Effective actions for fluids from holography

Based on:

arXiv:1405.4243 and arXiv:1504.07616

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Benasque, July 21, 2015
(see also arXiv:1504.07611 Crossley, Glorioso, Liu, Wang)

Motivation:

- Understanding the membrane paradigm and semi-holography
- Fluid/gravity duality versus effective actions for fluids, separation of dissipative from dissipationless effects
- Potential applications to quantum gravity, entanglement, surface states, etc

$$\nabla_\mu T^{\mu\nu} = 0$$

3 approaches to fluids

$$T = (\rho + p)u^\mu u^\nu + p g^{\mu\nu} + \pi^{\mu\nu}$$

gradient expansion

fluids

Partition function

(Banerjee et al, Jensen et al)

Effective actions (Nicolis et al)

$$\nabla_\mu T^{\mu\nu} = 0$$

3 approaches to fluids

$$T = (\rho + p)u^\mu u^\nu + p g^{\mu\nu} + \pi^{\mu\nu}$$

gravitational dual description

gradient expansion

Fluid/gravity: gradient expansion of Einstein equations

(Bhattacharyya et al)

fluids

Partition function
(Banerjee et al, Jensen et al)

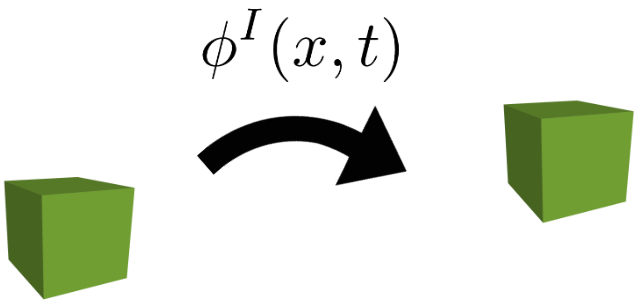
Euclidean AdS partition function

Effective actions (Dubovsky et al)

?? (But see Nickel, Son)

Effective actions for ideal dissipationless fluids

(Dubovsky et al, Bhattacharya et al)



Describes where a fluid element is at a given moment in time.

Symmetries: $\phi^I \to \xi^I(\phi)$ with $\det\left(\frac{\partial \xi^I}{\partial \phi^J}\right) = 1$

write $\vec{\phi} = \vec{x} + \vec{\pi}$

background Goldstones

Construct effective action in a derivative expansion consistent with the symmetries of the problem.

Volume preserving diff invariant building block:

$$s = \sqrt{\det\left(\partial_\mu \phi^I \partial_\nu \phi^J g^{\mu\nu}\right)}$$

Lowest order action:

$$S^{(0)} = \int d^d x \sqrt{-g} F(s)$$

- The conserved stress-energy tensor is the ideal fluid stress tensor

$$T^{(0)}_{\mu\nu} = p\,(g_{\mu\nu} + u_\mu u_\nu) + \rho\, u_\mu u_\nu$$

- provided that

$$\rho = -F, \quad p = -F' s + F, \quad T = -F'$$

$$J^\mu = {}^*(d\phi^1 \wedge d\phi^2 \wedge \ldots)$$

$$s = \sqrt{-J_\mu J^\mu}, \quad J^\mu = s\, u^\mu$$

- $\nabla_\mu J^\mu = 0$ identically, hence this construction is dissipationless

- Divide into longitudinal and transverse modes

$$\pi = \pi^T + \pi^L, \quad \text{such that} \quad \nabla \wedge \pi^L = 0, \quad \nabla \cdot \pi^T = 0$$

- The action up to quadratic order in an amplitude expansion

$$S^{(0)} \sim \int d^d x \left((\dot{\pi}^T)^2 + (\dot{\pi}^L)^2 - c_s^2 (\nabla \cdot \pi^L)^2 \right) + \dots$$

- The dispersion relations for the Goldstones are then

$$\pi^T : \quad \omega = 0$$

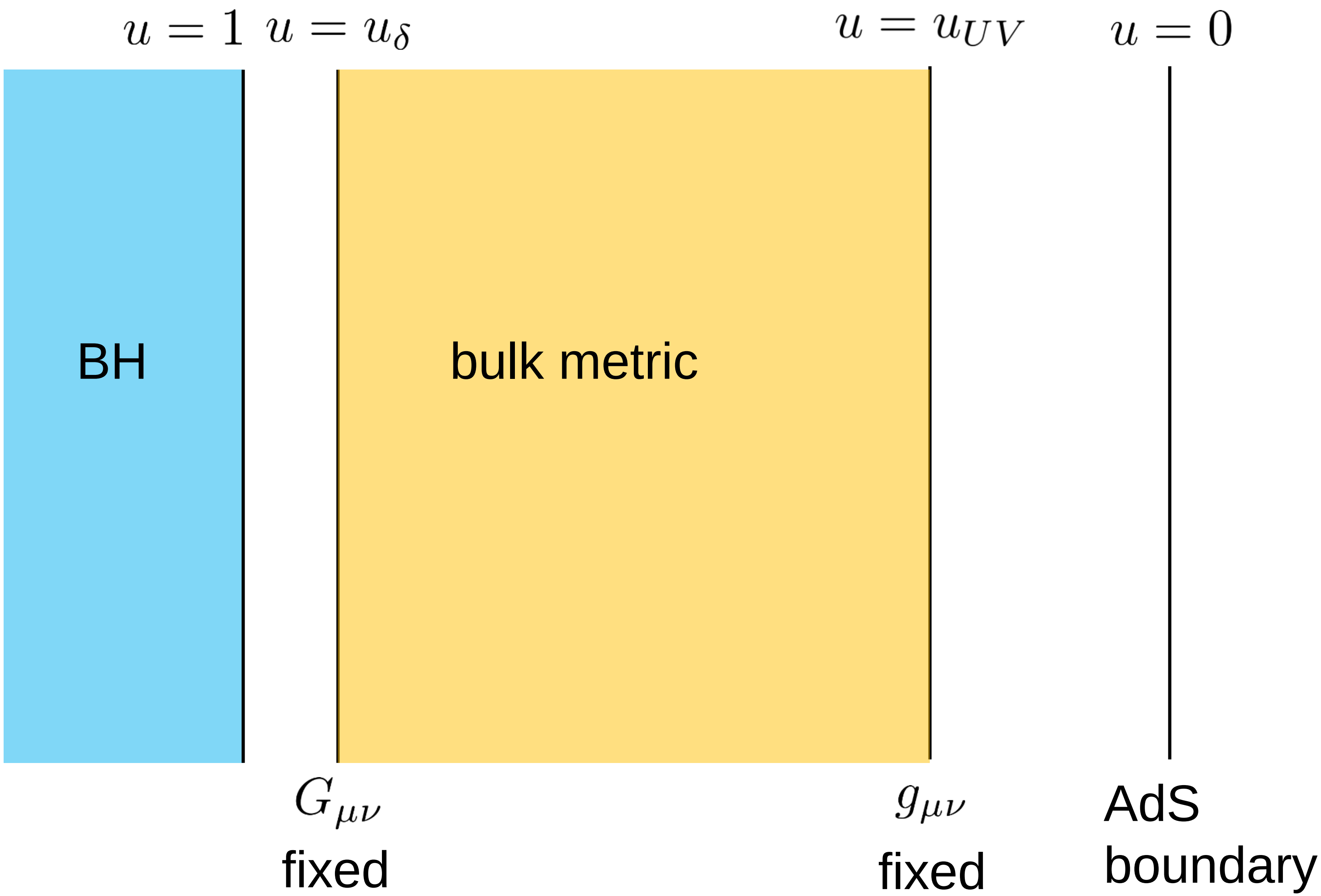
$$\pi^L : \quad \omega = c_s k$$

This effective action can be extended in many ways:

- It can be coupled to a dissipative sector (Endlich, Nicolis, Porto Wang)
- Can be extended to include anomalous transport (Haehl, Loganayagam, Rangamani)
- Effective action can accommodate fewer dissipationless transport coefficients than either standard approach to fluids or partition function approach (Bhattacharya^2, Rangamani) but this can be improved upon (Haehl, Loganayagam, Rangamani)

But a holographic interpretation of this effective action is unfortunately lacking. For example, how does volume preserving diffeomorphism invariance emerge?

Idea: consider a double Dirichlet problem for gravity



Naively computing the effective action as a function of the two metrics on the two boundaries yields something very non-local.

Imagine the metric on both boundaries is Poincare. Then a given bulk solution will break

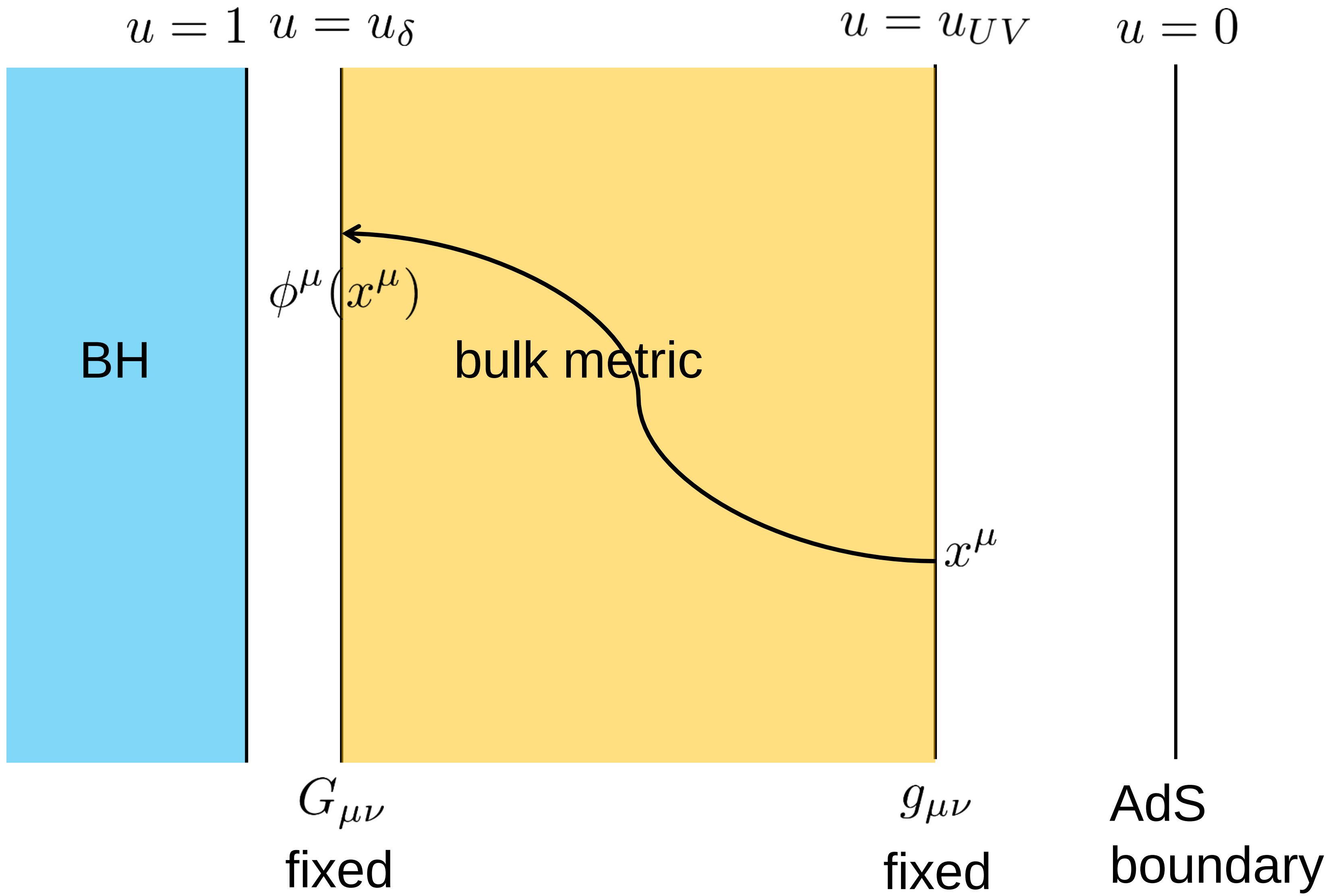
Poincare x Poincare → Poincare

Therefore there must be associated massless Goldstone bosons ϕ^μ.

Expect a local effective action $S[g_{\mu\nu}, G_{\mu\nu}, \phi^\mu]$

This action must be Diff x Diff invariant.

What is the geometric meaning of the Goldstone bosons? They are related to spatial geodesics connecting the two boundaries:



Change notation to make this clearer

$$S[g_{\mu\nu}(x^\mu), G_{MN}(\phi^M(x^\mu)), \phi^M(x^\mu)]$$

Could also have used other geodesics like null geodesics, probably related to these by field redefinition and choice of background.

Checked explicitly by perturbing the metric of a black brane background that these Goldstones naturally appear in the effective action.

Goal: compute $S[g_{\mu\nu}(x^\mu), G_{MN}(\phi^M(x^\mu)), \phi^M(x^\mu)]$ in a derivative expansion.

Define

$$h_{\mu\nu} = \phi^*(G) \equiv G_{MN} \frac{\partial \phi^M}{\partial x^\mu} \frac{\partial \phi^N}{\partial x^\nu}$$

This is a scalar of diffeomorphisms of the IR brane and a symmetric two-tensor on the UV brane. To lowest order

$$S[g_{\mu\nu}(x^\mu), G_{MN}(\phi^M(x^\mu)), \phi^M(x^\mu)] \equiv S[g_{\mu\nu}, h_{\mu\nu}]$$

If we also introduce

$$M_\mu^\lambda = h^{\nu\lambda} g_{\nu\mu}$$

then $S = \int dx^d \sqrt{-g} f(\mathrm{tr}(M), \mathrm{tr}(M^2), \ldots)$

We can determine the action explicitly for pure gravity with a negative cosmological constant. It is sufficient to find the most general solution of the Einstein equations with a diagonal metric with only radial dependence.

Result:

$$S = \int dx^d \sqrt{-g} f(\mathrm{tr}(\log M), \mathrm{tr}(\log^2 M))$$

$$f(t_1, t_2) = -\frac{2d\ell}{d-1} \frac{\cosh \ell u_2 - \cosh \ell u_1}{\sinh \ell u_2}$$

$$\log\left(\frac{\sinh \ell u_2}{\sinh \ell u_1}\right) = \frac{t_1}{2}$$

$$\left(\log\left(\frac{\tanh \frac{\ell}{2} u_2}{\tanh \frac{\ell}{2} u_1}\right)\right)^2 = \frac{d-1}{4(d-2)}\left(t_2 - \frac{t_1^2}{d-1}\right)$$

This result is valid in any dimension d.

Notice: no dependence on u_δ or u_{UV}

Also: no sign of volume preserving diffeomorphism invariance.

Send IR brane to horizon: make the IR metric degenerate

$$ds^2 = -\epsilon d\phi^t d\phi^t + G_{IJ} d\phi^I d\phi^J$$

Result:

$$S \sim \int dx^d \sqrt{-g} \left[1 - \exp\left(-\frac{d-1}{2(d-2)} \mathrm{Tr}' \log M \right) \right]^{\frac{1}{2}}$$

trace over finite eigenvalues

Recall that

$$h_{\mu\nu} = \phi^*(G) \equiv G_{MN} \frac{\partial \phi^M}{\partial x^\mu} \frac{\partial \phi^N}{\partial x^\nu}, \qquad M_\mu^\lambda = h^{\lambda\nu} g_{\mu\nu}$$

So that $\mathrm{Tr}'(\log M) = \log \det{}' M$ and the action only depends on $\det G_{IJ}$.

Volume preserving diffeomorphism invariance therefore arises by taking the IR boundary to the horizon.

Not clear why and whether it will still be valid when going to higher orders in the derivative expansion.

Taking the UV boundary to the boundary of AdS and dropping the divergence yields

$$S \sim \int dx^d \sqrt{-g} (\det{}' M)^{-\frac{d-1}{2(d-2)}}$$

which agrees exactly with the action of a conformal ideal fluid described before

$$S \sim \int dx^d \sqrt{-g} \left(\det g^{\mu\nu} \partial_\mu \phi^I \partial_\nu \phi^J G_{IK}\right)^{\frac{d-1}{2(d-2)}}$$

and the timelike Goldstone ϕ^t has completely disappeared from the theory.

The propagating degrees of freedom for the theory with finite cutoffs are shear modes and sound modes.

As we take the IR brane to the horizon, the velocity of the sheer mode ($\omega = c_T k$) goes to zero and it decouples.

There are two types of sound modes, one for the timelike Goldstone and one for the spatial directions. The timelike Goldstone disappears from the action in the near-horizon limit, although its dispersion relation remains finite.

All that remains in horizon limit: spatial sound mode with dispersion relation

$$\omega = \pm \frac{k}{\sqrt{d-1}} + b \log \epsilon \, k^3 + \mathcal{O}(\epsilon) + \mathcal{O}(k^4)$$

from gravity analysis, not from action

Besides the action for Goldstones discussed so far, one can also directly study gravitational perturbations of the background

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

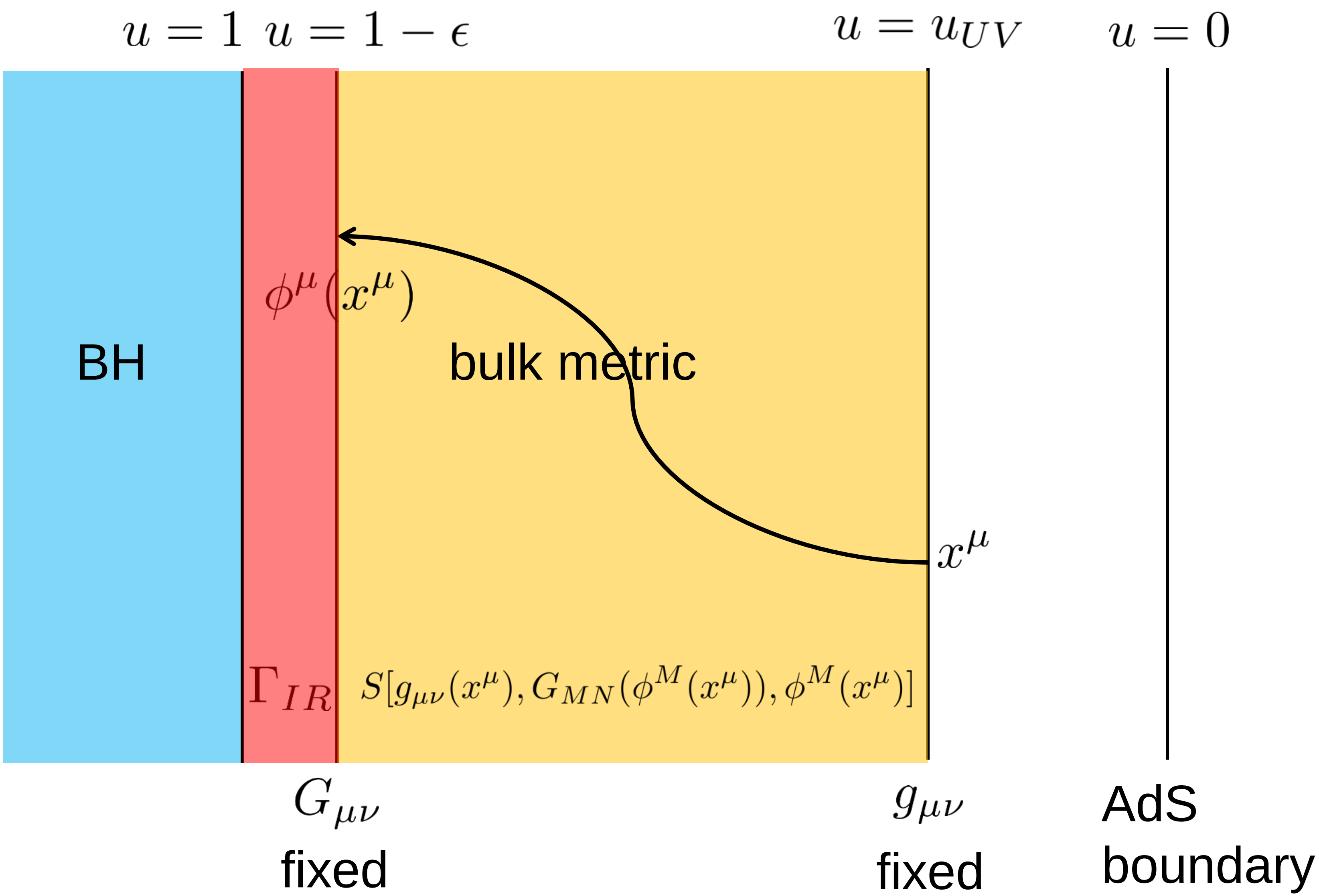
and expand the effective action to second order in $h_{\mu\nu}$ in a gradient expansion in k, ω. This is a different expansion compared to the Goldstone action where extra powers of k, ω are always accompanied by extra powers of $h_{\mu\nu}$.

In the study of gravitational perturbations, the Goldstone modes naturally appear as radial integrals of $h_{\mu\nu}$.

Summary so far:

- Determined the leading term in the effective action describing the double Dirichlet problem
- Taking the IR brane to the horizon results in a theory for a dissipationless ideal fluid which agrees with the action which was written previously
- Volume preserving diffeomorphism invariance appears only in this limit, but precise origin remains somewhat mysterious; alternative derivation of fluid/gravity duality.
- At higher orders in the derivative expansion, it does not seem consistent to send the IR brane to the horizon.

Including dissipation: coupling to the membrane.



Γ_{IR} is the effective action for the IR part of the metric, between the horizon and the IR brane.

It is computed with ingoing boundary conditions at the horizon and is the effective action of the membrane in the membrane paradigm.

Coupling then follows from $$\frac{\delta \Gamma_{IR}}{\delta G} + \frac{\delta S}{\delta G} = 0$$

Membrane paradigm: ignore near-horizon details and impose a simple boundary condition on a stretched horizon instead to model the black hole

$$2\epsilon \frac{\frac{\delta S}{\delta \Phi}}{i\omega\Phi} = \sigma \qquad \Phi_{\text{in}} \sim e^{-i\omega t}(1-u)^{-i\omega/2}$$

Membrane has σ=1, Dirichlet boundary condition is σ=0

Dispersion relations become

$$
\begin{aligned}
\omega_{shear} &= -\frac{i}{2}\sigma k^2 - \frac{i}{8}\sigma\left(2 + (1-\sigma^2)\log\epsilon - (1+\sigma^2)\log 2\right)k^4 + \mathcal{O}(\epsilon) \\
\omega_{sound} &= \pm\sqrt{\frac{1}{3}}k - \frac{i}{3}\sigma k^2 + \\
&\pm\left(\frac{1}{2\sqrt{3}} - \frac{\log 2}{3\sqrt{3}} + (1-\sigma^2)\frac{(1+\log 2+\log\epsilon)}{6\sqrt{3}}\right)k^3 + \mathcal{O}(\epsilon)
\end{aligned}
$$

For σ=0 recover previous result.

For σ=1 get the finite answers – same as one gets from fluid/gravity.

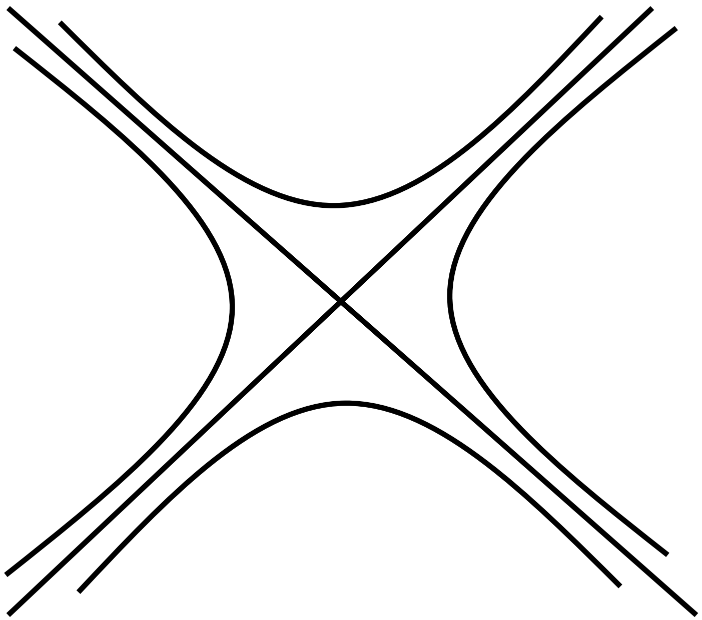
Cannot separate the dissipative (imaginary terms) from the non-dissipative ones (real terms) in a simple way.

Discussion

- We found a nice gravitational interpretation of the action for ideal dissipationless fluids at lowest order.
- Coupling to the membrane reproduces the standard AdS fluid.
- At higher order there are divergences if one takes the IR brane to the horizon. Cannot isolate the dissipationless part of a conformal fluid by this procedure and with these boundary conditions.
- The action can also be used to glue pieces of AdS together.
- By combining the action with different IR regimes can do a gravitational version of semi-holography. Would be interesting to e.g. couple to AdS2.
- The membrane paradigm as a boundary condition works fine, except that it fails to capture the massive quasi-normal modes.

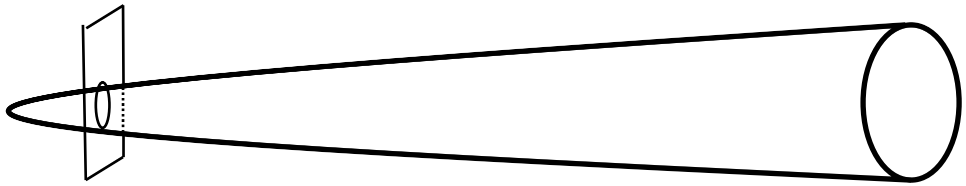
Discussion

- Why does volume-preserving diffeomorphism invariance appear?
- Will the story be modified when we use the Schwinger-Keldysh formalism to describe Γ_{IR} and S ? Cf recent work of Haehl, Loganayagam, Rangamani.



- Applications to cosmology?

Discussion



Connection to partition function approach:

Analyze double Dirichlet problem in Euclidean signature (time translation invariance) with periodic time.

Need to glue in an IR sector which corresponds to a smooth Euclidean tip where the thermal circle shrinks to a point.

No bulk contribution to IR action but non-trivial GH boundary term

$$\Gamma_{IR} = \beta^{-1} \int d^d x \sqrt{g} s$$

=Rindler partition function

Discussion

Complete action becomes

$$
\begin{aligned}
S_{Euc} &= \Gamma_{IR} + \Gamma_{DD} \\
&= \beta^{-1} \int d^d x \sqrt{g} s + \int d^d x \sqrt{g} F(s)
\end{aligned}
$$

Integrating out the IR metric is like integrating out s.
Result is the pressure

$$
S_{Euc} = \int d^d x \sqrt{g} p(\beta)
$$

In complete agreement with the partition function approach.

Discussion

Interpretation of entropy current?

$$T^{\mu\nu}_{\mathrm{UV}} = \frac{1}{\sqrt{-g}}\frac{\delta S}{\delta g_{\mu\nu}} \qquad T^{\mu\nu}_{\mathrm{UV}} u_\mu = -\rho u^\nu$$

$$T^{IJ}_{\mathrm{IR}} = \frac{1}{\sqrt{-G}}\frac{\delta S}{\delta G_{IJ}} \det\left(\frac{\partial x^\mu}{\partial \phi^I}\right) \qquad T^{IJ}_{\mathrm{IR}} u_I = -\rho_{\mathrm{IR}} u^J$$

β plays the role of inverse temperature

Entropy current is Noether current for $\delta\phi^I = \rho^{-1}_{\mathrm{IR}} u^I$ in case this is a symmetry.

Interpretation???

Discussion

Glueing pieces of AdS together is also what one would like to study if one is interested in the entanglement of spacetime itself as in holeography (Balasubramanian, Chowdhury, Czech, JdB, Heller)

If one associates Hilbert spaces to each of the Dirichlet boundaries of space-time, perhaps S can be interpreted as providing a density matrix in the product of these Hilbert spaces. Tracing over the interior Hilbert space would then give an entropy, could it be A/4G???

To be continued…..