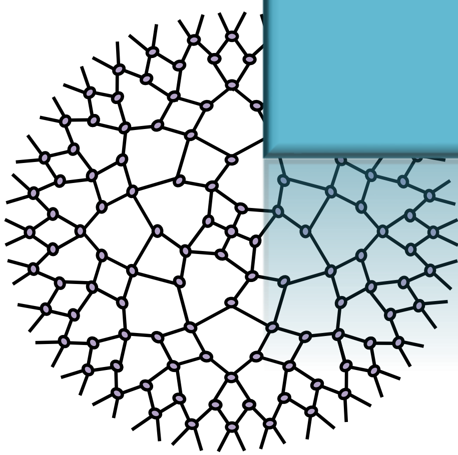
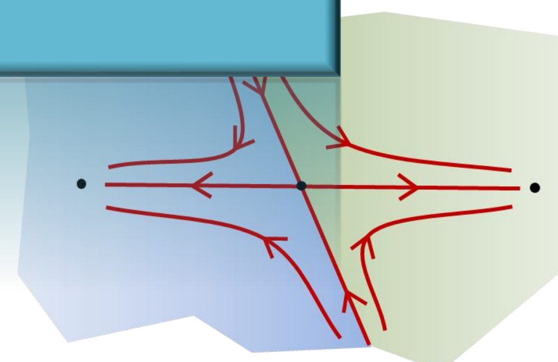
Centro de Ciencias de Benasque Pedro Pascual
Gravity – New perspectives from strings and higher dimensions
July 12th - 24th 2015

Multi-scale entanglement renormalization ansatz (MERA)





Guifre Vidal

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

SIMONS FOUNDATION

JOHN TEMPLETON FOUNDATION

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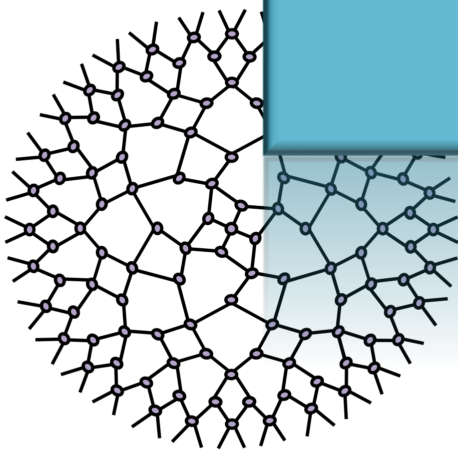

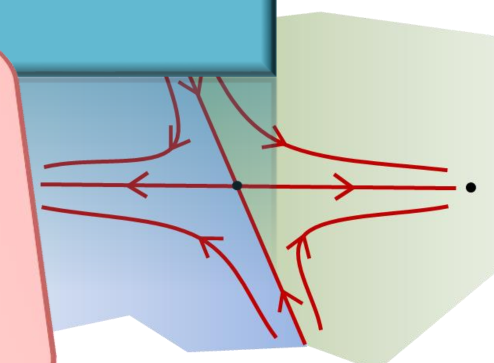

Glen Evenbly
UC Irvine

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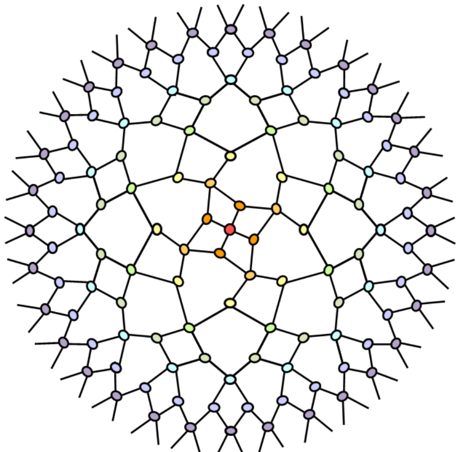
Why this talk, *here*?



MERA
(2005)

MERA ↔ AdS/CFT

Swingle, 2009

"Entanglement renormalization for quantum fields"
Haegeman, Osborne, Verschelde, Verstraete, 2011

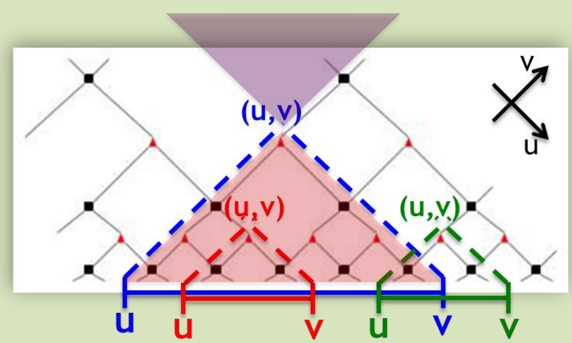
"Holographic Geometry of Entanglement Renormalization in Quantum Field Theories"
Nozaki, Ryu, Takayanagi, 2012

"Time Evolution of Entanglement Entropy from Black Hole Interiors"
Hartman, Maldacena, 2013

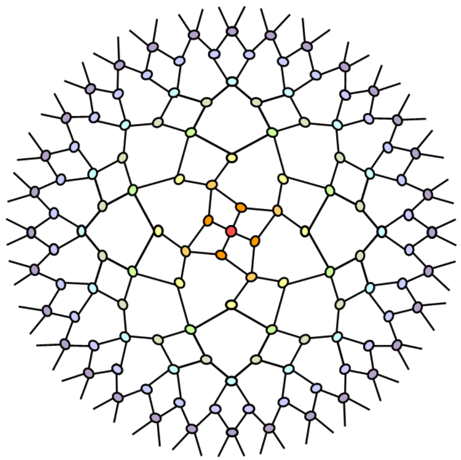
"Exact holographic mapping and emergent space-time geometry"
Xiaoliang Qi, 2013

"Holographic quantum error-correcting codes: Toy models for the bulk/boundary correspondence"
Pastawki, Yoshida, Harlow, Preskill, 2015

"Integral Geometry and Holography"
Czech, Lamprou, McCandlish, Sully, 2015



What can MERA do, *for sure*?



MERA
(2005)

input

1D quantum / 2D classical Hamiltonian

- **on the lattice**
- **at a critical point**

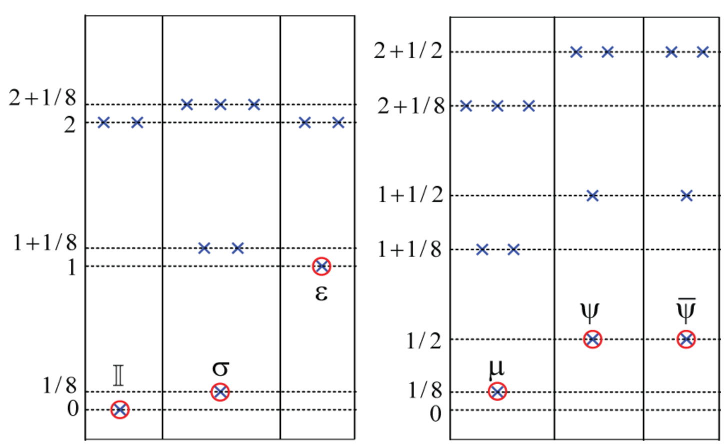
output

Numerical determination of conformal data:

- **central charge** c
- **scaling dimensions** $\Delta_\alpha \equiv h_\alpha + \bar{h}_\alpha$ **and conformal spins** $s_\alpha \equiv h_\alpha - \bar{h}_\alpha$
- **OPE coefficients** $C_{\alpha\beta\gamma}$

e.g. critical Ising model (approx. an hour on your laptop)

Pfeifer, Evenbly, Vidal 08



$(\Delta_{\mathbb{I}} = 0)$

$\Delta_\sigma \approx 0.124997$

$\Delta_\varepsilon \approx 0.99993$

$\Delta_\mu \approx 0.125002$

$\Delta_\psi \approx 0.500001$

$\Delta_{\bar{\psi}} \approx 0.500001$

$C_{\epsilon\sigma\sigma} = \frac{1}{2}$ $\quad C_{\epsilon\mu\mu} = -\frac{1}{2}$

$C_{\epsilon\psi\bar{\psi}} = i$ $\quad C_{\epsilon\bar{\psi}\psi} = -i$

$C_{\psi\mu\sigma} = \frac{e^{-\frac{i\pi}{4}}}{\sqrt{2}}$ $\quad C_{\bar{\psi}\mu\sigma} = \frac{e^{\frac{i\pi}{4}}}{\sqrt{2}}$

$(\pm 6 \times 10^{-4})$

outline

Part 1:
(old stuff)

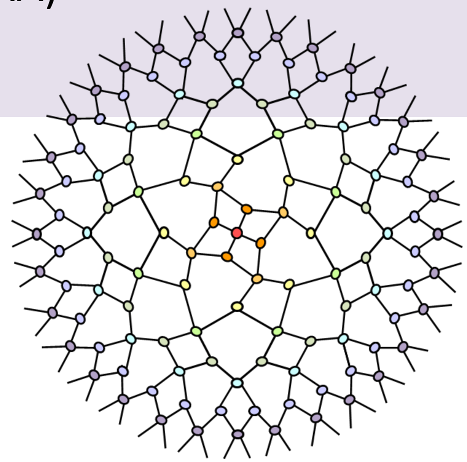
Tensor networks

+

Renormalization group

=

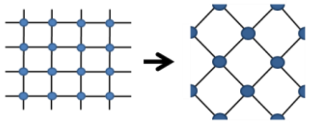
Multi-scale entanglement renormalization ansatz (MERA)



- quantum circuit
- RG transformation

Part 2:
(recent developments)

Tensor network renormalization (TNR)

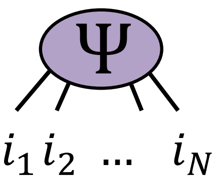


- RG flow in the space of tensors
- Local scale transformations on the lattice
  - ➢ plane to cylinder
  - ➢ hyperbolic plane (MERA)
  - ➢ thermal states / black holes

Many-body wave-function of N spins

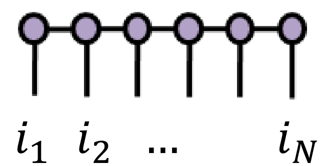
$$|\Psi\rangle = \sum_{i_1,i_2,\cdots,i_N} \Psi_{i_1 i_2 \cdots i_N} |i_1 i_2 \cdots i_N\rangle$$

2^N parameters

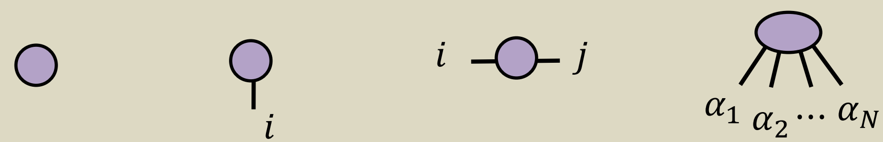


=

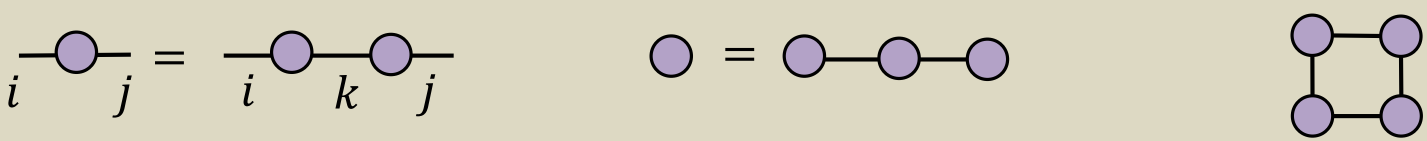
tensor network



graphical notation

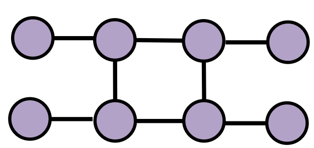


a $\quad b_{\mathrm{i}}$ $\quad c_{\mathrm{ij}}$ $\quad d_{\alpha_1 \alpha_2 \cdots \alpha_N}$



$$T_{ij} = \sum_k R_{ik} S_{kj} \qquad a = \vec{y}^{\dagger} \cdot M \cdot \vec{x} \qquad tr(ABCD)$$

why bother?

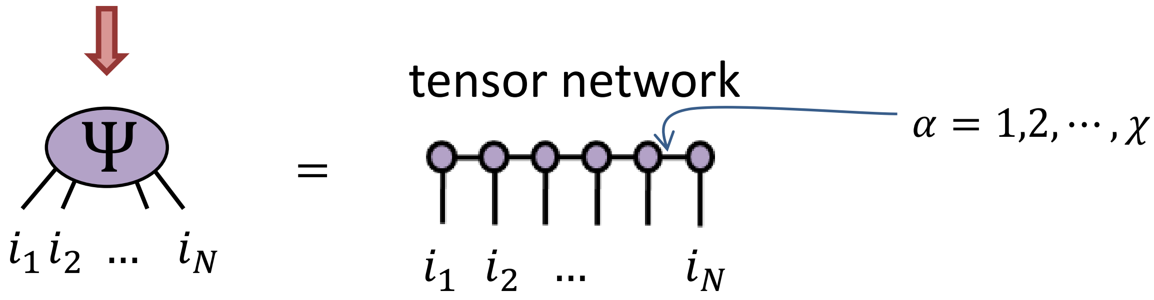


$$\sum_{ijklmnop} A_{ijk} B_{jlm} C_{nko} D_{kmr} x_i y_l z_n v_r$$

Many-body wave-function of N spins

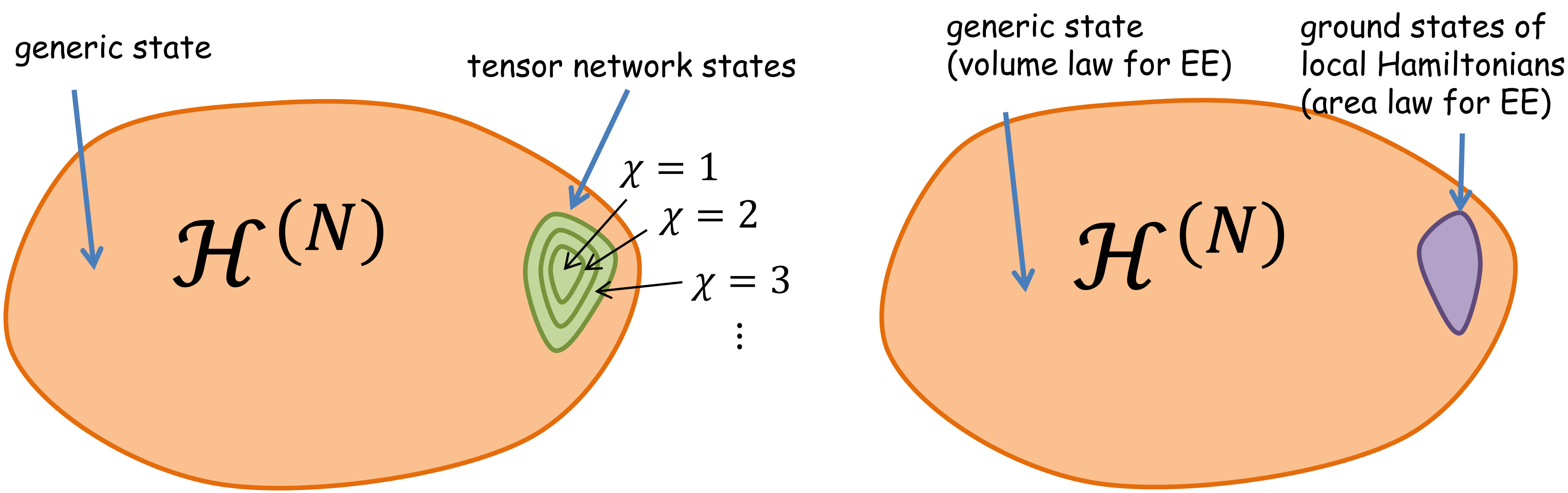
$$|\Psi\rangle = \sum_{i_1,i_2,\cdots,i_N} \Psi_{i_1 i_2\cdots i_N}\,|i_1 i_2\cdots i_N\rangle$$

2^N parameters

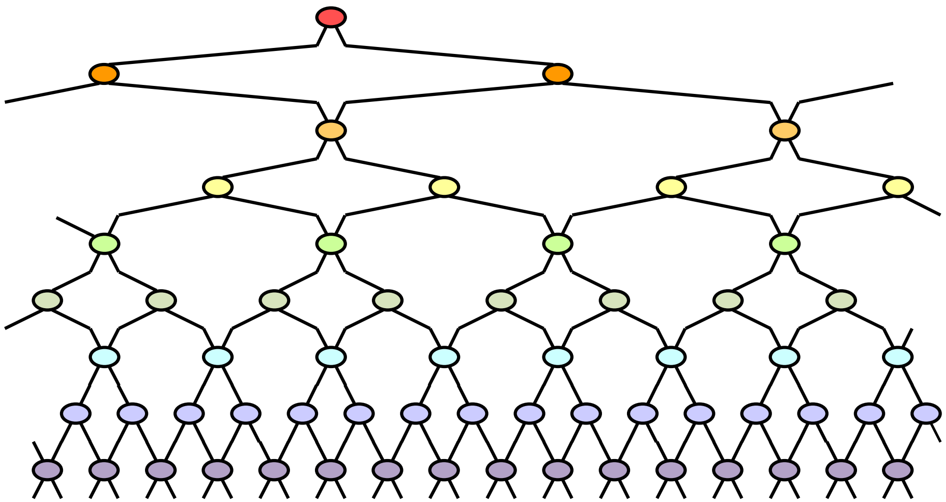


2^N parameters → $O(N)$ parameters

inefficient **efficient**

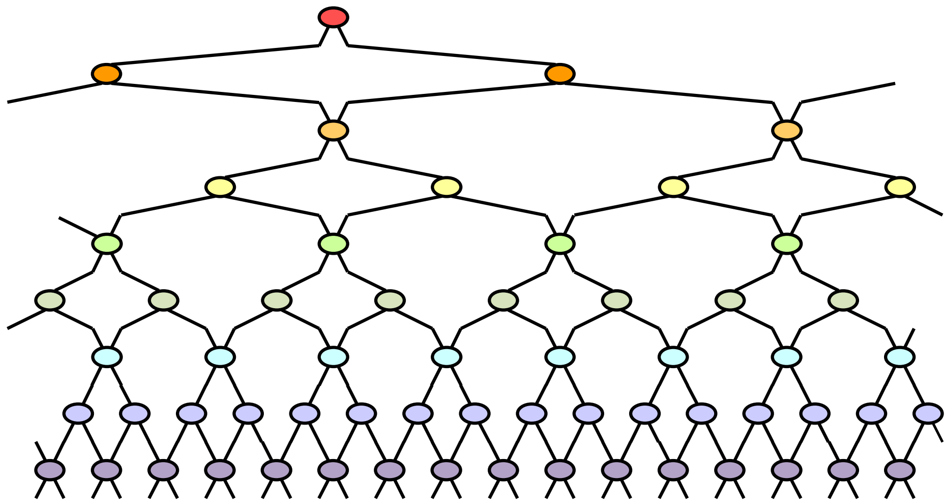


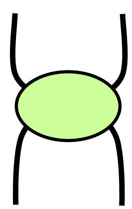
Multi-scale entanglement renormalization ansatz (MERA)



- Variational class of states for 1d systems, which extends in space and scale
- Variational parameters for different length scales stored in different tensors
- It is secretly a **quantum circuit** and an **RG transformation**

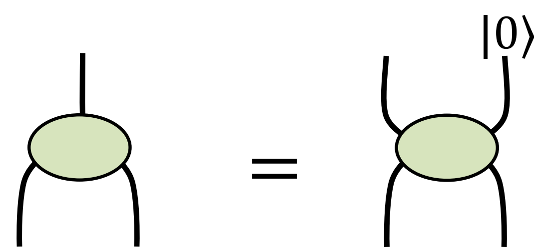
MERA as a quantum circuit





disentangler

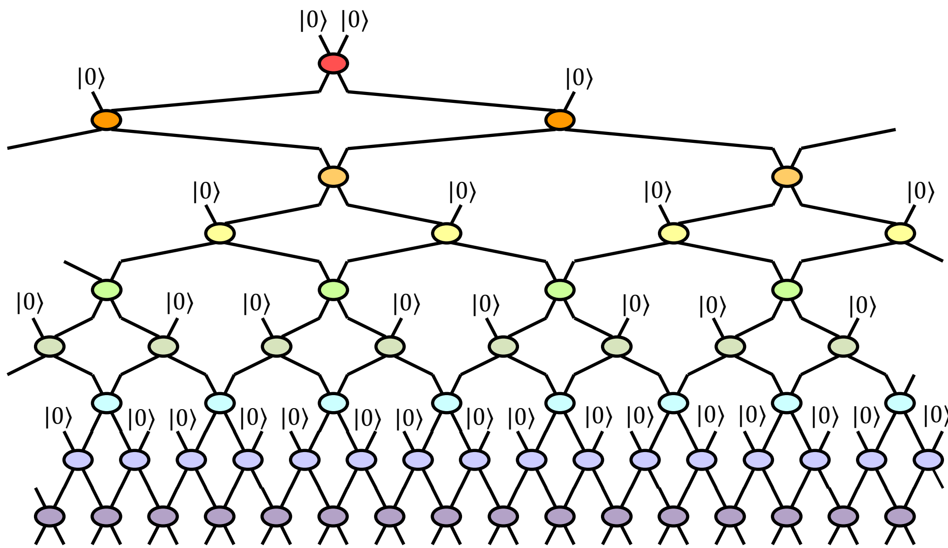
two-body unitary gate

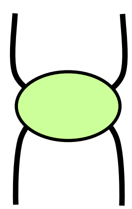


isometry

also a
two-body unitary gate

MERA as a quantum circuit





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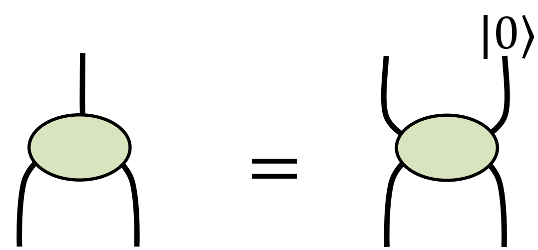
two-body unitary gate



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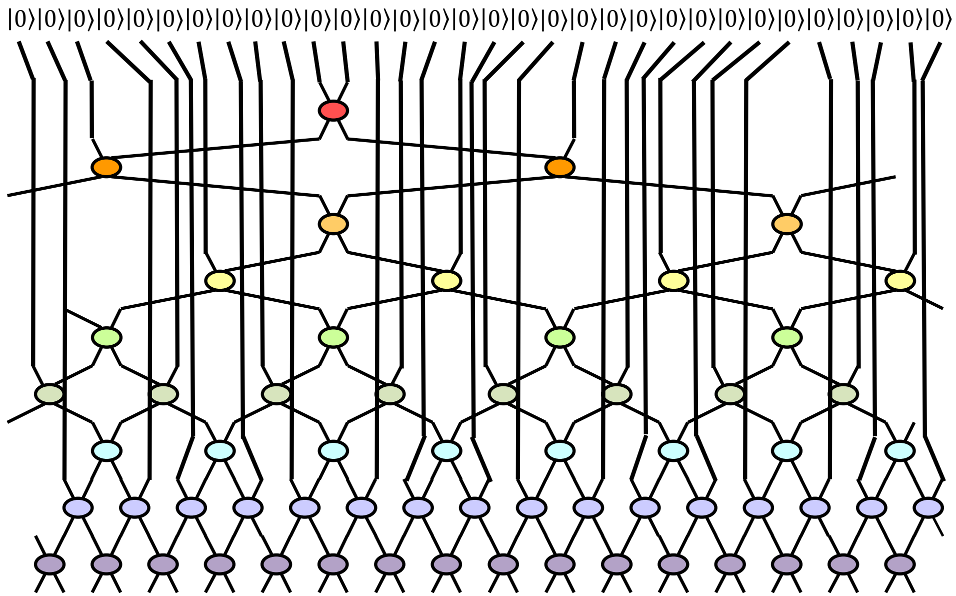
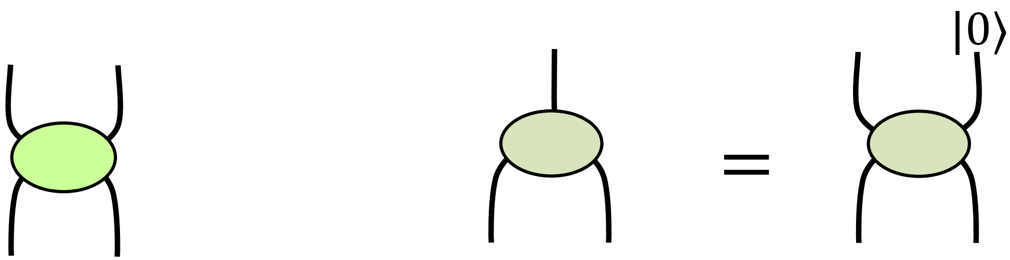
MERA as a quantum circuit





disentangler

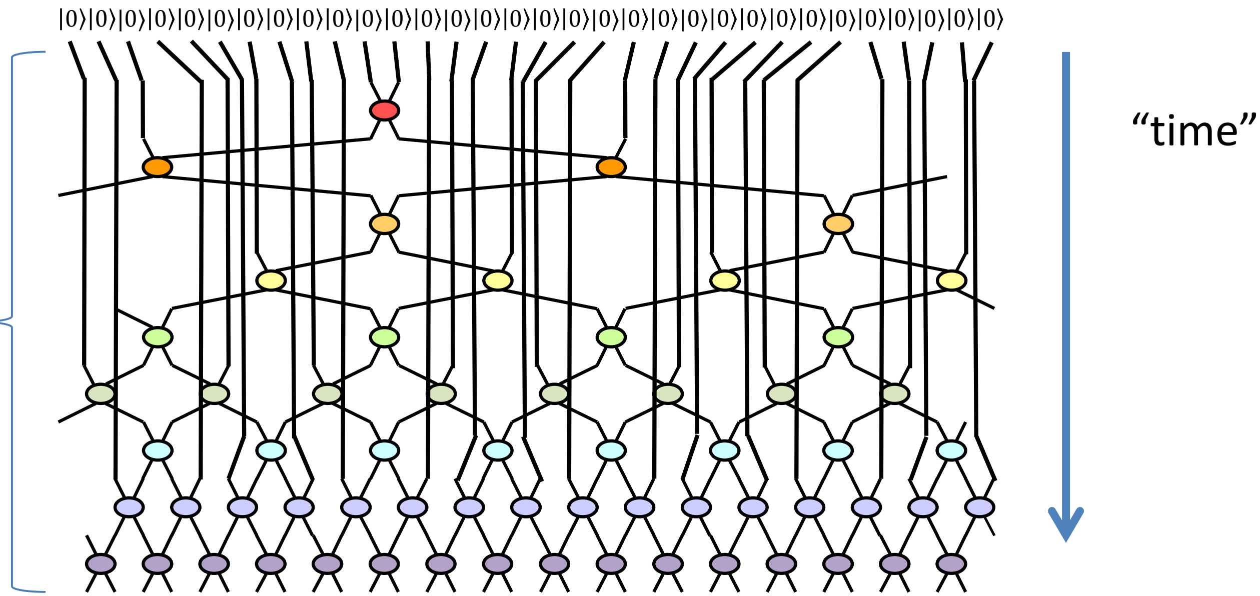
two-body unitary gate

isometry

also a
two-body unitary gate

MERA as a quantum circuit

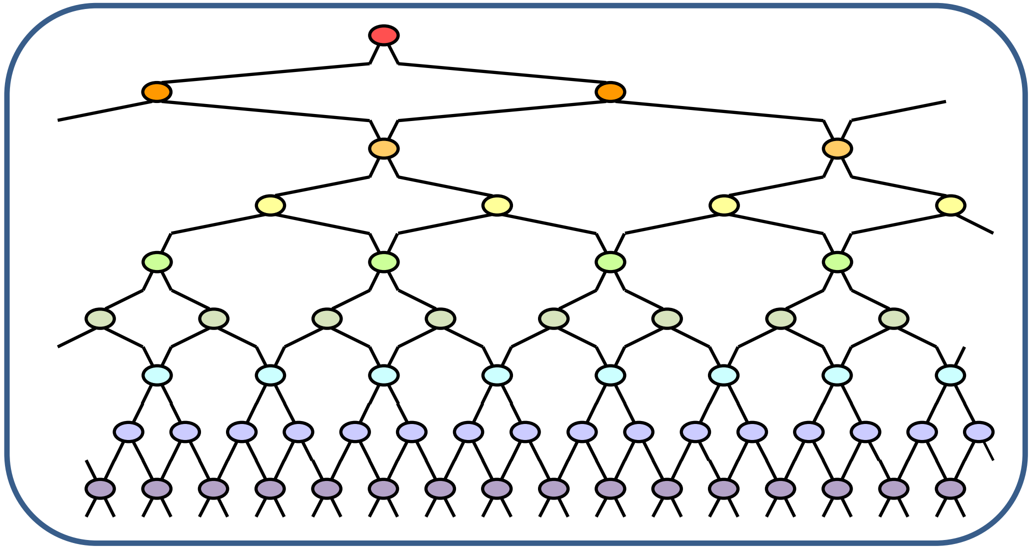
quantum circuit U



ground state ansatz $|\Psi\rangle = U\ |0\rangle^{\otimes N}$

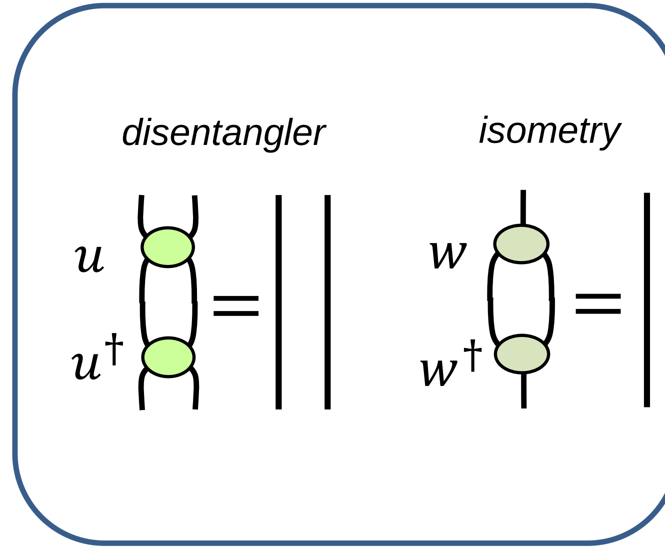
Entanglement introduced by gates at different "times" (= length scales)

MERA = tensor network + isometric/unitary constraints



~ hyperbolic plane?

(Swingle 2009)



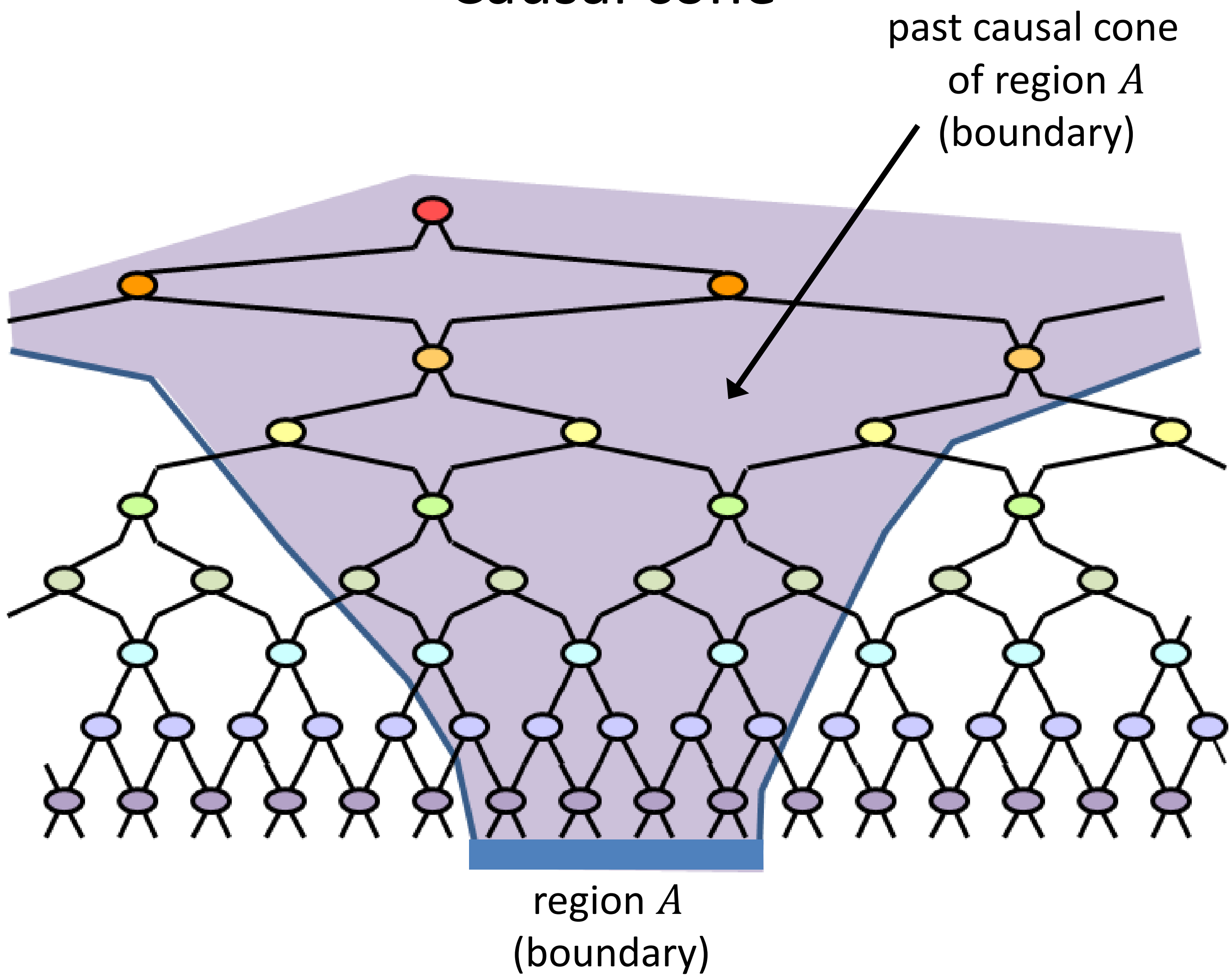
~ de Sitter space?

(Beny 2011, Czech 2015)

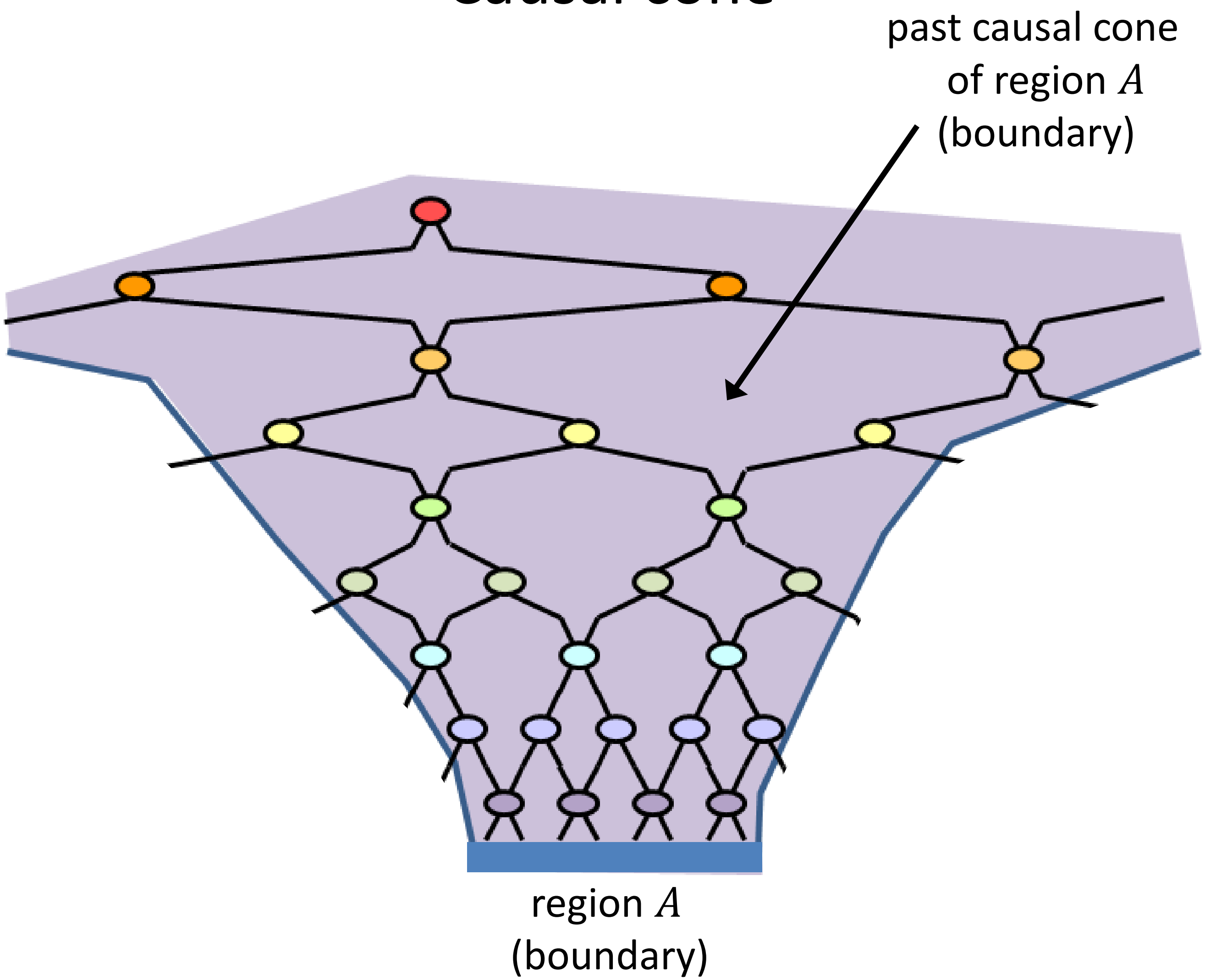
⇒ Causal structure

essential for many MERA properties
and computational efficiency

Causal cone



Causal cone

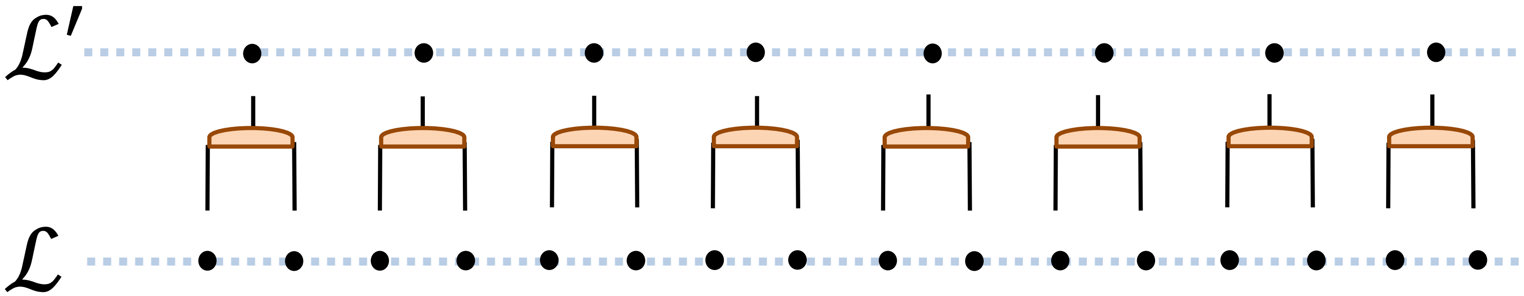


MERA as RG Transformation

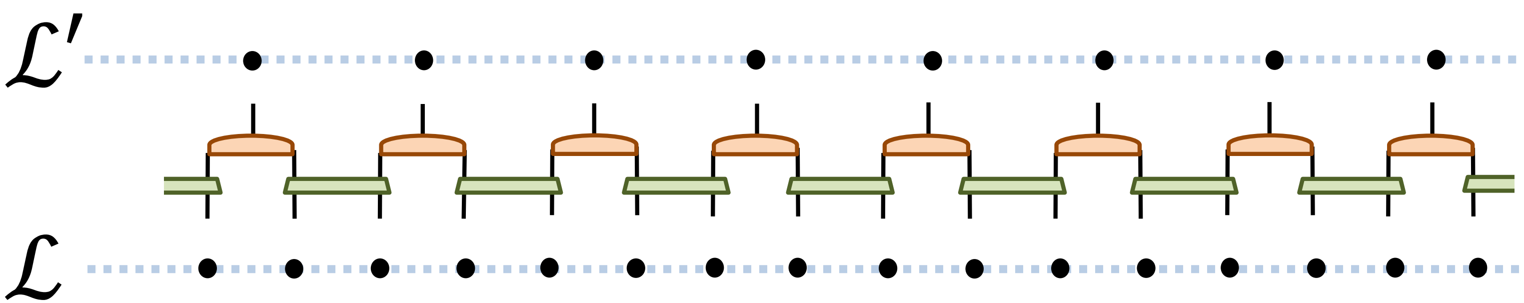
[Vidal 05]

Kadanoff (1966) + White (1992)

blocking variational optimization



Entanglement renormalization (2005)



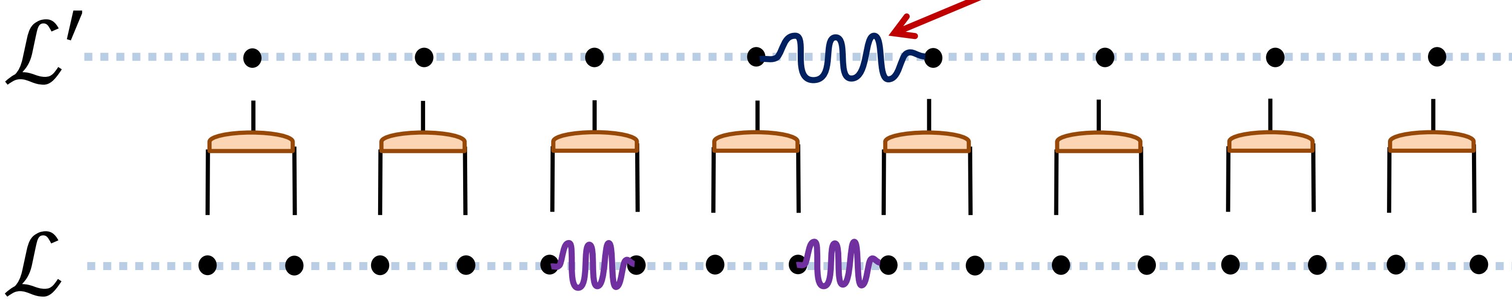
MERA as RG Transformation

[Vidal 05]

Kadanoff (1966) + White (1992)

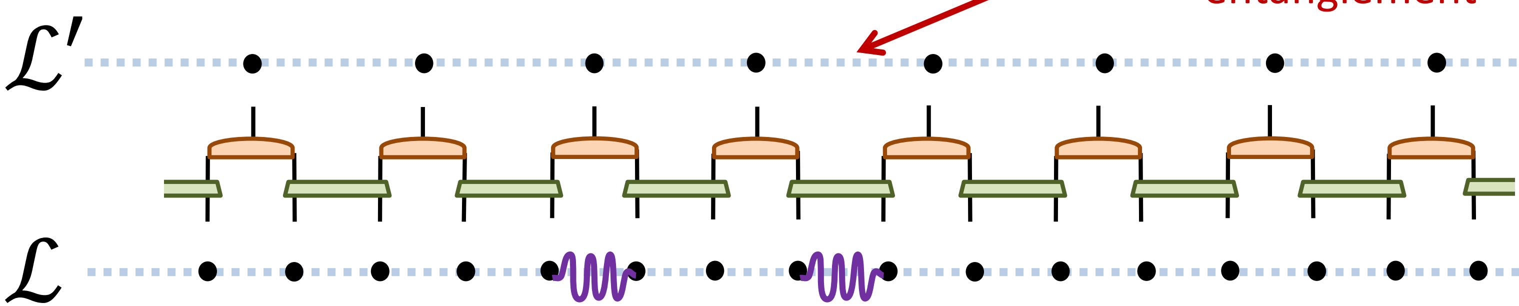
blocking variational optimization

failure to remove ***some*** short-range entanglement !

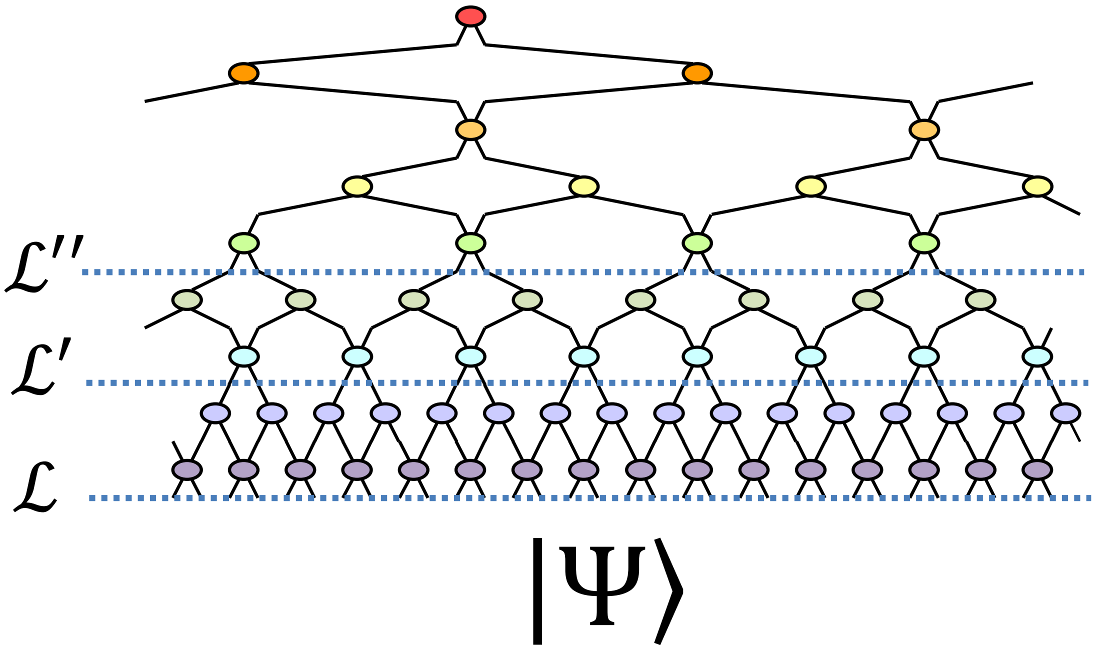


Entanglement renormalization (2005)

removal of ***all*** short-range entanglement

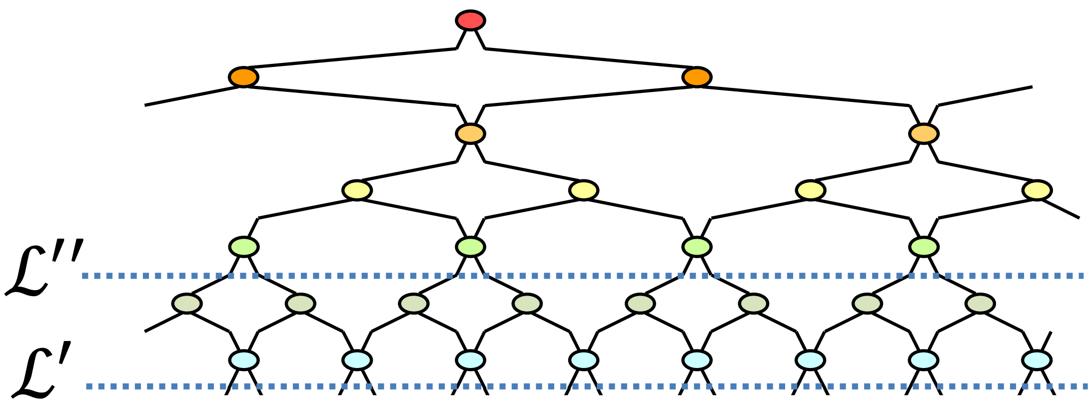


MERA as a sequence of ground state wave-functions



$$|\Psi\rangle \rightarrow |\Psi'\rangle \rightarrow |\Psi''\rangle \rightarrow \cdots$$

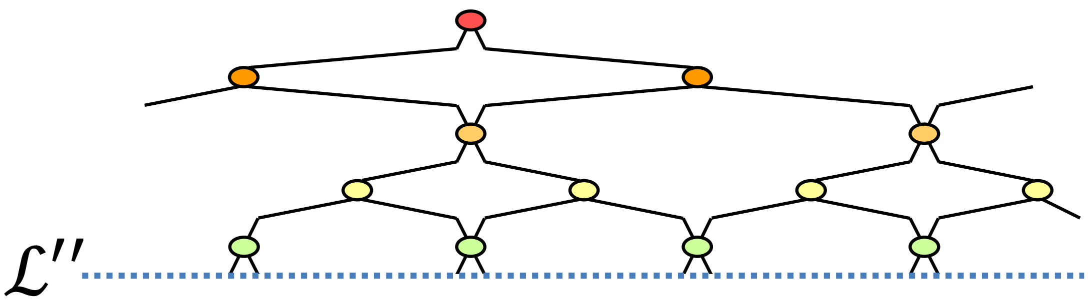
MERA as a sequence of ground state wave-functions



$$|\Psi'\rangle$$

$$|\Psi\rangle \rightarrow |\Psi'\rangle \rightarrow |\Psi''\rangle \rightarrow \cdots$$

MERA as a sequence of ground state wave-functions

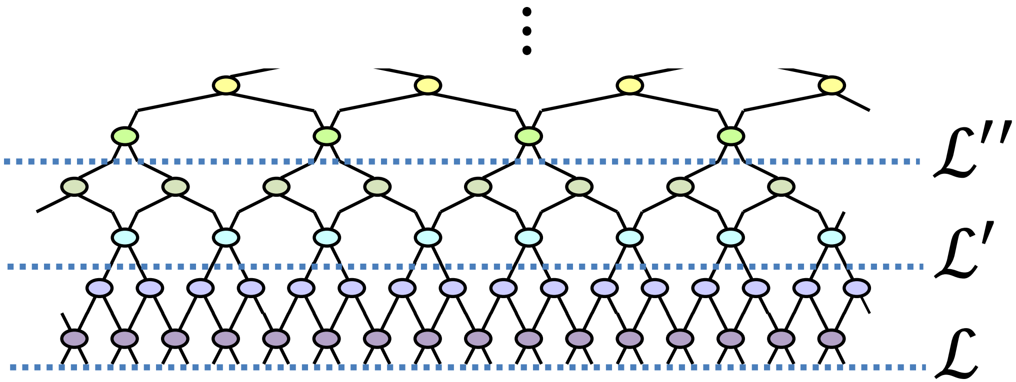


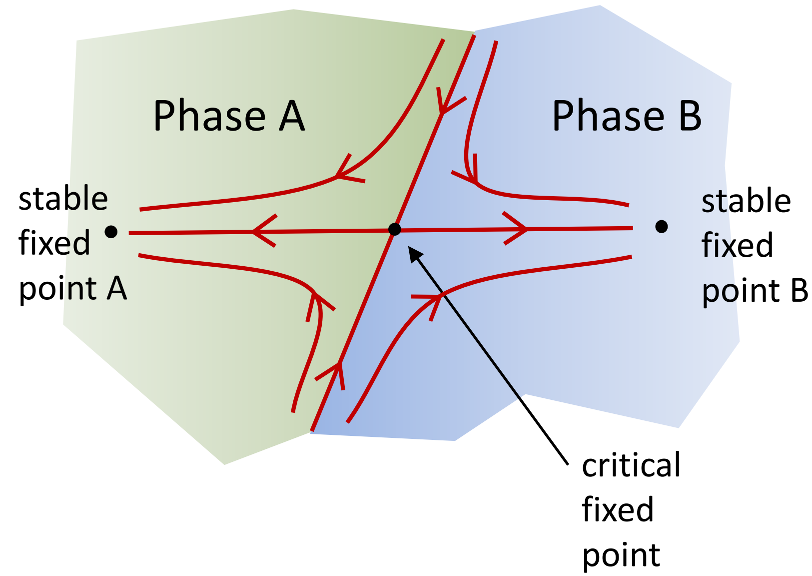
$$|\Psi''\rangle$$

$$|\Psi\rangle \to |\Psi'\rangle \to |\Psi''\rangle \to \cdots$$

MERA defines an RG flow
in the space of wave-functions

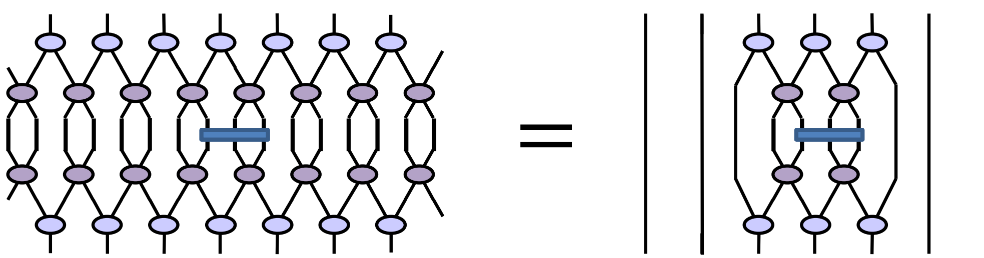
$$|\Psi\rangle \rightarrow |\Psi'\rangle \rightarrow |\Psi''\rangle \rightarrow \cdots$$





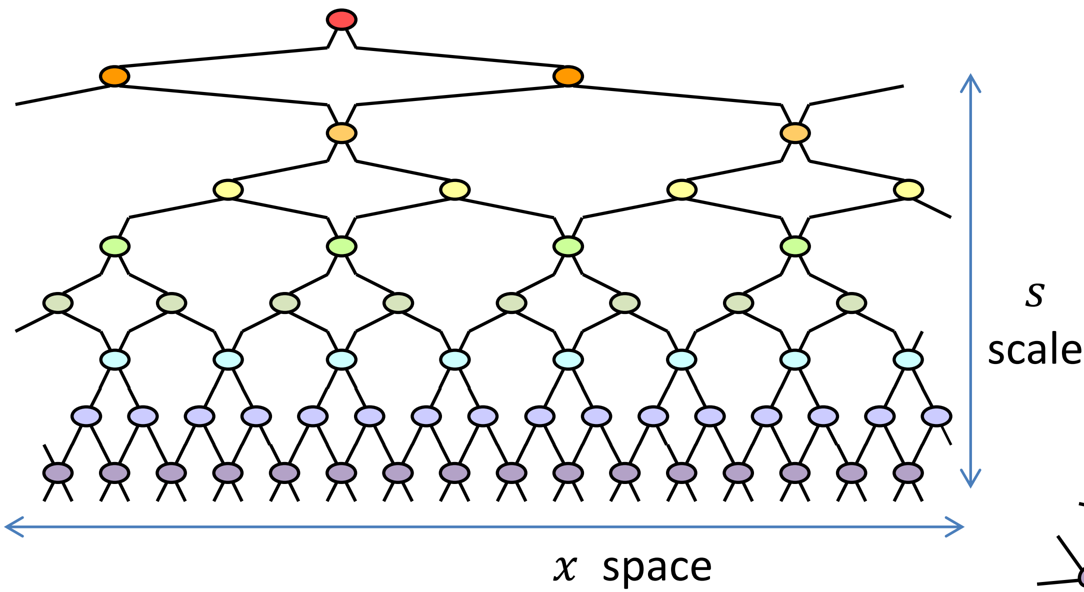
... and in the space of Hamiltonians

$$H \rightarrow H' \rightarrow H'' \rightarrow \cdots$$



local operators
are mapped into
local operators !

Entanglement entropy and correlations



- entanglement entropy

$$S_L \approx \log(L)$$

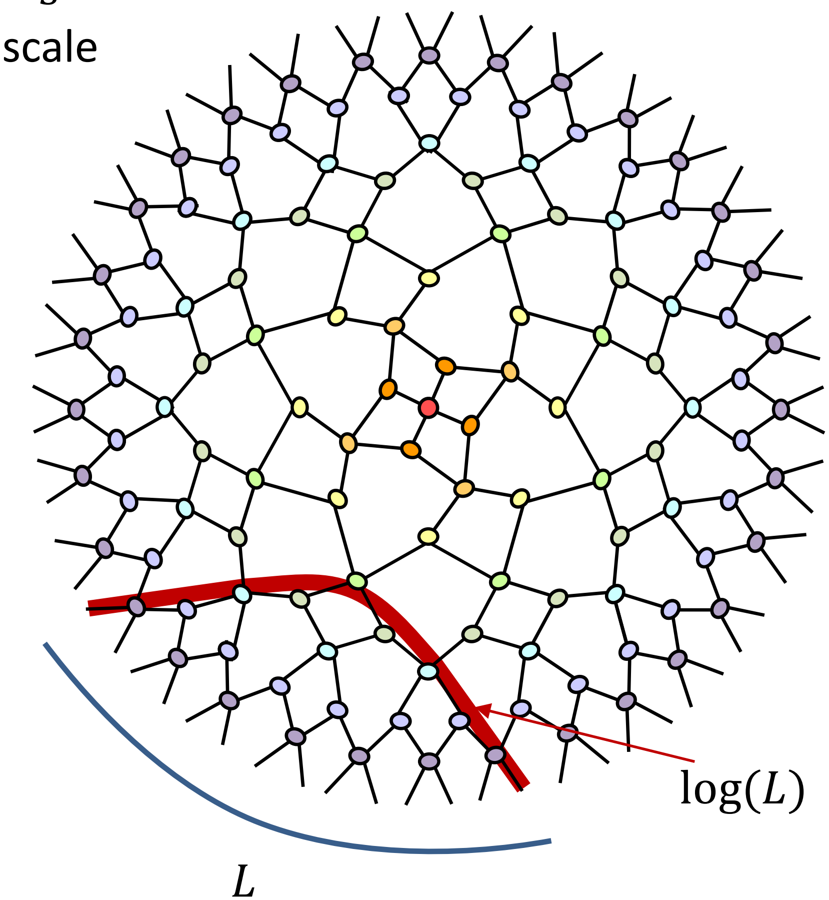
Computation of density matrix requires tracing out $\sim\log(L)$ indices

- two-point correlations

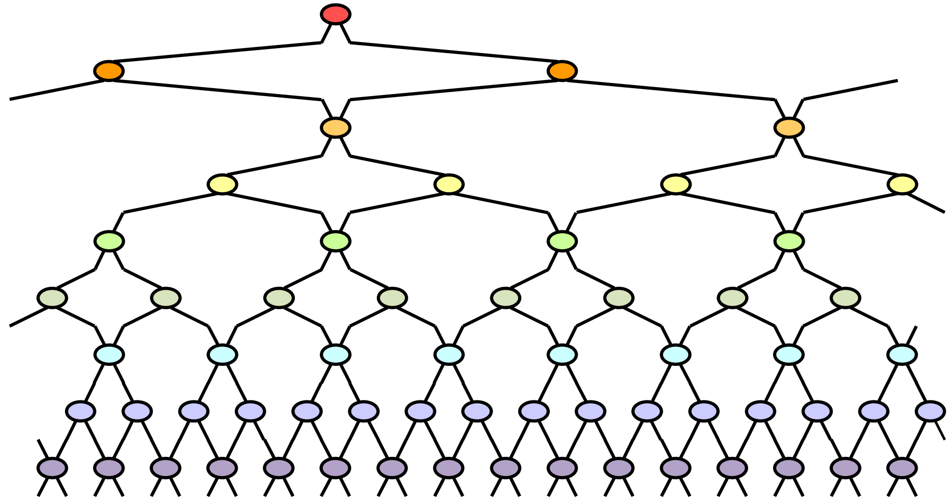
$$C(L) \approx L^{-2\Delta}$$

Geodesic distance $D \approx \log(L)$

$$C(L) \approx e^{-D} = e^{-2\Delta \log(L)} = L^{-2\Delta}$$



Summary so far



MERA

- Variational parameters for different length scales

- It is secretly a **quantum circuit** → *"entanglement at different length scales"*

  and an **RG transformation**

"removes short-range entanglement" $|\Psi\rangle \rightarrow |\Psi'\rangle$ $H \rightarrow H'$ *"preservation of locality"*

- Entanglement entropy and correlations as in 1+1 critical ground states

$$S_L \approx \log(L) \qquad C(L) \approx L^{-2\Delta}$$

blah, blah, blah… However, does it work?

[Given lattice Hamiltonian H,
optimize variational parameters by energy minimization]

input

1D quantum Hamiltonian

- **on the lattice**
- **at a critical point**

output

Numerical determination of

- **central charge** c
- **scaling dimensions**
  and conformal spi
- **OPE coefficients** C_{c}

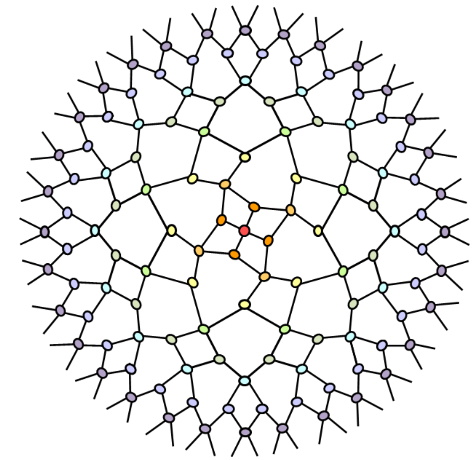
Also:
1+1 critical systems with

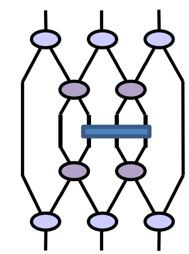
- impurities/defects
- boundaries
- interfaces

2+1 gapped phases with

- topological order
- frustrated spins
- interacting fermions

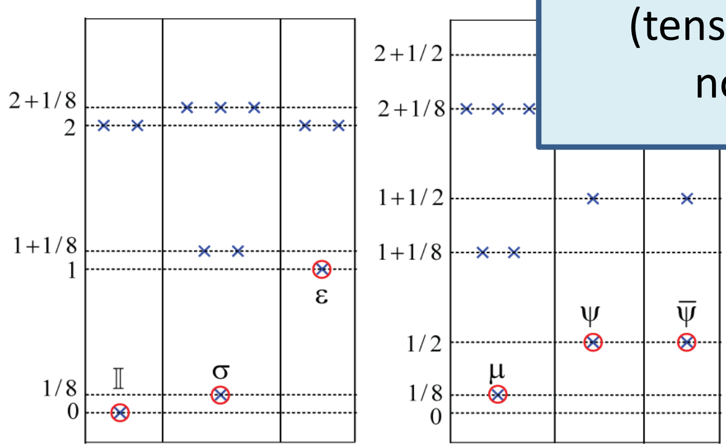
(tensor networks have no sign problem)





e.g. critical Ising model (appro

Pfeifer, Evenbly, Vidal 08



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$(\pm 6 \times 10^{-4})$

outline

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(old stuff)

Tensor networks

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Renormalization group

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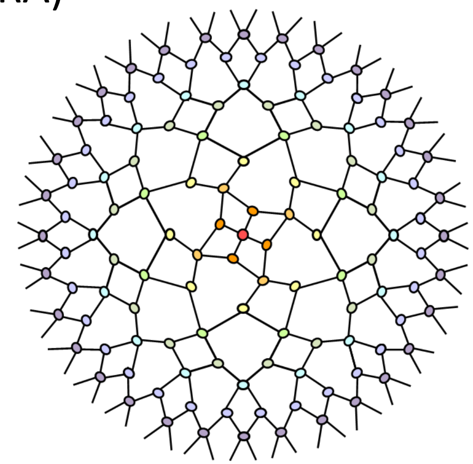
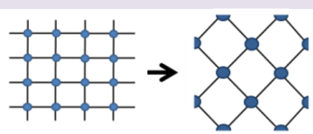


- quantum circuit
- RG transformation

Part 2:
(recent developments)

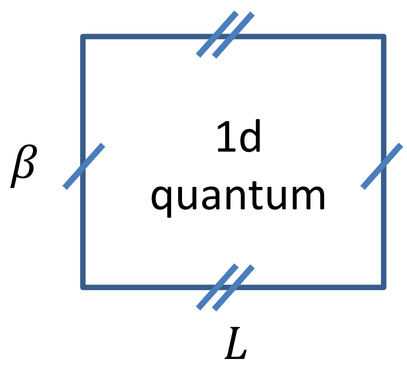
Tensor network renormalization (TNR)



- RG flow in the space of tensors
- Local scale transformations on the lattice
  - plane to cylinder
  - hyperbolic plane (MERA)
  - thermal states / black holes

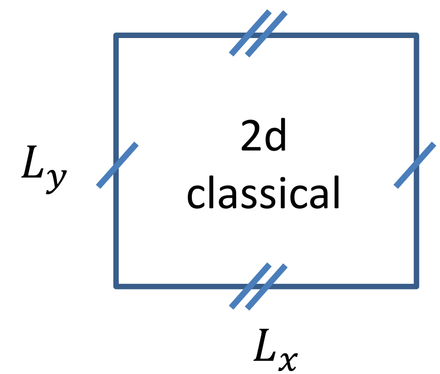
Euclidean path integral

$$Z(\lambda) = tr\, e^{-\beta H_q^{1d}}$$



Statistical partition function

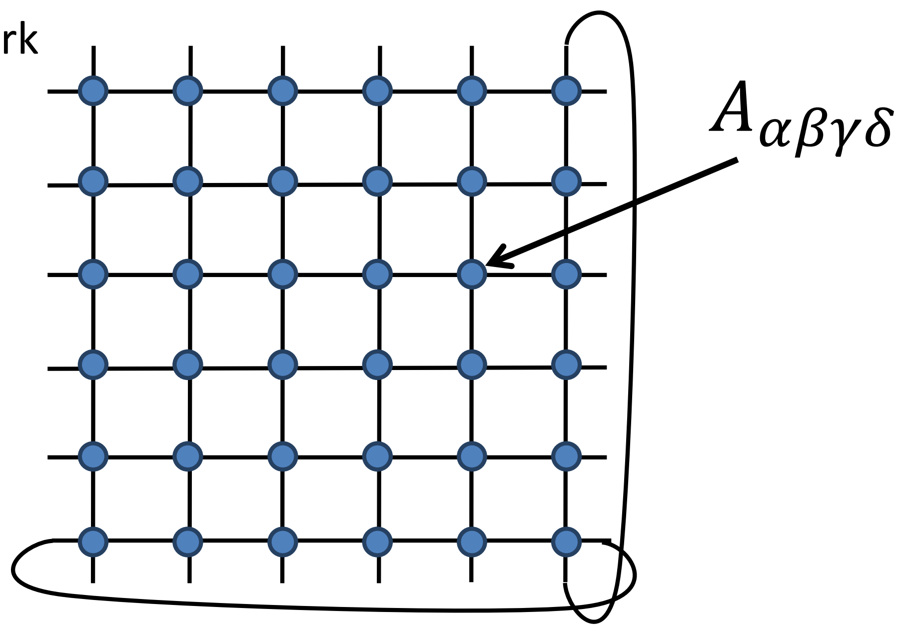
$$Z(T) = \sum_{\{s\}} e^{-\frac{1}{T} H_{cl}^{2d}}$$



$\sim$

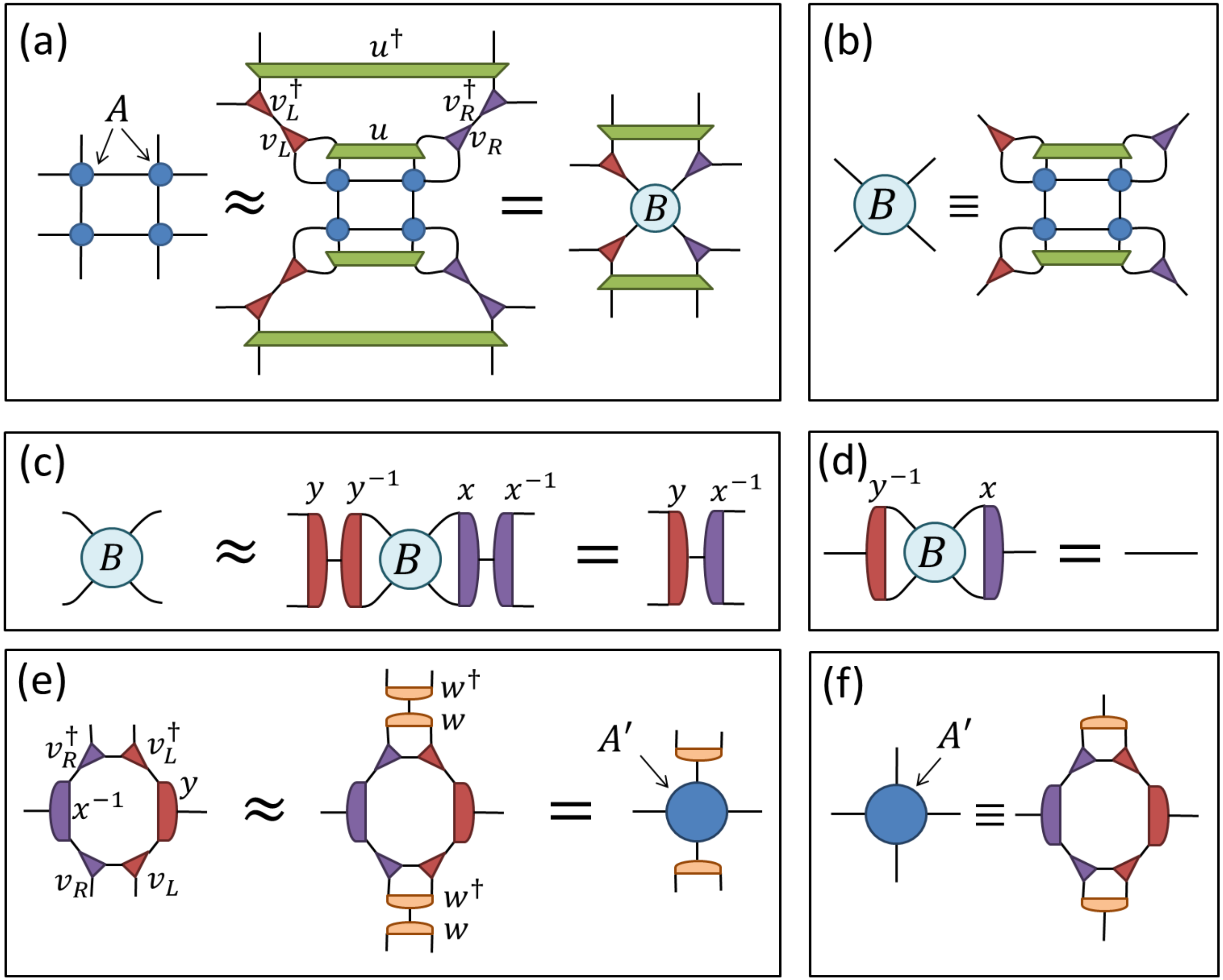
as a tensor network

$$Z =$$



Tensor Network Renormalization (TNR)

[Evenbly, Vidal 14-15]



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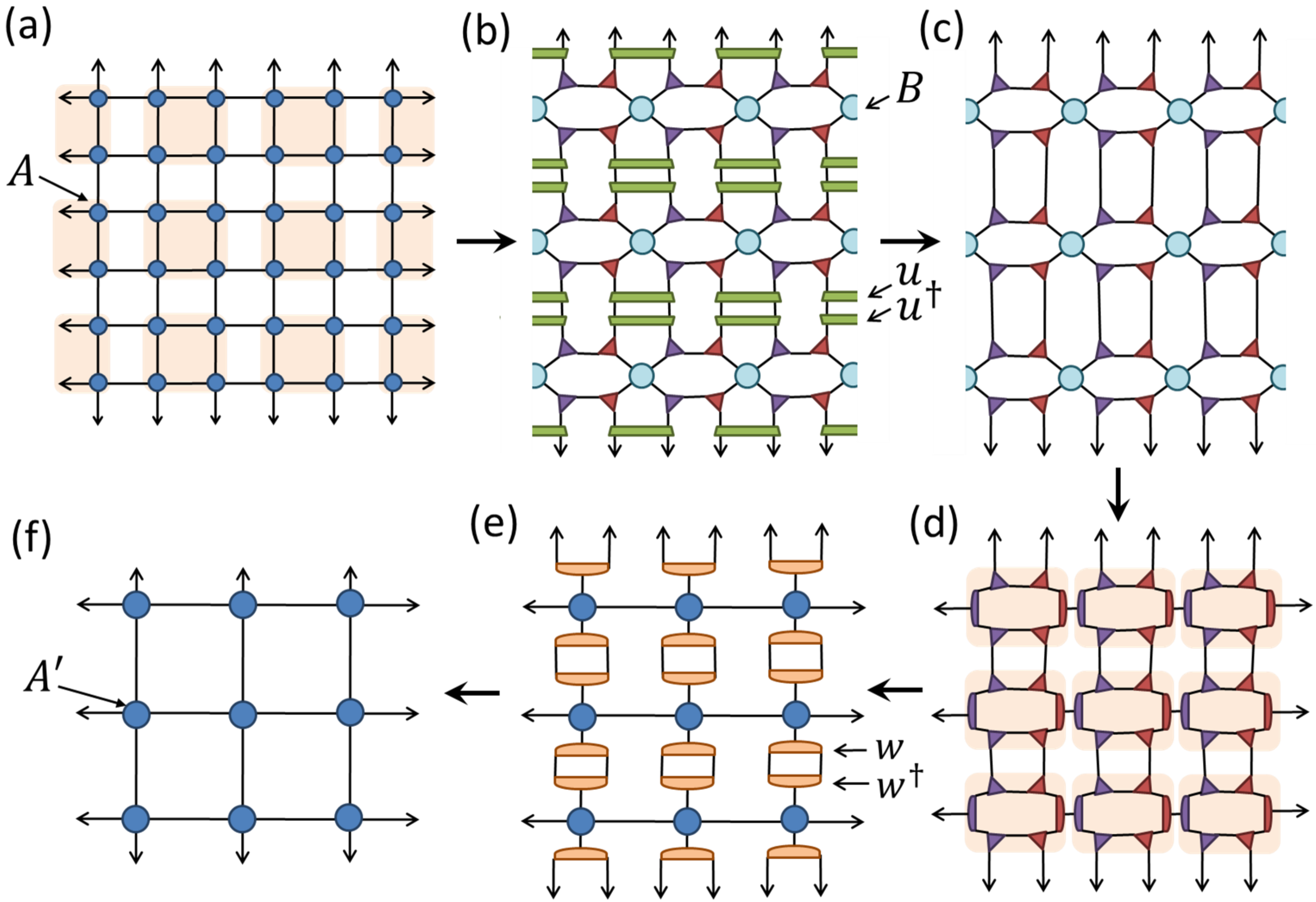
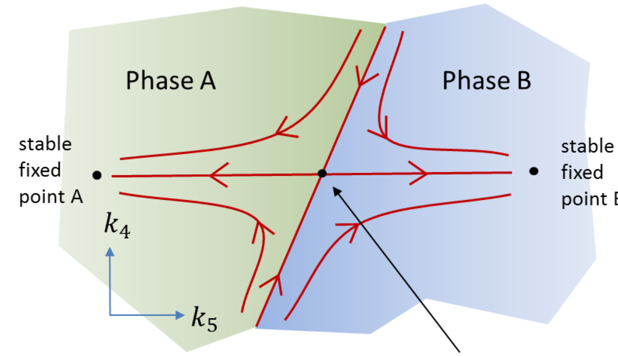
[Evenbly, Vidal 14-15]

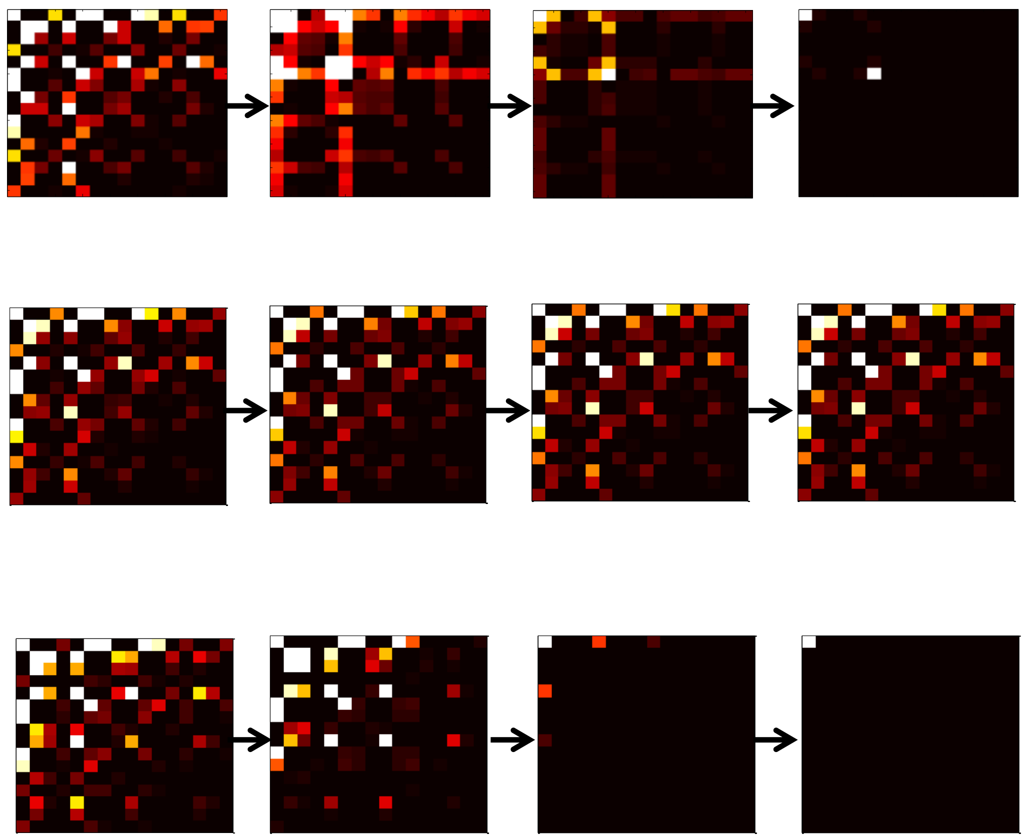


TNR -> proper RG flow
Example: 2D classical Ising



$$A \to A' \to A'' \to \cdots \to A^{fp}$$

$|A^{(1)}|$ $|A^{(2)}|$ $|A^{(3)}|$ $|A^{(4)}|$



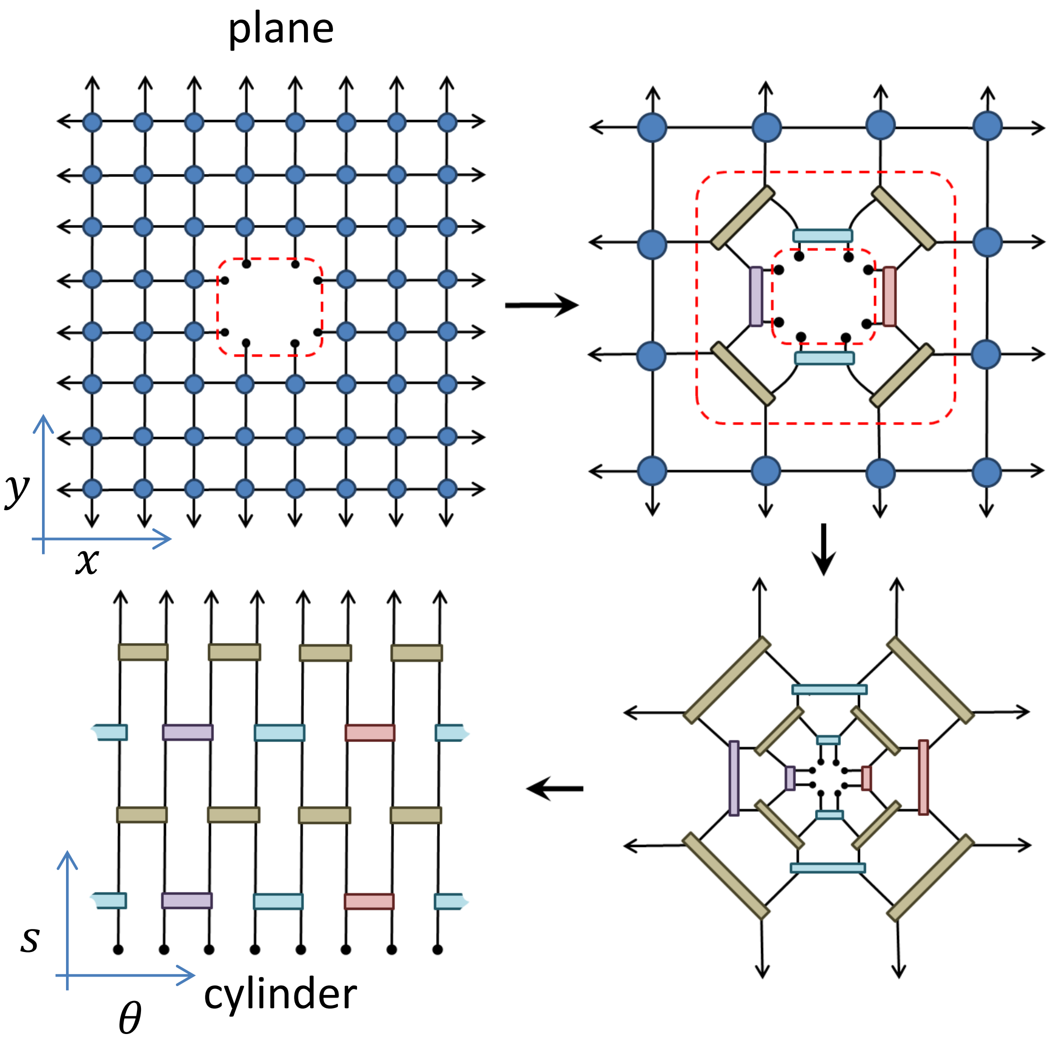
below critical
$T = 0.9\,T_c$
ordered (Z2) fixed point

critical
$T = T_c$
critical fixed point

above critical
$T = 1.1\,T_c$
disordered (trivial) fixed point

local scale transformations

[Evenbly et al, in preparation]



example 1: Plane to cylinder

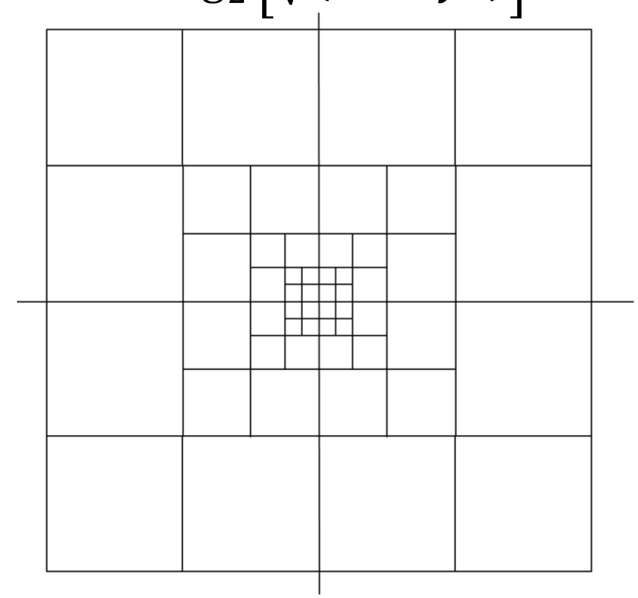
(radial quantization in CFT)

$$z \equiv x + iy$$

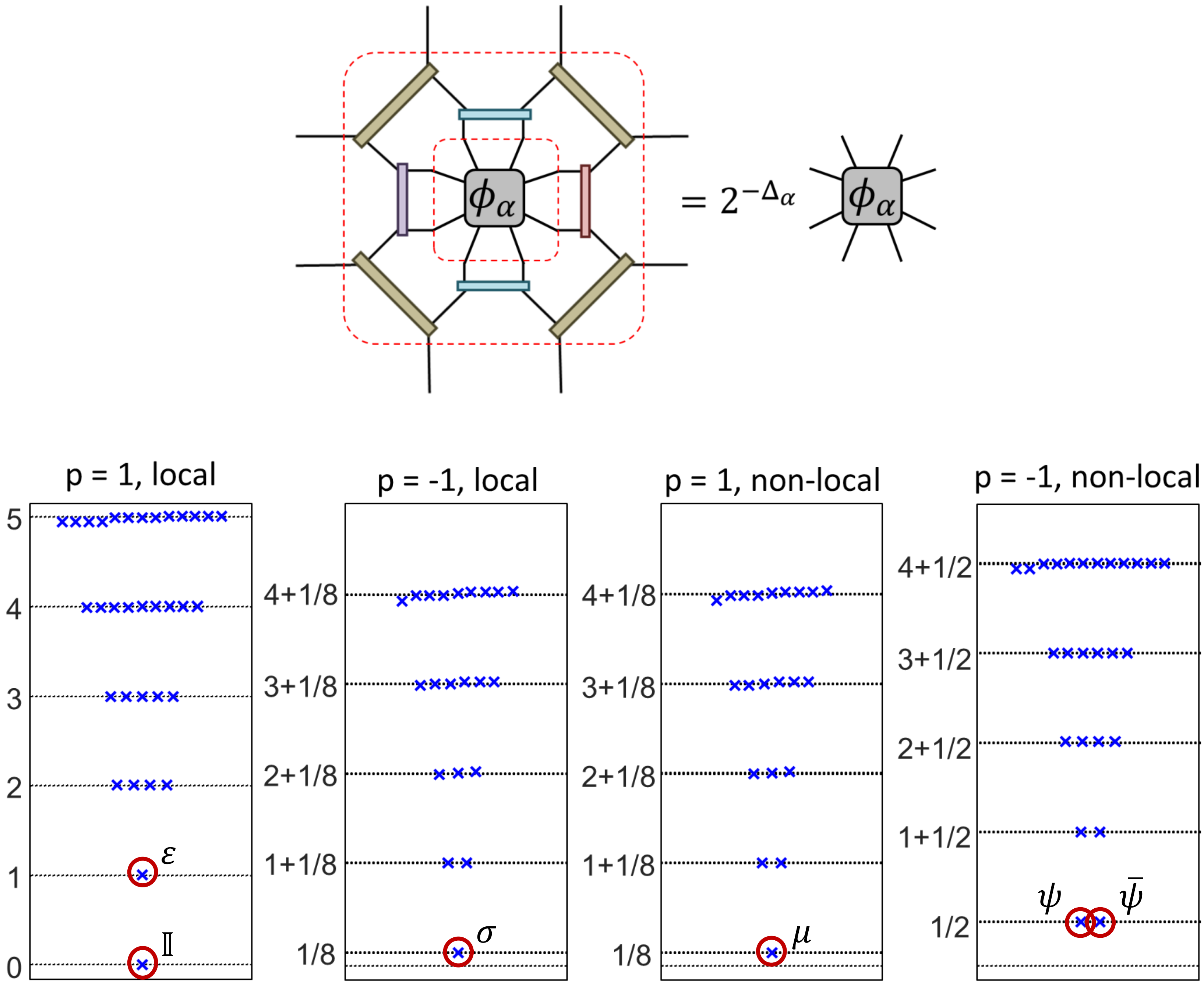
$$\boxed{z = 2^{w}}$$

$$w \equiv s + i\theta$$

$$s \equiv \log_2 \left[\sqrt{(x^2 + y^2)} \right]$$



- Extraction of scaling dimensions, OPE



ϕ_α
$= 2^{-\Delta_\alpha}$
ϕ_α
p = 1, local
p = -1, local
p = 1, non-local
p = -1, non-local
5
4
3
2
1
0
ε
$\mathbb{I}$
4+1/8
3+1/8
2+1/8
1+1/8
1/8
σ
μ
4+1/2
3+1/2
2+1/2
1+1/2
1/2
ψ
$\bar{\psi}$

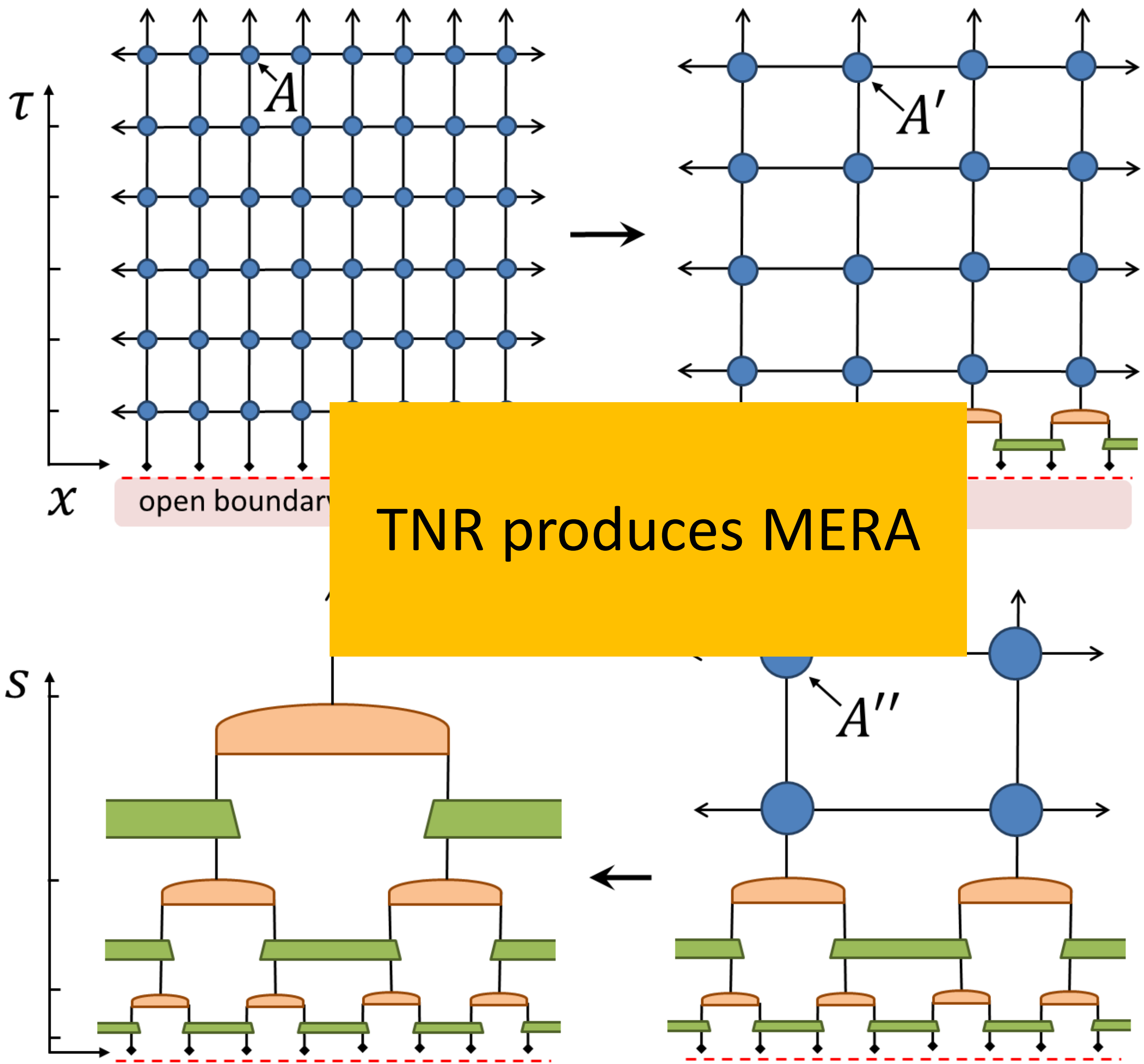
local scale transformations

example 2:

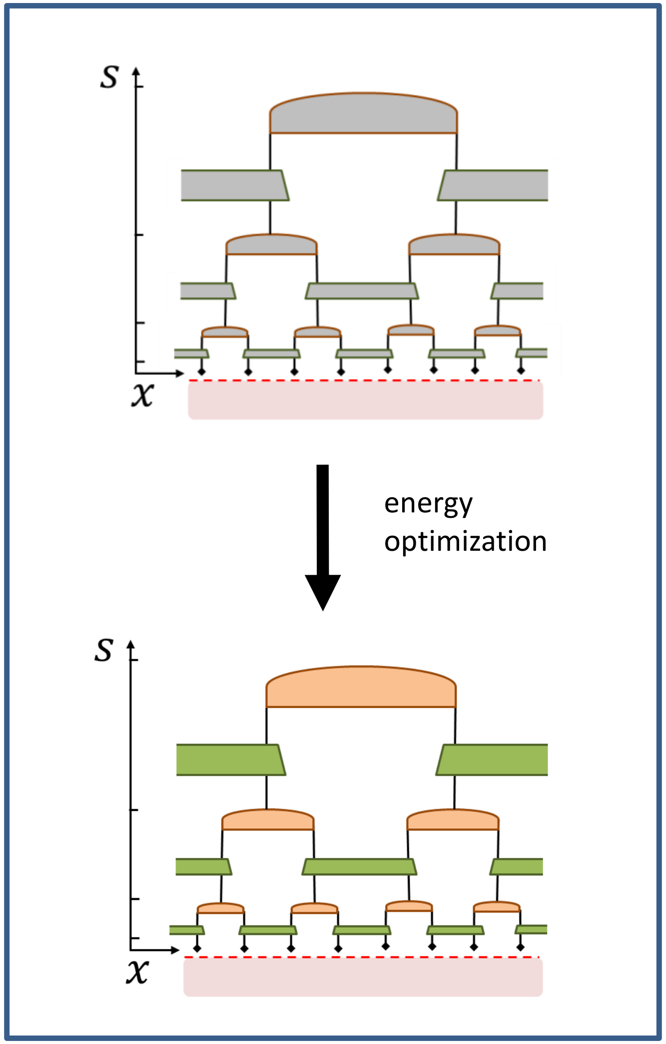
[Evenbly, Vidal, 15]

Upper half plane to cylinder

$|\Psi\rangle \sim e^{-\tau H}|\phi_0\rangle$



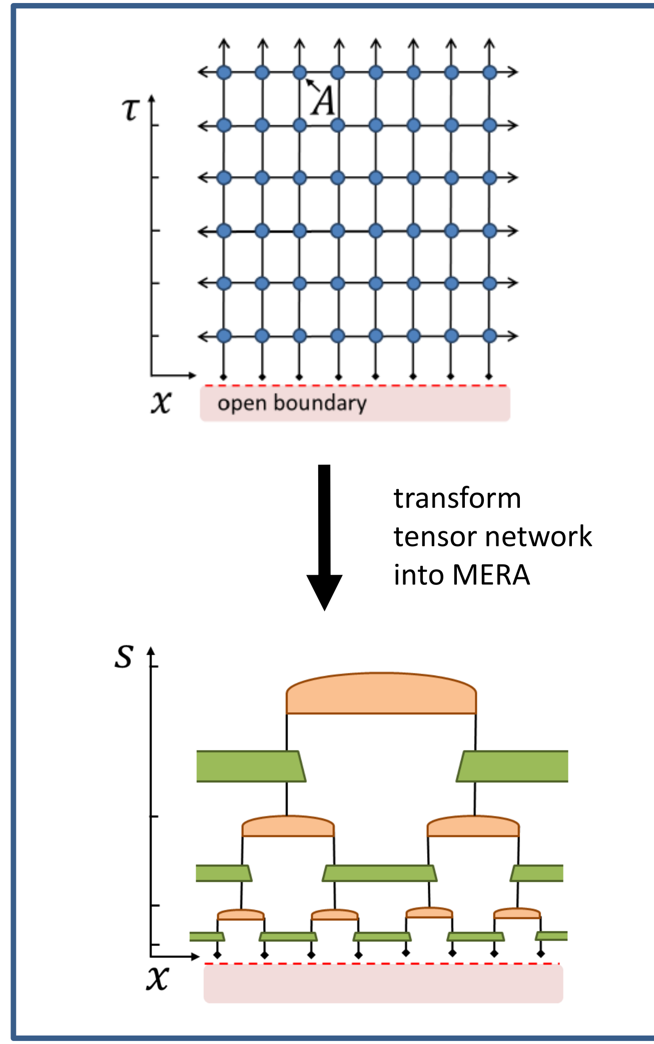
MERA = variational ansatz



energy minimization

- 1000s of iterations over scale
- local minima
- correct ground ?

MERA = by-product of TNR

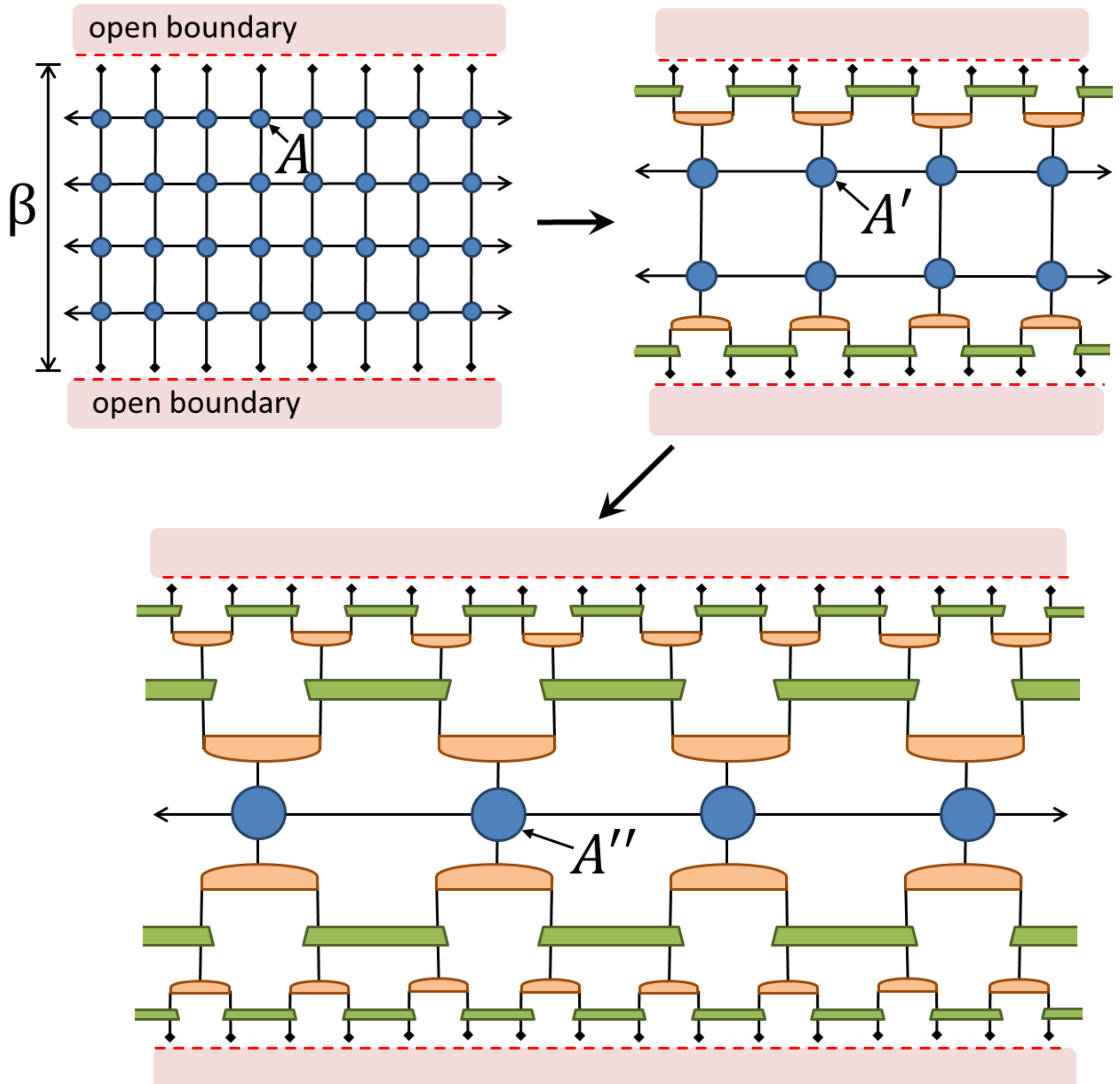


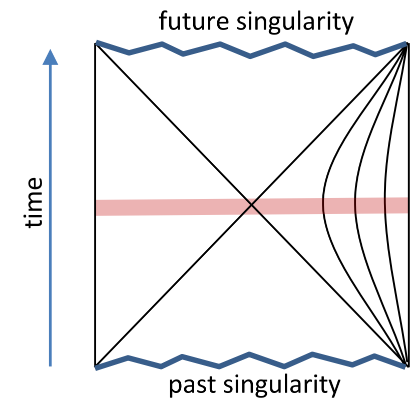
TNR -> MERA

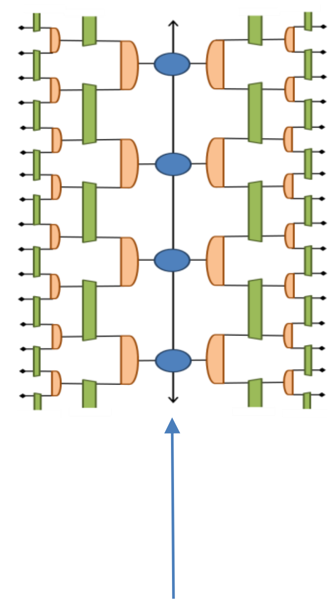
- single iteration over scale
- rewrite tensor network for ground state
- certificate of accuracy

MERA for a thermal state (or black hole in holography)

$$\rho_\beta \sim e^{-\beta H}$$



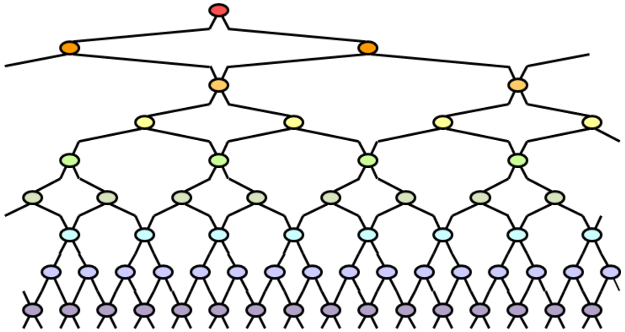




Einstein-Rosen bridge

Summary

Multi-scale entanglement
renormalization ansatz
(MERA)



- Part I: variational ansatz for ground states of CFTs (on the lattice)

MERA $\sim$ CFT

- Part II: by-product of coarse-graining the Euclidean path integral

TNR $\rightarrow$ MERA

MERA $\leftrightarrow$ holography

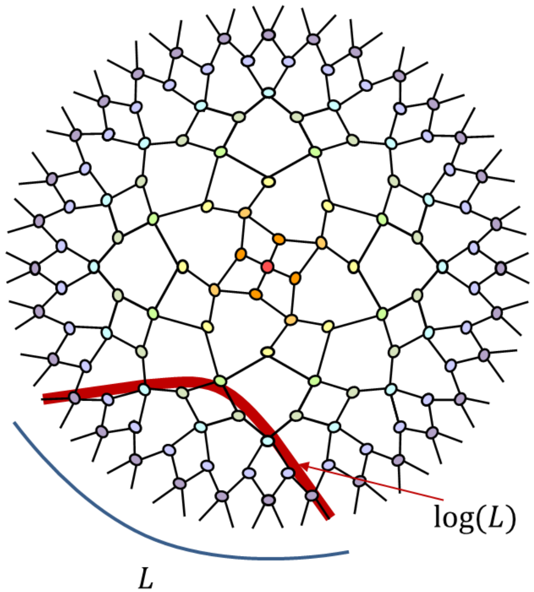
?

MERA = RG

Tensor network for ground state/Hilbert space of CFT, organized in extra dimension corresponding to scale

generic CFT (no large N, strong interactions)

e.g. for Ising model



dictionary	
boundary	bulk
state of CFT (or Hilbert space of CFT)	tensor network (?) (or unitary map, EHM)
scaling dimension Δ	mass $\sim\Delta$
entanglement entropy	"minimal connecting surface"
global on-site symmetry (e.g. Z_2)	local/gauge symmetry (e.g. Z_2)

MERA operates at scale of AdS radius
For smaller scale? → cMERA

Useful test bed

Generalized notion of *holographic* description?

THANK YOU!