Moduli of heterotic domain wall vacua

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X. de la Ossa, ML, E. Svanes (work in progress),
See also X. de la Ossa, ML, E. Svanes (1409.7539) and J. Gray, ML, D. Lüst (1205.6208)

Motivation and summary

Long history of heterotic string compactifications

- Minkowski 4D $\mathcal{N} = 1$ vacua with good particle physics from Calabi–Yau manifolds with vector bundles.
- Minkowski 4D $\mathcal{N} = 1$ vacua also from Strominger–Hull compactifications with flux and bundles.

Open problems

- Moduli stabilisation.
- Identification of moduli, both infinitesimal and finite.
- Moduli space geometry is Kähler. Determine Kähler potential?

This talk: compactifications on $SU(3)$ and G_2 structure manifolds

- Properties of heterotic 4D $\mathcal{N} = 1/2$ domain wall solutions.
- Moduli of integrable G_2 structure with bundles.
- Flow of SU(3) structures and interconnected moduli spaces (*time permitting*).

Outline

Heterotic supersymmetric vacua

Heterotic supergravity

- Bosonic fields: Metric G, B-field B, dilaton ϕ, gauge field A
- Fermionic fields: Gravitino, dilatino, gaugino

Fermionic SUSY variations vanish $\iff$

$$\left(\nabla_M + \frac{1}{8}\not{H}_M\right)\epsilon = 0$$
$$\left(\not{\nabla}\hat{\phi} + \frac{1}{12}\not{H}\right)\epsilon = 0$$
$$\not{F}\epsilon = 0$$

where $\not{\nabla} = \Gamma^M \nabla_M$, etc.

Compactifications

$\mathcal{M}_{10} = \mathcal{M}_E \times X$: SUSY $\iff$ nowhere vanishing spinor η on X: $\epsilon = \rho_E \otimes \eta$
$\iff$ X has reduced structure group

Hitchin:02, Gualtieri:04, Grana et al:05, ...

4D Heterotic $\mathcal{N}=1$ Minkowski vacua

Geometry: 6D manifold X with $SU(3)$ structure

Candelas, *et.al.*:85, Hull:86; Strominger:86, Ivanov, Papadopoulos:00; Gauntlett, *et.al.*:03,...

SUSY $\Rightarrow$ globally defined spinor η on X: $\nabla_H \eta = 0$

$\iff$ complex decomposable $(3,0)$-form Ψ and real $(1,1)$ form ω such that

$$\omega \wedge \Psi = 0 \ , \qquad \omega \wedge \omega \wedge \omega \sim \Psi \wedge \bar{\Psi}$$

- No H-flux $\iff$ X is Calabi–Yau $\qquad \mathrm{d}\Psi = 0 = \mathrm{d}\omega.$
- $H \neq 0$ * $\iff$ X is complex and conformally balanced $\qquad \mathrm{d}(e^{-2\phi}\Psi) = 0 = \mathrm{d}(e^{-2\phi}\omega \wedge \omega).$

* Need α' corrections to avoid no-go theorem for flux if X is compact without boundary

Gauge fields $\to$ vector bundle V

Candelas, *et.al.*:85,Donaldson:85, Uhlenbeck, Yau:86, Li, Yau:87,...

SUSY $\Longrightarrow$

- F holomorphic $F^{(0,2)} = F^{(2,0)} = 0$
- F satisfies Hermitian Yang-Mills equation $F \lrcorner \omega = 0$

polystable holomorphic V: $\exists !$ A satisfying HYM.

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4D Heterotic $\mathcal{N} = 1/2$ DW vacua

Lukas *et al*:10, 11; Gray, ML, Lüst:12, de la Ossa, ML, Svanes:14

Geometry

4D domain wall vacuum: $\mathcal{M}_{10} = \mathcal{M}_4 \times_W X(r) \equiv \mathcal{M}_3 \times Y$

$\mathcal{M}_4 = \mathcal{M}_3 \times \mathbb{R}$, $\mathcal{M}_3$ AdS or Minkowski

Embed $SU(3)$ in G_2: $\varphi = \mathrm{d}r \wedge \omega(r) + \mathrm{Re}(\Psi(r))$.

SUSY and BI at $\mathcal{O}(\alpha'^0)$

Restricts torsion, DW flow of the SU(3) structure, and the flux.

SU(3) torsion: $X(r)$ conformally balanced, but otherwise generic

SU(3) flow: $\partial_r \omega$ fixed in terms of ϕ and SU(3) torsion

$\partial_r \Psi$ fixed up to primitive (2,1)+(1,2)-form γ.

Flux $\widehat{H}$: fixed by SUSY in terms of ϕ and SU(3) torsion up to γ

$\Rightarrow$ can check Bianchi idenitity.

4D Heterotic $\mathcal{N} = 1/2$ DW vacua

Lukas *et al*:10, 11, 12, 13; Gray, ML, Lüst:12, de la Ossa, ML, Svanes:14

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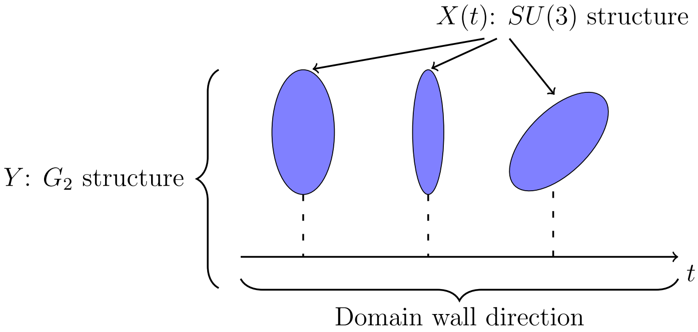

- H-flux components allowed by symmetry: $H_{\alpha\beta\gamma}$, H_{tmn} and H_{mnp}
- $H_{tmn} = 0 = H_{\alpha\beta\gamma}$ allows non-perturbative "uplift" to 4D AdS.

4D Heterotic $\mathcal{N} = 1/2$ DW vacua

Gray, ML, Lüst:12, de la Ossa, ML, Svanes:14, Fernandez–Gray:82, Chiossi–Salamon:02

SUSY $\iff$ Y has G_2 structure determined by 3-form φ ($\psi = *\varphi$)

$$\mathrm{d}\varphi = \tau_0\, \psi + 3\, \tau_1 \wedge \varphi + *\tau_3 \ ,$$
$$\mathrm{d}\psi = 4\, \tau_1 \wedge \psi + *\tau_2 \ .$$

with torsion classes (in G_2 reps)

$$\tau_0 = \mathrm{dvol}_{\mathcal{M}_3} \lrcorner H \ , \quad \tau_1 = \tfrac{1}{2}\, d\phi \ ,$$
$$\tau_2 = 0 \quad , \quad \tau_3 = -H + \tfrac{1}{6}\, \tau_0\, \varphi - \tau_1 \phi \lrcorner \psi$$

This is an integrable G_2 structure.

Gauge vector bundle

SUSY
- $F \wedge \psi = 0 \implies F$ is a G_2 instanton
- Embed $SU(3) \implies$ recover $F^{(2,0)} = 0 = F \lrcorner \omega$

4D Heterotic $\mathcal{N} = 1/2$ DW vacua

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Manifolds with G_2 structure

Fernandez–Gray:82, Chiossi–Salamon:02

Decomposition of forms

$\Lambda^k(Y)$ decomposes into $\Lambda^k_p(Y)$, p denotes G_2 irrep. Find these using φ:

Example: $\Lambda^1 = \Lambda^1_7 = T^*Y \cong TY$
$\Longrightarrow$ *any* $\beta \in \Lambda^2$ decomposes as $\beta = \alpha \lrcorner \varphi + \gamma$, where $\alpha \in \Lambda^1$ and $\gamma \lrcorner \varphi = 0$

$$\begin{aligned}
\Lambda^0 &= \Lambda^0_1 \ , \\
\Lambda^1 &= \Lambda^1_7 = T^*Y \cong TY \ , \\
\Lambda^2 &= \Lambda^2_7 \oplus \Lambda^2_{14} \ , \\
\Lambda^3 &= \Lambda^3_1 \oplus \Lambda^3_7 \oplus \Lambda^3_{27} \ .
\end{aligned}$$

Infinitesimal Moduli

Variational studies

- Moduli of heterotic $\mathcal{N} = 1$ vacua: deformations of conformally balanced complex 3-fold X with holomorphic gauge bundle V
- Moduli of heterotic $\mathcal{N} = 1/2$ vacua: deformations of 7D Y with integrable G_2 structure and an instanton gauge bundle V
- Flow of heterotic $\mathcal{N} = 1/2$ solutions at $\mathcal{O}(\alpha'^0)$: the moduli spaces of *different* $SU(3)$ structures may connect via flow along domain wall direction in Y with integrable G_2 structure.

Infinitesimal Moduli

Variational studies

- Moduli of heterotic $\mathcal{N} = 1$ vacua: deformations of conformally balanced complex 3-fold X with holomorphic gauge bundle V

  Atiyah:57, Kodaira, Spencer:58,60, Candelas, de la Ossa:91, Becker,*et.al*:05,06, Anderson,*et.al*:10,11,13, Fu, Yau:11,Anderson, Gray, Sharpe:14, de la Ossa, Svanes:14, Garcia-Fernandez,*et.al*:13,15,...

- **Moduli of heterotic $\mathcal{N} = 1/2$ vacua: deformations of 7D Y with integrable G_2 structure and an instanton gauge bundle V**

  de la Ossa, ML, Svanes: in progress

- Flow of heterotic $\mathcal{N} = 1/2$ solutions at $\mathcal{O}(\alpha'^0)$: the moduli spaces of *different* $SU(3)$ structures may connect via flow along domain wall direction in Y with integrable G_2 structure.

  de la Ossa, ML, Svanes:14

Geometric moduli: $\mathcal{N} = 1$

Naive moduli space

- $\partial_t \Psi$: Complex structure moduli $H_{\mathrm{d}}^{(2,1)}(X)$
- $\partial_t \omega$: $H = 0$ Kähler moduli $H_{\mathrm{d}}^{(1,1)}(X)$
  $H \neq 0$ Hermitian moduli
- $\partial_t A$: Vector bundle moduli

Candelas, de la Ossa:91, Becker, *et.al*:05,06

Geometric moduli: $\mathcal{N} = 1/2$

de la Ossa, ML, Svanes:in progress

Naive moduli space

- $\partial_t \varphi$ / $\partial_t \psi$: G_2 moduli
- $\partial_t A$: Vector bundle moduli

Geometric moduli

$$\partial_t \psi = \frac{1}{3!} M_t^a \wedge \psi_{bcda} \, \mathrm{d}x^{bcd} \ , \qquad M_t^a = M_{t\,b}{}^a \, \mathrm{d}x^b \ .$$

- Diffeomorphisms: $\boxed{\mathcal{L}_V \psi = -\frac{1}{3!} \left(D_\theta V^a \right) \wedge \psi_{bcda} \, \mathrm{d}x^{bcd}}$

  where $V \in TY$, $\quad D_\theta V^a = \mathrm{d}V^a + \theta_b{}^a \, V^b = \mathrm{d}V^a + \Gamma_{bc}{}^a \, \mathrm{d}x^c \, V^b$.

  $\Gamma_{bc}{}^a$: connection symbols of metric G_2 compatible ∇

- Preserve $\tau_2 = 0$: $\boxed{(\check{D}_\theta \Delta_t^a) \wedge \psi_{bcda} \, \mathrm{d}x^{bcd} = 0}$

  where $\Delta_{t\,b}{}^a = M_{t\,b}{}^a - \frac{1}{7} \left(\mathrm{tr} M_t \right)$

Geometric moduli: $\mathcal{N} = 1/2$

de la Ossa, ML, Svanes:in progress

Comment on connections on G_2 structure manifolds

- TY admits different connections.
- Two-parameter family of metric compatible connections. Bryant:03
- Connection with totally antisymmetric torsion $\iff \tau_2 = 0$ Friedrich:06
- Unique connection $\nabla^S = \nabla_{LC} + H$ with $Hol(\nabla^S) = G_2$ and totally antisymmetric torsion Bryant:03
  $H = \frac{1}{6}\tau_0\,\varphi - \tau_3 - (\tau_1 \lrcorner \psi)$.
- Variations of G_2 structure $\sim D_\theta$
  - $D_\theta V^a = \mathrm{d}V^a + \theta_b{}^a\, V^b$, $\qquad \theta_b{}^a = \Gamma_{bc}{}^a\, \mathrm{d}x^c$
  - $\Gamma_{bc}{}^a$: connection symbols of metric G_2 compatible ∇
  - D_θ is G_2 metric compatible only when torsion of ∇ is totally antisymmetric.
    $T = H \quad \Longrightarrow \quad D_\theta = \nabla_{LC} - H$
- Just as for forms, can project d, D_θ onto G_2 irreps.

Geometric moduli: $\mathcal{N} = 1/2$

de la Ossa, ML, Svanes:in progress

Decomposition of de Rham cohomology

Fernandez–Ugarte:98

Construct analogue of Dolbeault operator on a complex manifold

- The differential operator $\check{D}$ is defined by

$$\check{D}_0 = \mathrm{d} \ , \quad \check{D}_1 = \pi_7 \circ \mathrm{d} \ , \quad \check{D}_2 = \pi_1 \circ \mathrm{d} \ .$$

- $\tau_2 = 0 \iff \check{D}^2 = 0$. Then

$$0 \to \Lambda^0(Y) \xrightarrow{\check{D}} \Lambda^1(Y) \xrightarrow{\check{D}} \Lambda^2_7(Y) \xrightarrow{\check{D}} \Lambda^3_1(Y) \to 0$$

  is a differential, elliptic complex.

- Associated "Dolbeault" cohomology ring $\check{H}^*(Y)$

Geometric moduli: $\mathcal{N} = 1/2$

de la Ossa, ML, Svanes:in progress

Geometric moduli

- Infinitesimal moduli space (possibly infinite dimensional)

$$\mathcal{TM}_0(Y,\varphi) = \frac{\{\Delta \in \Lambda^1(Y, TY)\ :\ (\check{D}_\theta {\Delta_t}^a) \wedge \psi_{bcda}\, \mathrm{d}x^{bcd} = 0\}}{\{\Delta \in \Lambda^1(Y, TY)\ :\ \Delta^a = D_\theta V^a, \quad V \in TY\}}\ .$$

where $\check{D}_\theta {\Delta_t}^a = \pi_7(D_\theta {\Delta_t}^a)$.

- Variational constraints on torsion

G_2 holonomy

Joyce:96

No torsion $\implies \partial_t\varphi \in H^3_1(Y) \oplus H^3_{27}(Y)$, Y compact $\implies \dim \mathcal{TM}_0(Y,\varphi) = b^3$

Geometric moduli: $\mathcal{N} = 1/2$

de la Ossa, ML, Svanes:in progress

Geometric moduli

- Infinitesimal moduli space (possibly infinite dimensional)

$$\mathcal{TM}_0(Y,\varphi) = \frac{\{\Delta \in \Lambda^1(Y,TY) \;:\; (\check{D}_\theta \Delta_t{}^a) \wedge \psi_{bcda}\,\mathrm{d}x^{bcd} = 0\}}{\{\Delta \in \Lambda^1(Y,TY) \;:\; \Delta^a = D_\theta V^a, \quad V \in TY\}}$$
$$= \check{H}^1_{D_\theta}(Y,TY) \oplus \widetilde{\mathcal{TM}}_0(Y,\varphi).$$

- Variational constraints on torsion.

Constraint on D_θ

$\check{H}^1_{D_\theta}(Y,TY)$ is a cohomology $\iff (\check{D}^2_\theta V^a) = 0$

$\iff \boxed{R(\theta) \wedge \psi = 0}$ θ connection is an instanton

Bundle moduli: $\mathcal{N} = 1$

Atiyah:57, Anderson, *et.al*:10,11,13,...

Atiyah class stabilization

- Vector bundle moduli: $H^1_{\bar\partial}(\mathrm{End}(V))$

$F^{(2,0)} = 0 \iff F \wedge \Psi = 0$: couples bundle and complex structure moduli

- $\partial_t \Psi = \frac{1}{2}\alpha_t{}^a \wedge \Psi_{abc}\mathrm{d}x^{bc}$, $\partial_t F = \mathrm{d}_A \partial_t A$

$$\boxed{\partial_t(F \wedge \Psi) = 0 \iff \mathrm{d}_A\partial_t A \wedge \Psi = F_{ab}\mathrm{d}x^b \wedge \alpha_t{}^a \wedge \Psi}$$

- $\partial_t \Psi = 0 \implies \bar\partial_A(\partial_t A)^{(0,1)} = 0$; $\partial_t \Psi \neq 0 \implies \alpha_t$ constrained
- Atiyah map

$$\begin{array}{cccc} \mathcal{F}: & \Lambda^p(X, TX) & \longrightarrow & \Lambda^{p+1}(X, \mathrm{End}(V)) \\ & \alpha & \mapsto & \mathcal{F}(\alpha) = -F_{ab}\,\mathrm{d}x^b \wedge \alpha^a\ . \end{array}$$

  $\mathcal{F}$ is a map in cohomology:
  Bianchi identity $\mathrm{d}_A F = 0 \implies \mathcal{F}(\bar\partial\alpha) + \bar\partial_A(\mathcal{F}(\alpha)) = 0$

- Corrected moduli space for bundle and complex structure moduli:

$$\boxed{H^1_{\bar\partial}(\mathrm{End}(V)) \oplus \ker(\mathcal{F})}$$

Bundle moduli: $\mathcal{N} = 1/2$

de la Ossa, ML, Svanes:in progress

"Atiyah" class stabilization

Instanton condition $F \wedge \psi = 0$: couples bundle and geometric moduli

$$\boxed{0 = \partial_t(F \wedge \psi) \iff \check{D}_A(\partial_t A) = -\check{\mathcal{F}}(\Delta_t)\ .}$$

where $\Delta_t \in \Lambda^1(Y, TY)$, D_A covariant derivative, and $\check{D}_A$, $\check{\mathcal{F}}$: project to G_2 irreps

- "Atiyah" map

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  $\check{\mathcal{F}}$ is a map in cohomology:
  Bianchi identity $\mathrm{d}_A F = 0 \implies \check{\mathcal{F}}(\check{D}_\theta(\Delta)) + \check{D}_A(\check{\mathcal{F}}(\Delta)) = 0.$

  In fact, $\check{\mathcal{F}}$ maps **any** form $\Delta \in \mathcal{TM}_0(Y, \varphi)$ to a $\check{D}_A$-closed form

- Corrected moduli space for bundle and geometric moduli:

$$\boxed{H^1_{\check{D}_A}(\mathrm{End}(V)) \oplus \ker(\check{\mathcal{F}})}$$

- $\ker(\check{\mathcal{F}}) \subset \mathcal{TM}_0(Y, \varphi)$ may still be infinite-dimensional.

Infinitesimal Moduli: $\mathcal{N} = 1/2$

de la Ossa, ML, Svanes:in progress

Instanton condition $R(\theta) \wedge \psi = 0$: θ must vary with ψ

- Extra moduli for connection variations.
  $\mathcal{N} = 1$:
  Connection variations are related to field redefinitions. de la Ossa, Svanes:14
  SUSY constraints and BI $\implies$ EOM $\iff$ $R(\theta)$ is an instanton
  Ivanov:10, Martelli, Sparks:10
- Symmetry between TY connection θ and gauge connection A
  - $\mathcal{R}$ map

$$\begin{array}{cccc} \mathcal{R}: & \Lambda^p(Y, TY) & \longrightarrow & \Lambda^{p+1}(Y, \mathrm{End}(TY)) \\ & \Delta & \mapsto & \mathcal{R}(\Delta) = -R_{ab}\, \mathrm{d}x^b \wedge \Delta^a \ . \end{array}$$

  $\check{\mathcal{R}}$: map in cohomology that maps **any** $\Delta \in \mathcal{TM}_0(Y, \varphi)$ to a $\check{D}_\theta$-closed form
- Corrected moduli space for bundle, instanton and geometric moduli:

$$\boxed{H^1_{\check{D}_A}(\mathrm{End}(V)) \oplus H^1_{\check{D}_\theta}(\mathrm{End}(TY)) \oplus \ker(\check{\mathcal{F}} + \check{\mathcal{R}})}$$

- $\ker(\check{\mathcal{F}} + \check{\mathcal{R}}) \subset \mathcal{TM}_0(Y, \varphi)$ may still be infinite-dimensional.

Anomaly cancellation condition

de la Ossa, ML, Svanes:in progress

Corrected moduli space for bundle, instanton and geometric moduli:

$$\boxed{H^1_{\check{D}_A}(\mathrm{End}(V)) \oplus H^1_{\check{D}_\theta}(\mathrm{End}(TY)) \oplus \ker(\check{\mathcal{F}} + \check{\mathcal{R}})}$$

Last equation to bring in: anomaly cancellation condition

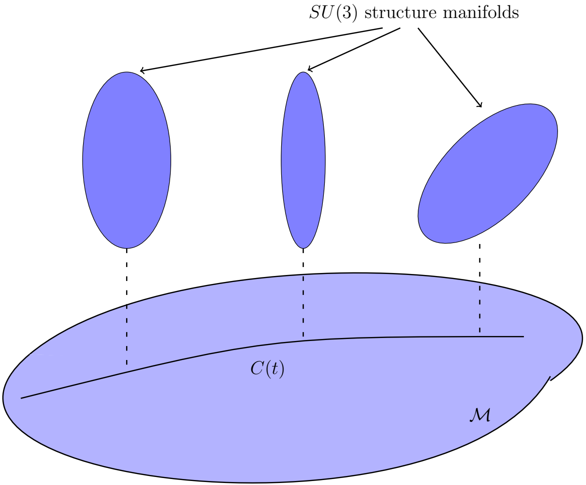
$$H = \mathrm{d}B + \frac{\alpha'}{4}\left(\mathcal{CS}[A] - \mathcal{CS}[\theta]\right) \qquad (\star)$$

- $\mathcal{N} = 1$: Anderson, Gray, Sharpe:14, de la Ossa, Svanes:14,,...
  Define extension bundle E for complex structure, bundle and connection:
  $0 \longrightarrow \mathrm{End}(V) \oplus \mathrm{End}(TX) \longrightarrow E \longrightarrow TX \longrightarrow 0$

  Vary $(\star)$ together with $H = \mathrm{d}^c\omega \rightsquigarrow$ map $\boxed{\mathcal{H} : \Lambda^p(X, E) \longrightarrow \Lambda^{p+1}(X, T^*X)}$
  - restricts E bundle moduli to lie in $\ker(\mathcal{H})$
  - finite-dim Hermitian moduli space
- $\mathcal{N} = 1/2$
  Vary $(\star)$ together with $H = \frac{1}{6}\,\tau_0\,\varphi - \tau_1 \lrcorner \psi - \tau_3 \implies$ map $\mathcal{H}$
  $\mathcal{H}$ well-defined in cohomology? Finiteness of moduli space? In progress...

Flow of SU(3) structures

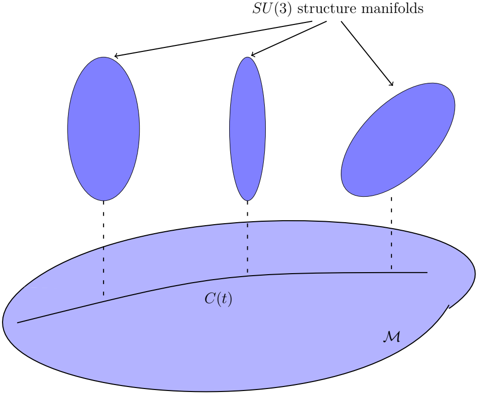
de la Ossa, ML, Svanes:14



t parametrizes a curve in the moduli space of SU(3) structures

Flow of SU(3) structures

de la Ossa, ML, Svanes:14



Two options:

- Fix torsion classes of SU(3) structure.
- Flow between different types of SU(3) structure.

Flow of SU(3) structures: Example

de la Ossa, ML, Svanes:14

Calabi–Yau with flux

- Assume X has $W_i = 0$ for $t = 0$.
- Embed $\varphi = N \wedge \omega + \mathrm{Re}(\Psi)$
- 7D flux (determined by SUSY): $H = c_1 \mathrm{d}t \wedge \omega + c_2 \mathrm{Re}(\Psi) + c_3 \mathrm{Im}(\Psi) + J(\gamma)$.

Torsion classes preserved by flow $\iff$ γ $SU(3)$-harmonic.

Non-harmonic γ: flow from CY to non-complex SU(3) structure ($\mathrm{Re} W_2 \neq 0$).

Conclusions and outlook

Conclusions

4D heterotic $\mathcal{N} = 1/2$ DW solutions from compactifications on $Y = \mathbb{R} \times_w X$:

- Y Integrable G_2 structure
- X Conformally balanced (non-complex) SU(3) structure

Infinitesimal moduli space of integrable G_2 structures with instanton bundle V:

$$H^1_{\check{D}_A}(\mathrm{End}(V)) \oplus H^1_{\check{D}_\theta}(\mathrm{End}(TY)) \oplus \ker(\check{\mathcal{F}} + \check{\mathcal{R}}),$$

$$\ker(\check{\mathcal{F}} + \check{\mathcal{R}}) \subset \mathcal{T}\mathcal{M}_0(Y, \varphi) \sim \check{H}^1_{D_\theta}(Y, TY) \oplus \widetilde{\mathcal{T}\mathcal{M}}_0(Y, \varphi)$$

- θ and A connections are instantons: $R \wedge \psi = 0 = F \wedge \psi$
- Variational constraints on G_2 torsion

Flow of X along DW direction:

- DW flow can change the torsion of the SU(3) structure

Conclusions and outlook

Conclusions

Infinitesimal moduli space of integrable G_2 structures with instanton bundle V:

$$H^1_{\check{D}_A}(\mathrm{End}(V)) \oplus H^1_{\check{D}_\theta}(\mathrm{End}(TY)) \oplus \ker(\check{\mathcal{F}} + \check{\mathcal{R}}),$$
$$\ker(\check{\mathcal{F}} + \check{\mathcal{R}}) \subset \mathcal{T}\mathcal{M}_0(Y, \varphi) = \check{H}^1_{D_\theta}(Y, TY) \oplus \widetilde{\mathcal{T}\mathcal{M}}_0(Y, \varphi)$$

Outlook

- Include constraint from Bianchi identity $\mathrm{d}H = \ldots$ (in progress).
  $\rightsquigarrow$ Determine conditions for finite-dimensional infinitesimal moduli space.
- Relation of $SU(3)$ and G_2 structure moduli spaces for domain wall solutions.
- Relevance for compactifications of M-theory and type II string theory.