

Jets at NNLO: status and use in PDF's

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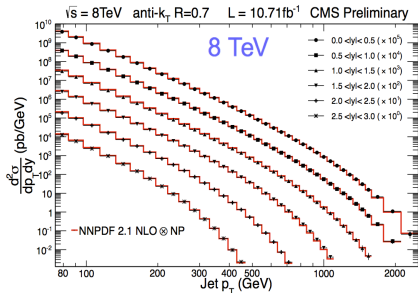
Inclusive jet and dijet cross sections

look at the **production** of **jets** of hadrons with large **transverse energy** in

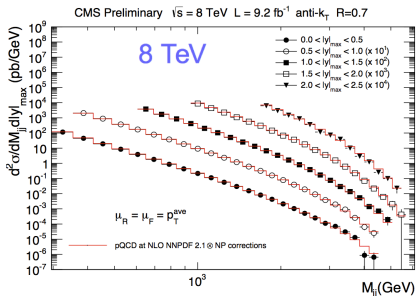
- inclusive jet events $pp \rightarrow j + X$
- exclusive dijet events $pp \rightarrow 2j$

cross sections measured as a function of the jet p_T , rapidity y and dijet **invariant mass** m_{jj} in **double differential** form

$$\frac{d^2\sigma}{dp_T dy} = \frac{1}{\epsilon \mathcal{L}_{\text{eff}}} \frac{N_{\text{jets}}}{\Delta p_T (2 \cdot \Delta|y|)}$$



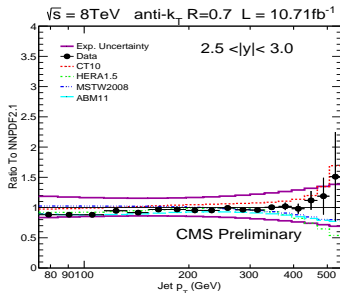
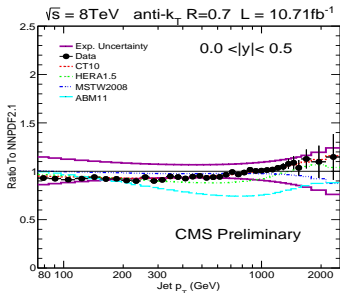
CMS PAS SMP-12-012



CMS-PAS-SMP-14-002

Inclusive jet cross section

- Jets up to $|y| = 3.0$, $p_T = 2.5$ TeV. Six rapidity bins of $\Delta|y| = 0.5$. @ 8TeV



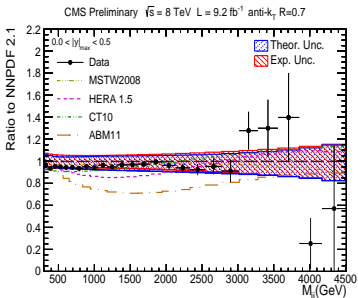
- theory: NLO QCD \otimes NP
- overall good agreement with data with similar predictions at low- p_T
 - except ABM11 \rightarrow not included jet data in their fit since NNLO corrections may be large
- significant mismatch in the predictions at high- p_T between all sets

$$\delta_{experimental} \sim 15 - 40\% \quad (\text{JES, luminosity, unfolding})$$

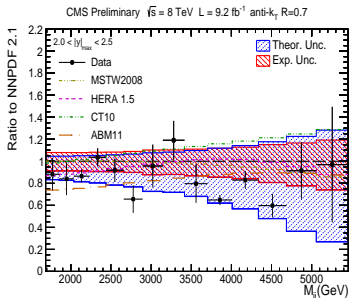
$$\delta_{theory} \sim 10 - 50\% \quad (\text{PDF, } \mu_R, \mu_F)$$

Dijet cross section

- Jets up to $|y| = 2.5$, $M_{jj} = 5.5$ TeV. Six rapidity bins of $\Delta|y_{\max}| = 0.5$. @ 8TeV



CMS-PAS-SMP-14-002



CMS-PAS-SMP-14-002

- overall good agreement with data within statistical/systematical uncertainties in all rapidity bins
- theory predictions show differences of $\mathcal{O}(10\%)$
- theoretical and experimental uncertainties are of comparable size even at high M_{jj}

$$\delta_{experimental} \sim 5 - 20\% \quad (\text{JES, luminosity, unfolding})$$

$$\delta_{theory} \sim 5 - 40\% \quad (\text{PDF, } \mu_R, \mu_F)$$

Towards NNLO QCD

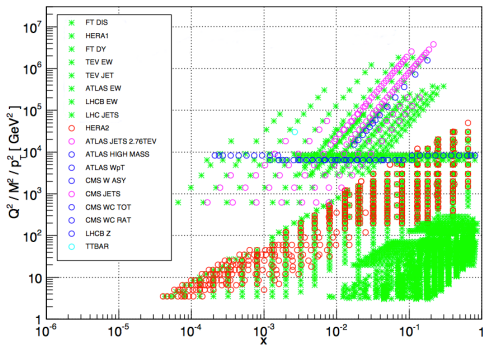
Motivation for NNLO

- to include higher-order effects → only way to reduce theoretical uncertainties in the fixed-order predictions used in experimental analysis
- to make reliable theory comparisons with LHC jet data
- to make jet data consistently included in NNLO PDF fits

Towards NNLO QCD

Motivation for NNLO

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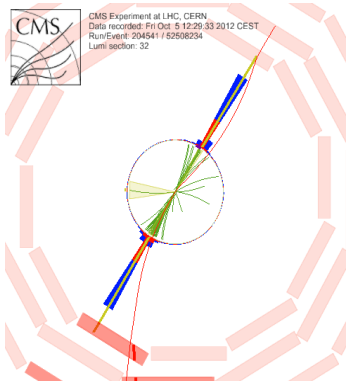
(NNPDF collaboration)

- particularly sensitive to gluon PDF at medium and large values of x → reduce PDF uncertainty
- different data constrain different parton combinations at different x
- check NNLO consistency with HERA, DIS and Tevatron data
- check consistency with $t\bar{t}$ data

Inclusive jet and dijet cross sections

State of the art:

- dijet production is known in NLO QCD
[Ellis, Kunszt, Soper '92]
[Giele, Glover, Kosower '94], [Nagy '02]
- NLO+Parton shower
[Alioli, Hamilton, Nason, Oleari, Re '11]
- NLO EW corrections
[Dittmaier, Huss, Speckner '12]
- approximate NNLO threshold corrections
[Kidonakis, Owens '00], [Florian, Hinderer, Mukherjee, Ringer, Vogelsang '13]



$M_{jj} = 5.5 \text{ TeV}$

Goal:

- obtain the jet cross sections at NNLO exact accuracy in double differential form

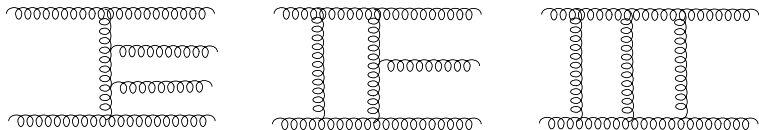
$$\frac{d^2\sigma}{dp_T d|y|} \quad \frac{d^2\sigma}{dm_{jj} dy^*}$$

NNLO ingredients

QCD jet **cross section** perturbative expansion at **hadron colliders**

$$d\sigma = \sum_{i,j} \int \left[d\hat{\sigma}_{ij}^{LO} + \left(\frac{\alpha_s}{2\pi}\right) d\hat{\sigma}_{ij}^{NLO} + \left(\frac{\alpha_s}{2\pi}\right)^2 d\hat{\sigma}_{ij}^{NNLO} + \mathcal{O}(\alpha_s^3) \right] f_i(x_1) f_j(x_2) dx_1 dx_2$$

NNLO **m-jet correction** contains three contributions:



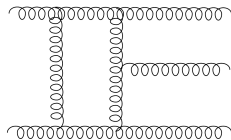
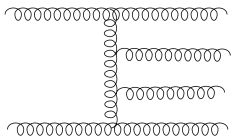
$$d\hat{\sigma}_{NNLO} = \int_{d\Phi_4} d\hat{\sigma}_{NNLO}^{RR} + \int_{d\Phi_3} d\hat{\sigma}_{NNLO}^{RV} + \int_{d\Phi_2} d\hat{\sigma}_{NNLO}^{VV}$$

- **explicit infrared poles** from loop integrations
- **implicit poles** in phase space regions for **single** and **double unresolved** gluon emission
- **procedure to extract the infrared singularities and assemble all the parts in a parton-level generator**
- **differential cross sections** → **kinematics** of the final state **intact** to apply **arbitrary** phase space observable cuts

NNLO antenna subtraction

$$\begin{aligned}d\hat{\sigma}_{NNLO} &= \int_{d\Phi_4} \left(d\hat{\sigma}_{NNLO}^{RR} - d\hat{\sigma}_{NNLO}^S \right) + \int_{d\Phi_3} \left(d\hat{\sigma}_{NNLO}^{RV} - d\hat{\sigma}_{NNLO}^T \right) \\ &+ \int_{d\Phi_2} \left(d\hat{\sigma}_{NNLO}^{VV} - d\hat{\sigma}_{NNLO}^U \right)\end{aligned}$$

- extract **singularities** keeping the **kinematics** of the final state **intact**
- IR pole cancellation analytic and local in phase space



- double unresolved configurations

- double soft
- triple collinear
- double collinear
- single soft and single collinear

- single unresolved configurations

- single soft
- single collinear

- remove **overlapping** of various single and double soft and/or collinear **limits**

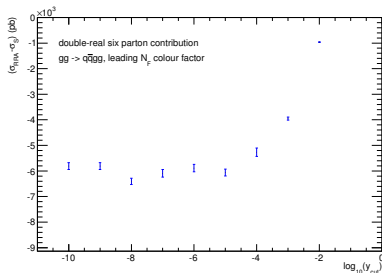
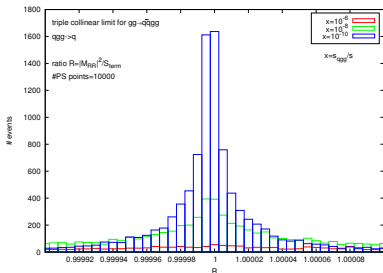
leading- N_F contributions at NNLO

NNLO contributions	perturbative order
$gg \rightarrow q\bar{q}gg$	tree-level (RR)
$gg \rightarrow q\bar{q}g$	one-loop (RV)
$gg \rightarrow ggg$	one-loop (RV)
$gg \rightarrow gg$	two-loop (VV)
$gg \rightarrow q\bar{q}$	two-loop (VV)

$$d\hat{\sigma}_{NNLO}^{RR} \rightarrow d\hat{\sigma}_{NNLO}^S$$

$$d\hat{\sigma}_{NNLO}^{RV} \rightarrow d\hat{\sigma}_{NNLO}^T$$

- 27 independent double/single unresolved singularities at RR level, e.g.,
 - triple collinear final-state $P_{q\bar{q}g} \rightarrow g, P_{qgg} \rightarrow q$
 - triple collinear initial-state $P_{\hat{g}q\bar{q}} \rightarrow \hat{g}, P_{\hat{g}qg} \rightarrow \hat{q}, P_{\hat{g}gg} \rightarrow \hat{g}$
- NLO and NNLO antenna functions correctly approximate the matrix elements in all unresolved configurations



NNLO antenna subtraction - VV process

- VV antenna subtraction term IR pole structure [Currie, Glover, Wells 2013]
 - integrated single unresolved emission from RV process \propto tree-level single soft function

$$\mathcal{Poles}(d\hat{\sigma}_{NNLO}^{U,a}) \sim \mathbf{J}^1(\epsilon, \hat{1}_g, \hat{2}_g, i_g, j_g) \left(A_4^1(\hat{1}_g, \hat{2}_g, i_g, j_g) - \frac{b_0}{\epsilon} A_4^0(\hat{1}_g, \hat{2}_g, i_g, j_g) \right)$$

- integrated iterated NLO emissions of RR process in analytic one-to-one correspondence with $(\mathbf{I}^1)^2$ operator of Catani

$$\mathcal{Poles}(d\hat{\sigma}_{NNLO}^{U,b}) \sim \mathbf{J}^1(\epsilon, \hat{1}_g, \hat{2}_g, i_g, j_g) \otimes \mathbf{J}^1(\epsilon, \hat{1}_g, \hat{2}_g, i_g, j_g) A_4^0(\hat{1}_g, \hat{2}_g, i_g, j_g)$$

- integrated double unresolved emission of RR process \propto tree-level double soft function
- integrated single unresolved emission of RV process \propto one-loop single soft function
- when added are in analytic one-to-one correspondence with \mathbf{I}^2 operator of Catani

$$\mathcal{Poles}(d\hat{\sigma}_{NNLO}^{U,c}) \sim \mathbf{J}^2(\epsilon, \hat{1}_g, \hat{2}_g, i_g, j_g) A_4^0(\hat{1}_g, \hat{2}_g, i_g, j_g)$$

- double virtual antennae subtraction term $d\hat{\sigma}_{NNLO}^U$ written compactly rederives the predicted Catani pole structure of the two-loop contribution in the antennae language

leading- N_F VV contribution $gg \rightarrow gg$

- all independent double/single unresolved singularities at RR, RV level, collapse to a simple structure once integrated down to the VV level

$$\begin{aligned}
 d\sigma_U &= \hat{\mathbf{J}}_4^1(1_g, 2_g, i_g, j_g) A_4^1(1_g, 2_g, i_g, j_g) \\
 &+ \mathbf{J}_4^1(1_g, 2_g, i_g, j_g) \hat{A}_4^1(1_g, 2_g, i_g, j_g) \\
 &+ \mathbf{J}_4^1(1_g, 2_g, i_g, j_g) \otimes \hat{\mathbf{J}}_4^1(1_g, 2_g, i_g, j_g) A_4^0(1_g, 2_g, i_g, j_g) \\
 &+ \hat{\mathbf{J}}_4^2(1_g, 2_g, i_g, j_g) A_4^0(1_g, 2_g, i_g, j_g)
 \end{aligned}$$

- allowing us to define integrated dipoles (in one-to-one correspondence with IR-Catani pole operators) with an analytic expansion in $d = 4 - 2\epsilon$ for all possible flavour combinations, e.g.,

$$\begin{aligned}
 \hat{\mathbf{J}}_4^2(1_g, 2_g, i_g, j_g) &= \hat{\mathbf{J}}_2^2(\hat{1}_g, \hat{2}_g) + \hat{\mathbf{J}}_2^2(\hat{2}_g, i_g) + \hat{\mathbf{J}}_2^2(i_g, j_g) + \hat{\mathbf{J}}_2^2(j_g, \hat{1}_g) \\
 &\rightarrow \text{leading-}N_F \text{ FF, IF, II gluon-gluon dipoles}
 \end{aligned}$$

$$\begin{aligned}
 \hat{\mathbf{J}}_2^2(\hat{1}_g, \hat{2}_g) &= \mathcal{G}_{4,gg}^0(s_{12}) + \mathcal{F}_3^1(s_{12}) + \frac{b_F}{\epsilon} \mathcal{F}_3^0(s_{12}) \left(\left(\frac{|s_{12}|}{\mu^2} \right)^{-\epsilon} - 1 \right) - \frac{1}{2} \bar{\Gamma}_{gg,F}^2(z_1) - \frac{1}{2} \bar{\Gamma}_{gg,F}^2(z_2) \\
 &+ \frac{1}{2} \frac{b_F}{\epsilon} \Gamma_{gg}^1(z_1) + \frac{1}{2} \frac{b_F}{\epsilon} \Gamma_{gg}^1(z_2) + \frac{1}{2} \frac{b_0}{\epsilon} \hat{\Gamma}_{gg}^1(z_1) + \frac{1}{2} \frac{b_0}{\epsilon} \hat{\Gamma}_{gg}^1(z_2) \\
 &+ S_{g \rightarrow q} \Gamma_{qg}(z_1) \mathcal{G}_{3,qg}^0(s_{12}) + S_{g \rightarrow q} \Gamma_{qg}(z_2) \mathcal{G}_{3,qg}^0(s_{12}) + \frac{1}{2} \Gamma_{gg}^1(z_1) \Gamma_{qg}^1(z_1) + \frac{1}{2} \Gamma_{qg}^1(z_2) \Gamma_{qg}^1(z_2)
 \end{aligned}$$

leading- N_F VV contribution $gg \rightarrow q\bar{q}$

- Similarly,

$$\begin{aligned}d\sigma_U &= \mathbf{J}_4^1(i_q, 1_g, 2_g, j_{\bar{q}}) B_2^1(i_q, 1_g, 2_g, j_{\bar{q}}) \\ &+ \frac{1}{2} \mathbf{J}_4^1(i_q, 1_g, 2_g, j_{\bar{q}}) \otimes \mathbf{J}_4^1(i_q, 1_g, 2_g, j_{\bar{q}}) B_2^0(i_q, 1_g, 2_g, j_{\bar{q}}) \\ &+ \mathbf{J}_4^2(i_q, 1_g, 2_g, j_{\bar{q}}) B_2^0(i_q, 1_g, 2_g, j_{\bar{q}}) \\ &- \bar{\mathbf{J}}_4^2(i_q, 1_g, 2_g, j_{\bar{q}}) B_2^0(i_q, 1_g, 2_g, j_{\bar{q}})\end{aligned}$$

- allowing us to define the integrated dipoles, $J_2^{(2)}(q, \hat{g})$, $\bar{J}_2^{(2)}(q, \hat{g})$ such that IR pole cancellation between real and virtual corrections at NNLO is achieved in transparent and analytic way

$$\mathcal{Poles} \left(d\hat{\sigma}_{NNLO}^{VV} - d\hat{\sigma}_{NNLO}^U \right) = 0 \quad \text{for } gg \rightarrow q\bar{q}$$

Advantages,

- integrated dipoles are process independent
- need to be derived once and for all and then can be recycled to compute other processes at NNLO
- calculation organized in a way that naturally leads to automation of the method

Jet production partonic channels

Fraction of jets per initial state contribution

LHC

- $gg \rightarrow gg$ dominates at low p_T
- $qg \rightarrow qg$ important in all p_T regions
- $qq \rightarrow qq$ dominant at high p_T

Tevatron

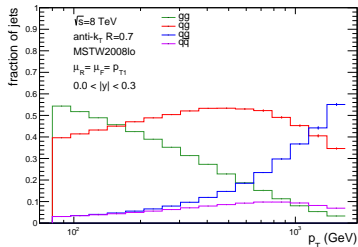
- qg and $q\bar{q}$ dominant

Numerical results at NNLO for

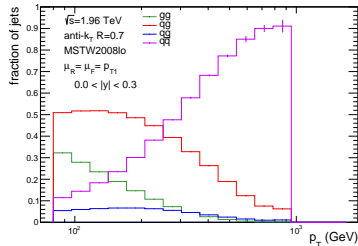
- $gg \rightarrow gg$ at leading colour
- $gg \rightarrow gg$ at subleading colour
- $q\bar{q} \rightarrow qg$ at leading colour

Ongoing work

- numerical implementation of qg and qq dipoles to extract predictions for qg and qq initial states



(LHC 8 TeV)

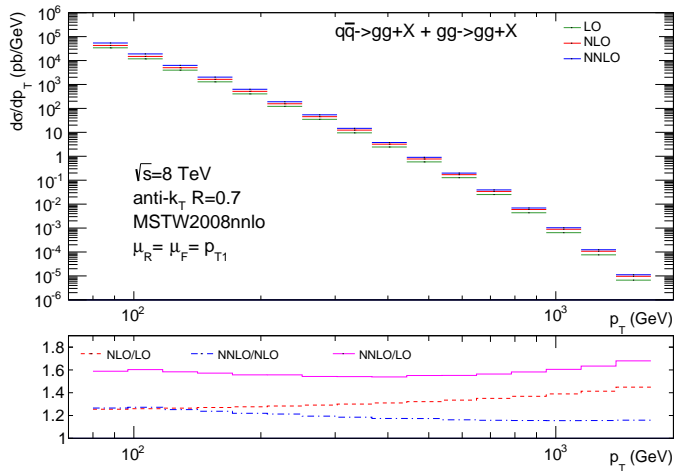


(Tevatron 1.96 TeV)

Numerical setup

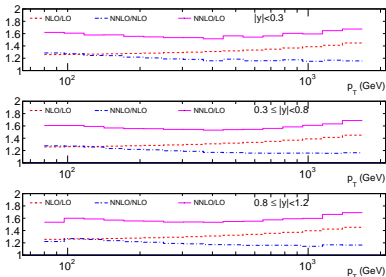
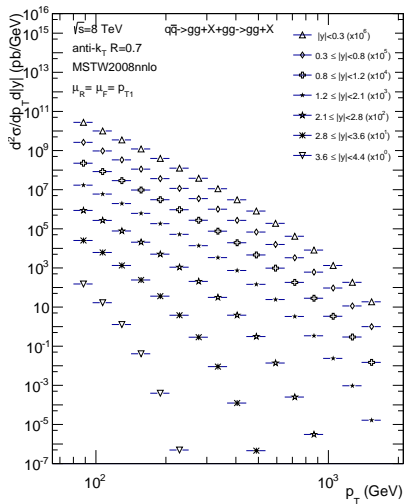
- pp collisions at $\sqrt{s} = 8$ TeV
- jets identified with the anti- k_T jet algorithm with resolution parameter $R = 0.7$
- jets accepted at rapidities $|y| < 4.4$
- leading jet with transverse momentum $p_T > 80$ GeV
- subsequent jets required to have at least $p_T > 60$ GeV
- MSTW2008nnlo PDF for all fixed-order predictions
- dynamical factorization and renormalization scales equal to the leading jet p_T
($\mu_R = \mu_F = \mu = p_{T1}$)
- present results for full colour $gg \rightarrow gg$ scattering and $q\bar{q} \rightarrow gg$ leading colour combined at NNLO

Inclusive jet p_T distribution at NNLO



- all jets in an event are binned
- NNLO correction stabilizes the NLO k-factor growth with p_T
- NNLO corrections 15 – 26% with respect to NLO

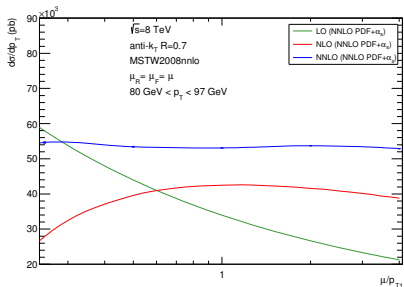
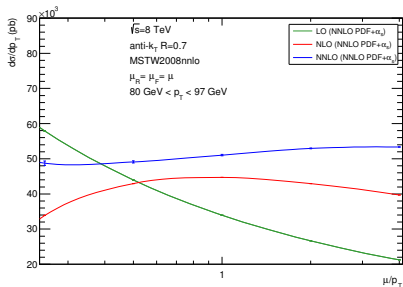
Double differential inclusive jet p_T distribution at NNLO



double differential k-factors

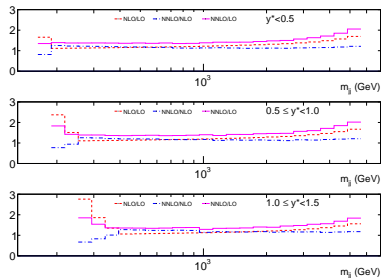
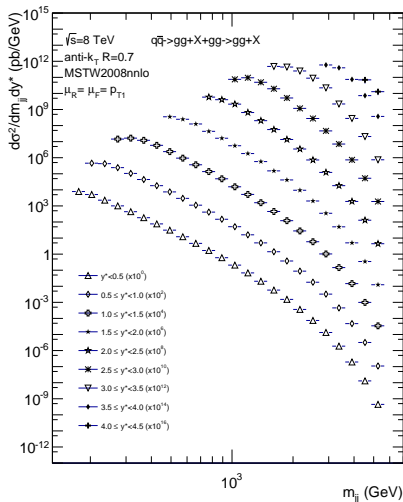
- NNLO prediction increases between 25% to 15% with respect to the NLO cross section
- similar behaviour between the rapidity slices

Scale choice for theory prediction



- scale dependence of the theory prediction gg -channel much reduced at NNLO
- size of the correction and uncertainty at low- p_T still depends on scale choice P_{T1} vs P_T

Double differential exclusive dijet mass distribution at NNLO



double differential k-factors

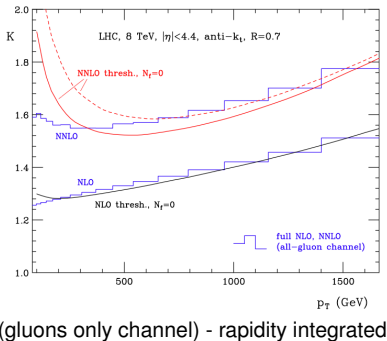
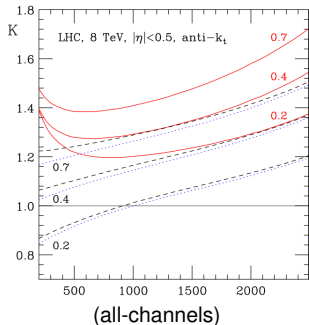
- NNLO corrections up to 20% with respect to the NLO cross section
- similar behaviour between the $y^* = 1/2|y_1 - y_2|$ slices
- asymmetric p_T cut for leading and subleading jet

Comparison with approximate NNLO predictions

- Approximate NNLO results from an improved threshold calculation for the single jet inclusive production [de Florian, Hinderer, Mukherjee, Ringer, Vogelsang '13]
 - $pp \rightarrow j + X$ with the **threshold limit** given by $s_4 = P_X^2 \rightarrow 0$
 - near **threshold phase space** available for **real-gluon** emission is **limited**
 - higher k th order **coefficient functions** dominated by **large logarithmic corrections**

$$\alpha_s^k w_{ab}^{(k)} \rightarrow \alpha_s^k \left(\frac{\log^m(z)}{z} \right)_+, \quad m \leq 2k - 1, \quad z = \frac{s_4}{s}$$

- $\delta(z)$ \times , 4th tower \times , $\mathcal{O}(z)$ \times



NNLO benchmark predictions for jet production

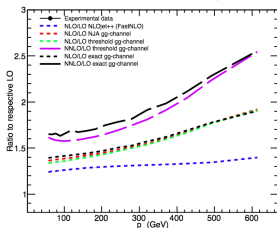
S. Carrazza, JP, arXiv:1407.7031

- understand and characterise the validity of the NNLO threshold approximation by comparing it with the exact computation using the gg -channel
- $\mu_R = \mu_F = p_T$ for both predictions
- comparison performed differential in p_T and rapidity following the exact experimental setups
- NNPDF23_nnlo_as_0118 set for all fixed order predictions
- NLO benchmark curves
 - green dashed curves → NLO-threshold gg -channel
 - black dashed curves → NLO-exact gg -channel
 - blue dashed curves → NLO-exact all channels
- NNLO benchmark curves
 - pink long-dashed curves → NNLO-threshold gg -channel → $d\sigma_{gg,NNLO}^{\text{thresh}}/d\sigma_{gg,LO}$
 - black long-dashed curves → NNLO-exact gg -channel → $d\sigma_{gg,NNLO}^{\text{exact}}/d\sigma_{gg,LO}$

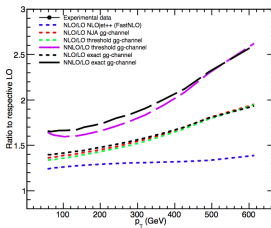
Tevatron CDF Run-II $\sqrt{s}=1.96$ TeV

S. Carrazza, JP, arXiv:1407.7031

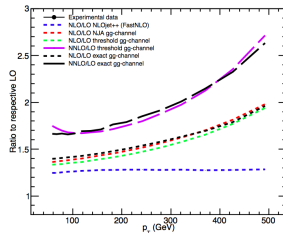
K-Factors - CDF Run-II kt, $|\eta|<0.1$



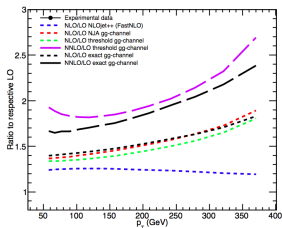
K-Factors - CDF Run-II kt, $0.1 < |\eta| < 0.7$



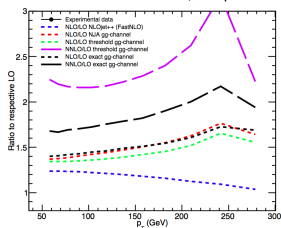
K-Factors - CDF Run-II kt, $0.7 < |\eta| < 1.1$



K-Factors - CDF Run-II kt, $1.1 < |\eta| < 1.6$



K-Factors - CDF Run-II kt, $1.6 < |\eta| < 2.1$

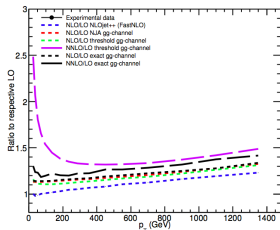


- differences $\leq 15\%$ at low- p_T in the central regions
- in the forward region differences $\geq 40\%$ for all p_T regions

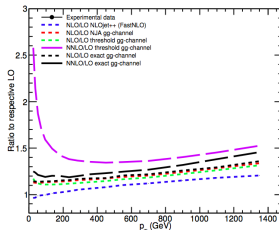
LHC ATLAS 2010 $\sqrt{s}=7$ TeV

S. Carrazza, JP, arXiv:1407.7031

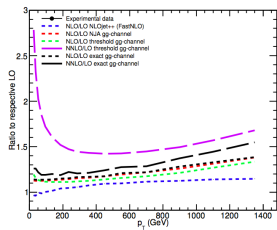
K-Factors - ATLAS 2010 7 TeV, $|\eta|<0.3$



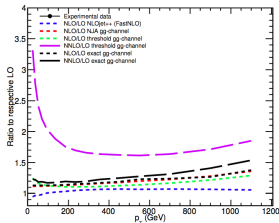
K-Factors - ATLAS 2010 7 TeV, $0.3<|\eta|<0.8$



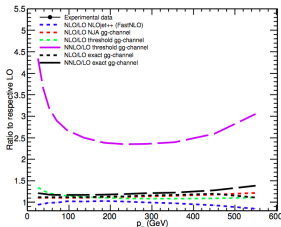
K-Factors - ATLAS 2010 7 TeV, $0.8<|\eta|<1.2$



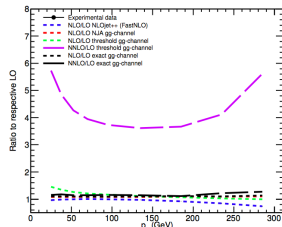
K-Factors - ATLAS 2010 7 TeV, $1.2<|\eta|<2.1$



K-Factors - ATLAS 2010 7 TeV, $2.1<|\eta|<2.8$



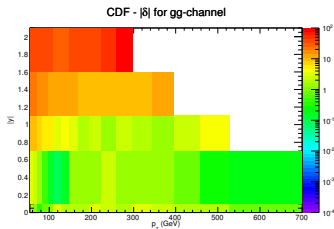
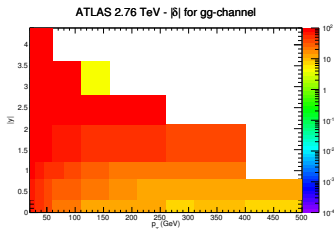
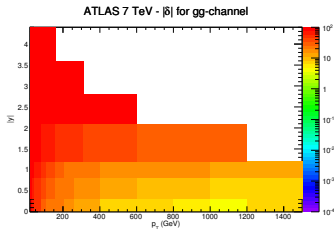
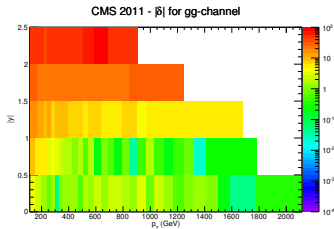
K-Factors - ATLAS 2010 7 TeV, $2.8<|\eta|<3.6$



- differences large at small p_T and increase with rapidity
- exact NNLO k-factor decreases with rapidity, NNLO threshold k-factor increases with rapidity

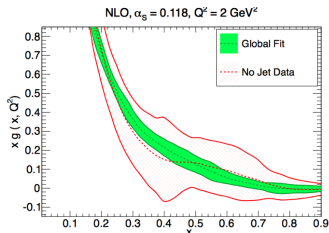
Threshold approximation - gg channel

S. Carrazza, JP, arXiv:1407.7031

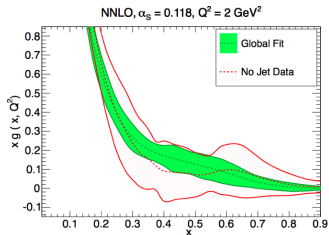


- relative difference $|\delta|$ between exact and approximate gg -channel k -factors as a function of p_T and $|y|$ for CMS, ATLAS 7 TeV and 2.76 TeV and CDF bins

Gluon-PDF



NNPDF collaboration



NNPDF collaboration

- jet data has a big impact on the medium to large- x gluon PDF reducing its uncertainty
- fit at NNLO obtained using a $|\delta| < 10\%$ criteria which excluded many jet data
- need exact NNLO all-channel prediction to include full jet dataset

Conclusions

- jet **cross sections** at the LHC delivered with **increasing experimental accuracy** making jet measurements **precision physics**
- **double-differential** jet **measurements** have a big **impact** on the extraction of the gluon PDF at medium to large- x
- **experimental** and **theory** errors of **comparable** size
- presented **exact** results for $gg \rightarrow gg + X$ and $q\bar{q} \rightarrow gg + X$ at NNLO
- leading- N_F gg RR, RV and VV corrections derived
- performed **comparison** between **exact** NNLO results and **approximate** NNLO results from **threshold resummation** in the gg -channel
 - **largest differences** arise at low- p_T for **central rapidities** and all p_T at **large rapidities**
 - **differences** are **smaller** at the Tevatron than at the LHC 7 TeV

Ongoing work:

- numerical implementation of qg and qq integrated dipoles to extract predictions for qg and qq initial states
- qg channel - most important at the LHC
- qq channel - important at high p_T

Back-up slides

Threshold approximation - gg channel

S. Carrazza, JP, arXiv:1407.7031

CMS 2011	N_{dat}	χ^2/dof	Exclusion regions (y, p_T)	
$ \delta < 15\%$	88	1.81	$1.0 < y < 1.5$ $ y > 1.5$	$p_T < 153$ GeV all p_T bins
$ \delta < 10\%$	83	1.89	$1.0 < y < 1.5$ $ y > 1.5$	$p_T < 272$ GeV all p_T bins
$ \delta < 7.5\%$	77	1.89	$0.5 < y < 1.0$ $1.0 < y < 1.5$ $ y > 1.5$	$p_T < 153$ GeV $p_T < 395$ GeV all p_T bins
$ \delta < 5\%$	59	1.83	$0.5 < y < 1.0$ $1.0 < y < 1.5$ $ y > 1.5$	$p_T < 220$ GeV $p_T < 737$ GeV, $p_T > 790$ GeV all p_T bins

ATLAS 2.76 TeV	N_{dat}	χ^2/dof	Exclusion regions (y, p_T)	
$ \delta < 15\%$	10	2.15	$0.0 < y < 0.3$ $0.3 < y < 0.8$ $ y > 0.8$	$p_T < 110$ GeV $p_T < 210$ GeV all p_T bins
$ \delta < 10\%$	3	0.35	$0.0 < y < 0.3$ $ y > 0.3$	$p_T < 260$ GeV all p_T bins
$ \delta < 7.5\%$	-	-	all $ y $ bins	all p_T bins
$ \delta < 5\%$	-	-	all $ y $ bins	all p_T bins

ATLAS 7 TeV	N_{dat}	χ^2/dof	Exclusion regions (y, p_T)	
$ \delta < 15\%$	16	1.82	$0.0 < y < 0.3$ $0.3 < y < 0.8$ $0.8 < y < 1.2$ $ y > 1.2$	$p_T < 260$ GeV $p_T < 400$ GeV $p_T < 1$ TeV all p_T bins
$ \delta < 10\%$	9	1.58	$0.0 < y < 0.3$ $0.3 < y < 0.8$ $ y > 0.8$	$p_T < 400$ GeV $p_T < 800$ GeV all p_T bins
$ \delta < 7.5\%$	5	2.02	$0.0 < y < 0.3$ $ y > 0.8$	$p_T < 500$ GeV all p_T bins
$ \delta < 5\%$	1	-	$0.0 < y < 0.3$ $ y > 0.3$	$p_T < 1$ TeV, $p_T > 1.2$ TeV all p_T bins

CDF	N_{dat}	χ^2/dof	Exclusion regions (y, p_T)	
$ \delta < 15\%$	60	2.32	$1.1 < y < 1.6$ $ y > 1.6$	$p_T < 96$ GeV all p_T bins
$ \delta < 10\%$	52	1.86	$1.1 < y < 1.6$ $ y > 1.6$	$p_T < 224$ GeV, $p_T > 298$ GeV all p_T bins
$ \delta < 7.5\%$	48	1.37	$0.7 < y < 1.1$ $ y > 1.1$	$p_T < 72$ GeV all p_T bins
$ \delta < 5\%$	45	1.28	$0.7 < y < 1.1$ $ y > 1.1$	$p_T < 110$ GeV all p_T bins

- Summary of exclusion regions in p_T and rapidity $|y|$ as a function of the relative difference between exact and threshold k-factors for the gluon-gluon channel
- χ^2/dof for aNNLO PDF fits as a function of exclusion criteria $|\delta|$