Large- and small-x resummations in PDF fits

Marco Bonvini

University of Oxford

Parton Distributions for the LHC, Benasque, Feb 18, 2015

Related to some work (in progress) with:

Richard Ball, Valerio Bertone, Stefano Forte, Simone Marzani, NNPDF,

Tiziano Peraro, Giovanni Ridolfi, Juan Rojo, Luca Rottoli, Maria Ubiali

Logarithmic enhancement: resummation

Single log or double log enhancements

$$\alpha_s^k \log^j \qquad 0 \le j \le (2)k$$

If/when

$$\alpha_s \log \sim 1 \qquad\qquad \alpha_s \log^2 \sim 1$$

all (such) terms in the perturbative series are equally important:

all order RESUMMATION

Example: DGLAP evolution: resummation of single $\log(Q^2/Q_0^2)$

Note:
if $\alpha_s \log^{(2)} < 1$, resummation can still be used to predict (the dominant (?) part of) higher orders.

Resummations in this talk

Large-x resummation:

- $x \to 1$
- associated with **soft** gluon emissions
- resums **double** $\log(1-x)$
- threshold resummation, a subset of soft-gluon (Sudakov) resummations
- in Mellin space, $\log N$ at $N \to \infty$

Small-x resummation

- $x \to 0$
- associated with **high-energy** gluon emissions
- resums **single** $\log x$
- high-energy (BFKL) resummation
- in Mellin space, poles $1/(N-1)$ in the limit $N \to 1$

Resum what?

Observable: $\sigma = \sigma_0 \, C(\alpha_s(\mu)) \otimes f(\mu) \, [\otimes f(\mu)]$

Evolution: $\mu^2 \frac{d}{d\mu^2} f(\mu) = P(\alpha_s(\mu)) \otimes f(\mu)$

Any object with a perturbative expansion and a log enhancement:

- coefficient functions $C(\alpha_s(\mu))$ (observable)
- splitting functions $P(\alpha_s(\mu))$ (evolution)

	observable coefficient functions $C(\alpha_s(\mu))$	evolution splitting functions $P(\alpha_s(\mu))$
large-x	?	?
small-x	?	?

Resummation in the evolution: large x

Singlet diagonal (P_{qq}, P_{gg}) and non-singlet ($P_{\rm ns}^{\pm}$):

$$P(x,\alpha_s) = \frac{A(\alpha_s)}{(1-x)_+} + B(\alpha_s)\delta(1-x) + C(\alpha_s)\log(1-x) + \dots$$

$$\gamma(N,\alpha_s) = -A(\alpha_s)\log N + [B(\alpha_s) - \gamma A(\alpha_s)] - C(\alpha_s)\frac{\log N}{N} + \dots$$

no log enhancement!

Singlet off-diagonal (P_{qg}, P_{gq}):

$$P(x,\alpha_s) = \sum_{n=0}^{\infty} \alpha_s^{n+1}\left[\sum_{k=0}^{2n} d_{nk}\log^k(1-x) + \dots\right]$$

$$\gamma(N,\alpha_s) = \sum_{n=0}^{\infty} \alpha_s^{n+1}\left[\sum_{k=0}^{2n} \tilde{d}_{nk}\frac{\log^k N}{N} + \dots\right]$$

Double log enhancement of the next-to-soft (NS) contributions [Vogt 1005.1606]
Can be resummed up to NNLL ($k=0,1,2$) [Almasy,Soar,Vogt 1012.3352]
Expected effect: negligible

Resummation in the evolution: small x

Singlet:

$$P(x,\alpha_s) = \sum_{n=0}^{\infty} \alpha_s^{n+1} \left[\sum_{k=0}^{n} a_{nk} \frac{\log^k x}{x} + \sum_{k=0}^{2n} b_{nk} \log^{2k} x + \dots \right]$$

$$\gamma(N,\alpha_s) = \sum_{n=0}^{\infty} \alpha_s^{n+1} \left[\sum_{k=0}^{n} \frac{a_{nk}}{(N-1)^{k+1}} + \sum_{k=0}^{2n} \frac{b_{nk}}{N^{k+1}} + \dots \right]$$

Single log enhancement at leading small x, in the singlet sector

$$P_{\rm singlet} = \begin{pmatrix} P_{gg} & P_{gq} \\ P_{qg} & P_{qq} \end{pmatrix} = \begin{pmatrix} \mathrm{LL} & \mathrm{LL} \\ \mathrm{NLL} & \mathrm{NLL} \end{pmatrix}$$

Non-singlet:

$$P(x,\alpha_s) = \sum_{n=0}^{\infty} \alpha_s^{n+1} \left[\sum_{k=0}^{2n} b_{nk} \log^k x + \dots \right]$$

is double log enhanced but subleading.

Small-x resummation: brief overview

DGLAP: $$\mu^2 \frac{d}{d\mu^2} f(x,\mu^2) = \int \frac{dz}{z} P\left(\frac{x}{z}, \alpha_s(\mu^2)\right) f(z,\mu^2)$$

BFKL: $$x \frac{d}{dx} f(x,\mu^2) = \int \frac{d\nu^2}{\nu^2} K\left(x, \frac{\mu^2}{\nu^2}, \alpha_s(\cdot)\right) f(x,\nu^2)$$

double Mellin transform $f(N,M) = \int dx\, x^N \int \frac{d\mu^2}{\mu^2} \left(\frac{\mu^2}{\mu_0^2}\right)^{-M} f(x,\mu^2)$

DGLAP: $M f(N,M) = \gamma(N, \alpha_s(\cdot)) f(N,M) +$ boundary

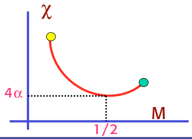
BFKL : $N f(N,M) = \chi(M, \alpha_s(\cdot)) f(N,M) +$ boundary

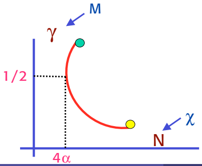
When both are valid (small x, large μ^2), consistency between the solutions gives (at fixed coupling)

$$\chi(\gamma(N,\alpha_s),\alpha_s) = N \qquad \leftrightarrow \qquad \gamma(\chi(M,\alpha_s),\alpha_s) = M$$

duality relation ("L rule")

For $\chi(M,\alpha_s) = \alpha_s \chi_0(M)$
the dual γ contains all orders in α_s/N



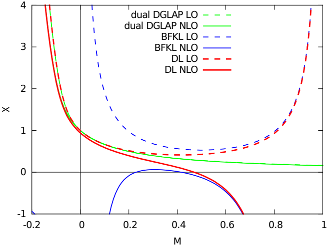


Small-x resummation: brief overview

What do we get?

- LL: strong growth at small x (not observed)
- NLL: no enhancement at small x (!!)

Totally unstable,
due to perturbative instability of the BFKL kernel



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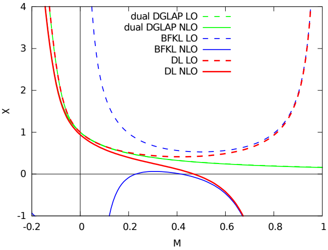

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ABF solution [Altarelli,Ball,Forte 1995,...,2008]

- use duality to resum BFKL kernel
- exploit symmetry $M \to 1 - M$ of χ
- impose momentum conservation
- reuse duality to get resummed anomalous dimensions

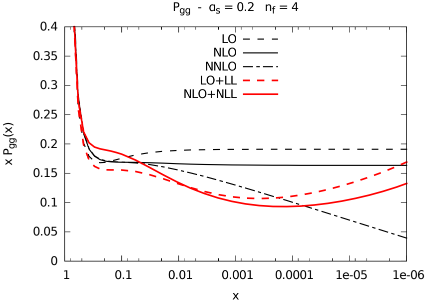
The result is perturbatively stable!
Finally

- resum running coupling contributions (changes the nature of the small-N singularity: branch-cut to pole)

Small-x resummation: the codes

Actual implementation rather complicated (several technical issues...)
Three independent codes:

- Richard's (orginal)
- Tiziano's
- mine



The codes are slow and require some tuning.
Fit the resummed splitting functions, and write the parameters on a α_s grid:

`HELL: High-Energy Large Logarithms`

© S.Marzani

fast interface, already successfully interfaced to `APFEL`

Still under development, the goal is to include resummed coefficient functions

Resummation in the coefficient functions: small x

High-energy (k_T) factorization:

$$\sigma \propto \int \frac{dz}{z} \int d^2\boldsymbol{k}\, \hat{\sigma}_g\left(\frac{x}{z}, \frac{Q^2}{\boldsymbol{k}^2}, \alpha_s(Q^2)\right) \mathcal{F}_g(z, \boldsymbol{k})$$

$$\begin{cases} \mathcal{F}_g(x, \boldsymbol{k}) & : \text{unintegrated PDF} \\ \hat{\sigma}_g\left(z, \frac{Q^2}{\boldsymbol{k}^2}, \alpha_s\right) & : \text{off-shell xs} \end{cases}$$

Defining

$$\mathcal{F}_g(N, \boldsymbol{k}) = U\left(N, \frac{\boldsymbol{k}^2}{\mu^2}\right) f_g(N, \mu^2)$$

we get

$$C_g(N, \alpha_s) = \int d^2\boldsymbol{k}\, \hat{\sigma}_g\left(N, \frac{Q^2}{\boldsymbol{k}^2}, \alpha_s\right) U\left(N, \frac{\boldsymbol{k}^2}{\mu^2}\right)$$

At LL accuracy, U has a simple form, in terms of small-x resummed anom dim γ

$$U\left(N, \frac{\boldsymbol{k}^2}{\mu^2}\right) \approx \boldsymbol{k}^2 \frac{d}{d\boldsymbol{k}^2} \exp \int_{\mu^2}^{\boldsymbol{k}^2} \frac{d\nu^2}{\nu^2} \gamma(N, \alpha_s(\nu^2))$$

- Only known at LL
- Just uses the off-shell cross sections $\hat{\sigma}(N, Q^2/\boldsymbol{k}^2, \alpha_s)$ (one for each proc)
- Can be included directly in `HELL`

Resummation in the coefficient functions: large x

Dressing the Born with soft gluon emissions leads to double log enhancement

$$C(N) = C_{\rm LO}(N) \left[1 + \sum_{n=1}^{\infty} \alpha_s^n \sum_{k=0}^{2n} c_{nk} \log^k N + \mathcal{O}(1/N) \right]$$

Known to NNNLL' for DIS, DY, Higgs: $k = 2n, 2n-1, \ldots, 2n-6$.

Well known formalism, can be derived in several ways (diagrammatic approach, factorisation methods, path-integral approach, SCET)

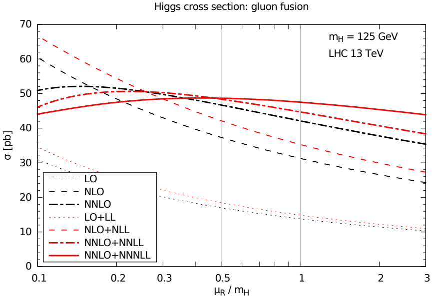
$$\frac{C(N)}{C_{\rm LO}(N)} = g_0(\alpha_s) \exp\left[\frac{1}{\alpha_s} g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \alpha_s^2 g_4(\alpha_s L) + \ldots \right]$$

Possible improvements $\qquad L = \log N$

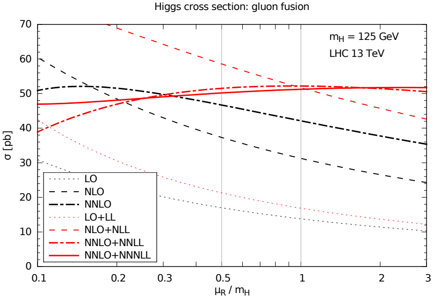
- $\log N \to \psi_0(N)$ (correct analytic structure)
- $N \to N+1$ (or something better, resums collinear contributions)
- resummation of constants (like π^2 resummation)
- extension to double-differential distributions $d\sigma/dM^2/dY$

A successful example: Higgs production

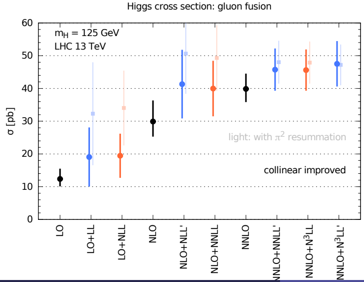
Standard resummation



Improved resummation



[MB,Marzani 1405.3654]



improved SCET resummation [MB,Rottoli 1412.3791]

Threshold resummation in DIS

The code ResHiggs [MB,Marzani 1405.3654] now includes DIS and DY

New name:

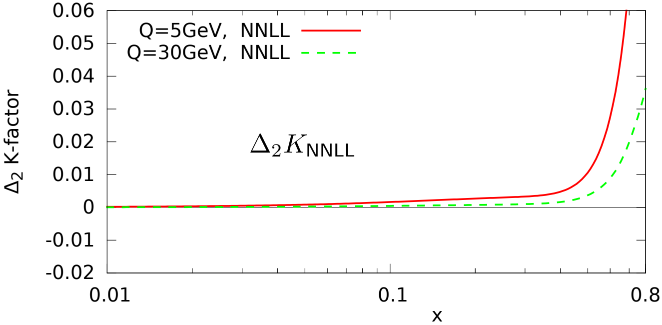
TROLL: ~~Threshold Resummation Of Large-x Logarithms~~
TROLL Resums Only Large-x Logarithms

© MB, L.Rottoli

to be made public at some point..

TROLL delivers $\Delta_j K_{\mathrm{N}^n\mathrm{LL}}$ to be used as $\sigma_{\mathrm{res}} = \sigma_{\mathrm{N}^j\mathrm{LO}} + \sigma_{\mathrm{LO}} \times \Delta_j K_{\mathrm{N}^n\mathrm{LL}}$

DIS:



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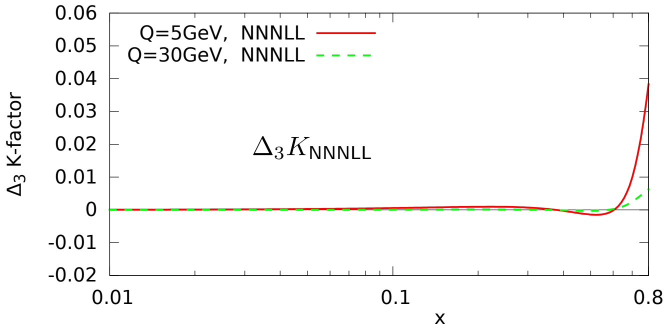
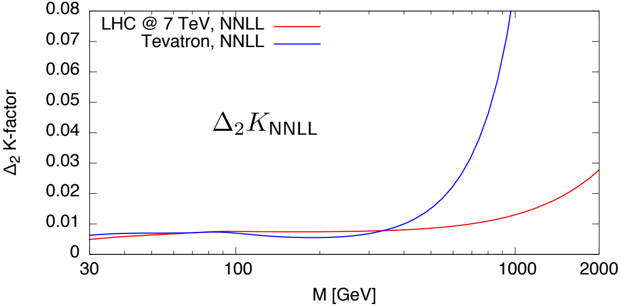
DIS:

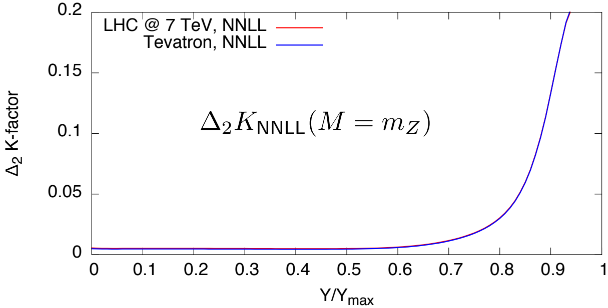


Threshold resummation in Drell-Yan

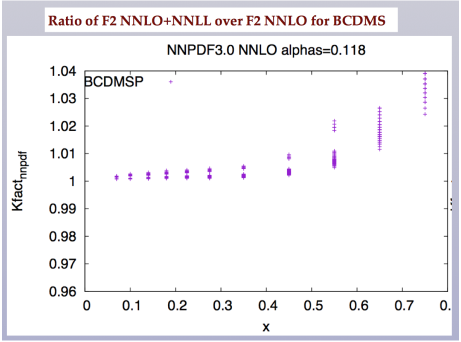
$$\frac{d\sigma_{\rm DY}}{dM^2}:$$

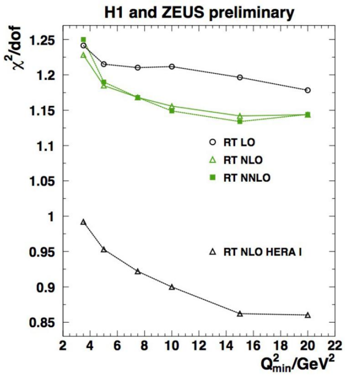


$$\frac{d\sigma_{\rm DY}}{dM^2 dY}:$$



Impact on PDF Fits





At small Q^2 all resummations are enhanced

A large-x resummed DIS-only fit is behind the corner...

Conclusions

	observable coefficient functions $C(\alpha_s(\mu))$	evolution splitting functions $P(\alpha_s(\mu))$
large-x	up to N^{3}LL$'$ for DIS, DY, Higgs ✓	—
small-x	only LL for DIS, DY, Higgs, $t\bar{t}$,... ✓	NLL ✓

- Impact of threshold resummation is only in the c.f., and moderate
- Sizeable effect only at very large x or large rapidity
- N^{3}LO DIS can improve the quality of the fit at large x, but includes some (possibly large) small-x contributions
- Small-x resummation in the evolution expected to be sizeable, and available soon
- Small-x resummation of the observable will be the last step for a fully resummed fit

Backup slides

Relevance of resummations

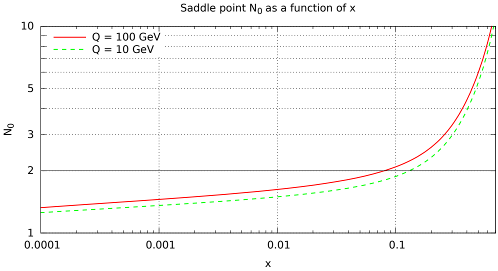
$$F_2(x)/x = \int_x^1 \frac{dz}{z}\, C(z,\alpha_s)\, f\left(\frac{x}{z}\right) = \int_x^1 \frac{dz}{z}\, C\left(\frac{x}{z},\alpha_s\right) f(z)$$

- Large z region always contributing
- Small z region accessible only when x is small

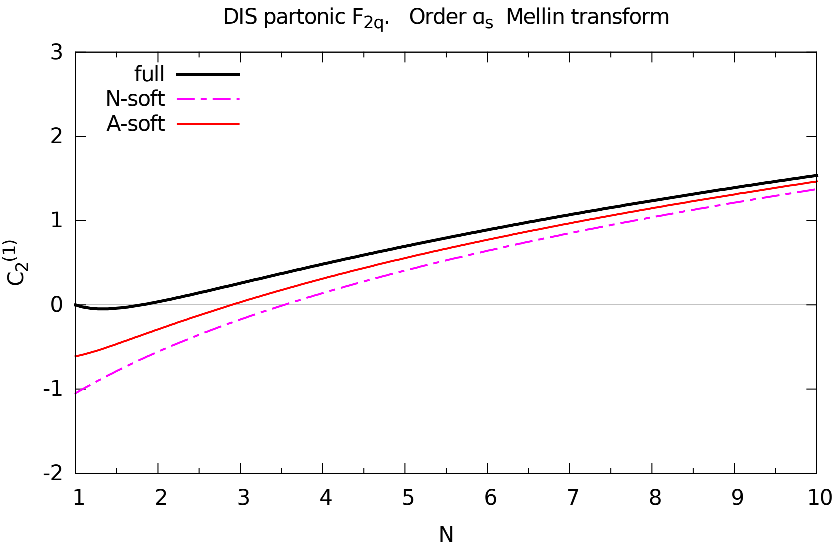
When shall we care about resummations?

$$F_2(x)/x = \int \frac{dN}{2\pi i}\, x^{-N}\, C(N,\alpha_s)\, f(N) = \int \frac{dN}{2\pi i}\, e^{N\log\frac{1}{x} + \log C(N,\alpha_s) + \log f(N)}$$

dominated by the saddle point N_0



Comparison to fixed order: DIS



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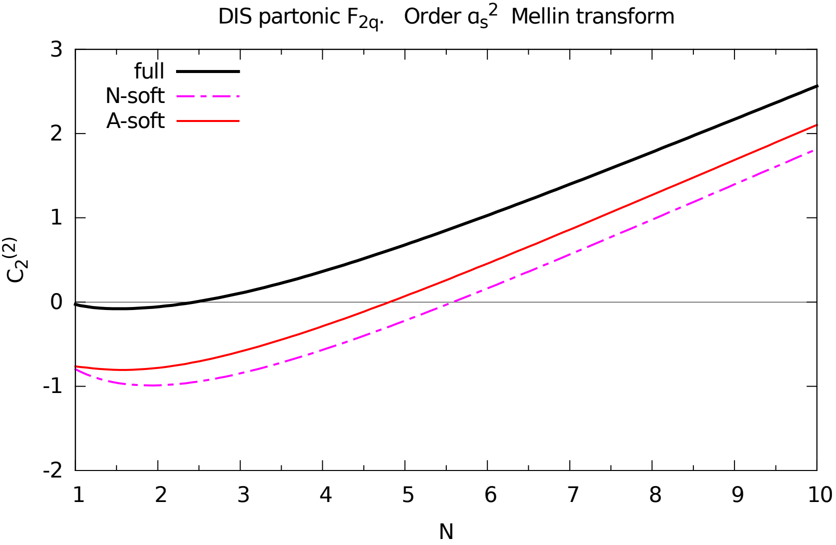

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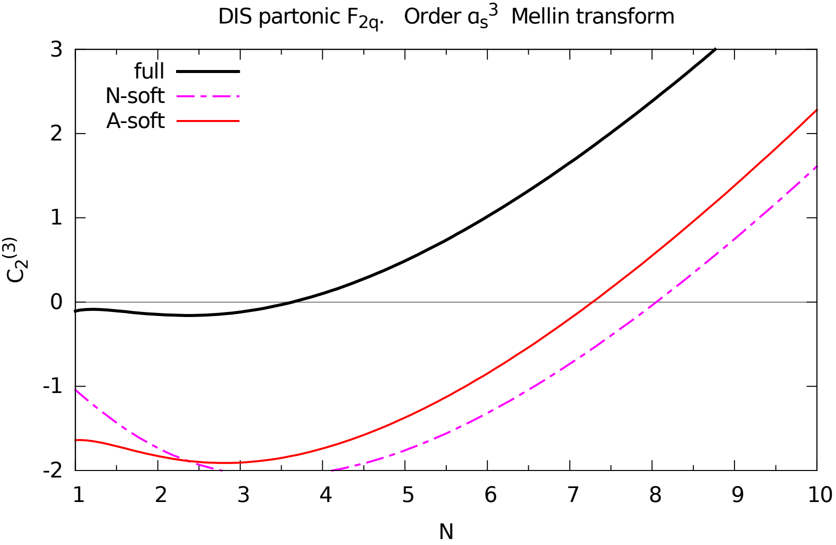
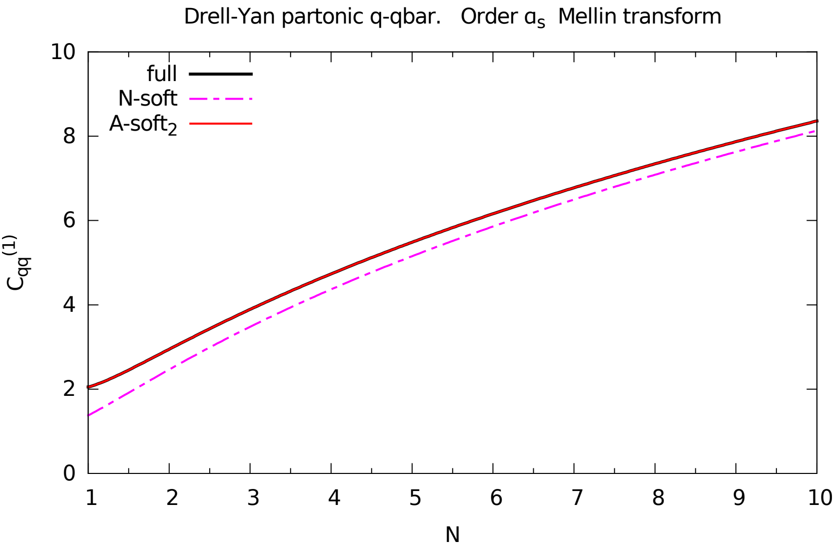


Comparison to fixed order: DY



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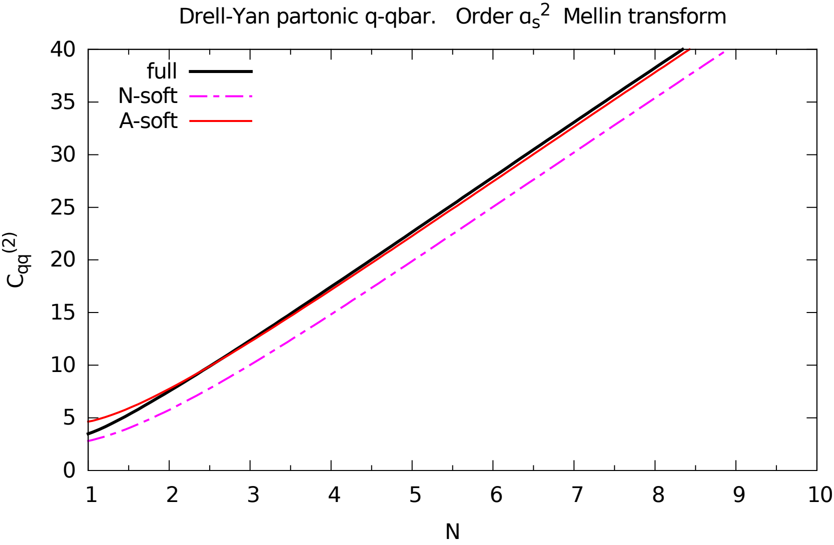
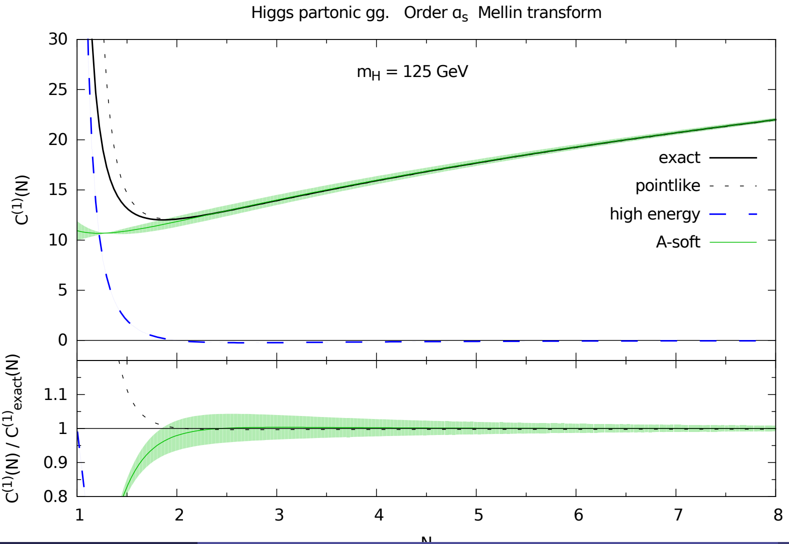


Comparison to fixed order: Higgs

$$C_{gg}(N,\alpha_s) = 1 + \alpha_s\, C^{(1)}_{gg}(N) + \alpha_s^2\, C^{(2)}_{gg}(N) + \dots$$



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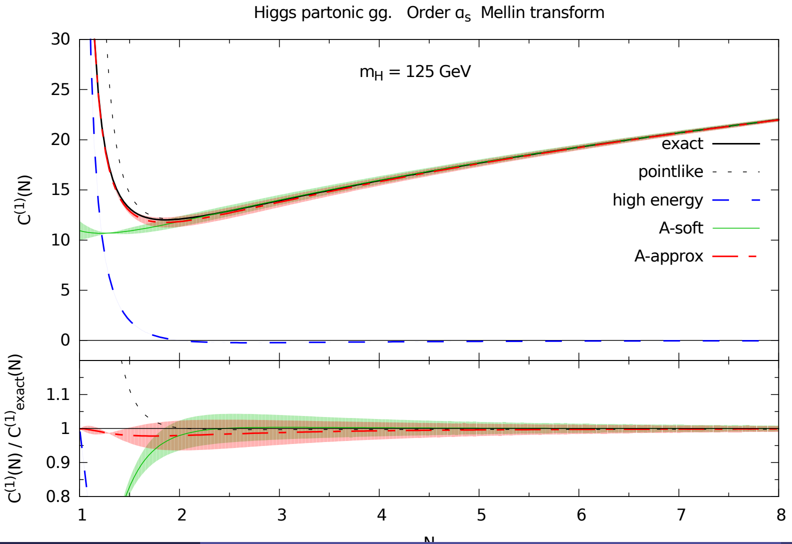

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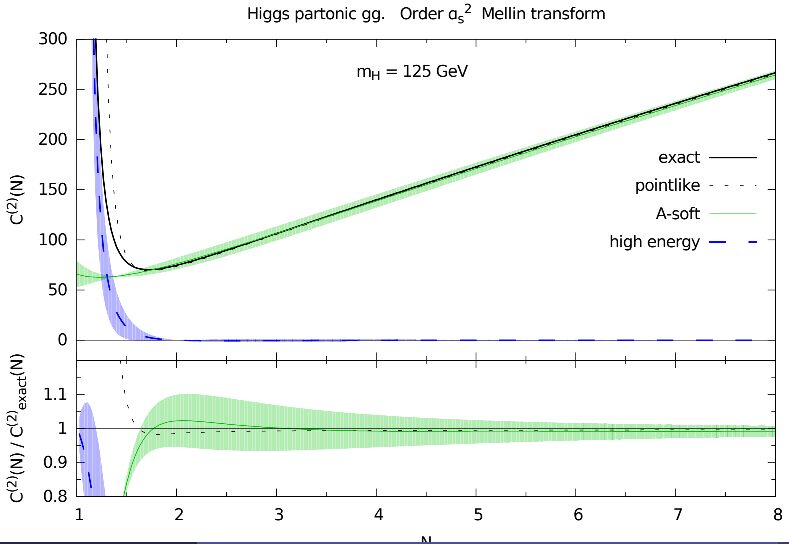

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