{\hat{\mu}\hat{\nu}} = 2 \partial_{[\hat{\mu}} \Xi_{\hat{\nu}]} \\ \delta A_{\hat{\mu}\hat{\nu}\hat{\rho}} = 3 \partial_{[\hat{\mu}} \theta_{\hat{\nu}\hat{\rho}]} - 3 A_{[\hat{\mu}\hat{\nu}} \partial_{\hat{\rho}]} \lambda - 3 m_R B_{[\hat{\mu}\hat{\nu}} \Xi_{\hat{\rho}]} \end{array}$$

- External diffeos, section condition and dictionary don't change,
- only the **generalised Lie derivative** does, let's see how

Deformed EFT (“XFT”)

- $\tilde{\mathbb{L}}_\Lambda$ now contains **non-derivative terms**:

$$\tilde{\mathbb{L}}_\Lambda V^M = \mathbb{L}_\Lambda V^M - X_{NP}{}^M \Lambda^N V^P$$

- X is a constant object encoding m_R , **which we know for all EFTs** :
 - It appears in the D-dimensional maximal gauged supergravity obtained by **torus reduction of massive IIA**.
 - It's a special case of *embedding tensor*.
(*this is not an assumption, it's a result.*)
- **Crucial:** we deduced this keeping only certain $\{y^m\}$ internal coords, we can now promote to $\{Y^M\}$ and find the **new section conditions**

Deformed EFT: consistency

- In **standard EFT closure** and **Jacobi** of \mathbb{L} require section condition

$$Y^{MN}{}_{PQ} \partial_M \otimes \partial_N = 0$$

- We need to repeat the same procedure for $\tilde{\mathbb{L}}$.

Several constraints:

- $Y^{MN}{}_{PQ} \partial_M \otimes \partial_N = 0$ is **unmodified** (good!)
- $X_{NP}{}^M$ has same constraints as **embedding tensor** in gauged sugra
- $\delta X_{NP}{}^M = 0$ under all gauge transformations (i.e.: it's not a tensor!)

- **Mixed condition:**

$$X_{NP}{}^M \partial_M = 0$$

(in particular: not a torsion!)

Deformed EFT: *interpretation*

$$\tilde{\mathbb{L}}_{\Lambda} V^M = \mathbb{L}_{\Lambda} V^M - X_{NP}^M \Lambda^N V^P$$

standard EFT
internal syms

Gauged SUGRA
internal syms

Deformed EFT (XFT)

- The structure of EFTs is that of gauged maximal sugras with infinitely many fields and internal symmetries
- It is only natural to superpose it with a standard gauging
- Just like in gauged sugra, X **breaks the global $E_{n(n)}$ explicitly** (solving the section constraint does, too)
- Romans mass was a deformation already in 10d, instead of finding its “origin”, we deformed EFT to implement it.

Deformed EFT: *interpretation*

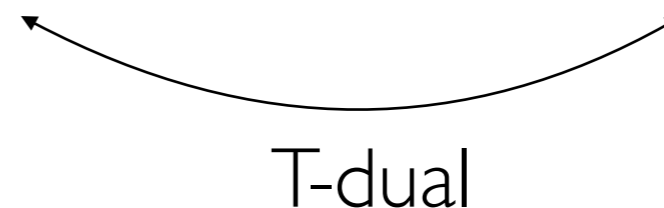
- Let's help us with an example: D=9 EFT, $E_{2(2)}=SL(2)\times R^+$
- Coordinates: external x^μ and $Y^M = \begin{pmatrix} y^{\alpha=1,2} \\ \text{IId} \end{pmatrix}, z$
type IIB
- Section condition $\partial_\alpha \times \partial_z = 0$: **IId sugra** or **type IIB**

$$\tilde{\mathbb{L}}_\Lambda V^M = \mathbb{L}_\Lambda V^M - X_{NP}{}^M \Lambda^N V^P, \quad \text{with } X_z{}^2{}^1 = m_R$$

- **New constraint:**

$$m_R \times \partial_1 = 0$$

kills IId coordinate, left with massive IIA, and IIB (with F_1 flux)



Deformed EFT: *interpretation*

- Same analysis in $D=4$:
- section condition + $X\partial$ -condition have **several 10d/11d solutions:**

————— T-duality —————→

massive IIA, type **IIB**, massless **IIA** (with constant RR flux)

and

11d SUGRA with Freund–Rubin parameter

↖ six “windings” of original massive IIA coordinates, plus a seventh “dual M-theory circle”

- except for mIIA, other constant fluxes can be removed by redefinition of gauge potentials & gauge-parameters

The XFT action in D=4

$$S_{\text{XFT}} = \int d^4x d^{56}Y e \left[\hat{R}(X) + \frac{1}{48} g^{\mu\nu} \mathcal{D}_\mu \mathcal{M}^{MN} \mathcal{D}_\nu \mathcal{M}_{MN} \right. \\ \left. - \frac{1}{8} \mathcal{M}_{MN} \mathcal{F}^{\mu\nu M} \mathcal{F}_{\mu\nu}{}^N + e^{-1} \mathcal{L}_{\text{top}}(X) - V_{\text{XFT}}(\mathcal{M}, g, X) \right]$$

$$\mathcal{F}_{\mu\nu}{}^M = F_{\mu\nu}{}^M - 12 [t^\alpha]^{MN} \partial_N B_{\mu\nu\alpha} - 2 Z^{M,\alpha} B_{\mu\nu\alpha} - \frac{1}{2} \Omega^{MN} B_{\mu\nu N}$$

(twisted-self-dual)

↑
satisfies sec. constraint algebraically,
and part of the X-constraint

$$\hat{R}_{\mu\nu}{}^{ab}(X) = R_{\mu\nu}{}^{ab}[\omega] + \mathcal{F}_{\mu\nu}{}^M e^{a\rho} \partial_M e_\rho{}^b$$

$$\mathcal{D}_\mu \equiv \partial_\mu - \tilde{\mathbb{L}}_{A_\mu}$$

This is always a direct superposition of EFT and Gauged N=8 SUGRA

The XFT scalar potential in D=4

$$V_{\text{XFT}}(\mathcal{M}, g, X) = V_{\text{EFT}}(\mathcal{M}, g) + V_{\text{SUGRA}}(\mathcal{M}, X) + V_{\text{cross}}(\mathcal{M}, X)$$

$$V_{\text{EFT}} = -\frac{1}{48} \mathcal{M}^{MN} \partial_M \mathcal{M}^{KL} \partial_N \mathcal{M}_{KL} + \frac{1}{2} \mathcal{M}^{MN} \partial_M \mathcal{M}^{KL} \partial_L \mathcal{M}_{NK} \\ - \frac{1}{2} g^{-1} \partial_M g \partial_N \mathcal{M}^{MN} - \frac{1}{4} \mathcal{M}^{MN} g^{-1} \partial_M g g^{-1} \partial_N g - \frac{1}{4} \mathcal{M}^{MN} \partial_M g^{\mu\nu} \partial_N g_{\mu\nu}$$

$$V_{\text{SUGRA}} = \frac{1}{168} \left[X_{MN}^P X_{QR}^S \mathcal{M}^{MQ} \mathcal{M}^{NR} \mathcal{M}_{PS} + 7 X_{MN}^P X_{QP}^N \mathcal{M}^{MQ} \right]$$

$$V_{\text{cross}} = \frac{1}{12} \mathcal{M}^{MN} \mathcal{M}^{KL} X_{MK}^R \partial_N \mathcal{M}_{RL} \quad \text{New cross-term!}$$

m_R from EFT and non-geometry

- In DFT m_R can be sourced by a RR $C_{(1)}$ potential with **linear dependence on a winding coordinate**

Hohm, Kwak - 2011

- The structure of EFT is more restrictive: violating section condition can be done (locally), but requires care.
- We **relax** the section condition of EFT, and factorise its fields:

$$\mathbf{V}_{\text{EFT}}^M(x, Y) = V_{\text{XFT}}^A(x, Y) E_A^M(Y)$$

where E are twist matrices such that $\mathbb{L}_{E_A} E_B^M = -X_{AB}{}^C E_C^M$

$$\underbrace{\mathbb{L}_\Lambda \mathbf{V}^M}_{\text{non-geom. EFT}} = \underbrace{(\tilde{\mathbb{L}}_\Lambda V^A) E_A^M}_{\text{XFT}}$$

- Notice that Λ , \mathbf{V} do **not satisfy sec. cnd.**, but Λ and V **satisfy XFT ones**
Thus, E can introduce non-geometric coordinate dependence in EFT

m_R from EFT and non-geometry

$$\mathbf{V}_{\text{EFT}}^M(x, Y) = V_{\text{XFT}}^A(x, Y) E_A^M(Y)$$

$$\mathbb{L}_\Lambda \mathbf{V}^M = (\tilde{\mathbb{L}}_\Lambda V^A) E_A^M$$

- This **non-geometric extension** of EFT allows e.g. to **source** m_R

$$E(Y)_A^M = E(Y^{\hat{P}})_A^M = \delta_A^M - \frac{1}{c'} Y^{\hat{P}} X_{\hat{P}A}^{\text{RR}M} \quad (\text{no sum over } \hat{P}).$$

↑
A single winding coord.

- **Crucial point:** we are extending to non-geometric coordinates, without truncating any of the physical ones (**not a SS reduction!**)
- !! Meaning of locally non-geometric backgrounds is still unclear, while XFT *solutions* are **globally well-defined (because mIIA is)**

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Concluding

- **Exceptional Field Theories** admit deformations which *still preserve 10- and 11-dim. solutions of section conditions*
- *one such deformation implements **massive IIA***
- This is now a **new tool** in its own right.
- These deformations are **more general than the Romans mass!!**
we have classified **all those allowing for 10d/11d solutions**
(we could easily go further and recover higher dim. gauged sugras)
- We can (locally) implement a “SS extension Ansatz”
to non-geometrically map EFT and XFT.
- Future: susy; D8 and 7-branes (& T/S-dualities); D=3;
and of course: more solutions!

Thank you!