Quantum correlations and entanglement in far-from-equilibrium spin systems

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PRL **110**, 075301 (2013), *Far from equilibrium quantum magnetism with ultracold polar molecules*

PRL **111**, 260401 (2013), *Breakdown of quasilocality in long-range quantum lattice models*

PRA **90**, 063622 (2014), *Quantum correlations and entanglement in far-from-equilibrium spin systems*

see also: Bachelor thesis K. Saka (U. Göttingen), PRL **107**, 115301 (2011); PRA **84**, 33619 (2011)

Quantum Many Body Systems: Entanglement

I) Superposition of states is *also* a possible state

II) Entanglement: (spin-1/2 particles, e.g., electrons)

possible states:

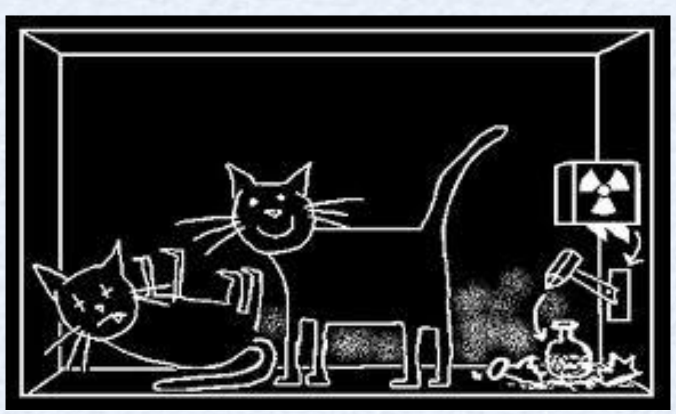
$$|\psi\rangle = \begin{cases} |\uparrow\rangle \otimes |\uparrow\rangle \\ |\uparrow\rangle \otimes |\downarrow\rangle \\ |\downarrow\rangle \otimes |\uparrow\rangle \\ |\downarrow\rangle \otimes |\downarrow\rangle \end{cases}$$

“classical”, “product state”

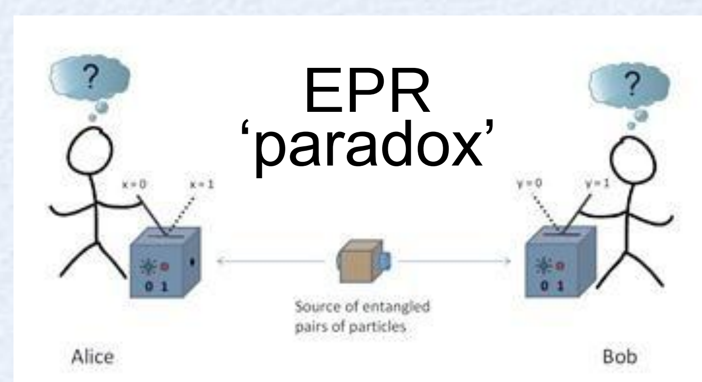
$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle \otimes |\uparrow\rangle + |\downarrow\rangle \otimes |\downarrow\rangle\right)$$

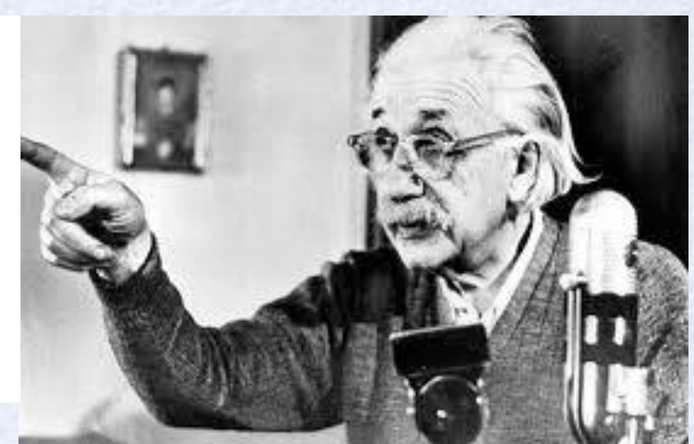
“entangled”: not a product state

Schrödingers cat



Einstein: “spooky action at a distance”





Quantum Many Body Systems: Correlation Effects

Correlated states:

"mean-field" picture of independent particles breaks down

$$\langle n_i n_j \rangle \neq \langle n_i \rangle \langle n_j \rangle$$

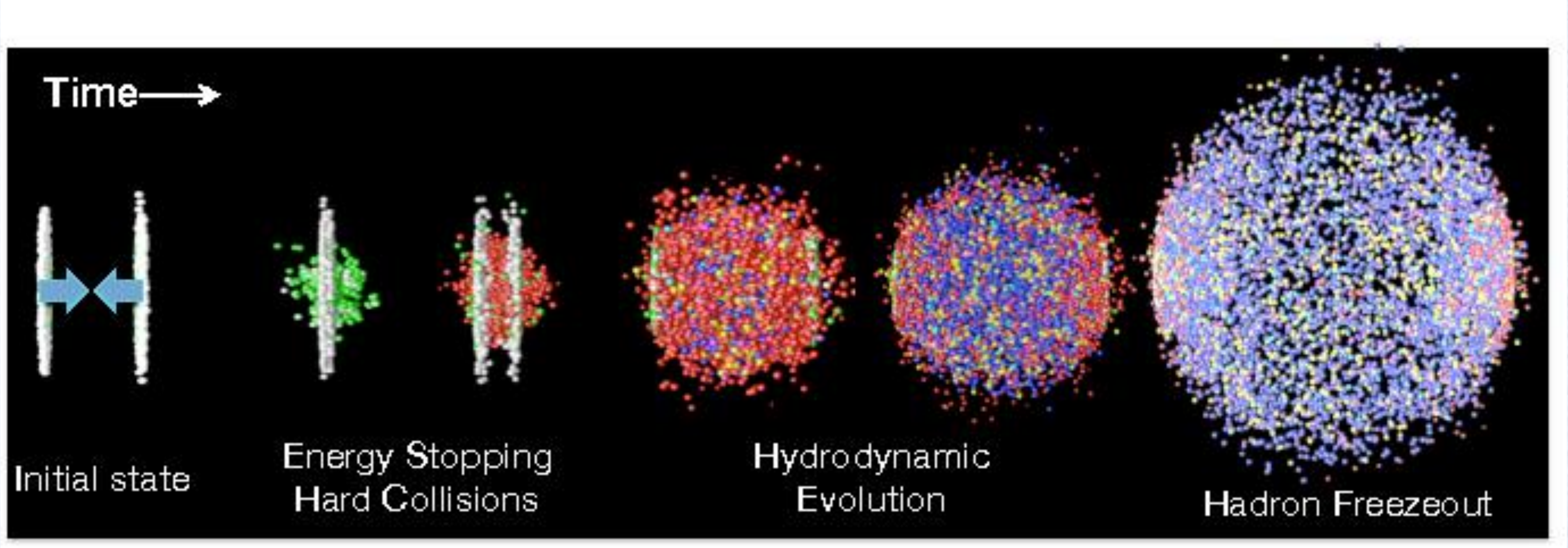
➠ Particles at sites *i* and *j* 'influence' each other

a) because of entanglement

b) because of mutual interactions.

Quantum Many Body Systems: Out-of-Equilibrium Dynamics

Example (high-energy physics):
heavy ion collisions



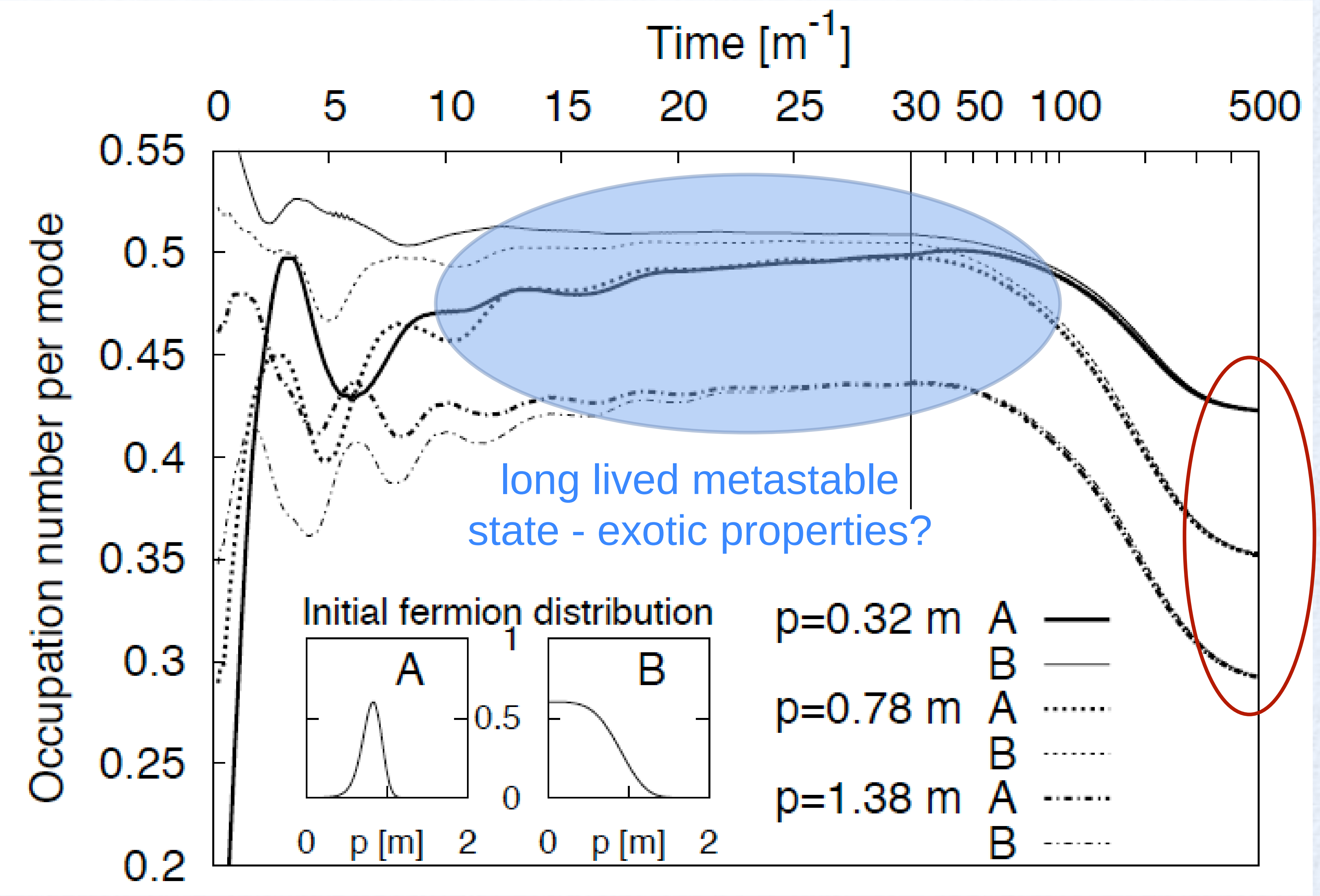
[from inspirehep.net]

Fundamental questions:

- How does the system ‘relax’ towards the ‘final state’?
- Temperature in the system?
- How do correlations and entanglement evolve in time?

Unconventional states: Out-of-Equilibrium Dynamics

"Prethermalization"

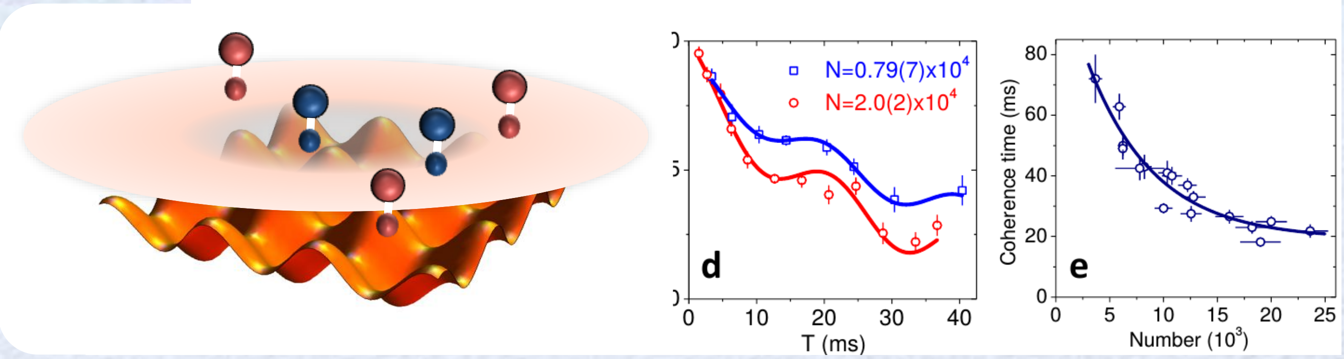


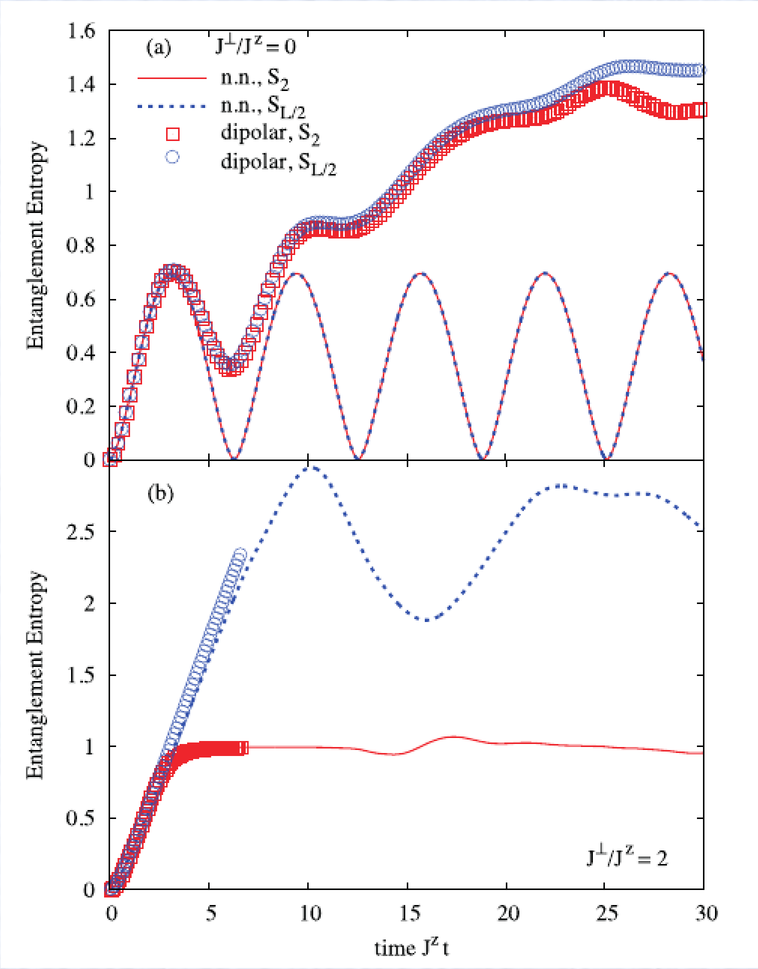
thermal
final state

[Berges *et al.*, PRL 2004]

Main Messages of the Talk:

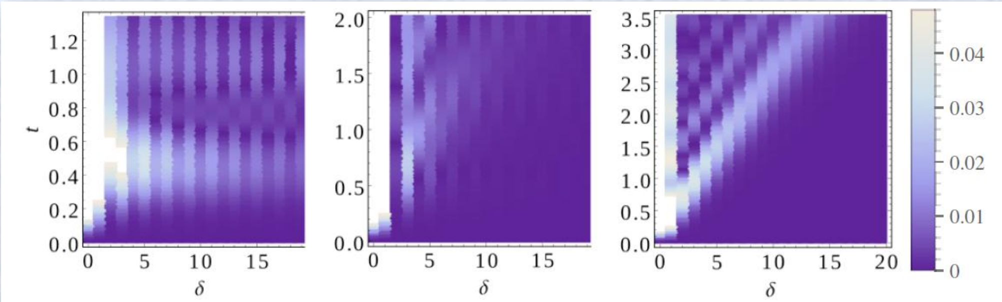
I) Experiments with ultracold polar molecules:
Dipolar Spin exchange interactions





II) Compare dynamics of correlations and entanglement of dipolar systems with short-range systems

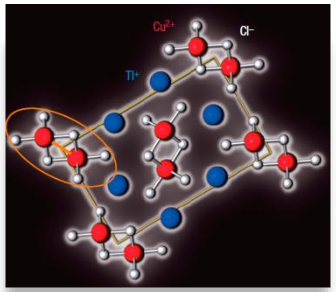
III) Generic algebraically decaying interactions:
Lieb-Robinson-type bounds for causality?



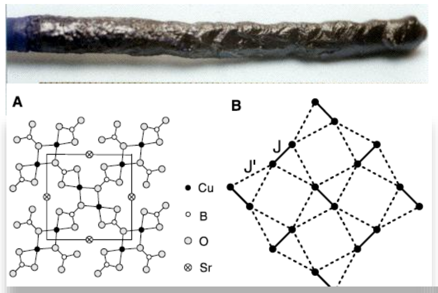
Correlated Systems in Nature: Quantum Magnets

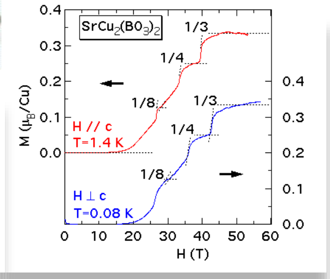
➠ Networks of static spins, realize collective quantum phenomena, e.g.

- $TlCuCl_3$ (S=1/2 ladder): Bose-Condensation of Triplet Excitations (Magnons)

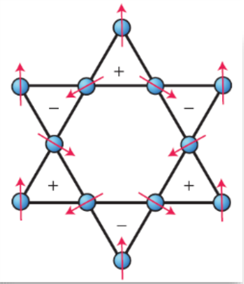


- $SrCu_2(BO_3)_2$ (S=1/2 Shastry-Sutherland lattice:
  fractional magnetization plateaux, magnetic superstructures
  (spin-supersolid?)





- Herbertsmithite $ZnCu_3(OH)_6Cl_2$ (S=1/2 kagome lattice):
  Spin liquid? (DMRG: yes!)



“Standard” Model to describe Quantum Magnetism: Heisenberg or XXZ Model

$$\mathcal{H} = \sum_{ij} J_{ij}^{\perp} \left(S_i^x S_j^x + S_i^y S_j^y \right) + J_{ij}^z S_i^z S_j^z$$

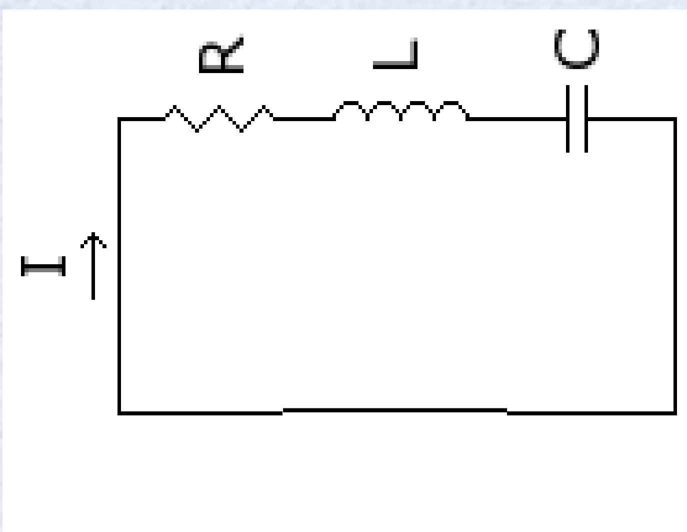
Here: Dynamics in systems with tuneable short and long-range interactions

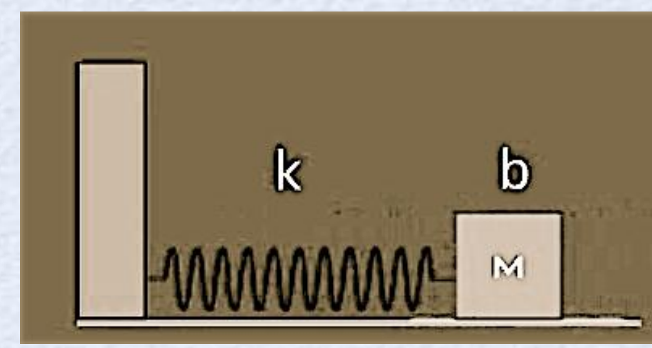
Quantum Simulators: Correlated Systems

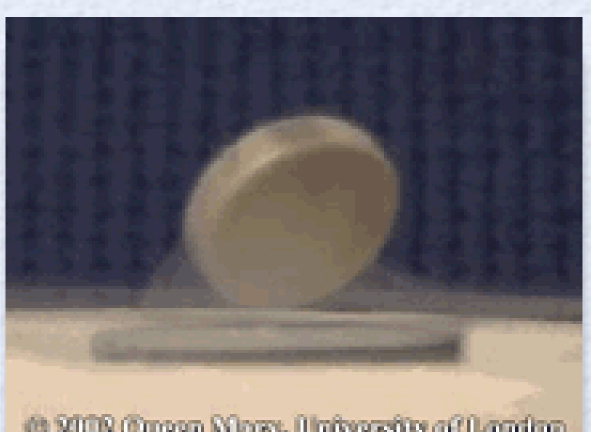
Idea: Use a well controlled quantum system to describe another, more difficult one (R.P. Feynman 1982, Y.I. Manin 1980)

➥ Quantum-Many-Body-*Models* via ultracold gases on optical lattices

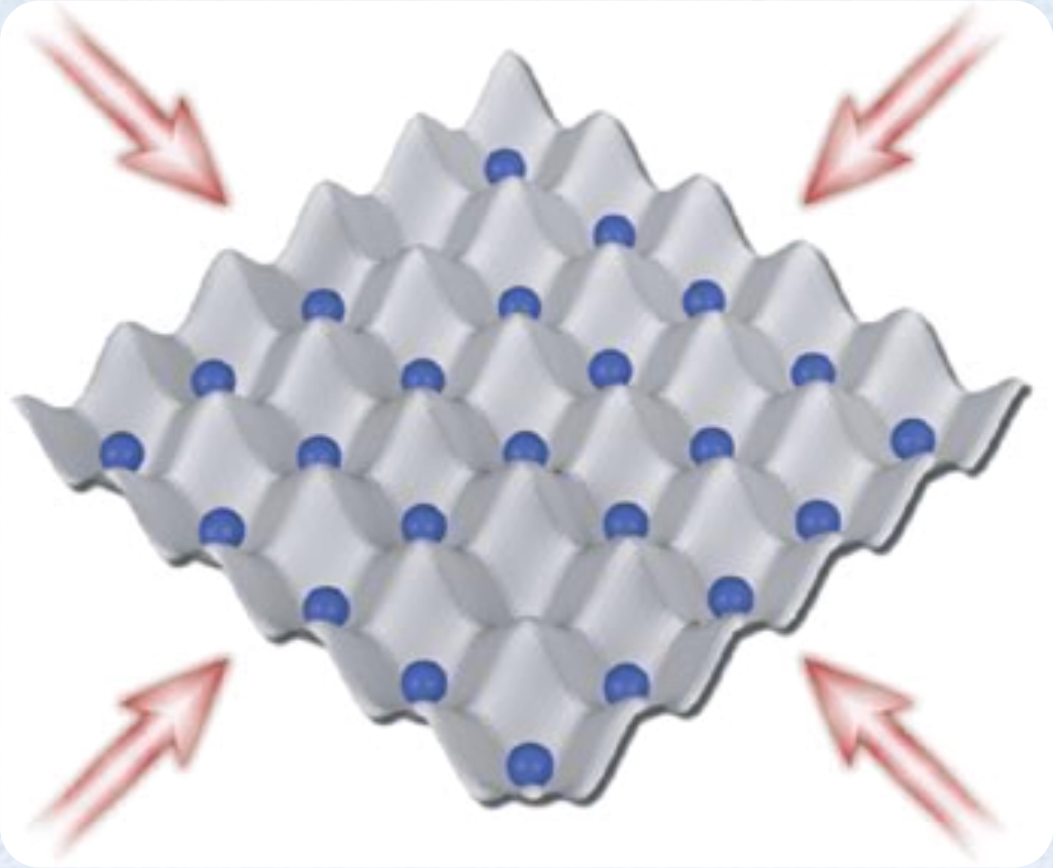
Similarity: compare electrical and mechanical networks







Optical Lattices



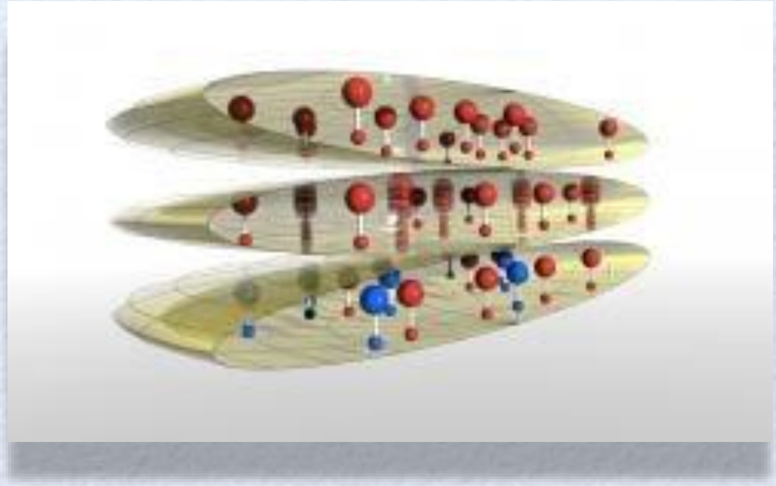
Standing waves of laser light: periodic structures

Mechanism: Stark-Effect

➠ Induced dipolemoment in neutral atoms leads to a trapping force in the periodic potential: “Crystals of Light”

Ultracold polar molecules: dipolar t-J and XXZ Model

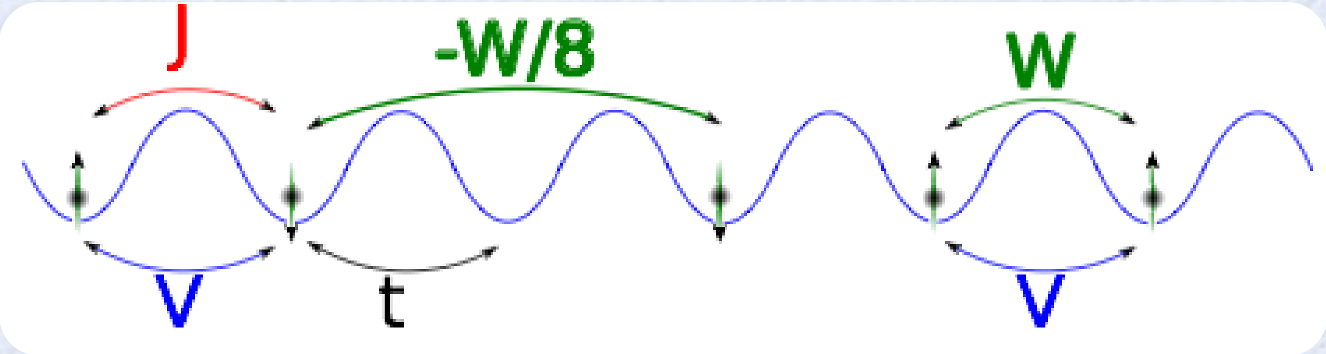
[A.V. Gorshkov, S.R. Manmana et al., PRL & PRA (2011)]



polar Molecules (e.g. KRb) in optical lattices:
2 rotational states ⇔ two spin states

Effective Model:

$$\mathcal{H} = -t\sum_{j,\sigma}\left[c^\dagger_{j,\sigma}c_{j+1,\sigma} + h.c.\right] + \sum_{i,j}\frac{1}{|i-j|^3}\left[\frac{J_\perp}{2}\left(S_i^+S_j^- + S_i^-S_j^+\right) + J_z S_i^z S_j^z + V n_i n_j + W\left(n_i S_j^z + S_i^z n_j\right)\right]$$



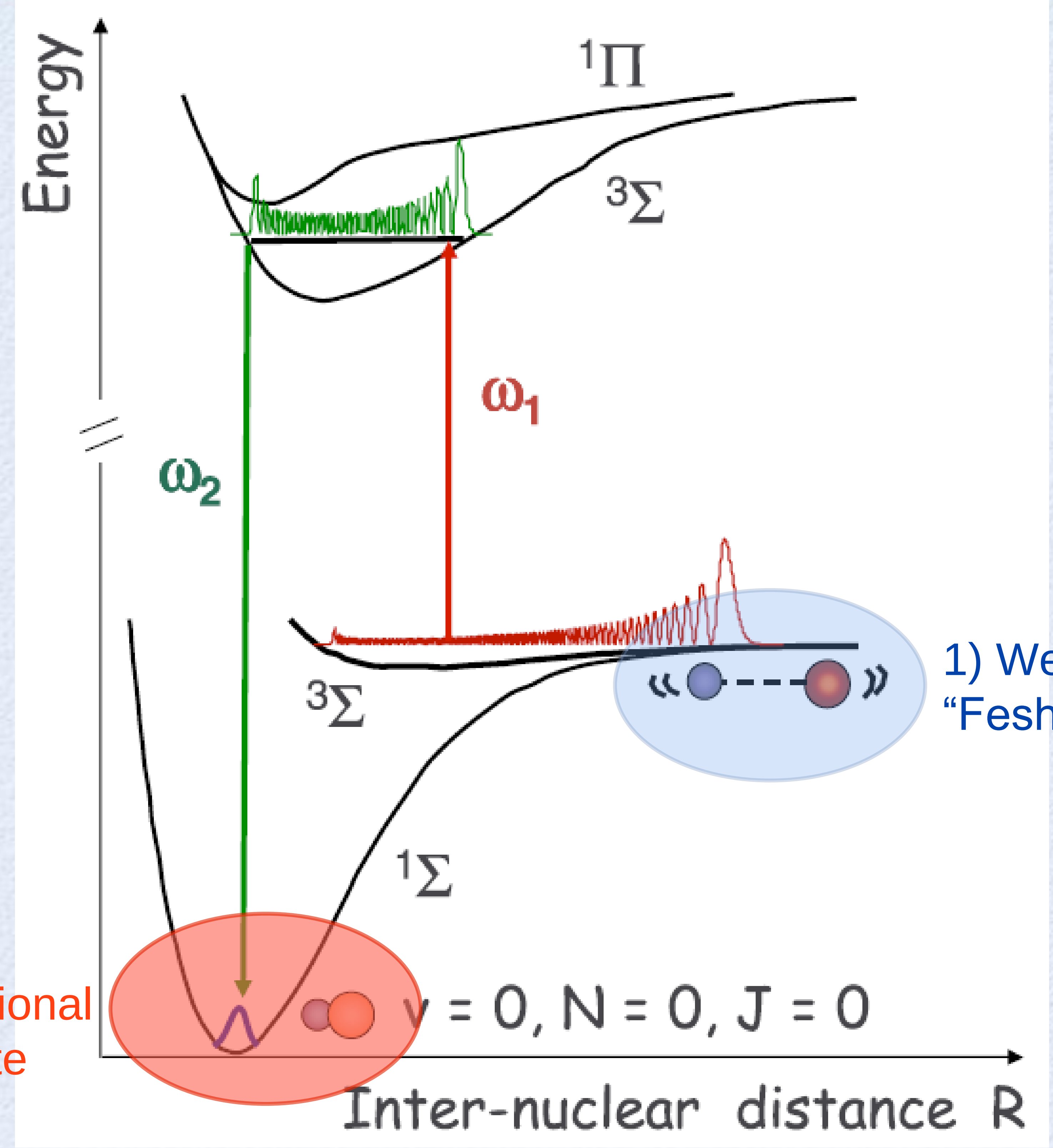
t: nearest-neighbor hopping
V: Coulomb-repulsion (long-range)
W: density-spin-interaction (long-ranged)
J: Heisenberg coupling (anisotropic, long-ranged)

Here: 1 particle per site, or very deep lattice
➠ t = V = W = 0, 1D for DMRG
➥ dipolar XXZ-chain

Ultracold polar molecules

[Review: L.D. Carr *et al.*, NJP **11**, 055049 (2009); see references therein]

2) Raman transfer scheme, "STIRAP":



1) Weakly bound "Feshbach molecule"

3) Rovibrational ground state

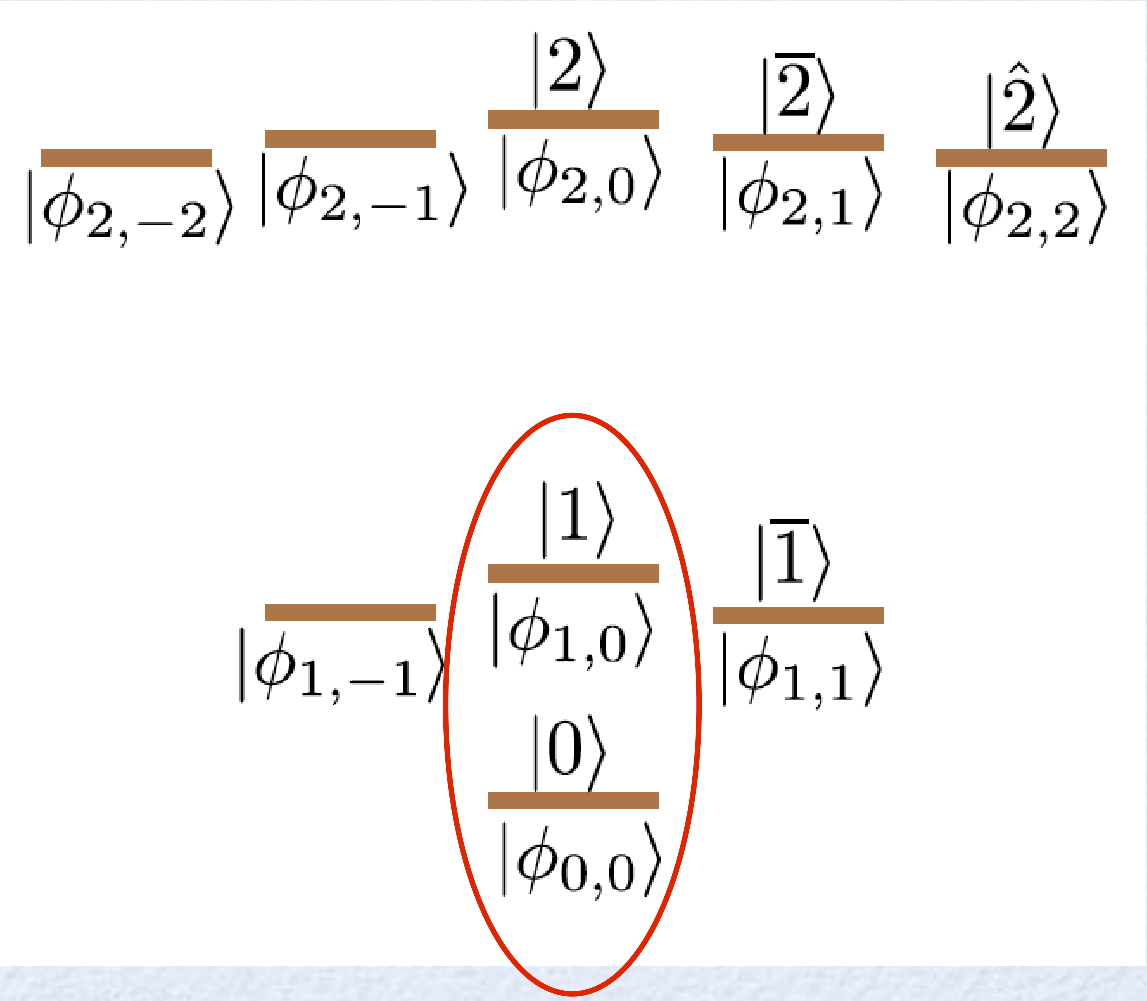
Polar molecules on optical lattices: effective models

[A.V. Gorshkov, S.R. Manmana et al., PRL & PRA (2011)]

2 basic observations:

- polar molecules are rigid rotors, e.g., in electric field: $H_0 = B\mathbf{N}^2 - d_0\vec{E}$
- dipolar, long-ranged interactions: $H_{\rm dd} = \frac{1}{2}\sum_{i\neq j}|\mathbf{R}_i - \mathbf{R}_j|^{-3}[d_0^{(i)}d_0^{(j)} + \frac{1}{2}(d_+^{(i)}d_-^{(j)} + d_-^{(i)}d_+^{(j)})]$

level scheme for
a rigid rotor in a field:



Idea: project dipolar operator onto two states ➠ effective S=1/2 system

How to verify the proposal? Dephasing of a fully polarized state

Simplification: neglect environment induced decoherence

➟ time evolution driven solely by the system's Hamiltonian
dynamics can be used to probe its properties

Example:
2 spin-1/2's polarized in x-direction: $|\psi\rangle_0 = |\rightarrow\rightarrow\rangle$ $H = J_\perp \left(S_1^x S_2^x + S_1^y S_2^y\right) + J_z\, S_1^z S_2^z$

$$= \frac{J_\perp}{2}\left(S_1^+ S_2^- + S_1^- S_2^+\right) + J_z\, S_1^z S_2^z$$

Mean field:

$H_{\mathrm{MF}} = J_\perp S_1^x \langle S_2^x \rangle$ "no field acting on the spin"

➟ $|\psi\rangle_0 = |\rightarrow\rightarrow\rangle$ eigenstate

No dynamics!

Exact many-body treatment:

spin flip terms come into play,
➟ $\langle S_i^x \rangle$ decreases

"Interaction induced dephasing",
depends on $J_\perp / J_z$
➥ directly probe many-body physics

Perturbation theory short times: $\langle S_i^x \rangle (t) \sim \langle S_i^x \rangle_0 - \alpha t^2$; $\langle S_i^y \rangle (t) \sim \langle S_i^y \rangle_0 - \beta t$

➟ apply to cold-molecules' XXZ model

Verifying the model: dephasing of a fully polarized state

What can be done now? ➠ Time evolution of simple initial states

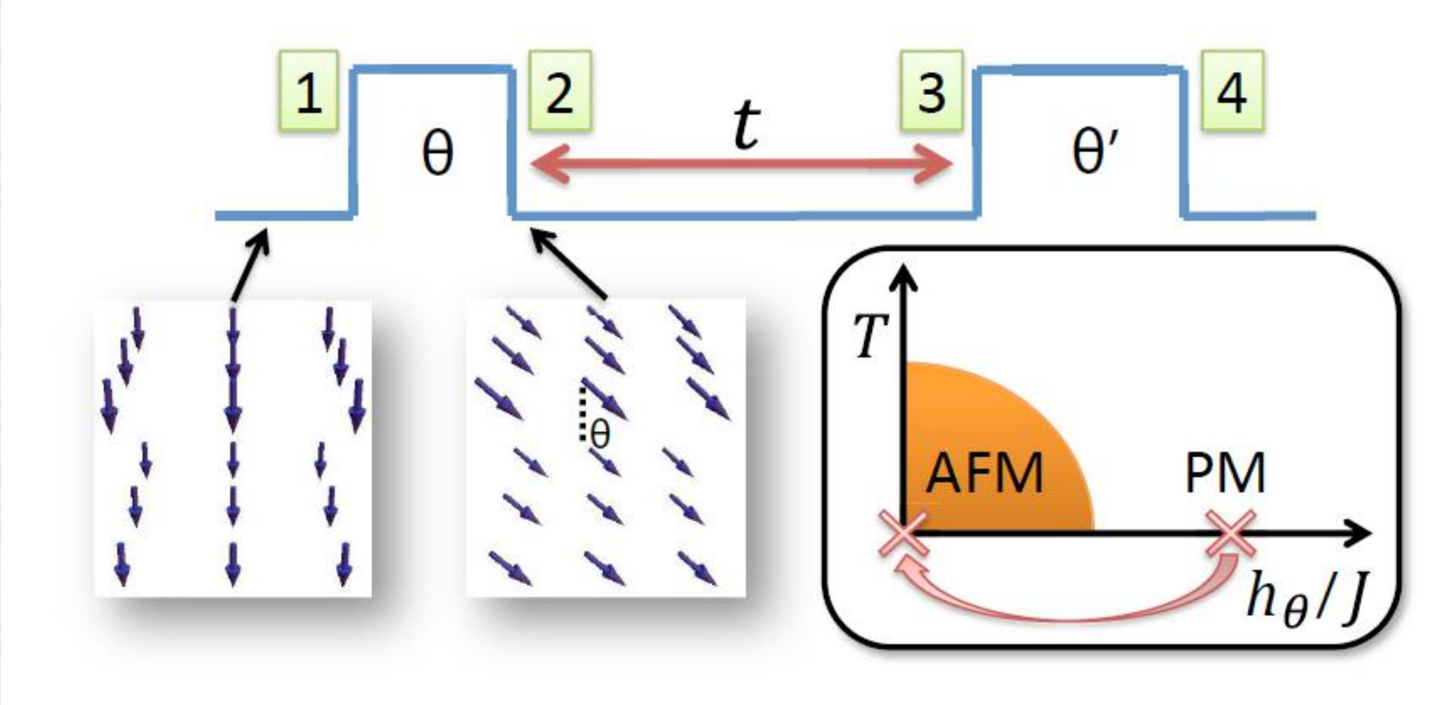
Specific case: $$\mathcal{H} = \sum_{i>j} \frac{1}{|i-j|^3} \left[\frac{J_\perp}{2} (S_i^+ S_j^- + S_i^- S_j^+) + J_z S_i^z S_j^z \right]$$

✓ one molecule per site
✓ or very deep lattice

Idea:

- apply a strong short pulse to generate fully polarized state at variable angle θ from z axis
- let evolve
- measure 〈$S_x(t)$〉, 〈$S_y(t)$〉

"Ramsey spectroscopy"...



...or "infinite quantum quench"

Verifying the model: 1) Short time limit

[K.R.A. Hazzard, S.R. Manmana, M. Foss-Feig, and A.M. Rey, PRL (2013)]

Perturbation theory: analytical results for short time behavior

$$\langle S_a^x(t)\rangle = \langle S_a^x\rangle - it\,\langle[S_a^x, H]\rangle - \frac{t^2}{2}\langle[[S_a^x, H], H]\rangle$$

➠ $\langle S^x(t)\rangle = \langle S^x(0)\rangle - \alpha t^2 + O(\{J_z, J_\perp\}\,t^4)$

with

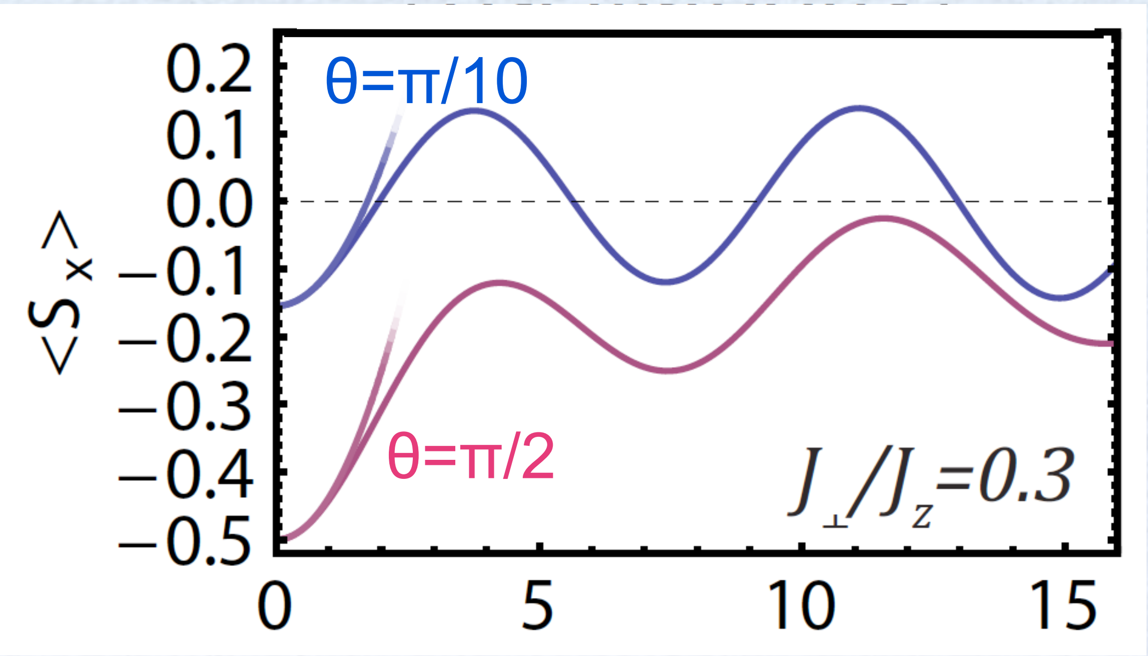
$$\alpha = \frac{1}{8}(J_z - J_\perp)^2 \sin\theta\left(2\,\zeta(6) + \cos^2\theta\left[4\,\zeta(3)^2 - \frac{2\,\pi^6}{945}\right]\right)$$

- Temperature does not play a role
- Characteristic $J_\perp/J_z$ and θ-dependence
- Also possible for other observables (e.g. $\langle S_y\rangle$, $\langle S_x^2\rangle$, $\langle S_y^2\rangle$)
- Need only very short times

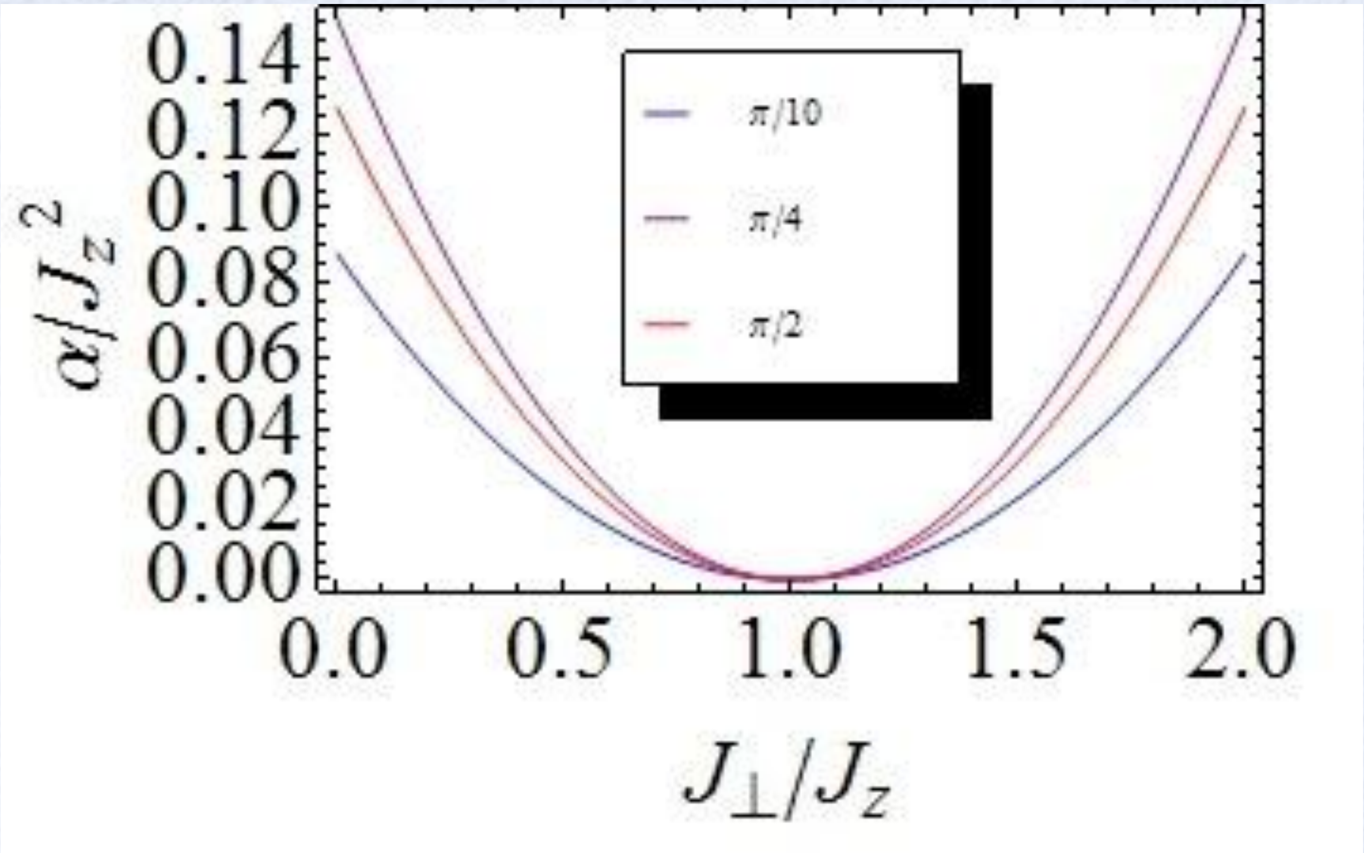
➠ Verify XXZ model in ***ongoing*** experiments

Verifying the model:
1) Short time limit

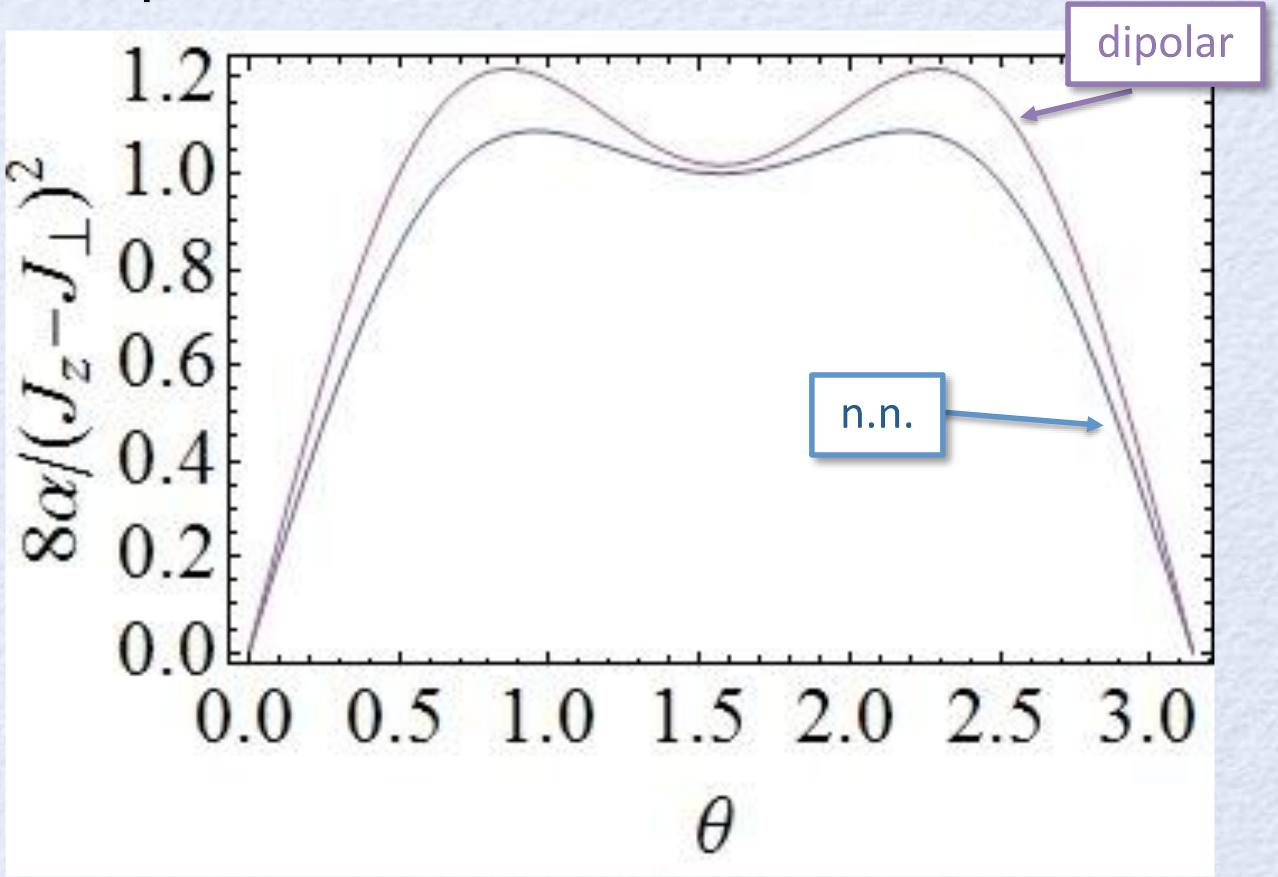
Validity of the
short time description:



J-dependence:



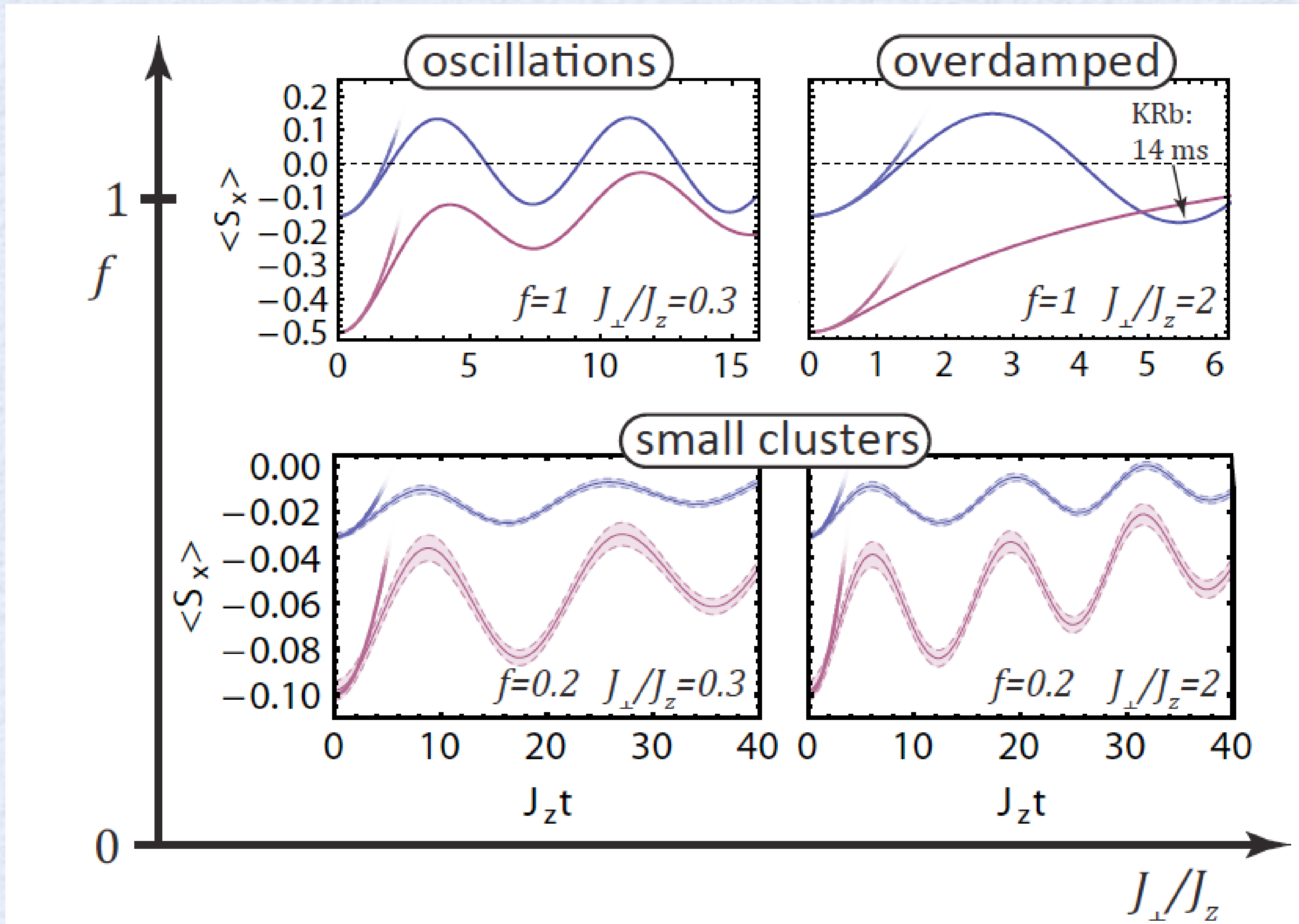
θ-dependence:



Verifying the model: dephasing of a fully polarized state

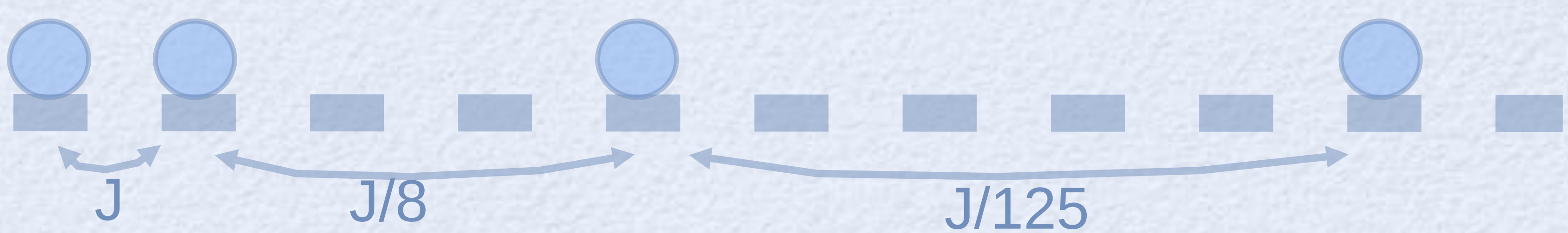
[K.R.A. Hazzard, S.R. Manmana, M. Foss-Feig, and A.M. Rey, PRL (2013)]

‘dynamical phase diagram’ (J,f):



small f:

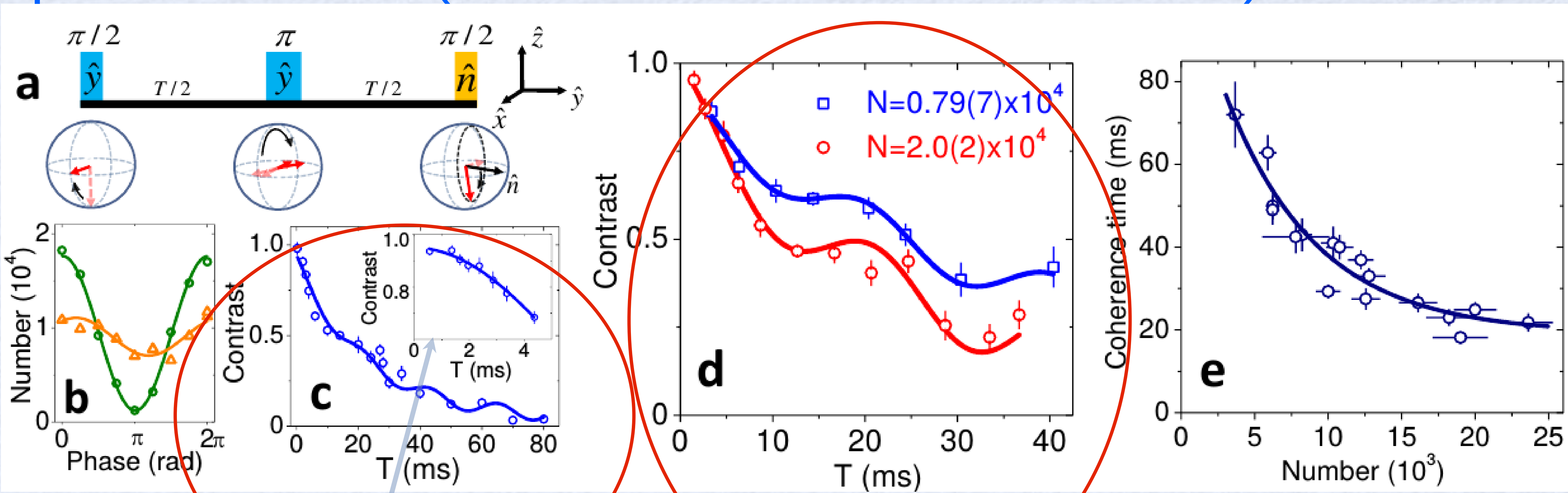
$1/r^3$ interactions:



‘2-particle cluster’ most important

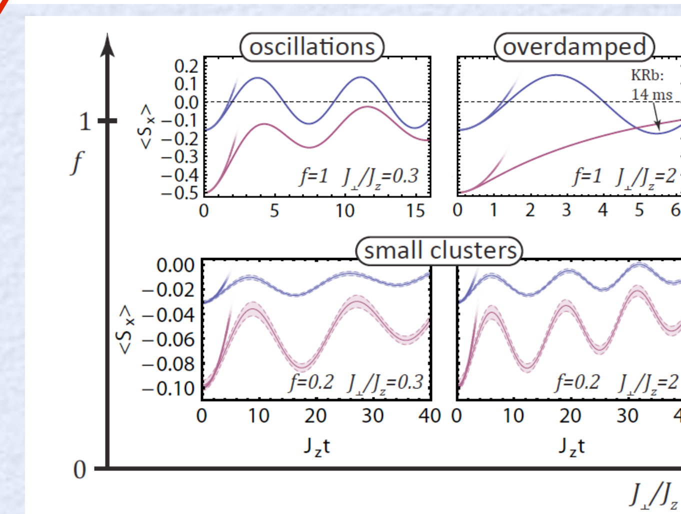
Experiments with polar molecules

Experimental test (JILA [B. Yan et al., Nature **501**, 521 (2013)]):



quadratic short time behavior

oscillations on top of decay,
filling dependence

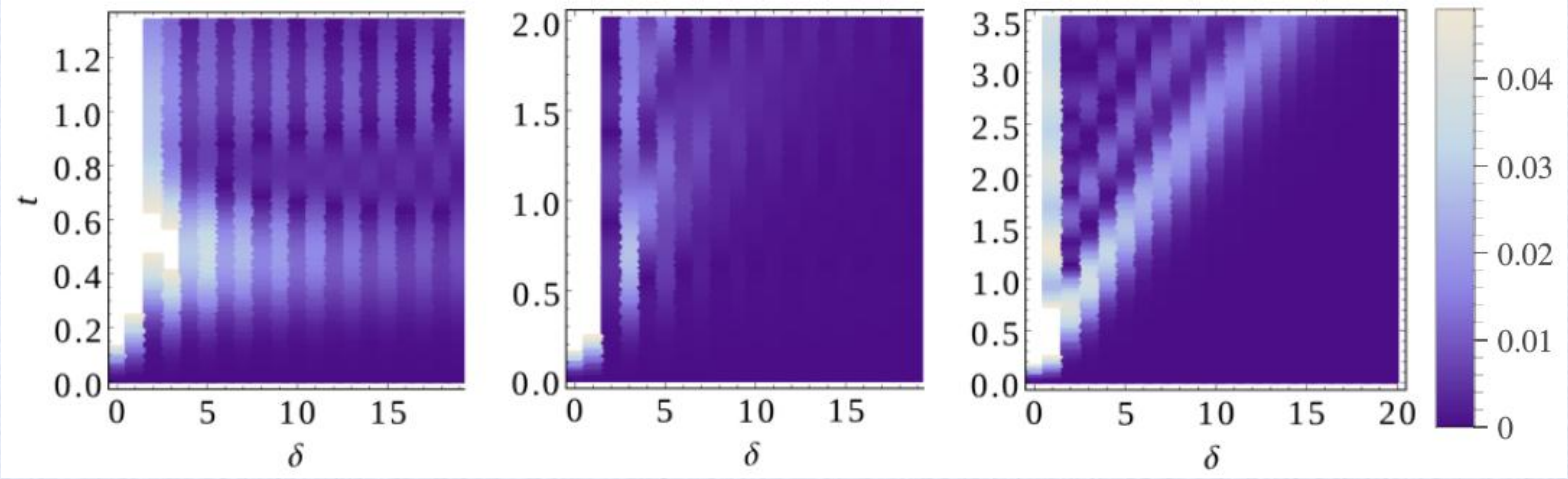


➠ evidence for dipolar interactions, points towards spin-systems ($J_\perp/2$ = 52 Hz; measured frequency: 48 ± 2 Hz)

Two natural questions:

I. Time evolution of correlations and entanglement?

II. Vary the exponent of the long-range interaction?



„Light-cone“and emergence of a causal region vs. Instantaneous propagation of information

Quasilocality: Lieb-Robinson bound

QM nonrelativistic:

local perturbations can have immediate effect everywhere

But: very small for short-range, finite-d systems:

light-cone, quasilocality & Lieb-Robinson-bound:

$$\|[O_A(t), O_B(0)]\| \leq C\|O_A\|\|O_B\| \min(|A|, |B|) \mathrm{e}^{[v|t|-d(A,B)]/\xi}$$

Long-range interactions ~ $r^{-\alpha}$?

$$\|[O_A(t), O_B(0)]\| \leq C\|O_A\|\|O_B\| \frac{\min(|A|, |B|)(e^{v|t|}-1)}{(\mathrm{d}(A, B)+1)^{\alpha}}$$

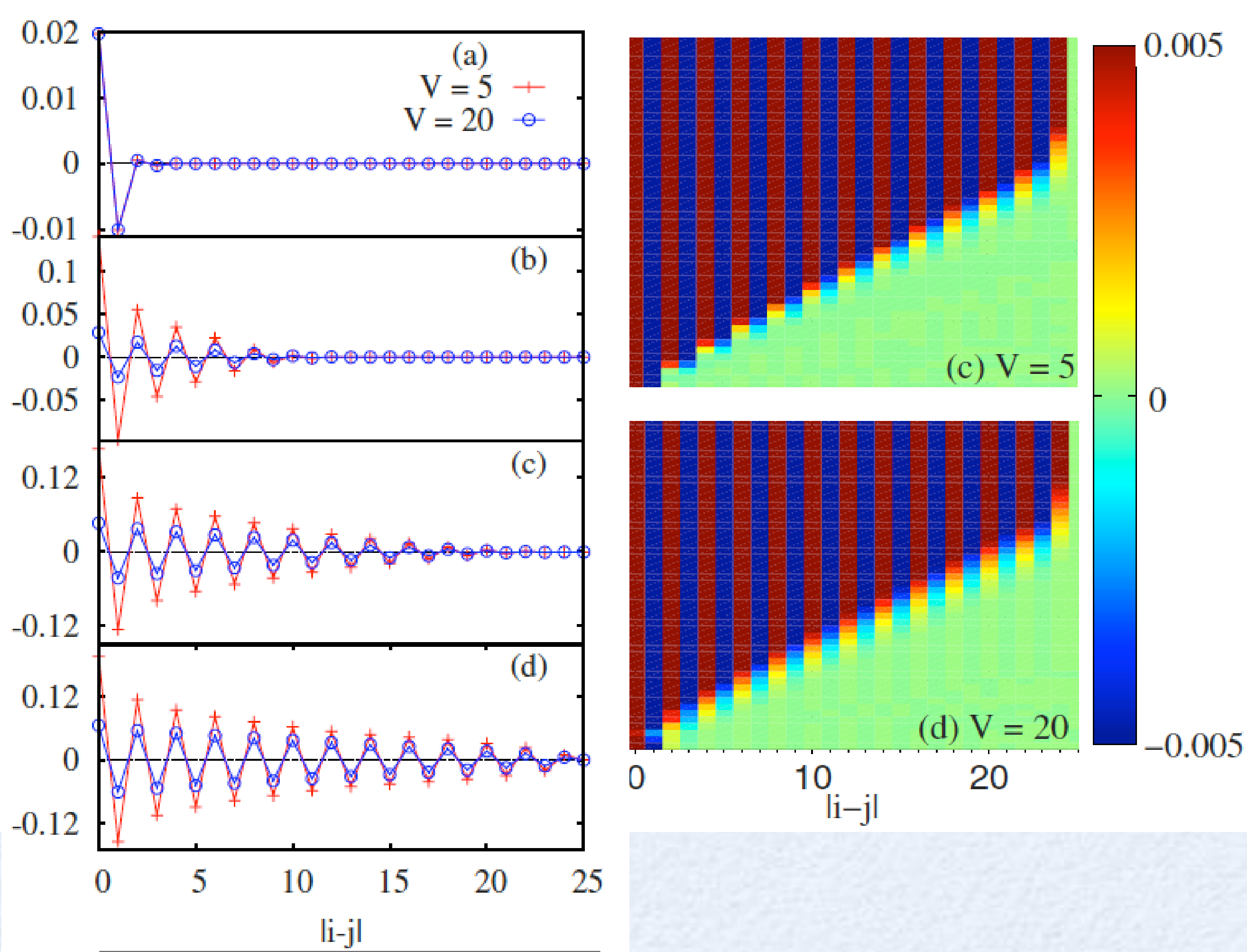
(Koma&Hastings 2006)

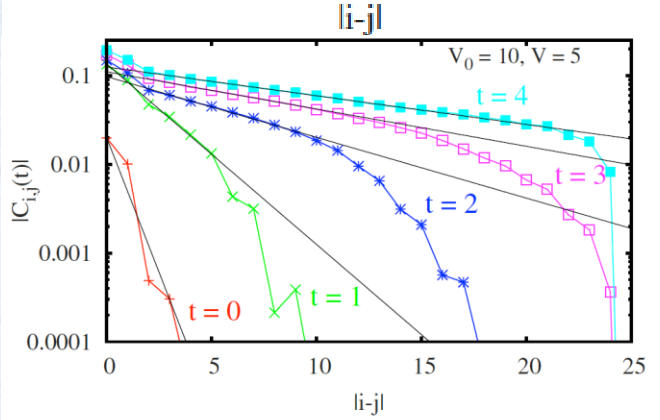
Logarithmic behaviour $v|t| > \ln\left[1 + \frac{\epsilon[1+\mathrm{d}(A, B)]^{\alpha}}{\min(|A|, |B|)}\right]$

Short-range systems: Nature of the light-cone

[S.R. Manmana, S. Wessel, R.M. Noack, and A. Muramatsu, PRB **79**, 155104 (2009)]

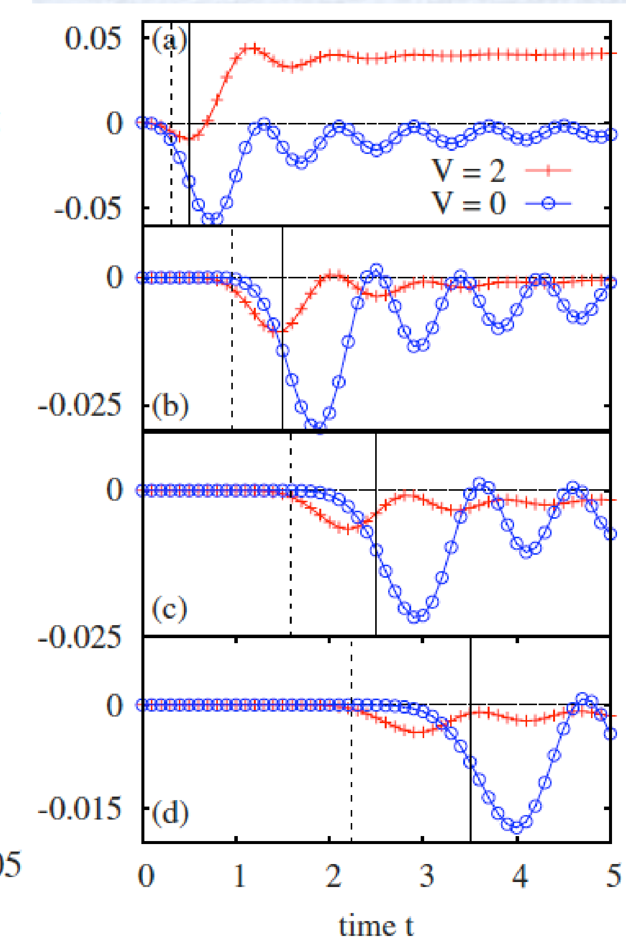
$$\langle n_i n_j \rangle - \langle n_i \rangle \langle n_j \rangle$$





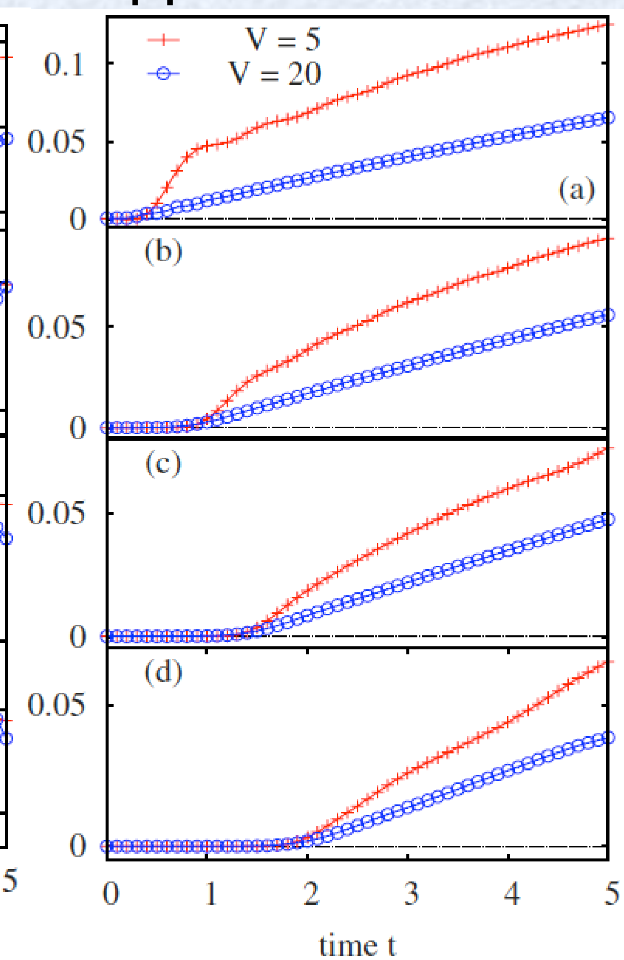
➠ Correlation length *growing* in time

Metallic Phase:



➠ Dip moving *ballistically* through the system

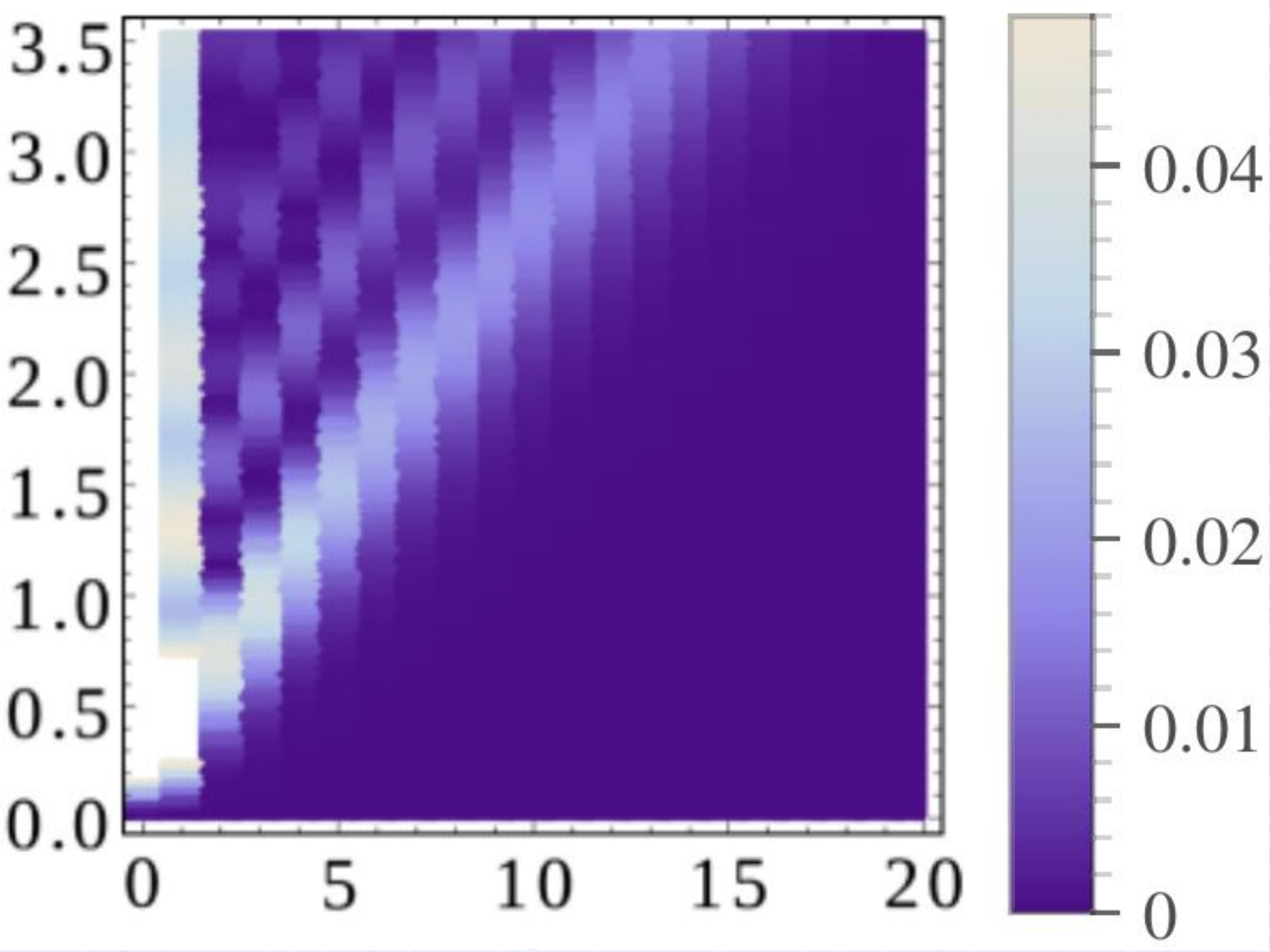
Gapped Phase:



➠ *Onset* of correlations moving "*ballistically*"

Light cone with Dipolar interactions

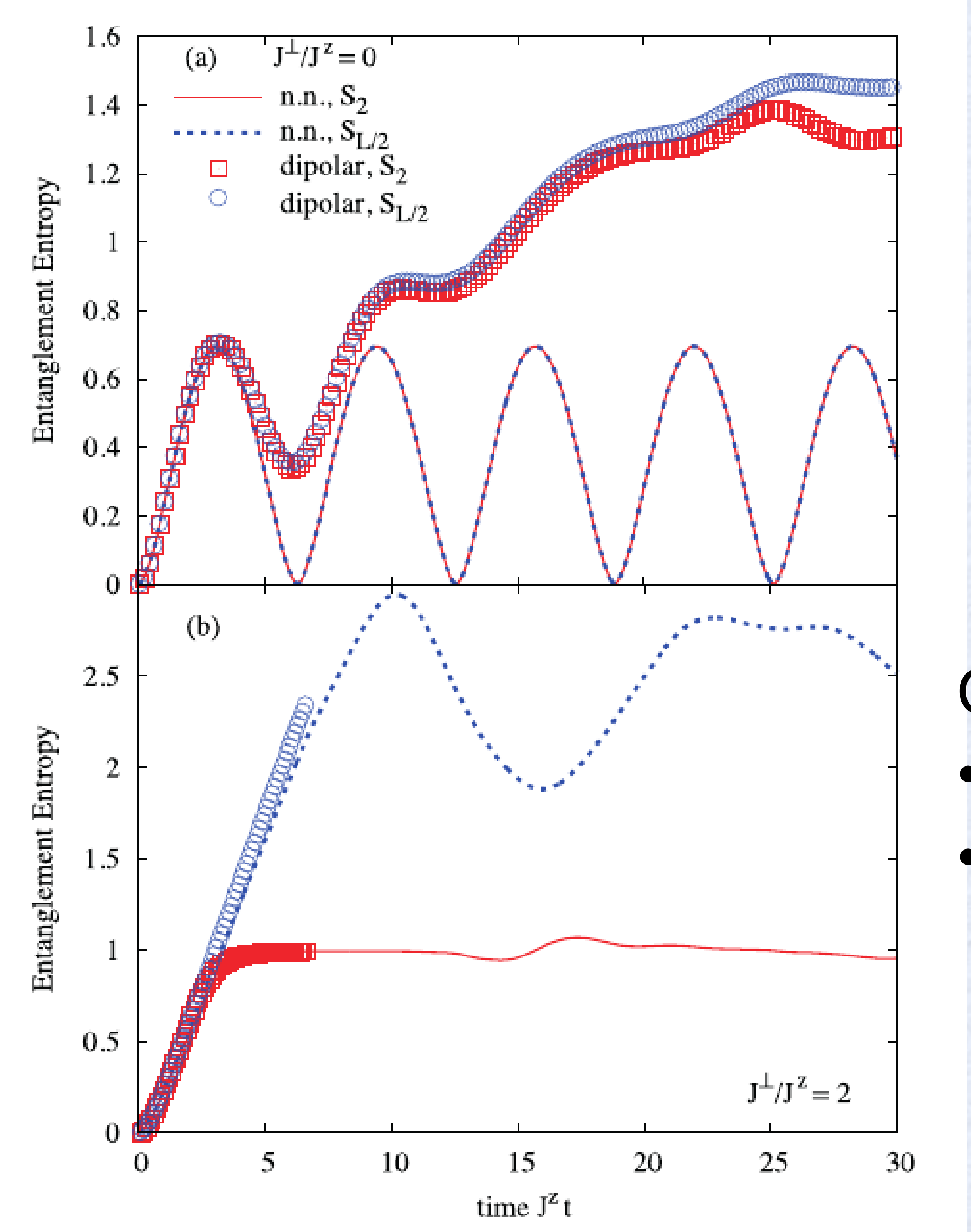
$\alpha=3$:



Looks quite linear!

Entanglement Dynamics?

Entanglement entropy: $S_{\mathcal{R}} = -\mathrm{Tr}\left(\rho_{\mathcal{R}} \ln \rho_{\mathcal{R}}\right)$



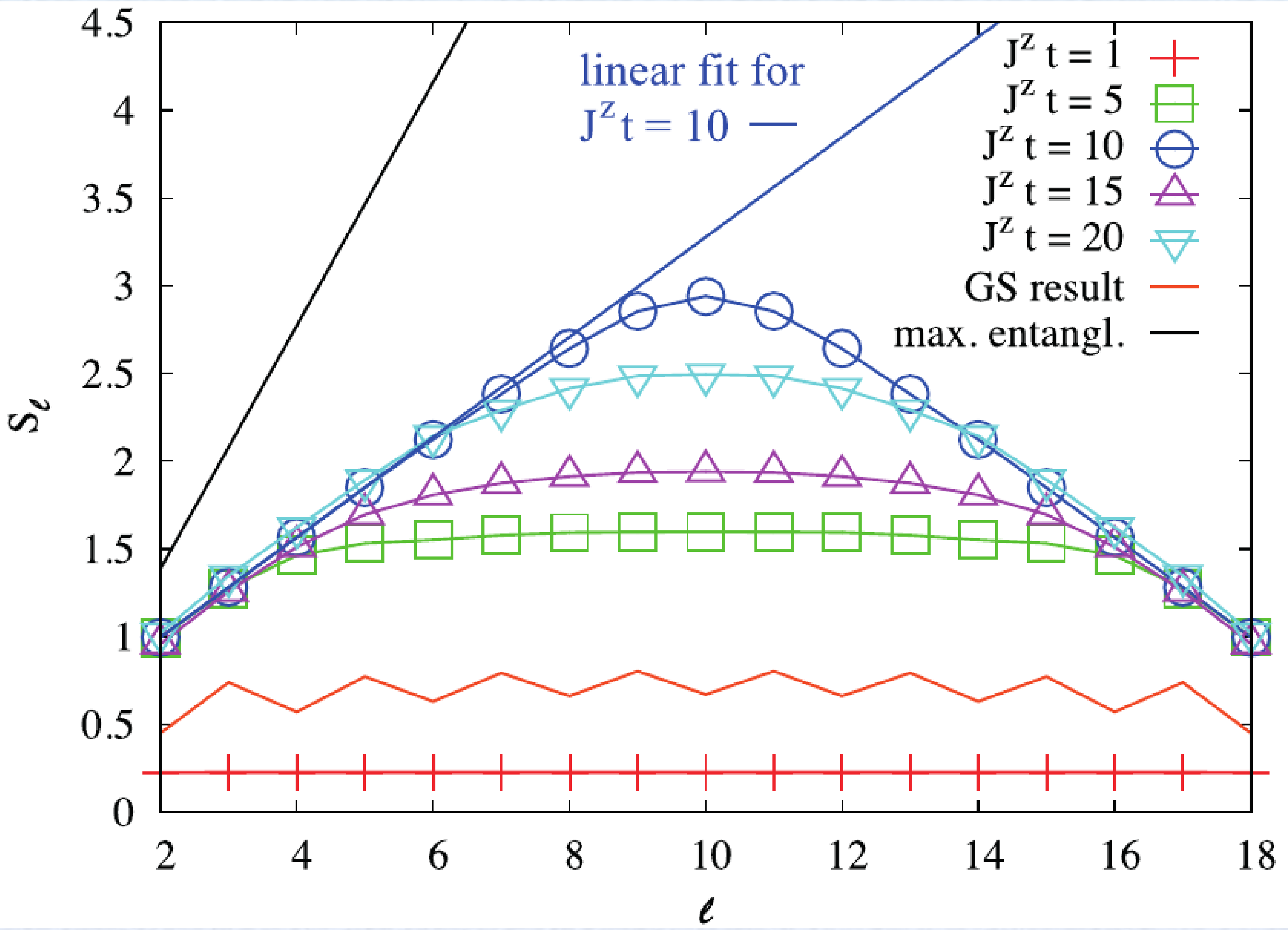
Ising limit:

- Bulk and edge behavior similar
- Dipolar case: richer behavior

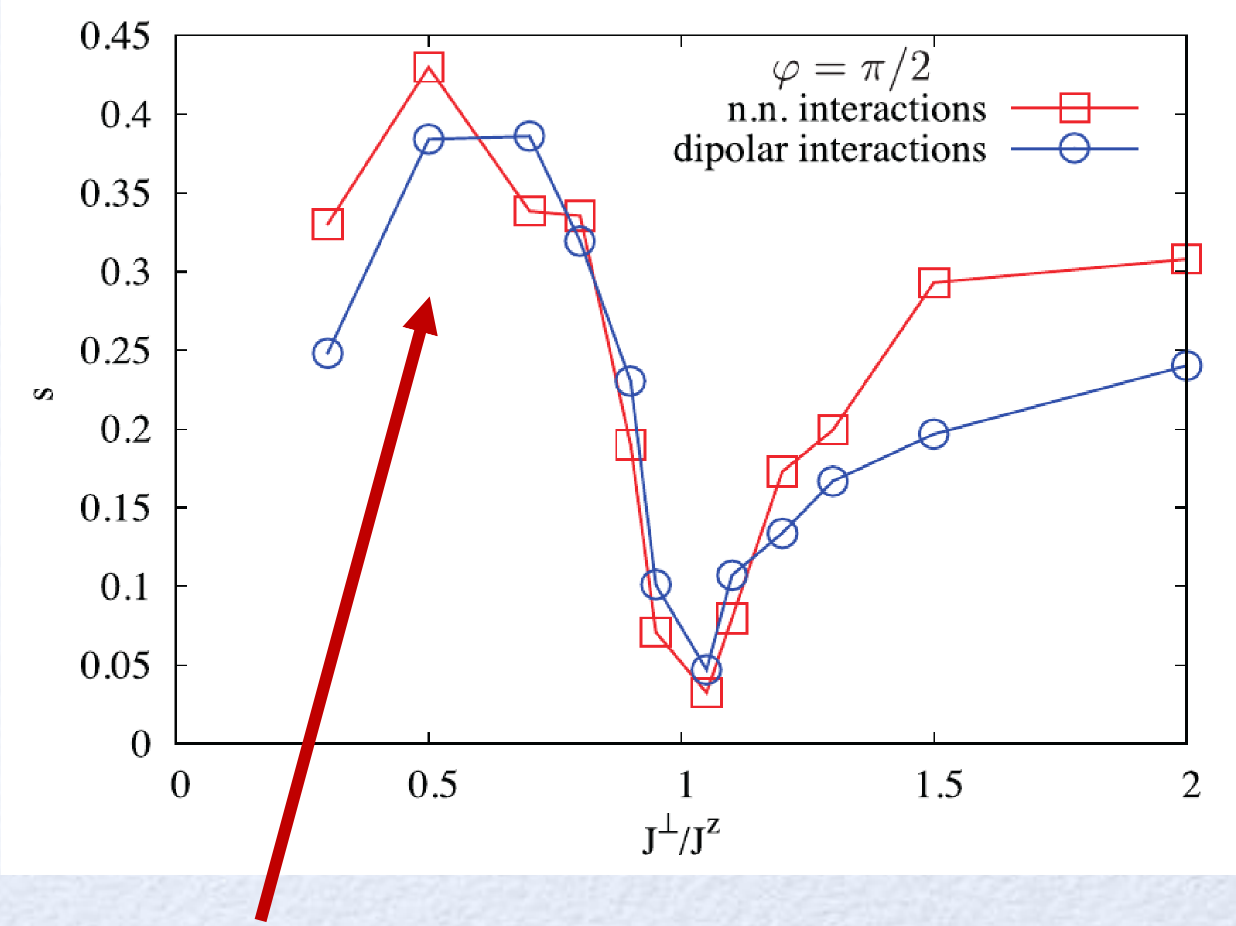
Gapless case:

- Bulk and edge behavior very different
- Dipolar case similar to n.n. interactions

Entanglement: Emerging volume-law behavior



Slope:



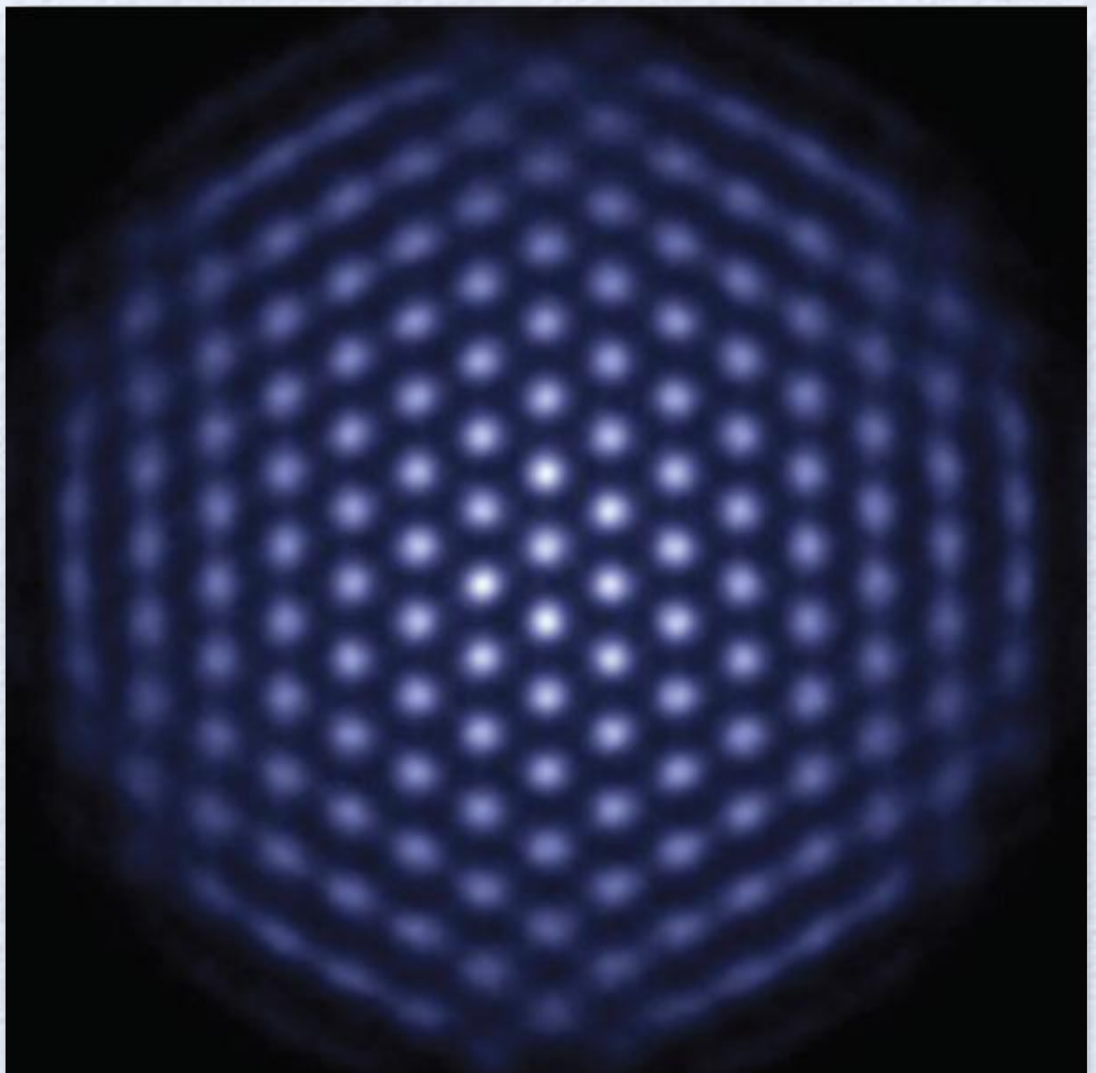
Bulk and edge behavior similar

- Linear growth of entanglement entropy: volume law behavior
- Comparison to maximally entangled state: slope about half that strong

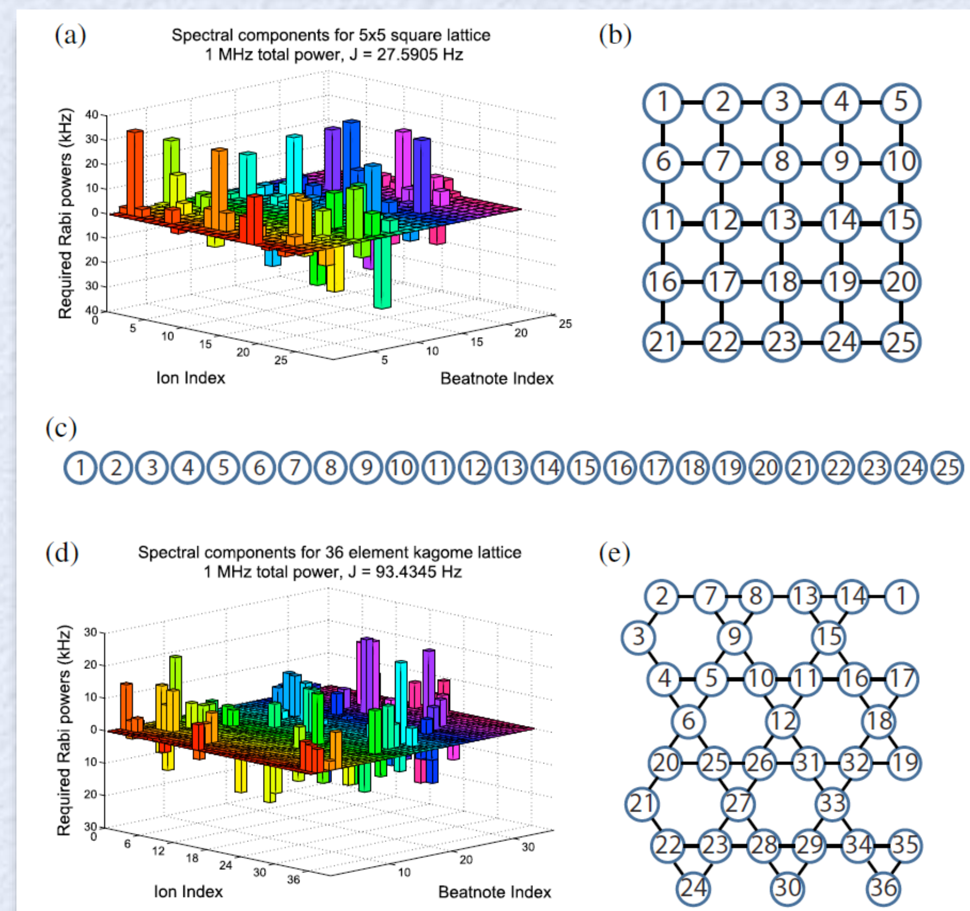
Maximal entanglement for similar parameters as n.n. system

Increase the interaction range: Ions in a Trap

$^9Be^+$ ions in a Penning trap (NIST Boulder)
[J.W. Britton et al., Nature **484**, 489 (2012)]



$^{171}Yb^+$ ions (JQI/NIST Maryland)
[K. Kim et al., Nature **465**, 590 (2010);
R. Islam et al., Nature Comm. **2**,377 (2011);
NJP and more...]



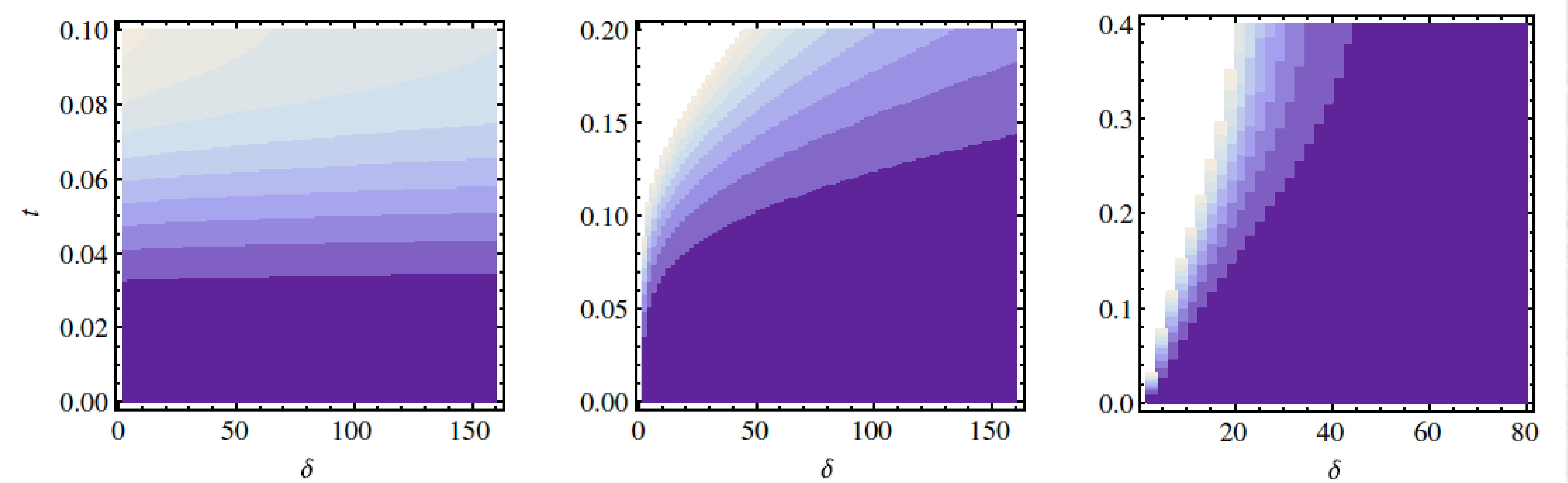
Realization of Ising models with transverse field on variety of lattices:

Interactions ~ $1/r^{\alpha}$

Long-range Interactions: Causal Horizon vs. Immediate Spread

[J. Eisert, M. v.d. Worm, S.R. Manmana, and M. Kastner, PRL **111**, 260401 (2013)]

α = 1/4 α = 3/4 α = 3/2

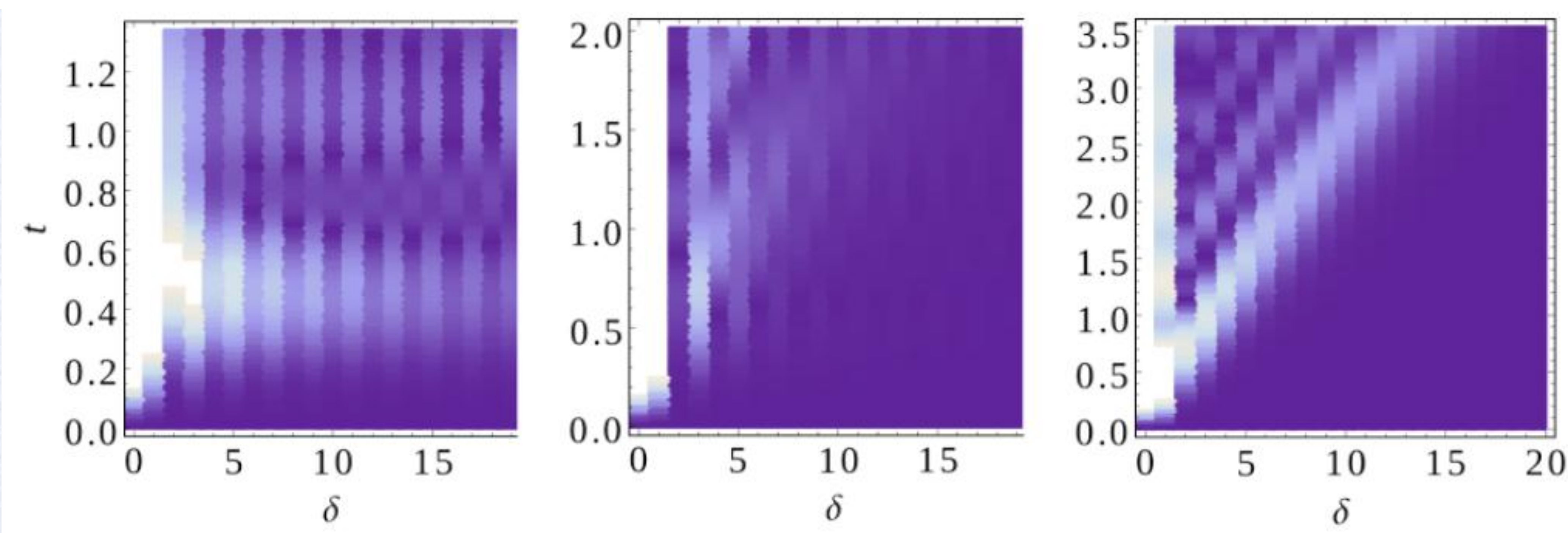


Ising,
L=1001

$$v|t| > \epsilon\delta^q$$

power law shape

α = 3



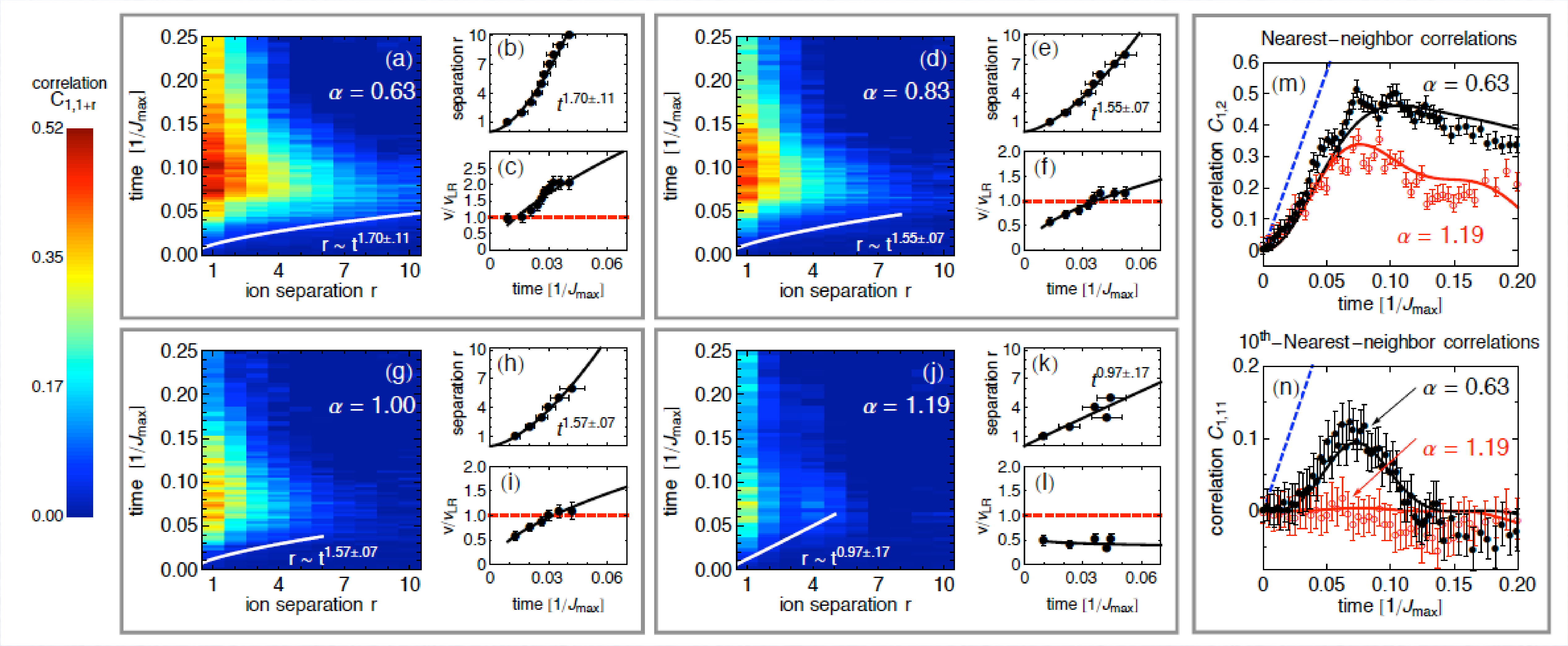
XXZ,
L=40

generic initial state: causal region appears for α > D
product initial state: causal region appears for α > D/2

Ion-Trap-Experiments

[P. Richerme et al., Nature **511**,198 (2014)]

Interactions ~ $1/r^{\alpha}$

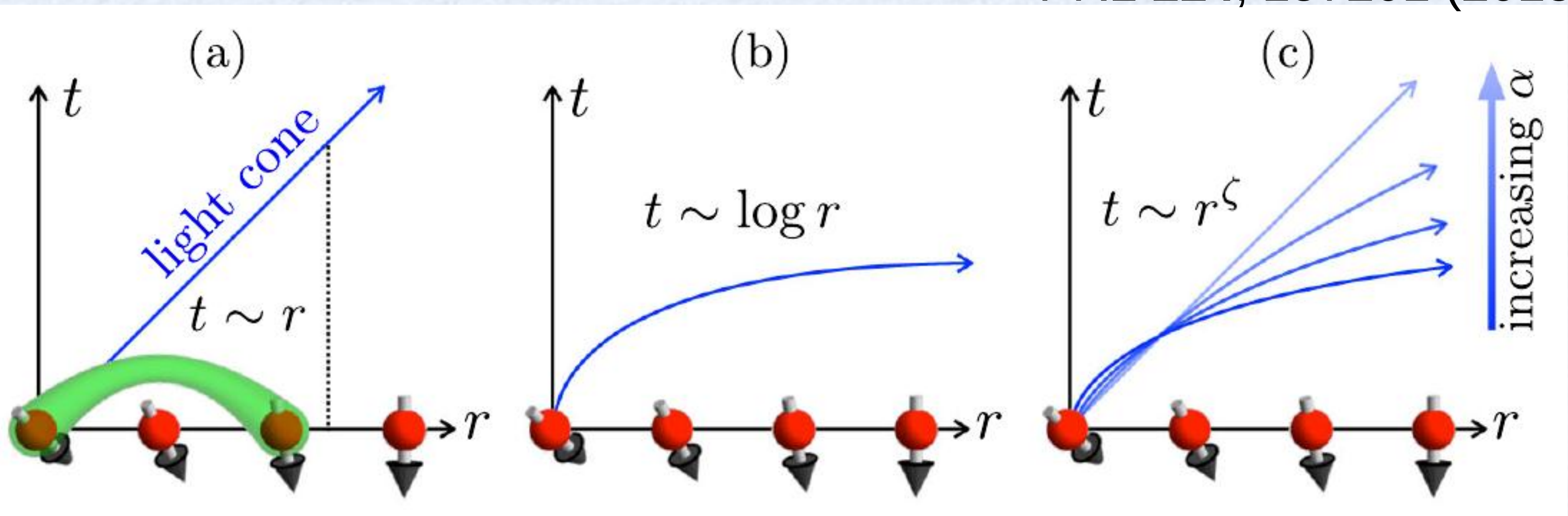


Not a linear ‘region of causality’, but curved!

Algebraic bounds for causality?

Proposed behaviours:

Lieb-Robinson | Hastings-Koma | Foss-Feig et al. PRL **114**, 157201 (2015)



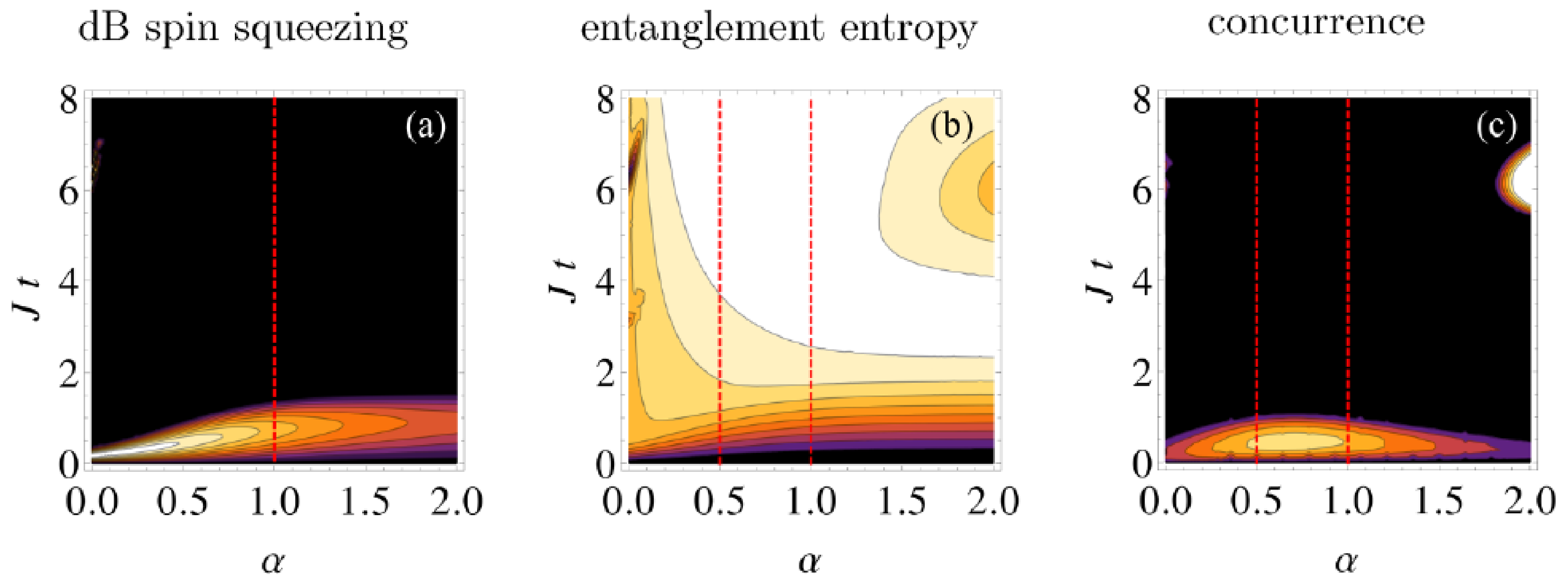
$$C_r(t) \leq \mathscr{C}_r(t) \equiv 2c\kappa\left(e^{vt-r/\chi} + 2\kappa\frac{e^{v_\chi t}}{[r/R(t)]^\alpha}\right) \longrightarrow \mathscr{C}_r(t) \sim \exp[vt - r/t^\gamma] + \frac{t^{\alpha(1+\gamma)}}{r^\alpha}$$

- α>2D: *algebraic* shape of the light-cone rather than logarithmic
- Becomes increasingly linear as α grows

Entanglement for variable α

Long-range Ising model with arbitrary α:

$\varphi = \pi/2$



$$\xi = \min_{\hat{n}} \frac{\sqrt{N}\sqrt{\langle(\mathbf{S}\cdot\hat{n})^2\rangle - \langle\mathbf{S}\cdot\hat{n}\rangle^2}}{|\langle\mathbf{S}\rangle|}$$

$$S_{\mathcal{R}} = -\mathrm{Tr}\left(\rho_{\mathcal{R}} \ln \rho_{\mathcal{R}}\right)$$

$$\mathcal{C}(\rho) \equiv \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}$$

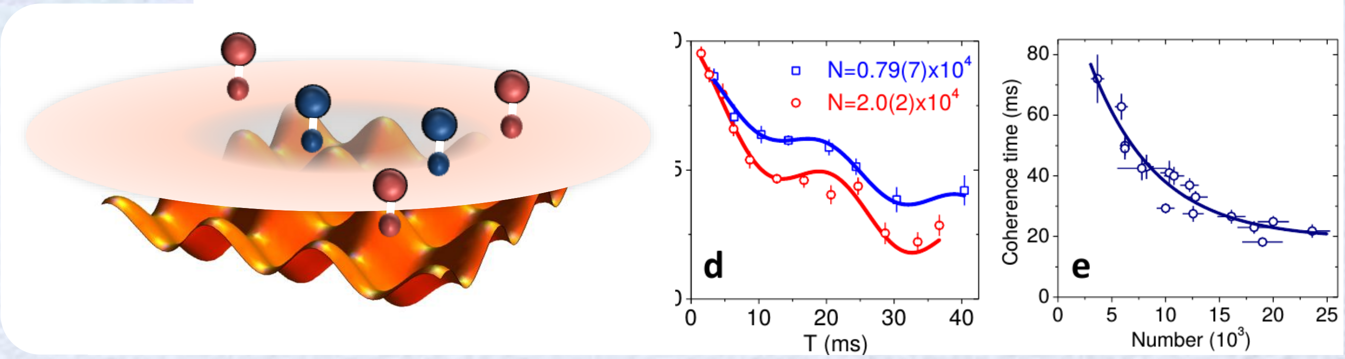
Ramsey spectroscopy

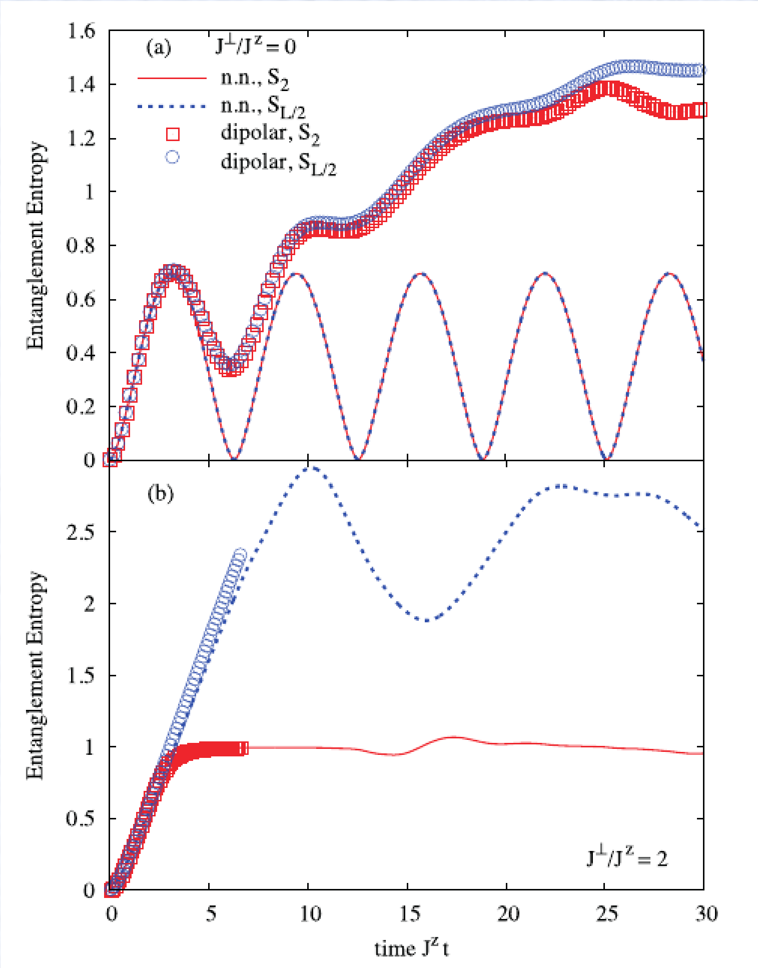
Pure states

Mixed states

Main Messages of the Talk:

I) Experiments with ultracold polar molecules:
Dipolar Spin exchange interactions





II) Compare dynamics of correlations and entanglement of dipolar systems with short-range systems

III) Generic algebraically decaying interactions:
Lieb-Robinson-type bounds for causality?