Non-perturbative charm and VFNS

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Parton Distributions for the LHC, Benasque, Feb 18, 2015

Mostly based on:

MB, Andrew Papanastasiou, Frank Tackmann

"Resummation and Matching of b-quark Mass Effects in $b\bar{b}H$ Production"

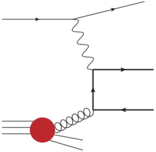
to appear soon

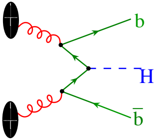
Heavy quarks

Heavy quarks $\Leftrightarrow m \gg \Lambda$

$$m_c \sim 1.3\ \mathrm{GeV}, \qquad m_b \sim 4.7\ \mathrm{GeV}, \qquad m_t \sim 173\ \mathrm{GeV}$$

m regulates IR divergences: gluon splittings are finite





Collinear region of gluon splitting: $\mu_{\mathrm{H}} \sim Q$: Hard Scale

$$\left(\alpha_s \log \frac{m}{\mu_{\mathrm{H}}}\right)^k$$

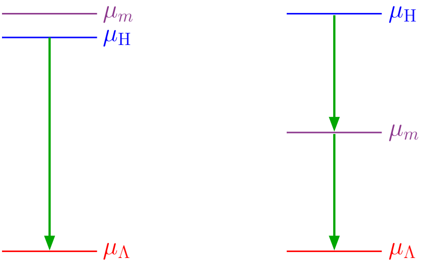
At high energies $\mu_{\mathrm{H}} \gg m$ these logs are large $\Rightarrow$ **Resummation**

We use an EFT approach

We introduce a new scale

$$\mu_m \sim m$$

and we consider two scale hierarchies



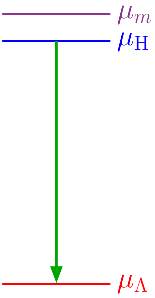
The cross section will be

$$\langle P|O_{\rm DIS}|P\rangle = \sigma = \begin{cases} \sigma^{(n_l)} & \text{for } \mu_m \geq \mu_{\rm H} \\ \sigma^{(n_l+1)} & \text{for } \mu_m \leq \mu_{\rm H} \end{cases}$$

$O_{\rm DIS} = j^\mu j^\nu L_{\mu\nu}$, $|P\rangle =$ proton

EFT factorization: $\mu_m \geq \mu_{\rm H}$ (std QCD massless fact)

EFT matching at $\mu_{\rm H}$



$$O_{\rm DIS} = \sum_{\ell=g,q,\bar q} \boldsymbol{C}_\ell^{(n_l)}(\mu_{\rm H}, m)\, O_\ell^{(n_l)}(\mu_{\rm H})$$

$O_\ell^{(n_l)}$: collinear quark/gluon (or PDF) operators
$\to$ evolve with DGLAP anomalous dimensions

$$\sigma^{(n_l)} = \sum_{\ell,\ell'=g,q,\bar q} \boldsymbol{C}_\ell^{(n_l)}(\mu_{\rm H}, m)\, U_{\ell\ell'}^{(n_l)}(\mu_{\rm H}, \mu_\Lambda)\, f_{\ell'}^{(n_l)}(\mu_\Lambda)$$

$$f_\ell^{(n_l)}(\mu_\Lambda) = \langle P | O_\ell^{(n_l)}(\mu_\Lambda) | P \rangle$$

Evolved fixed flavor number (FFN) PDF:

$$f_\ell^{(n_l)}(\mu) = \sum_{\ell'=g,q,\bar q} U_{\ell\ell'}^{(n_l)}(\mu, \mu_\Lambda)\, f_{\ell'}^{(n_l)}(\mu_\Lambda)$$

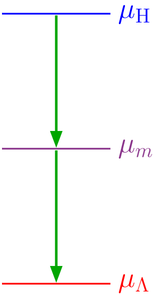
Heavy Flavor integrated out at $\mu_{\rm H}$: only contributes to $\boldsymbol{C}_\ell^{(n_l)}(\mu_{\rm H}, m)$

EFT factorization: $\mu_m \le \mu_{\rm H}$ (resummation)

EFT matching at $\mu_{\rm H}$

$$O_{\rm DIS} = \sum_{i=g,q,\bar q,Q,\bar Q} C_i^{(n_l+1)}(\mu_{\rm H}, m)\, O_i^{(n_l+1)}(\mu_{\rm H}, m)$$

$O_i^{(n_l+1)}$: collinear quark/gluon operators, including heavy quark
$\to$ evolve with DGLAP anomalous dimensions, with $n_f = n_l + 1$



EFT matching at μ_m

$$O_i^{(n_l+1)}(\mu_m, m) = \sum_{\ell=g,q,\bar q} \mathcal{M}_{i\ell}^{(n_l)}(\mu_m, m)\, O_\ell^{(n_l)}(\mu_m)$$

Resummed cross section:

$$\sigma^{(n_l+1)} = C_i^{(n_l+1)}(\mu_{\rm H}, m)\, U_{ij}^{(n_l+1)}(\mu_{\rm H}, \mu_m) \mathcal{M}_{j\ell}^{(n_l)}(\mu_m, m)\, U_{\ell\ell'}^{(n_l)}(\mu_m, \mu_\Lambda)\, f_{\ell'}^{(n_l)}(\mu_\Lambda)$$

Evolved variable flavor number (VFN) PDF:

$$f_i^{(n_l+1)}(\mu, \mu_m, m) = \sum_j^{n_l+1} U_{ij}^{(n_l+1)}(\mu, \mu_m) \sum_{\ell,\ell'}^{n_l} \mathcal{M}_{j\ell}^{(n_l)}(\mu_m, m)\, U_{\ell\ell'}^{(n_l)}(\mu_m, \mu_\Lambda)\, f_{\ell'}^{(n_l)}(\mu_\Lambda)$$

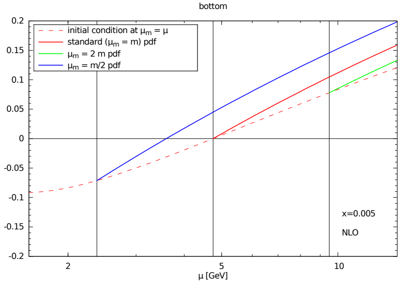
The matching scale μ_m

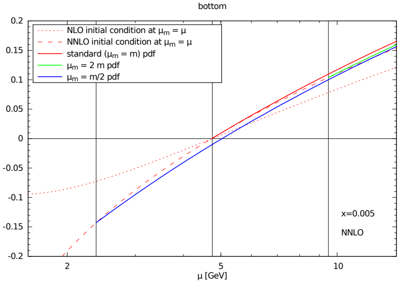
$$\sigma = \begin{cases} \boldsymbol{C}_\ell^{(n_l)}(\mu_{\rm H}, m)\, U_{\ell\ell'}^{(n_l)}(\mu_{\rm H}, \mu_\Lambda)\, f_{\ell'}^{(n_l)}(\mu_\Lambda) \\ C_i^{(n_l+1)}(\mu_{\rm H}, m)\, U_{ij}^{(n_l+1)}(\mu_{\rm H}, \mu_m) \boldsymbol{\mathcal{M}}_{j\ell}^{(n_l)}(\mu_m, m)\, U_{\ell\ell'}^{(n_l)}(\mu_m, \mu_\Lambda)\, f_{\ell'}^{(n_l)}(\mu_\Lambda) \end{cases}$$

To all orders in α_s, σ is $\mu_m, \mu_{\rm H}$ **independent**

The residual dependence at finite order is formally higher order

Example: bottom PDF $f_b^{(5)}(\mu, \mu_m, m)$





The Charm quark

$$f_c^{(4)}(\mu,\mu_m,m) = U_{cj}^{(4)}(\mu,\mu_m)\,\boldsymbol{\mathcal{M}}_{j\ell}^{(3)}(\mu_m,m)\;U_{\ell\ell'}^{(3)}(\mu_m,\mu_\Lambda)\,f_{\ell'}^{(3)}(\mu_\Lambda)$$

The charm PDF is generated perturbatively at the scale $\mu_m \sim m_c \sim 1.3\,\mathrm{GeV}$ through the matching condition $\boldsymbol{\mathcal{M}}_{c\ell}^{(3)}(\mu_m,m)$, known to order $\alpha_s^2(\mu_m)$

Do we trust this truncated perturbative expansion?

- if yes, then we fit g,u,d,s at $\mu_{\rm fit} < \mu_m$, and let the matching condition generate the charm PDF
- if no, we can fit a non-perturbative charm PDF, simply letting $\mu_{\rm fit} > \mu_m$, such that the charm PDF is always active

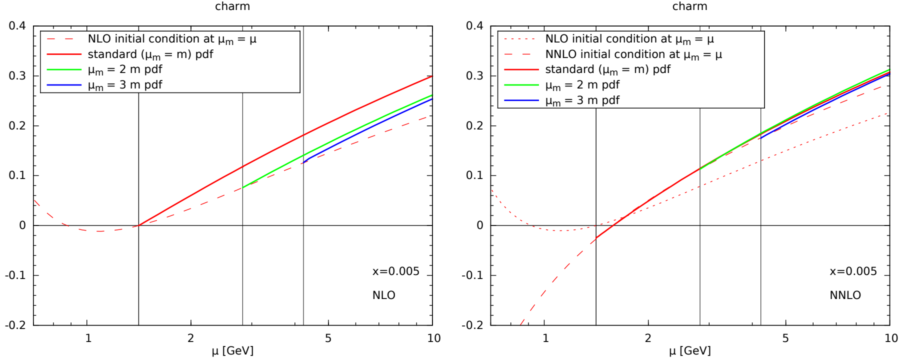
Ok but, do we?

$$\alpha_s(1\,\mathrm{GeV}) \sim 0.4 \qquad\qquad \alpha_s(4\,\mathrm{GeV}) \sim 0.2$$

Depends on the actual value of μ_m...

Non-perturbative Charm PDF?

$$f_c^{(4)}(\mu,\mu_m,m) = U_{cj}^{(4)}(\mu,\mu_m)\,\boldsymbol{\mathcal{M}}_{j\ell}^{(3)}(\mu_m,m)\;U_{\ell\ell'}^{(3)}(\mu_m,\mu_\Lambda)\,f_{\ell'}^{(3)}(\mu_\Lambda)$$



A larger μ_m $(2m_c$ or $3m_c)$ is safer!! Perhaps no need to fit...

Do not cross the bottom threshold!

Conclusions

Discussed in the talk:

- We have derived using an EFT framework a generic VFNS, which reduces to ACOT, S-ACOT and FONLL in special cases
- We suggest to use the (unphysical) scale μ_m to study the perturbative stability of the matching condition, and also as an additional source of uncertainty
- A larger scale $\mu_m > m$ is advisable (α_s is smaller)
- Perhaps there is no need to fit a non-perturbative charm
- If we want to fit, we simply let the charm quark active at each scale (formally $\mu_m < \mu_{\text{fit}}$), and use always the cross section above threshold
- If we think there is an "intrinsic" charm, then we should not cross μ_m (we can't fully integrate out the charm)!

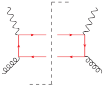
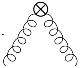
Not discussed:

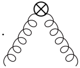
- Perturbative counting: heavy quark PDF $\sim \mathcal{O}(\alpha_s)$
- bbH production: perturbatively stable cross section

Backup slides

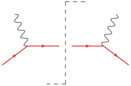
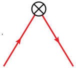
Computation of Wilson coefficients

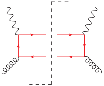
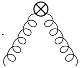
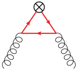
The case $\mu_m > \mu_{\rm H}$

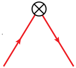
 $= \boldsymbol{C}_g^{(n_l)(1)}(\mu_{\rm H}, m) \cdot$ 

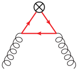
 $= 1$

The case $\mu_m < \mu_{\rm H}$

 $= C_Q^{(n_l+1)(0)}(\mu_{\rm H}, m) \cdot$ 

 $= C_g^{(n_l+1)(1)}(\mu_{\rm H}, m) \cdot$  $+ C_Q^{(n_l+1)(0)}(\mu_{\rm H}, m)$ 

 $= 1$

 $= P_{qg}^{(1)} \log \frac{\mu^2}{m^2}$

$$C_g^{(n_l+1)(1)}(\mu_{\rm H}, m) = \boldsymbol{C}_g^{(n_l)(1)}(\mu_{\rm H}, m) - C_Q^{(n_l+1)(0)}(\mu_{\rm H}, m) P_{qg}^{(1)} \log \frac{\mu^2}{m^2}$$

Perturbative counting: DIS heavy quark production

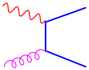
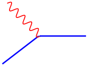
$$f_Q^{(n_l+1)}(\mu,\mu_m,m) = \Big[U_{QQ}^{(n_l+1)}(\mu,\mu_m)\,\mathcal{M}_{Q\ell''}^{(n_l)}(\mu_m,m) + U_{Q\ell'}^{(n_l+1)}(\mu,\mu_m)\,\mathcal{M}_{\ell\ell'}^{(n_l)}(\mu_m,m)\Big] f_{\ell'}^{(n_l)}(\mu_m)$$

$$= \qquad \mathcal{O}(1) \qquad \mathcal{O}(\alpha_s) \quad + \mathcal{O}\left(\alpha_s \log\frac{\mu}{\mu_m}\right) \qquad \mathcal{O}(1)$$

For $\mu/\mu_m \gtrsim 300$ then $f_Q^{(n_l+1)} \sim \mathcal{O}(1)$, but for $\mu/\mu_m \lesssim 300$ then $f_Q^{(n_l+1)} \sim \mathcal{O}(\alpha_s)$!

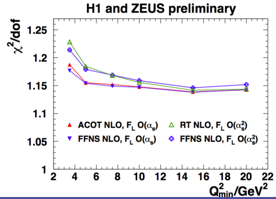
DIS heavy quark production at LO

$$\sigma = \qquad C_g^{(n_l+1)} \otimes f_g^{(n_l+1)} + C_Q^{(n_l+1)} \otimes f_Q^{(n_l+1)}$$

$= \Big[$  $-$ coll $\Big] \otimes f_g^{(n_l+1)} +$  $\otimes f_Q^{(n_l+1)}$

$$= \qquad \mathcal{O}(\alpha_s) \qquad \mathcal{O}(1) \quad + \quad \mathcal{O}(1) \qquad \mathcal{O}(\alpha_s)$$

Everything is order α_s:
we use NLO matching and evolution

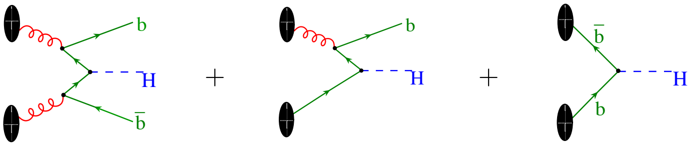


bbH and power counting

$$\sigma = \underbrace{C_{ij}^{(n_l+1)}(\mu_{\rm H}, m)\left[U^{(n_l+1)}(\mu_{\rm H}, \mu_m)\boldsymbol{\mathcal{M}}^{(n_l)}(\mu_m, m)\ U^{(n_l)}(\mu_m, \mu_\Lambda)}_{\text{perturbative part: } \alpha_s \text{ counting here}}\ f^{(n_l)}(\mu_\Lambda)\right]^2_{ij}$$

Hence the LO in bbH is given by

LO =



$$= \quad \mathcal{O}(1 \cdot 1 \cdot \alpha_s^2) \quad + \quad \mathcal{O}(1 \cdot \alpha_s \cdot \alpha_s) \quad + \quad \mathcal{O}(\alpha_s \cdot \alpha_s \cdot 1)$$