Towards an effective action for hydrodynamics

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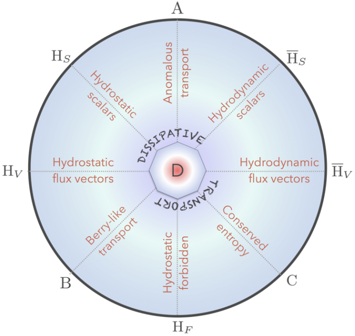
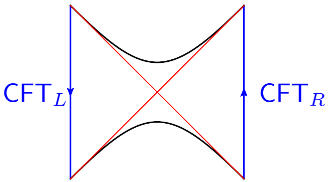


Benasque — 16 July 2015

FH, R. Loganayagam, M. Rangamani
[1412.1090], [1502.00636], [1507.xxxxx]

Motivation

- How to do low energy effective field theory (in a Wilsonian sense) for **mixed states**?
  - Path integral contains both bras and kets
    $\Rightarrow$ **Schwinger-Keldysh** doubling: $\mathcal{H}_{phys} \subset \mathcal{H}_R \otimes \mathcal{H}_L$
- Many applications, e.g., **black hole dynamics:**
  - Double copy somehow encodes physics behind horizon
  - The two copies are coupled (entanglement, dissipation, ...)
  - Unitarity? ...



- Preview: doing the doubling properly seems to give a lot of illuminating structure and powerful formalism (incl. new emergent symmetries)

Motivation

- These are tough questions. Start with something more tractable to learn about the general structure
- **Hydrodynamics:** generic description of near-equilibrium dynamics of mixed states on length scales $L \gg \ell_{\text{mfp}}$
  - This talk: how to make progress on these problems by formulating hydrodynamics as a Wilsonian effective field theory

The hydrodynamic gradient expansion

- Hydrodynamics: near-equilibrium EFT for long wavelength fluctuations about Gibbsian density matrix

$$\boxed{\text{microscopic theory}}$$

$$\downarrow L \gg \ell_{\mathsf{mfp}}$$

macroscopic fluid variables:	$u^\mu(x),\ T(x),\ \mu(x)$	$(u^2=-1)$
background sources:	$g_{\mu\nu}(x),\ A_\mu(x)$	

$$\downarrow \text{ phenomenology}$$

Constitutive relations:	Dynamics:
$T^{\mu\nu} = T^{\mu\nu}_{(0)} + T^{\mu\nu}_{(1)} + \ldots$	$D_\nu T^{\mu\nu} \simeq F^{\mu\nu} J_\nu$
$J^\mu = J^\mu_{(0)} + J^\mu_{(1)} + \ldots$	$D_\mu J^\mu \simeq 0$

- E.g. (charged) ideal fluid: $T^{\mu\nu}_{(0)} = \varepsilon\, u^\mu u^\nu + p\, P^{\mu\nu}\,, \qquad J^\alpha_{(0)} = q\, u^\alpha$

The hydrodynamic gradient expansion

- Hydrodynamics: near-equilibrium EFT for long wavelength fluctuations about Gibbsian density matrix

microscopic theory

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$\downarrow$ phenomenology

Constitutive relations:	Dynamics:
$T^{\mu\nu} = T^{\mu\nu}_{(0)} + T^{\mu\nu}_{(1)} + \ldots$	$D_\nu T^{\mu\nu} \simeq F^{\mu\nu} J_\nu + \boxed{\mathrm{T}^{\perp\mu}_{H}}$
$J^{\mu} = J^{\mu}_{(0)} + J^{\mu}_{(1)} + \ldots$	$D_\mu J^{\mu} \simeq \boxed{\mathrm{J}^{\perp}_{H}}$ (cov. anomalies)

- E.g. (charged) ideal fluid: $T^{\mu\nu}_{(0)} = \varepsilon\, u^\mu u^\nu + p\, P^{\mu\nu}\,, \qquad J^{\alpha}_{(0)} = q\, u^\alpha$

The hydrodynamic gradient expansion

- Our goal **is not**...

  ... to determine transport coefficients $(\varepsilon, p, q, \ldots)$ for any particular microscopic system

  ... to solve the fluid equations for $\{u^\mu, T, \mu\}$

- Our goal **is**: to provide all symmetry-allowed constitutive relations order by order in ∇_μ which are consistent with:

Second law constraint:

$\exists\, J_S^\mu = s\, u^\mu + J_{S,(1)}^\mu + \ldots$ with $D_\mu J_S^\mu \gtrsim 0$ (on-shell)

  - Gives quite non-trivial constraints on physically allowed constitutive relations, e.g.:
    - Neutral 1st order: viscosities $\eta, \zeta \geq 0$ *Bhattacharyya '12*
    - Neutral 2nd order: 5 relations among 15 a-priori independent transport coefficients *Son-Surowka '09*
    - Anomaly induced transport completely fixed *Jensen-Loganayagam-Yarom '13*

So what's the problem?

- This 'current algebra' approach is phenomenlogically very well understood & tested
- But: doesn't make much sense from point of view of **Wilsonian field theory**

Phenomenological hydrodynamics	Natural for field theorist
• "current algebra": provide all tensor structures $T^{\mu\nu}$, J^μ ad hoc	• fields & symmetries $\Rightarrow$ eff. action S • $T^{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g_{\mu\nu}}\,, \quad J^\mu = \frac{1}{\sqrt{-g}} \frac{\delta S}{\delta A_\mu}$
• dynamics = conservation laws	• dynamics: $\delta S = 0$
• second law constraint: $D_\mu J^\mu_S \gtrsim 0$	• ??
• ??	• Schwinger-Keldysh path integral
• ??	• dual black hole description

Outline

- ✓ Review of hydrodynamics
- → Adiabaticity and dissipation
- ∘ Effective actions I: Simple Lagrangians
- ∘ Classification of transport
- ∘ Effective actions II: Doubling and emergent symmetry
- ∘ Outlook: Effective actions III

Disclaimer

From now on I will only discuss **neutral fluids**.
Adding an arbitrary number of abelian or non-abelian flavours is mainly a technical task without new conceptual ideas, see [1502.00636].

Off-shell entropy production and adiabaticity

- Inequality constraint $\nabla_\mu J_S^\mu \gtrsim 0$ is much more conveniently incorporated if we don't have to simplify it using equations of motion.
- Use Lagrange multiplier $\boldsymbol{\beta}^\mu$ and consider **off-shell statement**:

$$\nabla_\mu J_S^\mu + \boldsymbol{\beta}_\mu \left(\nabla_\nu T^{\mu\nu} - \mathrm{T}_H^{\mu\perp}\right) \equiv \Delta \geq 0$$

Loganayagam '11

- Natural Lagrange multiplier:
  - $\boldsymbol{\beta}^\mu = \frac{1}{T}\, u^\mu$ (local thermal vector)

Task: solve for $\{J_S^\mu, T^{\mu\nu}\}$ as functionals of $\{\boldsymbol{\beta}^\mu, g_{\mu\nu}\}$

- ▷ Ideally: Find Wilsonian effective action which defines all solutions off-shell

- Marginal case $\Delta = 0$: **'adiabaticity equation'**
  - Particularly rich structure! $\Rightarrow$ focus on this first

Aside: adiabaticity equation for free energy current

$$\boxed{\nabla_\mu J_S^\mu + \boldsymbol{\beta}_\mu \left(\nabla_\nu T^{\mu\nu} - \mathrm{T}_H^{\mu\perp} \right) = 0}$$

- Can trade entropy current J_S^μ for **free energy current** $\mathcal{G}^\sigma$:

$$-\frac{\mathcal{G}^\sigma}{T} \equiv J_S^\sigma - (J_S^\sigma)_{canonical} \qquad \text{with} \qquad (J_S^\sigma)_{canonical} = -\boldsymbol{\beta}_\nu T^{\nu\sigma}$$

- Grand-canonical version of adiabaticity equation:

$$\boxed{-\left[\nabla_\sigma \left(\frac{\mathcal{G}^\sigma}{T} \right) - \frac{\mathcal{G}_H^\perp}{T} \right] = \frac{1}{2} T^{\mu\nu} \, \pounds_{\boldsymbol{\beta}} \, g_{\mu\nu}}$$

  - Solve for $\{\mathcal{G}^\sigma, T^{\mu\nu}\}$ as functionals of $\{\boldsymbol{\beta}^\mu, g_{\mu\nu}\}$

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Effective actions I: Lagrangians

- Consider most obvious effective action:
  - Fields: hydrodynamic fields + sources $= \{\boldsymbol{\beta}^\mu,\, g_{\mu\nu}\}$
  - Symmetries: diffeomorphism invariance

$$S = \int \sqrt{-g}\, \mathcal{L}[\boldsymbol{\beta}^\mu, g_{\mu\nu}]$$

  - Basic variation defines hydrodynamic currents:

$$\delta S = \int \sqrt{-g}\, \Big[\tfrac{1}{2}\, T^{\mu\nu}\, \delta g_{\mu\nu} + T\, \mathfrak{h}_\sigma\, \delta\boldsymbol{\beta}^\sigma + \underbrace{\nabla_\mu (\not{\delta}\Theta_{\text{PS}})^\mu}_{\text{surface term}}\Big]$$

  - This defines the stress tensor $T^{\mu\nu}$
  - Does it solve **adiabaticity equation?**
  - Does it exhibit **correct dynamics** ($=$ conservation)?

Effective actions I: Lagrangians

- To check that $T^{\mu\nu}$ is a solution of adiabaticity equation, need to define entropy current in terms of effective action S:

$$J_S^\mu = s\, u^\mu \qquad \text{with} \qquad s \equiv \left[\frac{1}{\sqrt{-g}} \frac{\delta S}{\delta T} \right]_{\{u^\mu, g_{\mu\nu}\}\ \text{fixed}} = -\mathfrak{h}_\sigma \boldsymbol{\beta}^\sigma$$

- Demand invariance of S under arbitrary diffeos. This gives **Bianchi identity:**

$$\nabla_\nu T^{\mu\nu} = \frac{g^{\mu\nu}}{\sqrt{-g}} \pounds_{\boldsymbol{\beta}} \left(\sqrt{-g}\; T\, \mathfrak{h}_\nu \right)$$

- Get indeed solution to adiabaticity equation:

$$\nabla_\mu J_S^\mu + \boldsymbol{\beta}_\mu \nabla_\nu T^{\mu\nu} = -\nabla_\mu \left[(\boldsymbol{\beta}^\sigma \mathfrak{h}_\sigma) T\, \boldsymbol{\beta}^\mu \right] + \frac{1}{\sqrt{-g}} \boldsymbol{\beta}^\nu \pounds_{\boldsymbol{\beta}} \left(\sqrt{-g}\, T\, \mathfrak{h}_\nu \right) = 0$$

Remark: interpretation of entropy current

- The free energy current derived from our definition of J_S^μ is

$$\mathcal{G}^\sigma \equiv -\mathcal{L}\, u^\sigma + T \underbrace{(\oint_{\mathcal{B}} \Theta_{\text{PS}})^\sigma}_{\substack{\text{surface term w/} \\ \delta g_{\mu\nu} \mapsto \pounds_{\boldsymbol{\beta}} g_{\mu\nu}}}$$

  - $\mathrm{N}^\mu \equiv \mathcal{L}\, \boldsymbol{\beta}^\mu - (\oint_{\mathcal{B}} \Theta_{\text{PS}})^\mu$ is the **Noether current for diffeomorphisms along $\boldsymbol{\beta}^\mu$**

Consistency check: hydrostatic equilibrium

- To get a feeling for why our $\mathcal{G}^\sigma$ is sensible, consider **hydrostatic equilibrium**
  - Spacetime manifold: Euclidean $\Sigma_{\mathcal{M}} \times S^1$
  - $\exists$ timelike Killing vector $K^\mu = \boldsymbol{\beta}^\mu|_{equil.}$ with $\pounds_K g_{\mu\nu} = 0$

$$\mathcal{G}^\sigma|_{equil.} = -\mathcal{L} u^\sigma$$

... is a **Landau-Ginzburg** free-energy current! Indeed:

$$S|_{equil.} = \int_{\Sigma_{\mathcal{M}} \times S^1} \mathcal{L}[K^\mu, g_{\mu\nu}]\, d^d x = -\int_{\Sigma_{\mathcal{M}}} \left(\frac{\mathcal{G}^\sigma}{T}\right) d^{d-1} S_\sigma$$

... is an **equilibrium partition function**

Dynamics

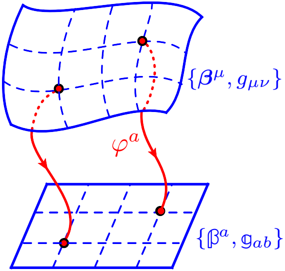
- To get correct dynamics, formulate problem as a σ-model
  - Physical fields are pullbacks of a **reference configuration**:

$$\varphi^a : \quad \text{physical fluid} \longrightarrow \text{worldvolume reference manifold}$$

$$g_{\mu\nu} = \frac{\partial\varphi^a}{\partial x^\mu}\frac{\partial\varphi^b}{\partial x^\nu}\, \mathbb{g}_{ab}[\varphi(x)]\,, \qquad \boldsymbol{\beta}^\mu = \frac{\partial x^\mu}{\partial\varphi^a}\, \mathbb{\beta}^a[\varphi(x)]$$

  - Vary pullback fields φ^a, while holding the reference configuration $\mathbb{\beta}^a$ fixed

$$\frac{\delta S}{\delta\varphi^a} = 0 \qquad \Rightarrow \qquad \frac{1}{\sqrt{-g}}\,\pounds_{\boldsymbol{\beta}}\left(\sqrt{-g}\;T\,\mathfrak{h}_\nu\right) \simeq 0$$



- Reminder: Bianchi identity: $\nabla_\nu T^{\mu\nu} = \frac{g^{\mu\nu}}{\sqrt{-g}}\,\pounds_{\boldsymbol{\beta}}\left(\sqrt{-g}\;T\,\mathfrak{h}_\nu\right)$
  - Hence get the dynamics expected from phenomenology

So are we done?

- Is this a complete Lagrangian theory of hydrodynamics?

So are we done?

- Is this a complete Lagrangian theory of hydrodynamics?
  - No. A lot of transport is not captured by this construction.

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Example: neutral, conformal fluid at $\mathcal{O}(\partial^2)$

- Most general 2nd order (neutral, Weyl-invariant) stress tensor:

$$\begin{aligned} T^{\mu\nu}_{(2)} = &(\lambda_1 - \kappa)\, \sigma^{<\mu\alpha}\sigma_\alpha{}^{\nu>} \\ &+ (\lambda_2 + 2\tau - 2\kappa)\, \sigma^{<\mu\alpha}\omega_\alpha{}^{\nu>} \\ &+ \tau\, \left(u^\alpha \mathcal{D}^{\mathcal{W}}_\alpha \sigma^{\mu\nu} - 2\sigma^{<\mu\alpha}\omega_\alpha{}^{\nu>}\right) \\ &+ \lambda_3\, \omega^{<\mu\alpha}\omega_\alpha{}^{\nu>} \\ &+ \kappa\, \left(C^{\mu\alpha\nu\beta}u_\alpha u_\beta + \sigma^{<\mu\alpha}\sigma_\alpha{}^{\nu>} + 2\sigma^{<\mu\alpha}\omega_\alpha{}^{\nu>}\right) \end{aligned}$$

Example: neutral, conformal fluid at $\mathcal{O}(\partial^2)$

- Most general 2nd order (neutral, Weyl-invariant) stress tensor:

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\begin{aligned}
T^{\mu\nu}_{(2)} = &(\lambda_1 - \kappa)\, \sigma^{<\mu\alpha}\sigma_\alpha{}^{\nu>} \\
&+ (\lambda_2 + 2\tau - 2\kappa)\, \sigma^{<\mu\alpha}\omega_\alpha{}^{\nu>} \\
&+ \tau \left(u^\alpha \mathcal{D}^{\mathcal{W}}_\alpha \sigma^{\mu\nu} - 2\sigma^{<\mu\alpha}\omega_\alpha{}^{\nu>} \right) && \to \text{Class } \overline{\mathrm{H}}_S \\
&+ \lambda_3\, \omega^{<\mu\alpha}\omega_\alpha{}^{\nu>} && \to \text{Class } \mathrm{H}_S \\
&+ \kappa \left(C^{\mu\alpha\nu\beta} u_\alpha u_\beta + \sigma^{<\mu\alpha}\sigma_\alpha{}^{\nu>} + 2\sigma^{<\mu\alpha}\omega_\alpha{}^{\nu>} \right) && \to \text{Class } \mathrm{H}_S
\end{aligned}
$$

τ, λ_3, κ

Are all derivable from a Lagrangian

$$
\mathcal{L}^{\mathcal{W}}_2[\boldsymbol{\beta}^\mu, g_{\mu\nu}] = \frac{1}{4} \left[-\frac{2\,\kappa}{(d-2)} ({}^{\mathcal{W}}R) + 2\,(\kappa - \tau)\,\sigma^2 + (\lambda_3 - \kappa)\,\omega^2 \right]
$$

Note: λ_3 and κ are hydrostatic, τ is genuinely hydrodynamic

Example: neutral, conformal fluid at $\mathcal{O}(\partial^2)$

- Most general 2nd order (neutral, Weyl-invariant) stress tensor:

$$
\begin{aligned}
T^{\mu\nu}_{(2)} = &\ (\lambda_1 - \kappa)\, \sigma^{<\mu\alpha}\sigma_\alpha{}^{\nu>} && \to \text{Class D} \\
&+ (\lambda_2 + 2\tau - 2\kappa)\, \sigma^{<\mu\alpha}\omega_\alpha{}^{\nu>} && \\
&+ \tau\, \left(u^\alpha \mathcal{D}^{\mathcal{W}}_\alpha \sigma^{\mu\nu} - 2\sigma^{<\mu\alpha}\omega_\alpha{}^{\nu>}\right) && \to \text{Class } \overline{\mathrm{H}}_S \\
&+ \lambda_3\, \omega^{<\mu\alpha}\omega_\alpha{}^{\nu>} && \to \text{Class } \mathrm{H}_S \\
&+ \kappa\, \left(C^{\mu\alpha\nu\beta}u_\alpha u_\beta + \sigma^{<\mu\alpha}\sigma_\alpha{}^{\nu>} + 2\sigma^{<\mu\alpha}\omega_\alpha{}^{\nu>}\right) && \to \text{Class } \mathrm{H}_S
\end{aligned}
$$

$(\lambda_1 - \kappa)$

Leads to $\Delta \simeq -(\lambda_1 - \kappa)\, \frac{1}{T}\, \sigma^\mu{}_\nu \sigma^\nu{}_\rho \sigma^\rho{}_\mu$

$\Rightarrow$ Dissipative ($\Rightarrow$ not Lagrangian)

(but unconstrained by second law, since $\sigma^3 \ll \sigma^2$)

Example: neutral, conformal fluid at $\mathcal{O}(\partial^2)$

- Most general 2[nd] order (neutral, Weyl-invariant) stress tensor:

$$
\begin{aligned}
T^{\mu\nu}_{(2)} = {} & (\lambda_1 - \kappa)\, \sigma^{<\mu\alpha}\sigma_\alpha{}^{\nu>} && \to \text{Class D} \\
& + (\lambda_2 + 2\tau - 2\kappa)\, \sigma^{<\mu\alpha}\omega_\alpha{}^{\nu>} && \to \text{Class B} \\
& + \tau \left(u^\alpha \mathcal{D}^{\mathcal{W}}_\alpha \sigma^{\mu\nu} - 2\sigma^{<\mu\alpha}\omega_\alpha{}^{\nu>} \right) && \to \text{Class } \overline{\mathrm{H}}_S \\
& + \lambda_3\, \omega^{<\mu\alpha}\omega_\alpha{}^{\nu>} && \to \text{Class } \mathrm{H}_S \\
& + \kappa \left(C^{\mu\alpha\nu\beta} u_\alpha u_\beta + \sigma^{<\mu\alpha}\sigma_\alpha{}^{\nu>} + 2\sigma^{<\mu\alpha}\omega_\alpha{}^{\nu>} \right) && \to \text{Class } \mathrm{H}_S
\end{aligned}
$$

$(\lambda_2 + 2\tau - 2\kappa)$

Looks schematically like a Berry curvature (Class B):

$$
(T^{\mu\nu})_{\mathrm{B}} \equiv -\frac{1}{4} \left(\mathcal{N}^{(\mu\nu)(\alpha\beta)} - \mathcal{N}^{(\alpha\beta)(\mu\nu)} \right) \pounds_{\boldsymbol{\beta}}\, g_{\alpha\beta}
$$

(cannot be obtained from Lagrangian)

Example: neutral, conformal fluid at $\mathcal{O}(\partial^2)$

- Out of 5 transport coefficients, 3 come from a Lagrangian: τ, λ_3 and κ
- For fluids described by $\mathcal{L}[\boldsymbol{\beta}^\mu, g_{\mu\nu}]$, the other 2 combinations are zero:

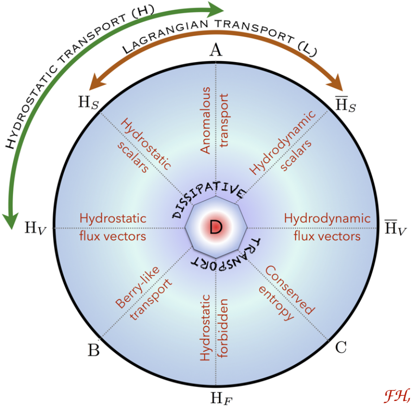
$$(\lambda_1 - \kappa) = 0 \quad \text{and} \quad (\lambda_2 + 2\tau - 2\kappa) = 0$$

  - These relations have been **observed in Einstein gravity** *Haack-Yarom '08*
    - ⋆ Our simple Lagrangians seem to know about holography
    - ⋆ Derive $\mathcal{L}[\boldsymbol{\beta}^\mu, g_{\mu\nu}]$ from gravity directly? *Nickel-Son '10*

      de Boer et al. '15, Crossley et al. '15
  - First relation ensures **no entropy production at subleading order** (this is not required by second law!)
    $\rightarrow$ **"Principle of minimum dissipation"** in holography?

    FH, Loganayagam, Rangamani '14

Summary of eight classes of transport



S and V refer to:
$\mathcal{G}^\sigma = \mathcal{S}\,\boldsymbol{\beta}^\sigma + \mathcal{V}^\sigma$

FH, Loganayagam, Rangamani '15

Theorem: The eightfold way of hydrodynamic transport

There are eight classes of hydrodynamic transport consistent with the second law. Two of them are describable by Lagrangians $\mathcal{L}[\boldsymbol{\beta}^\mu, g_{\mu\nu}]$. Further, Class H_F constitutive relations are forbidden by the second law.

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Effective actions II: Schwinger-Keldysh mystery

- Non-equilibrium effective field theory should involve **Schwinger-Keldysh doubling**:

$$\mathcal{H}_{phys} \subset \mathcal{H}_R \otimes \mathcal{H}_L$$

- Integrating out high energy modes from SK path integral leads to coupling between R and L (**"influence functionals"**)

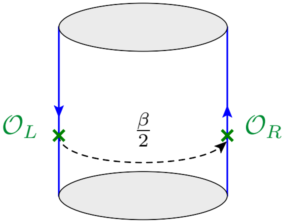
> Just doubling everything gives too much freedom (easy to write influence functionals which violate microscopic unitarity)

  - Important obstacle for systematic understanding of non-equilibrium physics (mixed states, dissipation, fluctuations, noise...)

- But: fluids are heavily constrained by second law and a lot of nice structure in the classification
  - Can solve the problem of influence functionals in the case of hydrodynamics

Effective actions II: KMS condition

- In thermal equilibrium: microscopic theory should satisfy **KMS invariance**
  - Non-local boundary conditions on SK correlators to make sure they are analytic continuations of Euclidean correlators



- Proposal:

At long distances (hydro regime) the non-local KMS relations turn into an emergent local $U(1)_\mathsf{T}$ symmetry.

Effective actions II: KMS condition

- How to make this explicit in hydrodynamics?
  - Reminder I: $\nabla_\mu J_S^\mu = 0$ was mysterious from Wilsonian point of view
  - Reminder II: For 'Lagrangian' classes of transport, $\mathrm{N}^\sigma \equiv J_S^\sigma - (J_S^\sigma)_{canonical}$ is **Noether current for diffeomorphisms along $\boldsymbol{\beta}^\mu$**
  - Elevate these **thermal translations** to a $U(1)_\mathsf{T}$ gauge symmetry with gauge field $\mathsf{A}^{(\mathsf{T})}{}_\mu$
  - Demand effective action be invariant under this symmetry (s.t. adiabaticity $\Leftrightarrow$ $U(1)_\mathsf{T}$ conservation equation)

Effective actions II: adiabatic master Lagrangian

Proposed field content:

- Hydrodynamic field: $\boldsymbol{\beta}^\mu$
- Background source: $g_{\mu\nu}$
- SK copy of source: $\tilde{g}_{\mu\nu}$
- $U(1)_\mathsf{T}$ gauge field: $\mathsf{A}^{(\mathsf{T})}{}_\mu$

Proposed symmetries:

- Diffeo invariance
- $U(1)_\mathsf{T}$ KMS gauge invariance
  (act like twisted diffeomorphism along $\boldsymbol{\beta}^\mu$)

- **Any constitutive relations** $\{T^{\mu\nu}, \mathcal{G}^\sigma\}$ **which satisfy adiabaticity equation** can be obtained from a diffeo and $U(1)_\mathsf{T}$ invariant Lagrangian (and vice versa):

FH-Loganayagam-Rangamani '14-'15

$$\mathcal{L}_\mathsf{T}[\boldsymbol{\beta}^\mu, g_{\mu\nu}, \tilde{g}_{\mu\nu}, \mathsf{A}^{(\mathsf{T})}{}_\mu] = \frac{1}{2}\, T^{\mu\nu}\tilde{g}_{\mu\nu} - \frac{\mathcal{G}^\sigma}{T}\, \mathsf{A}^{(\mathsf{T})}{}_\sigma$$

Effective actions II: some compelling features

- Field content and symmetries are such that we **get precisely the 7 adiabatic classes** and nothing more (no Class H_F)
  - $U(1)_{\mathsf{T}}$ keeps Schwinger-Keldysh doubling under control
  - Adiabaticity equation is consequence of $U(1)_{\mathsf{T}}$ Bianchi identity
  - Conserved entropy current is gauge current of emergent $U(1)_{\mathsf{T}}$ symmetry
- Summary: this construction provides a complete EFT explanation of phenomenological axioms of (adiabatic) hydrodynamics

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Outlook: Effective actions III

- $\mathcal{L}[\boldsymbol{\beta}^\mu, g_{\mu\nu}]$: 2 out of 7+1 classes
- $\mathcal{L}[\boldsymbol{\beta}^\mu, g_{\mu\nu}, \tilde{g}_{\mu\nu}, \mathsf{A}^{(\mathsf{T})}{}_\mu]$: effective action for all 7 classes of adiabatic transport
- Plan to get the 8th dissipative class:
  - Understand in detail the structure of SK path integrals and KMS condition
    - Surprising features: right way to formulate is in terms of **hidden BRST symmetries**
  - Derive $U(1)_\mathsf{T}$ emergent symmetry from first principles
  - Formulate the hydrodynamic σ-model more systematically
  - Action principle for all 8 classes

FH-Loganayagam-Rangamani [1507.xxxxx] and [w.i.p.]

- Investigate consequences for holography, black holes etc.