Hair-Brane Ideas on the Horizon

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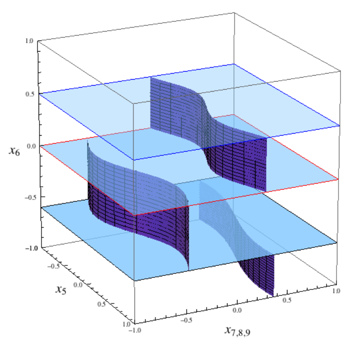
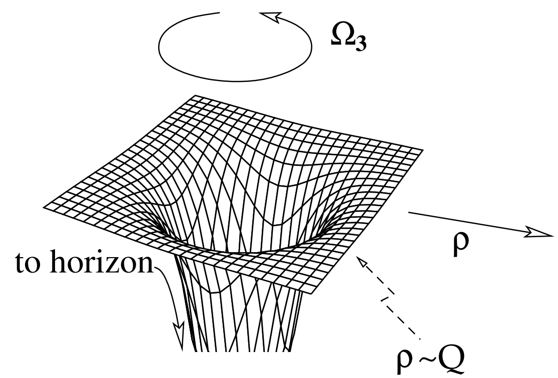
based on arXiv:1409.6017, and to appear

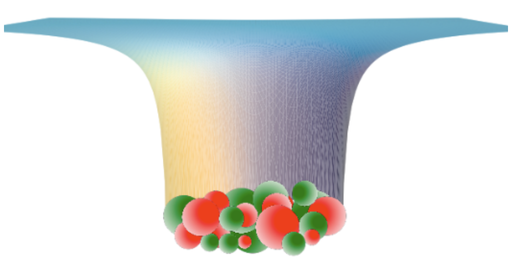
Branes, Geometry and Entropy

- Conventional view: Brane picture valid at weak coupling, states map to orbifold CFT $(T^4)^N/S_N$; geometry valid at strong coupling



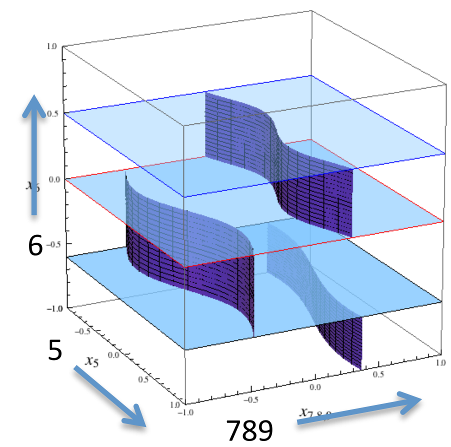


- Microstate Geometries Program (Lunin-Mathur '01, Bena-Warner '04): Each BH μstate associated to a distinct horizonless solution of supergravity (+ stringy effects?)



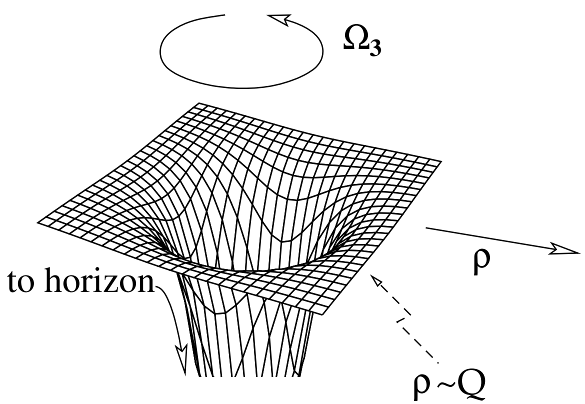
Branes, Geometry and Entropy

- Compactify type IIA on T^5 along (56789);
- Wrap n_2 D2-branes along (56);
- Wrap n_4 D4-branes along (5789);
- Excite n_p units of momentum along (5)

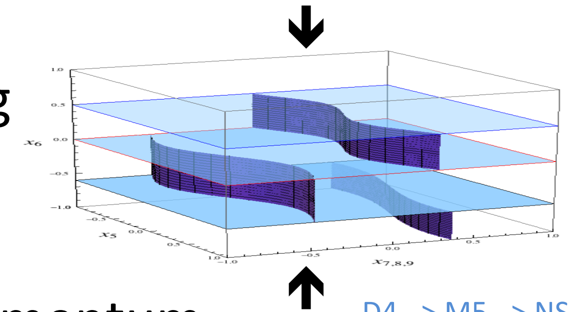


- Near-horizon geometry is $AdS_3 x S^3 x T^4$. Horizon at $\rho=0$ has associated entropy

$$S_{BH} = 2\pi\left(\sqrt{n_2 n_4 n_p} + \sqrt{n_2 n_4 \bar{n}_p}\,\right)$$



Branes, Geometry and Entropy

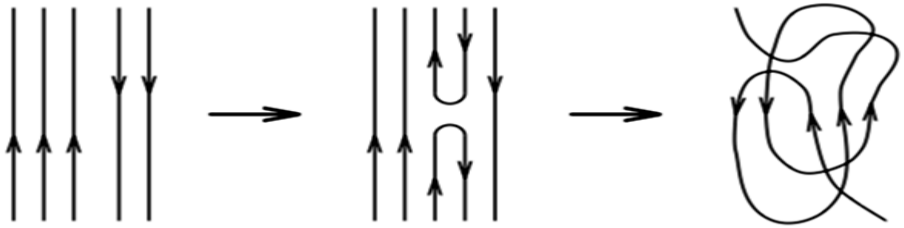


- Performing a 6-11 flip w/different torus scaling leads to ***little string theory*** (LST)

- Scaling such that both anti-winding & anti-momentum excitations are relevant yields (Maldacena '96)

$$S_{BH} = 2\pi\sqrt{n_5}\left(\sqrt{n_1} + \sqrt{\bar{n}_1}\right)\left(\sqrt{n_p} + \sqrt{\bar{n}_p}\right)$$

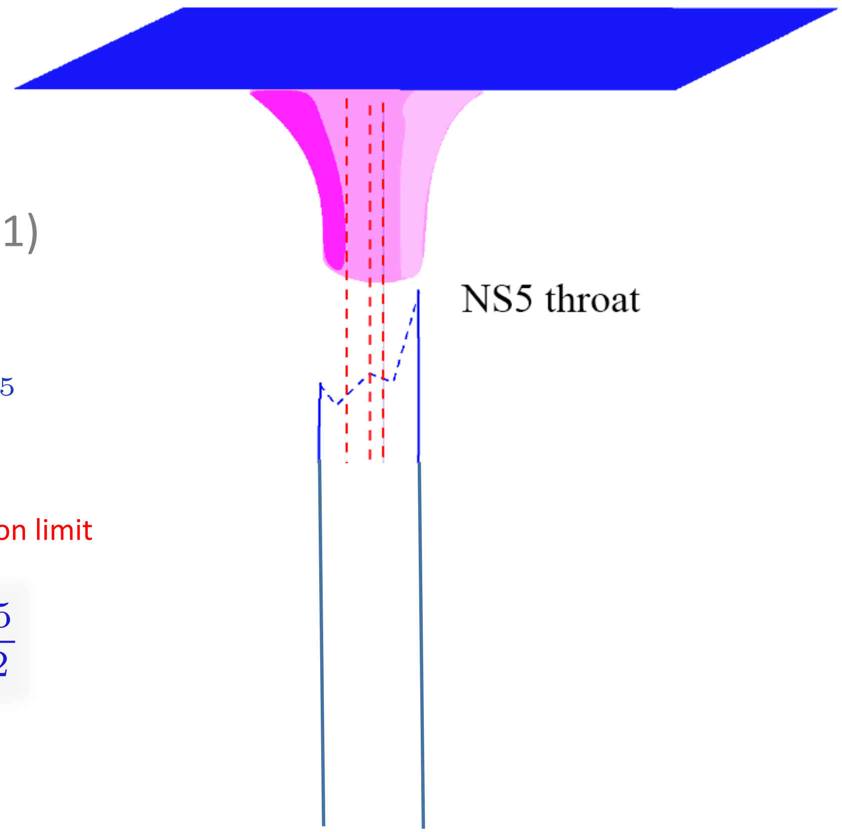
$$= 2\pi\left(\sqrt{n_5 N_L} + \sqrt{n_5 N_R}\right)$$



BPS Fivebrane Geometries

- Thinking about NS5 dynamics and its associated Little String Theory (LST) provides useful intuition:

- The (original) near-horizon limit is a linear dilaton × SU(2)$_{n5}$ CFT on the worldsheet (Callan-Harvey-Strominger ’91)



$$ds^2 = -dt^2 + H\left[dr^2 + r^2 d\Omega_3^2\right] + ds^2_{\mathbb{T}^5}$$

$$H_{ijk} = \epsilon_{ijk}{}^{l}\partial_l log(H)$$

$$e^{2\Phi} = H$$

Near-horizon limit

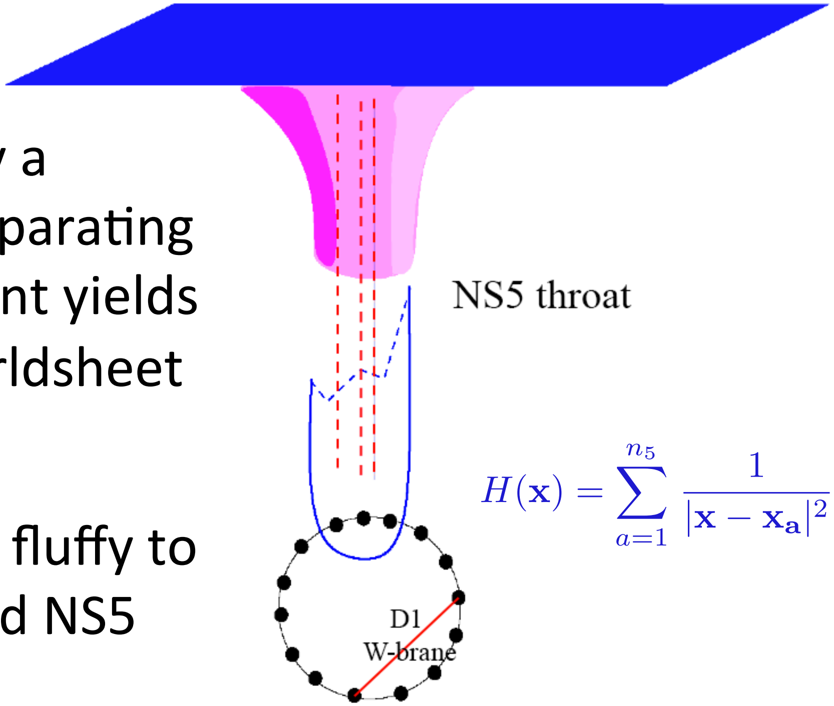
$$H(r) = \cancel{1} + \frac{n_5}{r^2}$$

BPS Fivebrane Geometries

- Coincident fivebranes are singular in perturbative string theory; $g_s \to \infty$ down the throat.

- The Coulomb branch is described by a **nonsingular** worldsheet CFT, *e.g.* separating poles into Z_{n5} symmetric arrangement yields $[SL(2,R)/U(1) \times SU(2)/U(1)]/Z_{n5}$ worldsheet dynamics (Giveon-Kutasov '99)

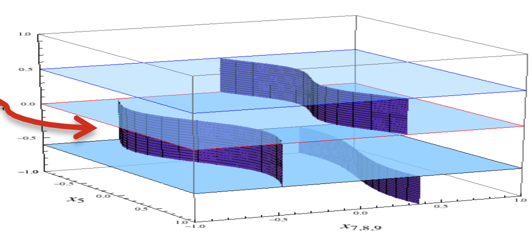
- Nonsingular because strings are too fluffy to resolve the throat of a single isolated NS5 (D-branes can see it, however)



BPS Fivebrane Geometries

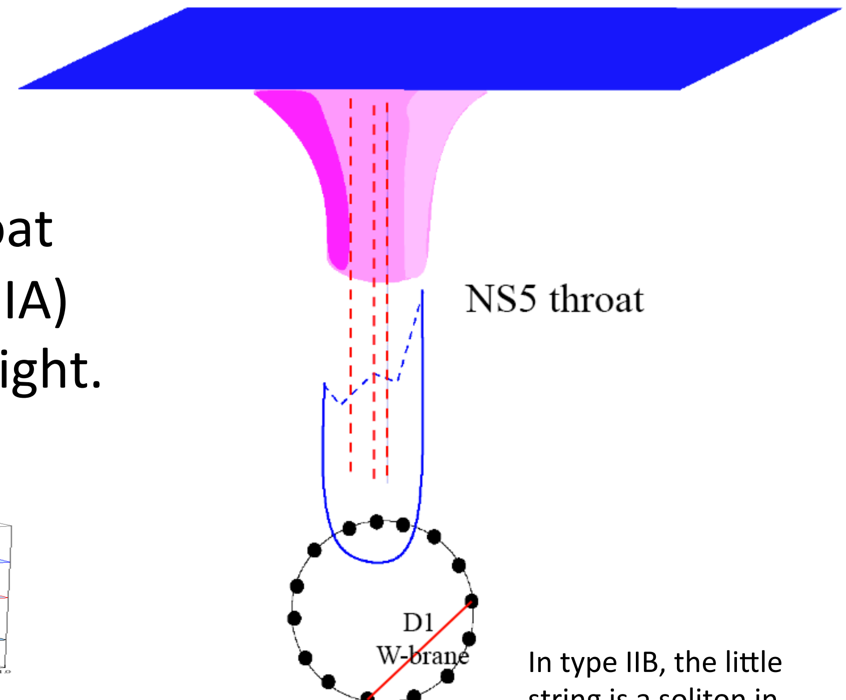
- The little string is related to the fractionated D-branes that stretch between NS5's

- Separations of NS5-branes govern the depth of a capped throat.
  As separations scale down, the throat deepens; D1-branes (in IIB; D2's in IIA) stretching between NS5's become light.



D2 W-string

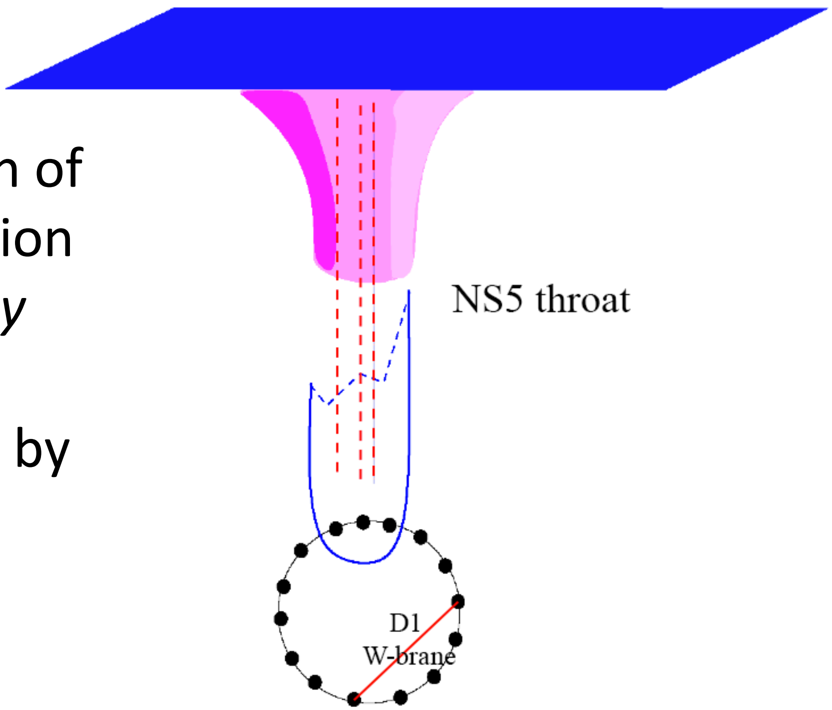
(directly related to little string; sources tensor gauge field on IIA fivebranes)



In type IIB, the little string is a soliton in NS5 gauge theory

BPS Fivebrane Geometries

- When D-branes stretched between NS5's become lighter than F1 strings, the worldsheet description breaks down



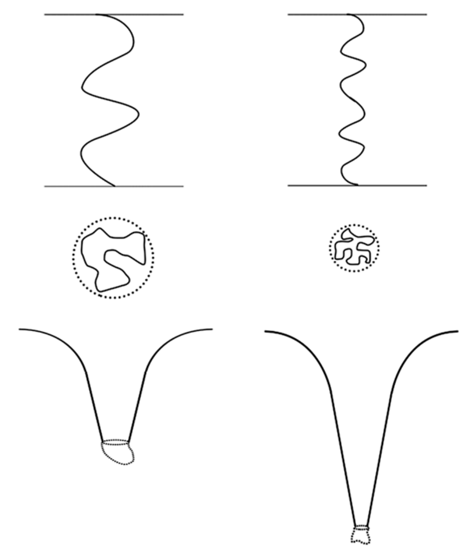
- Dynamics passes to the Higgs branch of fractionated (little) strings with tension *$O(1/n_5)$ which dominate the entropy*

- 3-charge BPS state counting is given by the elliptic genus of little strings
(F1 elliptic genus has less entropy)

BPS Microstate Geometries

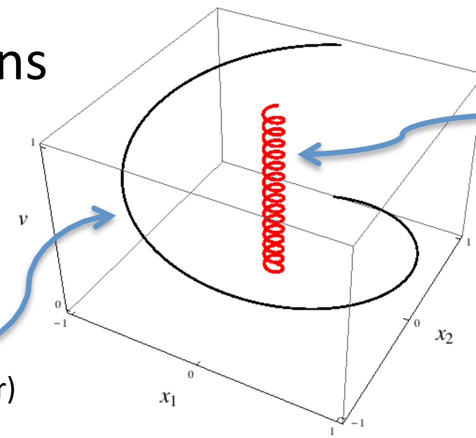
- 2-charge BPS µstate geometries enumerated (Lunin-Mathur '01): U-duality maps D2-D4 (or D1-D5) to F1-P; construct F1-P geometry and map back. A BPS F1-P is simply a string with only left-moving oscillator excitations (total level $n_1 n_p$).

- Separation of F1-P source strands governs depth of capped AdS throat; source strands are forced to separate onto the `Coulomb branch' by centrifugal force of large angular momentum



BPS Microstate Geometries

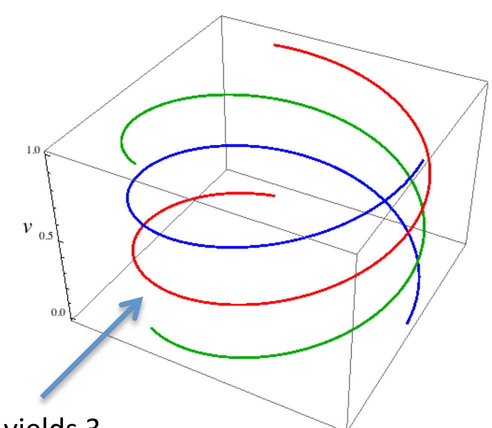
- Example: $(AdS_3 \times S^3)/Z_k$ U-dual source configuration: F1-P with $n_1 n_5/k$ excitations of k^{th} oscillator mode (Lunin-Mathur '01). The total angular momentum is the number of oscillator quanta.



k=$n_1 n_5$ yields deepest throat

k=1 yields global AdS (spectral flowed to R sector)

- Can also consider $(SL(2,R) \times SU(2))/Z_k$ WZW model as an F1-NS5 WS background. There are 4(k-1) moduli from twisted sectors, describing motion of fivebranes on the Coulomb branch, *i.e.* the NS5 version of the long string sector (EJM-McElgin '01,'02). Pushing fivebranes together is again a singular limit of the worldsheet theory



k=3 yields 3 NS5-F1 clusters

BPS Microstate Geometries

- To obtain 3-charge BH µstate, need to add third charge n_p
- Proposed microstate geometries are more sophisticated version of 2-charge examples (Mathur etal, Bena-Warner etal)

$$ds^2 = \frac{1}{\sqrt{Z_1 Z_5}}\Big[\frac{(-dt+k)^2}{Z_p} + Z_p(dz+\beta)^2 + Z_1 Z_5\, ds^2_{\mathcal{B}_4} + Z_1\, dx^2_{\mathbf{T}^4}\Big]$$

- Metric coeffs $Z_{1,5,p}$, k, β are harmonic functions/forms; simple, symmetric choice of hyperKähler base $\mathcal{B}_4$ is

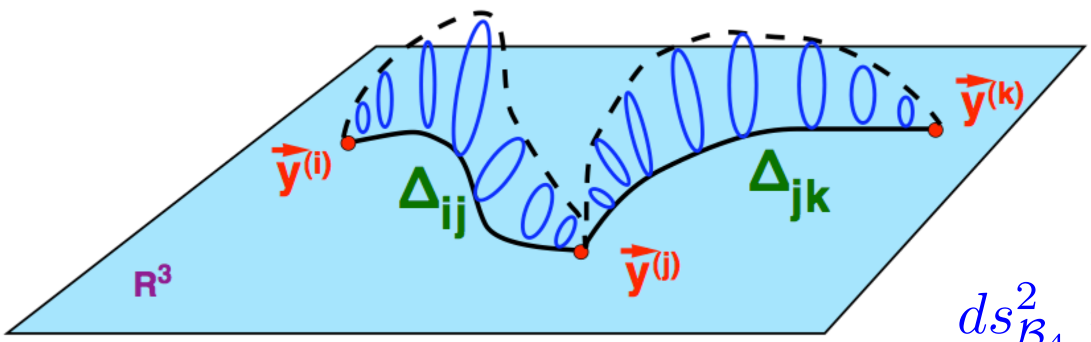
$$ds^2_{\mathcal{B}_4} = V^{-1}(d\psi + A)^2 + V\, ds^2_{\mathbf{R}^3}$$ (Gibbons-Hawking geometry)

BPS Microstate Geometries

- Simple choice for harmonic fn V (with $\nabla V = \nabla \times A$)

$$V = \epsilon_0 + \sum_a \frac{q_a}{|\mathbf{y} - \mathbf{y}_a|}$$

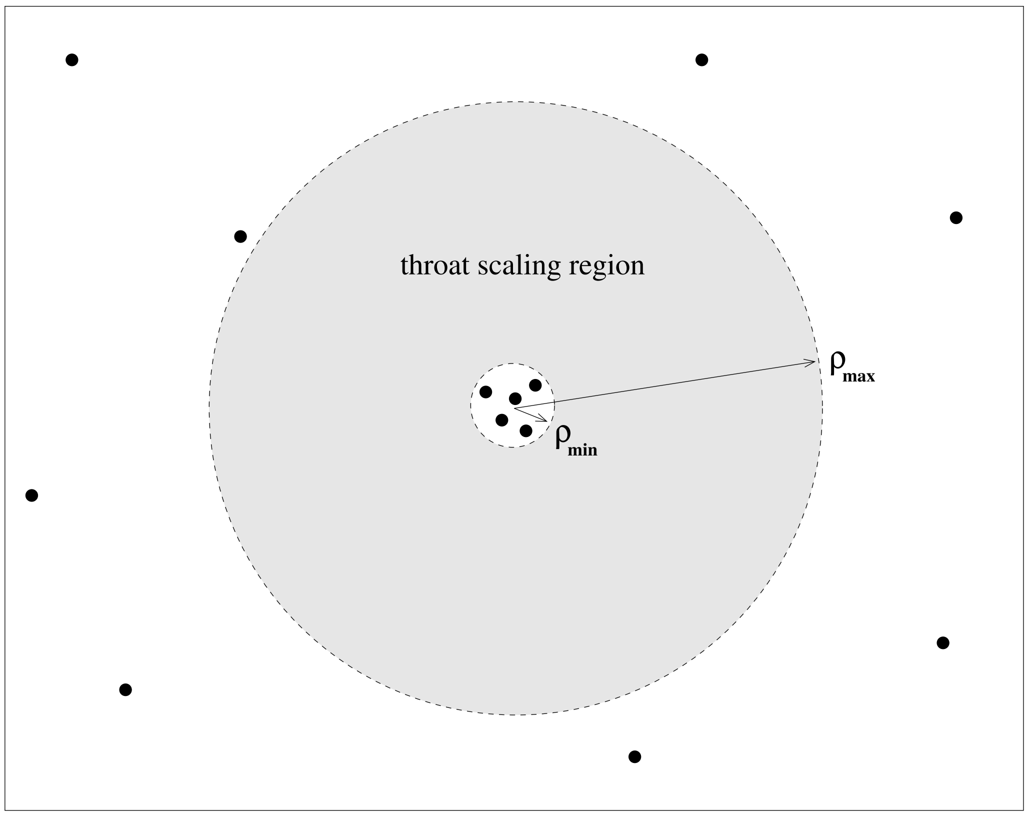
- Other harmonic fns/forms have poles at y_a whose residues conspire to make full geometry smooth; charge sources localize at y_a



$$ds^2_{\mathcal{B}_4} = V^{-1}(d\psi + A)^2 + V ds^2_{\mathbf{R}^3}$$

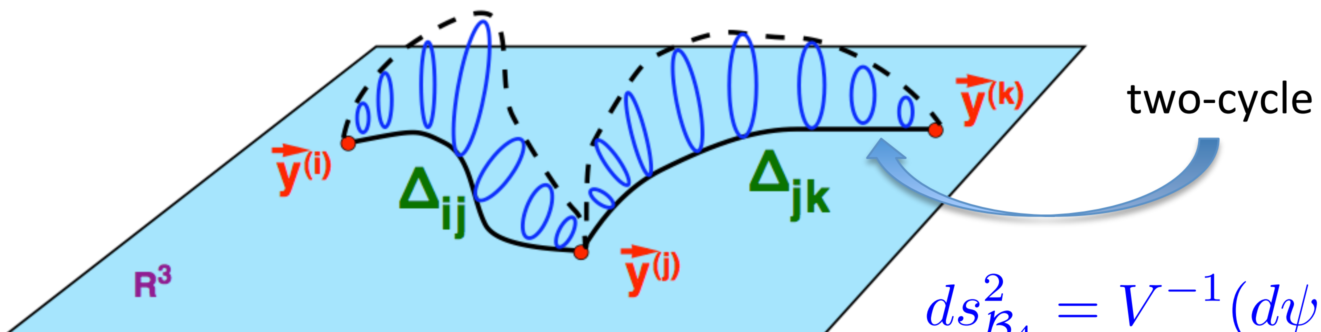
BPS Microstate Geometries

- Source separation controls depth of throat (as in 2-charge geometries, and little string theory, *etc* …)



BPS Microstate Geometries

- In μstate geometries, separation of sources (poles in harmonic fns) controls size of two-cycles in GH base $\mathcal{B}_4$



$$ds^2_{\mathcal{B}_4} = V^{-1}(d\psi + A)^2 + V ds^2_{\mathbf{R}^3}$$

- When source separation scales down, wrapped branes stretching between charge centers become light; eventually these condense and dynamics passes to Higgs branch. Does the geometrical description break down as in LST?

BPS Microstate Geometries

- Three relevant duality frames:

M-theory

Charge	Dipole charge
M2: 56	M5: 78 9$\underline{10}$ ψ
M2: 78	M5: 56 9$\underline{10}$ ψ
M2: 9$\underline{10}$	M5: 56 78 ψ
J: ψ	KKM: 123 $\boldsymbol{\psi}$
M2:	y_{ij} ψ

$g_s \rightarrow \infty$

Charge	Dipole charge
D2: 56	D4: 78 9$\underline{10}$
D2: 78	D4: 56 9$\underline{10}$
D2: 9$\underline{10}$	D4: 56 78
D0:	D6: 56 78 9$\underline{10}$
F1:	y_{ij}

IIA

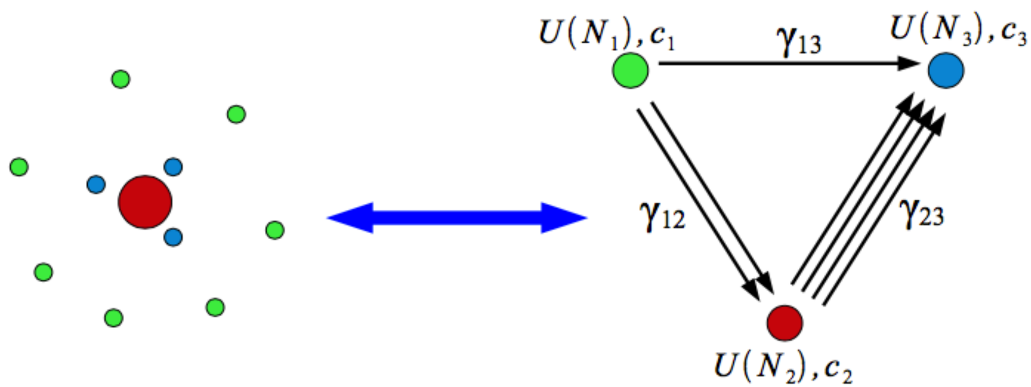
shrink $(T^2)_{9\underline{10}}$

Charge	Dipole charge
D3: 56 v	D3: 78 ψ
D3: 78 v	D3: 56 ψ
P: v	KKM: 123 $\boldsymbol{v}$
J: ψ	KKM: 123 $\boldsymbol{\psi}$
D3:	y_{ij} ψv

IIB

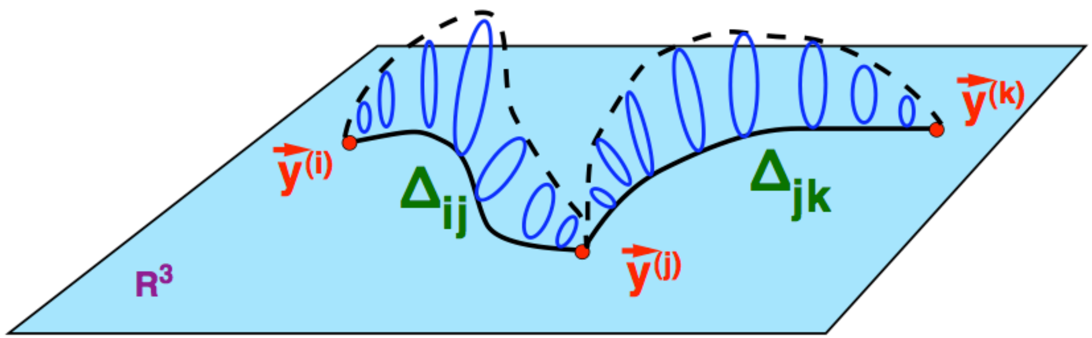
Geometries and Quivers

- Multicenter dynamics captured by QM of collective modes of brane bound states, including lightest stretched branes (à la Matrix Theory).
- Position of centers described by vector multiplets of quiver nodes; stretched branes are quanta of quiver link hypermultiplets



Geometries and Quivers

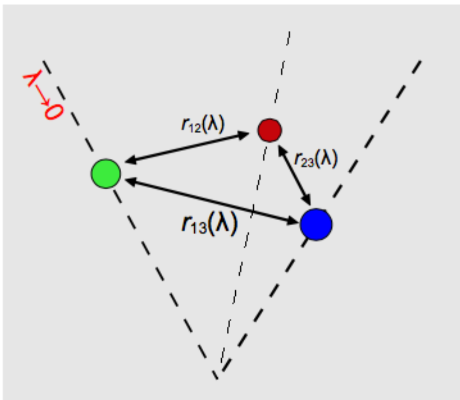
- Stretched strings in IIA frame lift to M2-branes stretching between KK monopole centers (descending to D3-branes in IIB); they are hypermultiplet quanta in quiver QM

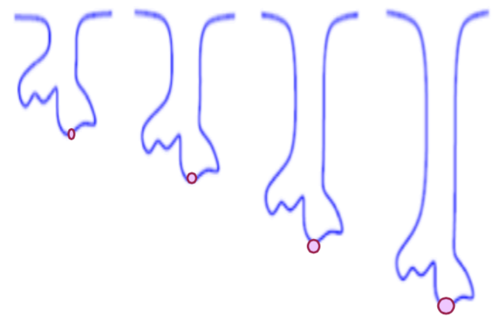


- The BPS condition guarantees that the mass of the wrapped brane is $\mu|\mathbf{y}_{(i)}-\mathbf{y}_{(j)}|$ in all frames

Geometries and Quivers

- Closed quivers admit scaling solutions where centers coalesce

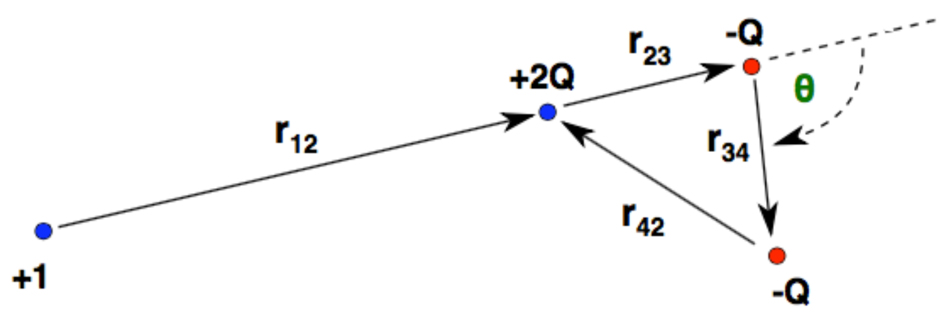




- On the geometry side, a local throat forms and deepens. In the limit,
  - Hypermultiplets become massless and condense
  - The geometry develops a horizon
- Horizon dynamics involves a *wrapped brane condensate*

Geometries and Quivers

- This approach is not expected to fully capture horizon dynamics; the horizon that forms should not carry details of how it was assembled, *e.g.* the data of the specific quiver used. Nonabelian dof's that redistribute charges/fluxes are missing.
- But hopefully embodies qualitatively correct horizon physics
- Consider simplest three-node scaling cluster (Bena-Wang-Warner '07):



Geometries and Quivers

Charge	Dipole charge
M2: 56	M5: 78 9$\underline{10}$ ψ
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M2: 9$\underline{10}$	M5: 56 78 ψ
J: ψ	KKM: 123 $\boldsymbol{\psi}$

- Total charge of cluster:

$$\{KKM, M5, M2, J_\psi\} = \{1, d^I, Q_I, J_\psi\}$$

- In terms of constituent charges, one has

$$
\begin{aligned}
d^I &= \textstyle\sum_a d^I_a \\
Q_I &= \textstyle\sum_a Q_{Ia} = \frac{1}{2} C_{IJK} \sum_a \underline{d^J_a d^K_a} / q_a \\
J_\psi &= \textstyle\sum_a \underline{d^1_a d^2_a d^3_a} / q^2_a \\
J_T &= 4 \left| \textstyle\sum_{a,I} d^I_a \, \mathbf{y}_a \right|
\end{aligned}
$$

Requiring smooth geometry relates conserved charges to dipole charges

Geometries and Quivers

- Entropy is given by $E_{7(7)}$ invariant $S_{BH} = \pi\sqrt{\mathcal{I}_4}$

$$\mathcal{I}_4 = [(2d^1 d^2 Q_1 Q_2) - (d^3 Q_3)^2 \; + \; \text{cyclic}] \\ - \; d^1 d^2 d^3 [4J_T + 2(d^I Q_I) - 3d^1 d^2 d^3]$$

Note that S_{BH} scales like $d^1 d^2 d^3/q$ (since Q_I scales like $C_{IJK}\, d^J\, d^K/q$)

- On the other hand, the entropy of the "pure Higgs" states of the triangular quiver has been computed to be (Denef-Moore '07, Bena etal '12)

$$S_\triangle \sim \alpha\,(|\Gamma_{12}| + |\Gamma_{23}| + |\Gamma_{31}|) \, + \ldots$$

where Γ_{ij} is the number of ij links on the quiver, and $\alpha \sim O(1)$

Geometries and Quivers

- The number of links Γ_{ij} of the quiver has a physical interpretation in the M-theory frame. M2-branes wrapping the flux cycles Δ_{ab} experience effective magnetic field in T^6 from G_4 flux along $\Delta_{ab} \times T^2$

- Degeneracy of lowest Landau level (*c.f.* Gaiotto-Strominger-Yin '04, '07)

$$\Gamma_{ab} = \; q_a q_b \, \Pi^{(1)}_{ab} \Pi^{(2)}_{ab} \Pi^{(3)}_{ab}$$

$$\Pi^{(I)}_{ab} = \frac{1}{4\pi} \int_{\Delta_{ab}} F^{(I)} = \left(\frac{k^I_b}{q_b} - \frac{k^I_a}{q_a} \right)$$

is the number of phase space cells on T^6 available to the M2-brane center-of-mass motion (here $G_4^{(I)} = F^{(I)} \wedge \omega_I$ with ω_I along T^2);
this number is cubic in dipole charges d^I_a

Geometries and Quivers

- Interestingly, both entropies scale as

$$S_{BH} \sim S_{\triangle} \sim d^1 d^2 d^3/q \sim \sqrt{q\, Q_1 Q_2 Q_3}$$

- For the example charge assignments in BWW '07 one finds

$$S_{\triangle}/S_{BH} \sim 0.04$$

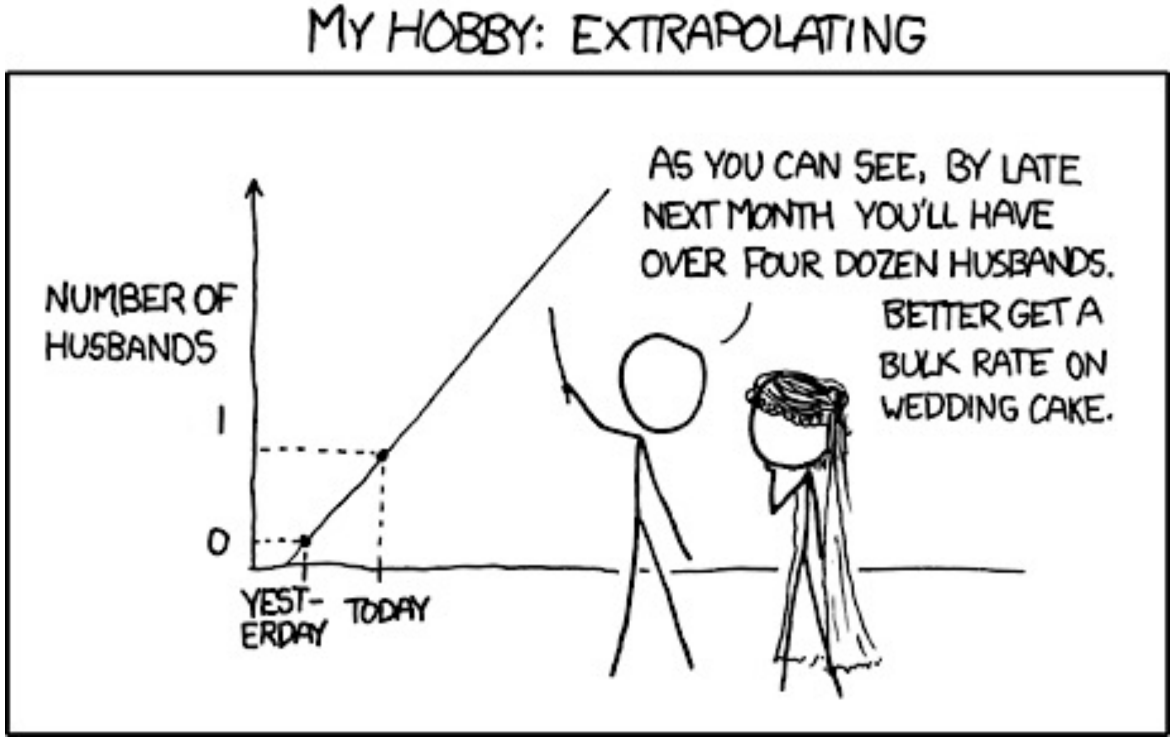
- These results indicate that a significant fraction of the BH dof's are related to fractionated branes wrapping the horizon. Of course, a realistic treatment must incorporate the indistinguishability of constituents of the merged cluster

The Hair-Brane Idea

- These stretched branes are direct analogues of the stretched branes on the Coulomb branch of LST. There, the little string appears when these objects condense (rather directly in IIA, or as a soliton in IIB). KK monopoles are T-dual to NS5, so “W-string” objects appear in IIB, are indirect in IIA/M-theory frame

- **Conjecture**: *Proper treatment of the assembly of black holes from the Coulomb branch of μstate geometries will exhibit the long string that carries the entropy of AdS_3 black holes as a collective excitation of the branes that become light at the entrance to the Higgs branch. Note that in the IIB frame, the stretched branes are D3’s wrapping* $\Delta_{ab} \times S^1_v$ *which are ideal candidates to wind into the effective long string as Δ_{ab} shrinks and they become light.*

Beyond the Extremal Limit



Beyond the Extremal Limit

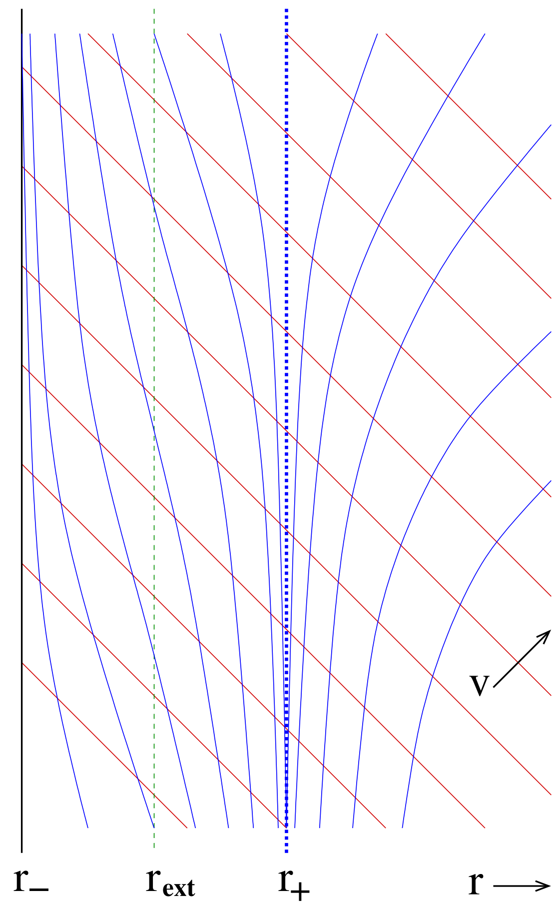
- Naïve BH geometry is $AdS_3 \times S^2$ or $AdS_3 \times S^3$ rotating BTZ; in EF coords

$$ds^2 = -f\,dv^2 + 2dvdr + r^2(d\varphi + \beta dv)^2$$

$$f = \frac{r^2}{\ell^2} - M_3 + \frac{16\,G_3^2 J_3^2}{r^2} \qquad \beta = \frac{4\,G_3 J_3}{r^2}$$

- Inner/outer horizons: *f(r)=0* has 2 solutions $r_\pm$; both horizons have a connection to thermodynamics (Cvetic-Larsen '96-98)

$$dM = T_\pm dS_\pm + \Omega^\pm dJ$$

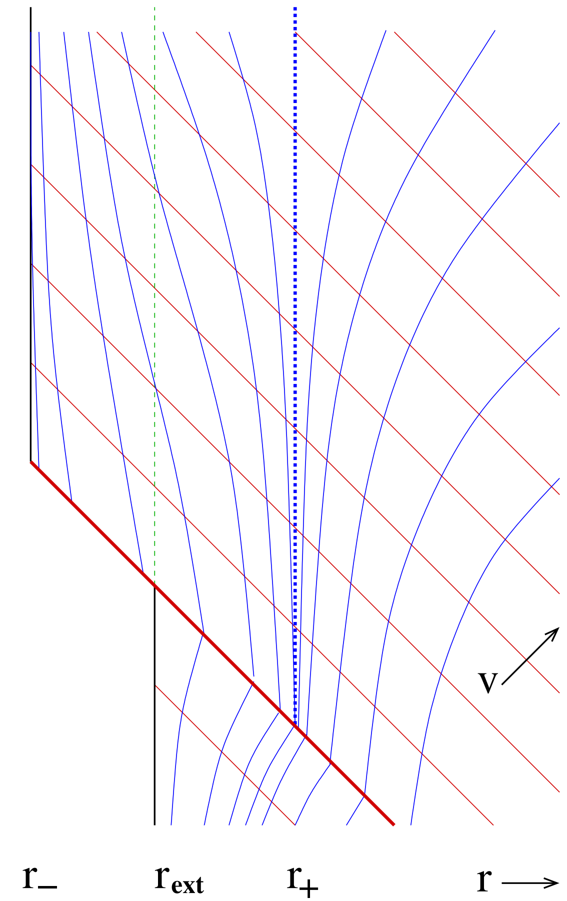


Beyond the Extremal Limit

- At (BPS) extremality, $r_+ = r_-$; infalling matter splits the two horizons

- Inner horizon is **unstable and singular** due to blue-shifting of perturbations
  (Marolf-Ori '11, Murata-Reall-Tanahashi '13)

- Expect that structure of topology, fluxes, condensing branes, *etc*, exists to regularize the would-be singularity
  *at the **inner** horizon*



Beyond the Extremal Limit

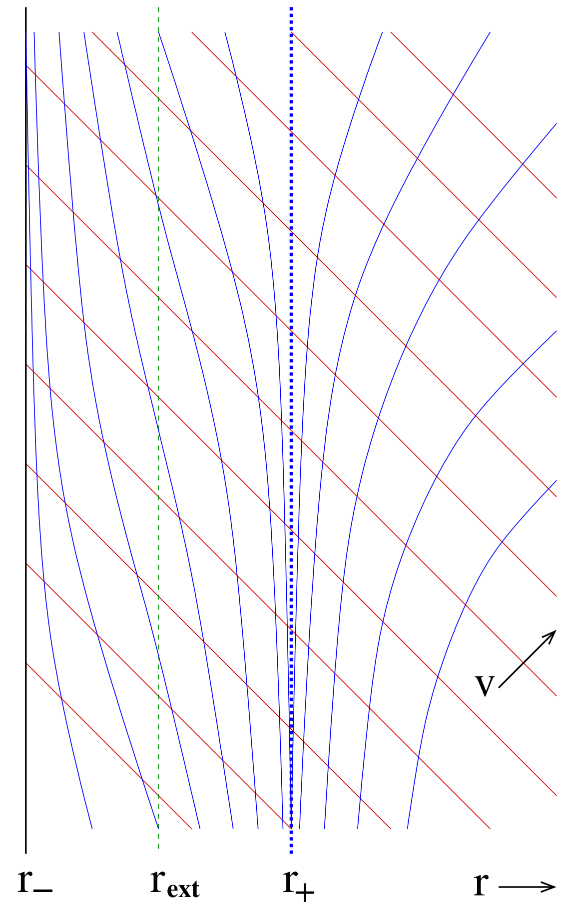
- Areas of inner/outer horizons are related to left/right entropy of the effective long string (Cvetic-Larsen '96-98)

$$S_\pm = S_L \pm S_R$$

- Area difference of inner/outer horizons

$$\frac{\Delta A}{4G} = \frac{A_+ - A_-}{4G} = 2S_R$$

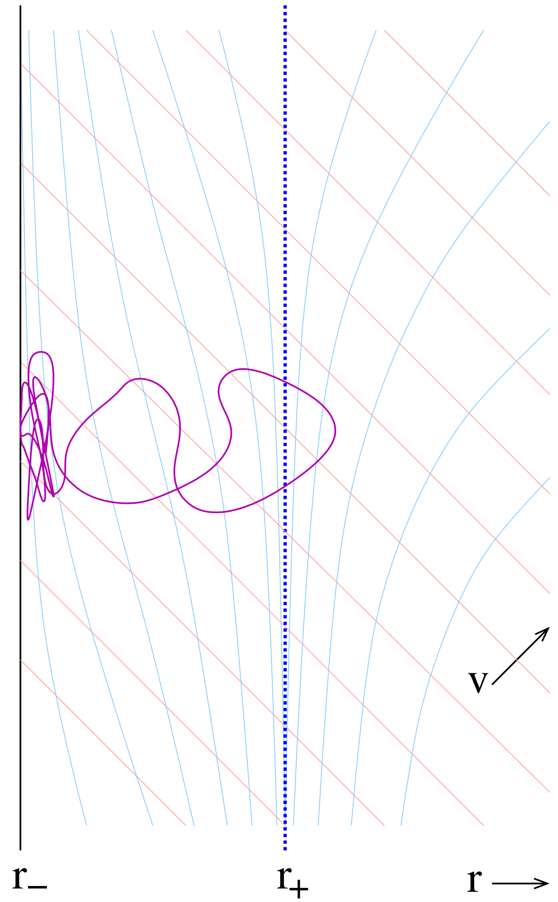
suggests that right-movers of the long string, and an equal number of left-movers, seem to be `floating' between the two horizons



Beyond the Extremal Limit

- Near extremality $S_R << S_L$ most of the dof's sit at the inner horizon, and (if extremal geometries are a guide) resolve the singularity there

- Excitations of the long string above extremality form an "atmosphere" whose average outer extent is expected to be the outer horizon

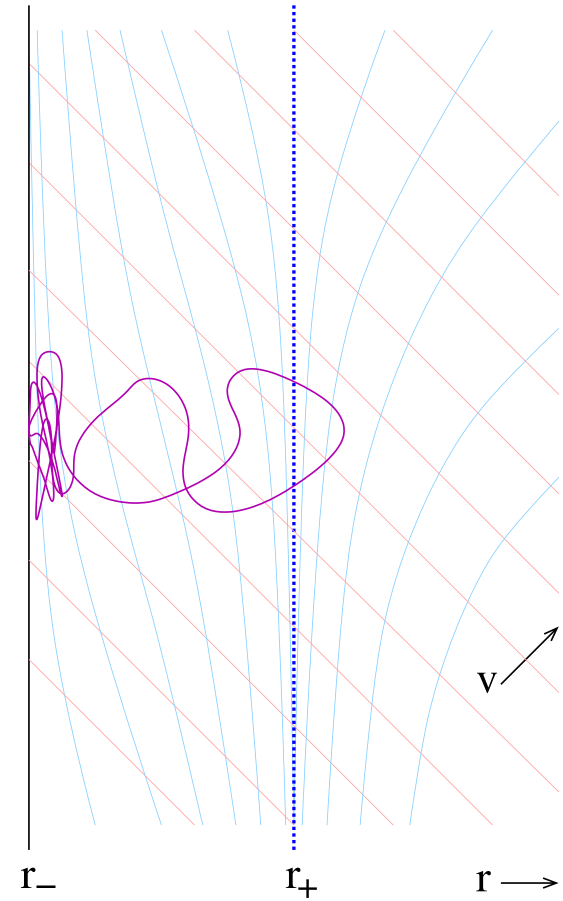


Beyond the Extremal Limit

- Suggests horizons are phase boundaries

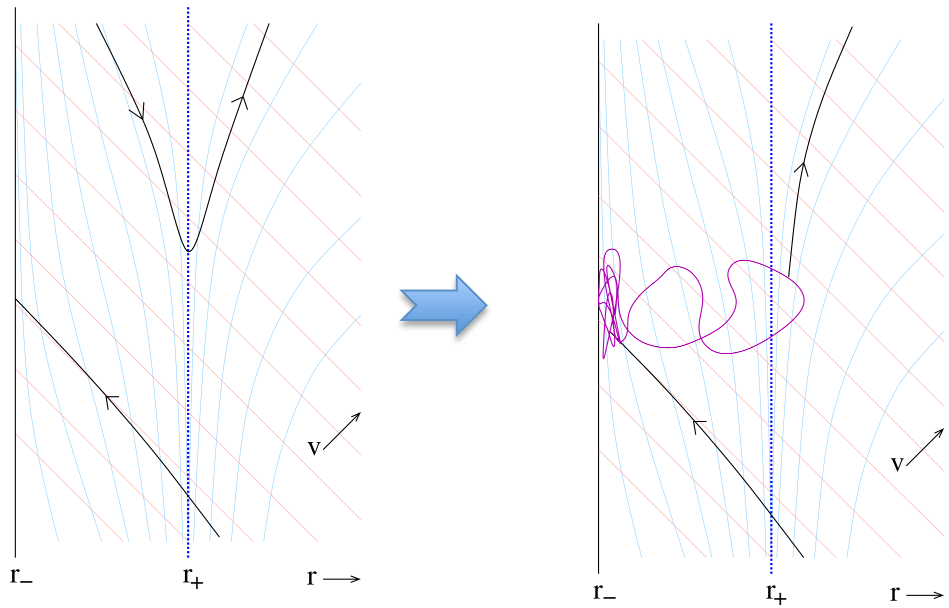
- Outer horizon is where one first encounters Higgs phase dof's as non-virtual excitations

- If there is a physical realization of the BH interior, the inter-horizon region is then a mixed phase, with both geometrical (Coulomb) and entropic (Higgs) dof's; while inner horizon completes the phase transition & Coulomb branch disappears



Paradox Lost?

- In the long string picture, the BH interior and Hawking radiation are modified:



But how? Why doesn't the long string fall in?

How does it causally communicate information from the resolved null singularity at r_- to Hawking radiation emitted near r_+ ?

Paradox Lost?

- A conjectural answer: *The long string interacts* ***differently*** *with geometry than ordinary (non-fractionated) matter,* (perhaps similar to the way F1's don't fit in single NS5 throats)

- Clues about the fractionated string may come from thinking about similar situations for fundamental strings:

- Black holes appear in the spectrum above the *correspondence point* (Horowitz-Polchinski '96, Giveon-Kutasov-Rabinovici-Sever '05), where the fundamental string entropy matches the BH entropy. The correspondence transition is an intrinsically quantum effect.

Paradox Lost?

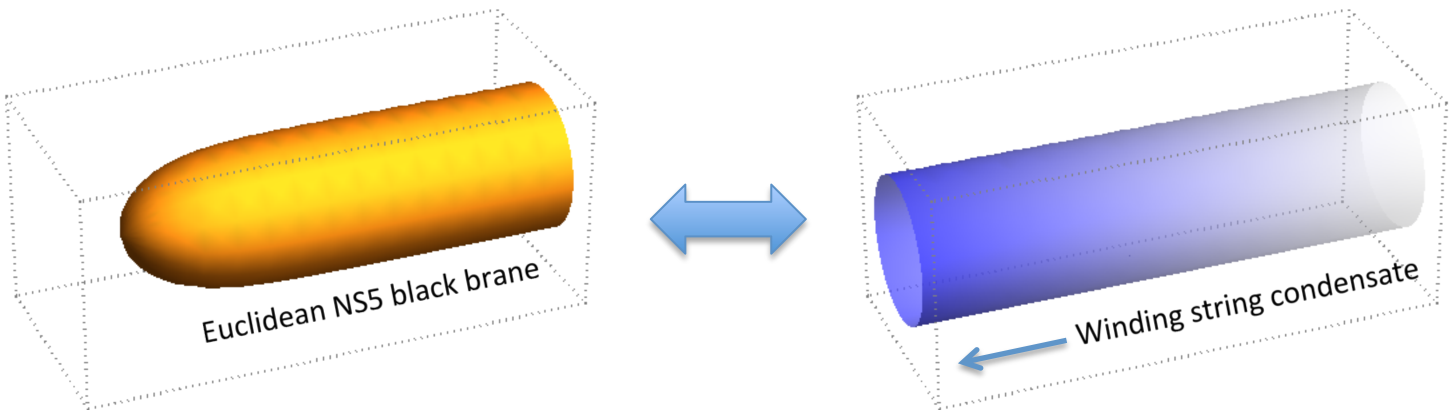
- At weak coupling or string scale ambient curvature, there are no black hole states, only `fundamental' strings.

- In worldsheet string theory on AdS_3, or in an NS5 throat, when the curvature scale is less than string scale, *all high energy states are Hagedorn strings*; there are no BTZ BH's or black NS5-branes (GKRS '05)

- The correspondence point in AdS_3 or LST occurs where the Hagedorn entropy matches the black hole entropy; this occurs when the string worldsheet dynamics has $c_{eff} = 6$

Paradox Lost?

- NS5 thermo involves an *r-t* plane SL(2,R)/U(1) worldsheet σ-model. There is exact weak/strong duality of this `cigar' σ-model of the Euclidean BH geometry with the Sine-Liouville CFT describing a condensate of strings winding the Euclidean time circle:



- Winding condensate is Euclidean description of Hagedorn gas thermo (Atick-Witten '88); at $c_{eff} = 6$, descriptions of the state as a BH, or as horizonless spacetime filled with string, are equally valid

Paradox Lost?

- Entropy formulae for nonextremal BTZ or NS5

$$S_{long} = 2\pi\left(\sqrt{n_2 n_4 n_p} + \sqrt{n_2 n_4 \bar{n}_p}\,\right)$$
$$S_{little} = 2\pi\left(\sqrt{n_5 N_L} + \sqrt{n_5 N_R}\right)$$

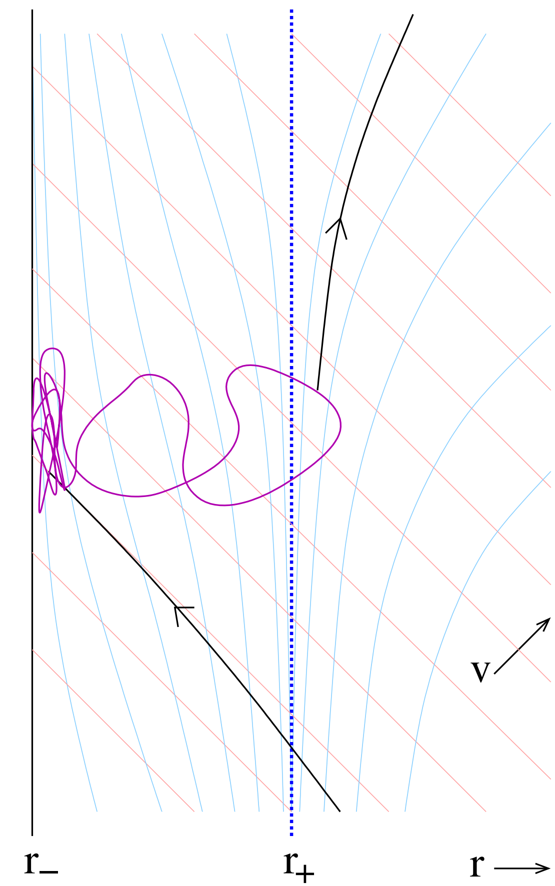
  can be interpreted as the density of states on a fractional tension long/little string whose excitations have $c_{eff} = 6$ and whose inverse tension scale is of order the ambient curvature.

- This scale ($c_{eff} = 6$) is precisely the correspondence scale of GKRS. If the long/little string behaves like a fundamental string at its correspondence point, it would not see BH structure. The long/little string would not collapse under self-gravity, precisely because its tension is too low.

Paradox Lost?

- The question of what happens to an infalling observer is governed by the response function of the fractionated branes

- If fractionated branes interact sufficiently violently w/ordinary matter, the equivalence principle could fail as soon as one hits fractionated brane matter at the outer horizon. If so, perhaps a physical realization of a firewall structure?



Paradox Lost?

- However, the disparity in tension scales suggests a much softer interaction, like a D-brane plowing through a Hagedorn gas of fundamental strings

- **We should accept what AdS/CFT has been telling us for ~20 yrs:** ***[Fractional branes fill the BH interior!]()***