# Dynamical fields in de Sitter from CFT entanglement

**Michał P. Heller**
Perimeter Institute for Theoretical Physics, Canada

based on **1509.00113**
with J. de Boer, R. Myers and Yasha Neiman

# Introduction

# Differential entropy and hole-ography

Balasubramanian, de Boer, Chowdhury, Czech & Heller 1310.4204

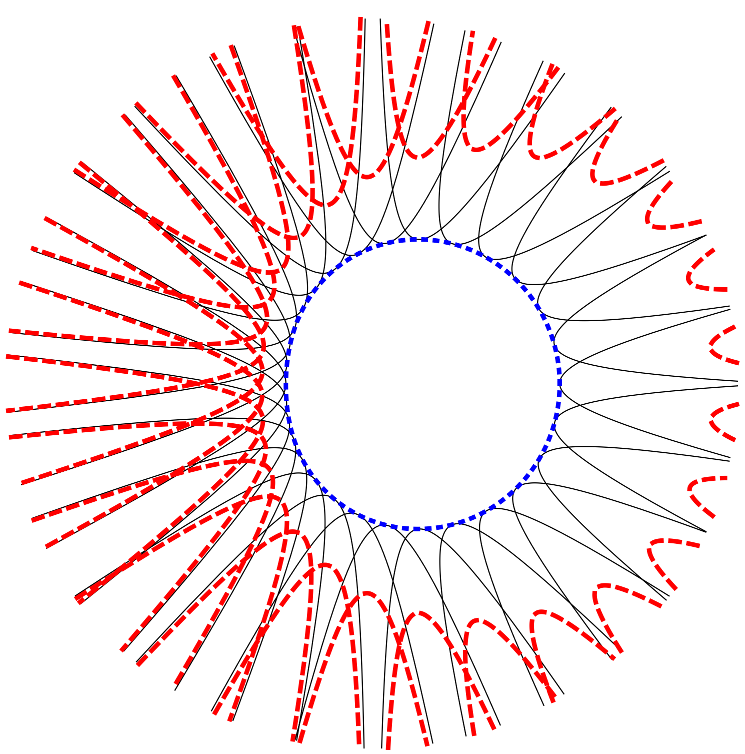
Consider a $CFT_2$ in the vacuum (say on $S^1$ of length $L$) and entanglement entropy of an interval of an angular opening $2\alpha$

$$S(\alpha) = \frac{c}{3}\log(\frac{2L}{\mu}\sin\alpha)$$

Consider now an infinite family of intervals given by $\alpha(\theta)$ and compute

$$E = \int_0^{2\pi} d\theta\, S'(\alpha(\theta))$$

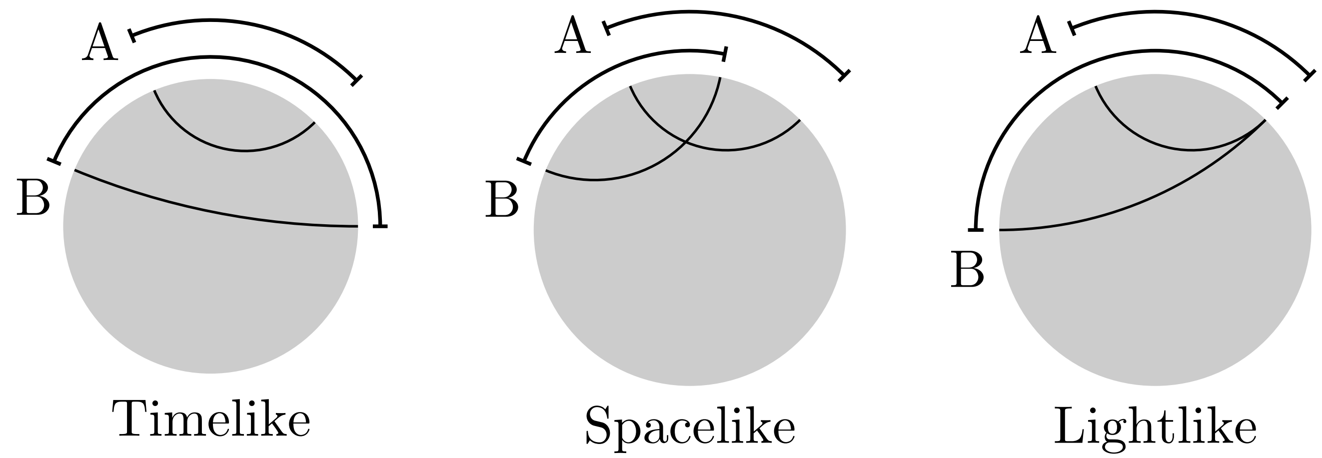
This novel UV-finite quantity is called differential entropy.



In a holographic $CFT_2$ it measures areas of closed curves on a constant-t slice of $AdS_3$

# Kinematic space Czech, Lamprou, McCandlish & Sully 1505.05515

One can introduce a partial order on the set of intervals for which we calculate EE




This (partially) motivates introducing the light-cone coordinates $u = \theta - \alpha$ and $v = \theta + \alpha$ and considering space with the volume form $\omega = \partial_u \partial_v S \, du \wedge dv$

SSA turns out to guarantee $\partial_u \partial_v S \geq 0$. For the vacuum we obtain

$$\omega = \frac{c}{12 \sin^2(\alpha)} du \wedge dv$$

The minimal metric compatible with this volume form and the partial order is

$$ds^2 = \frac{c}{12 \sin^2(\alpha)} du \, dv = \frac{c}{12 \sin^2(\alpha)} \left(-d\alpha^2 + d\theta^2\right)$$ ⟵ de Sitter$_2$ !

# Question behind this work

Is there more to this Lorentzian structure
than merely ordering intervals on a constant time slice of a $CFT_2$?

# Dynamics in de Sitter

# Entanglement first law

Consider small perturbations of some reference density matrix $\rho = \rho_0 + \delta\rho$

The change in the entropy is equal to the change in the modular Hamiltonian

$$\delta S = -\mathrm{tr}\,(\rho \log \rho) - S_0 = \delta \langle H_{mod} \rangle$$

In general, $H_{mod} = \log \rho$ is unknown, but for reduced density matrices for spherical entangling surfaces in the $\mathrm{CFT_D}$ vacuum it turns out to be fixed by conformality

$$H_{mod} = c' - 2\pi \int_{|\vec{x}|^2 < \alpha^2} d^{(D-1)}x \, \frac{\alpha^2 - |\vec{x}|^2}{2\alpha} \, T_{\tau\tau}$$

Casini, Huerta & Myers 1102.0440

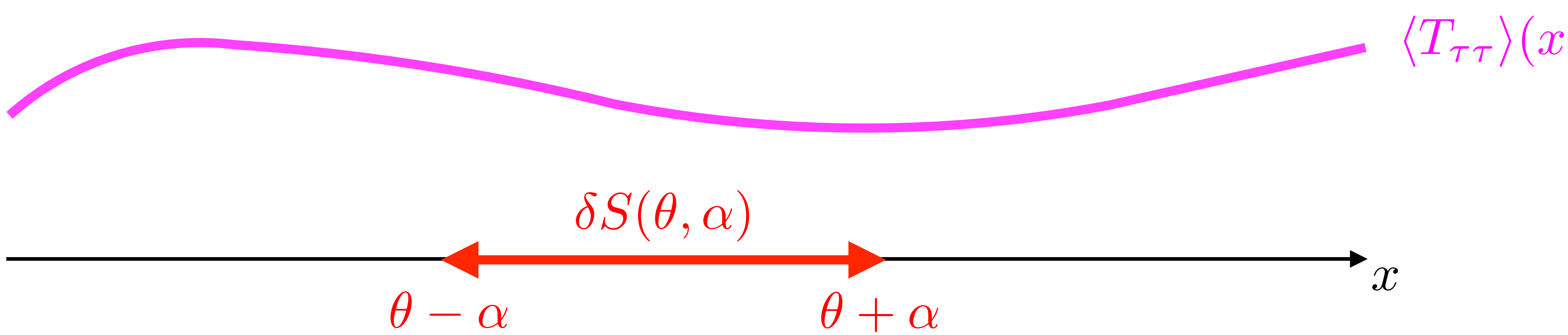
As a result, the change in the entanglement entropy for small perturbations of $|0\rangle$ is

$$\delta S = -2\pi \int_{|\vec{x}|^2 < \alpha^2} d^{(D-1)}x \, \frac{\alpha^2 - |\vec{x}|^2}{2\alpha} \, \delta \langle T_{\tau\tau} \rangle$$

# Entanglement first law yields dynamics in de Sitter

In two dimensions (on a plane) we simply have

$$\delta S(\theta,\alpha) = -2\pi \int_{\theta-\alpha}^{\theta+\alpha} dx \, \frac{\alpha^2-(\theta-x)^2}{2\alpha} \langle T_{\tau\tau}\rangle(x) \quad \text{at fixed} \quad \tau$$




It is straightforward to check that

$$\frac{1}{\sqrt{-g}}\partial_a\left(\sqrt{-g}\,g^{ab}\,\partial_b\,\delta S\right) - \frac{2}{L^2}\,\delta S = 0$$

with $g_{ab}\,dx^a dx^b = \frac{L^2}{\alpha^2}\left(-d\alpha^2+d\theta^2\right)$ being the inflationary patch of de Sitter$_2$ !

# Specific example: universal excited state

Any CFT has $T_{\mu\nu}$ operator, hence $T_{\tau\tau}(\tau_0,\theta_0)|0\rangle$ is an universal excitation in CFT$_2$'s

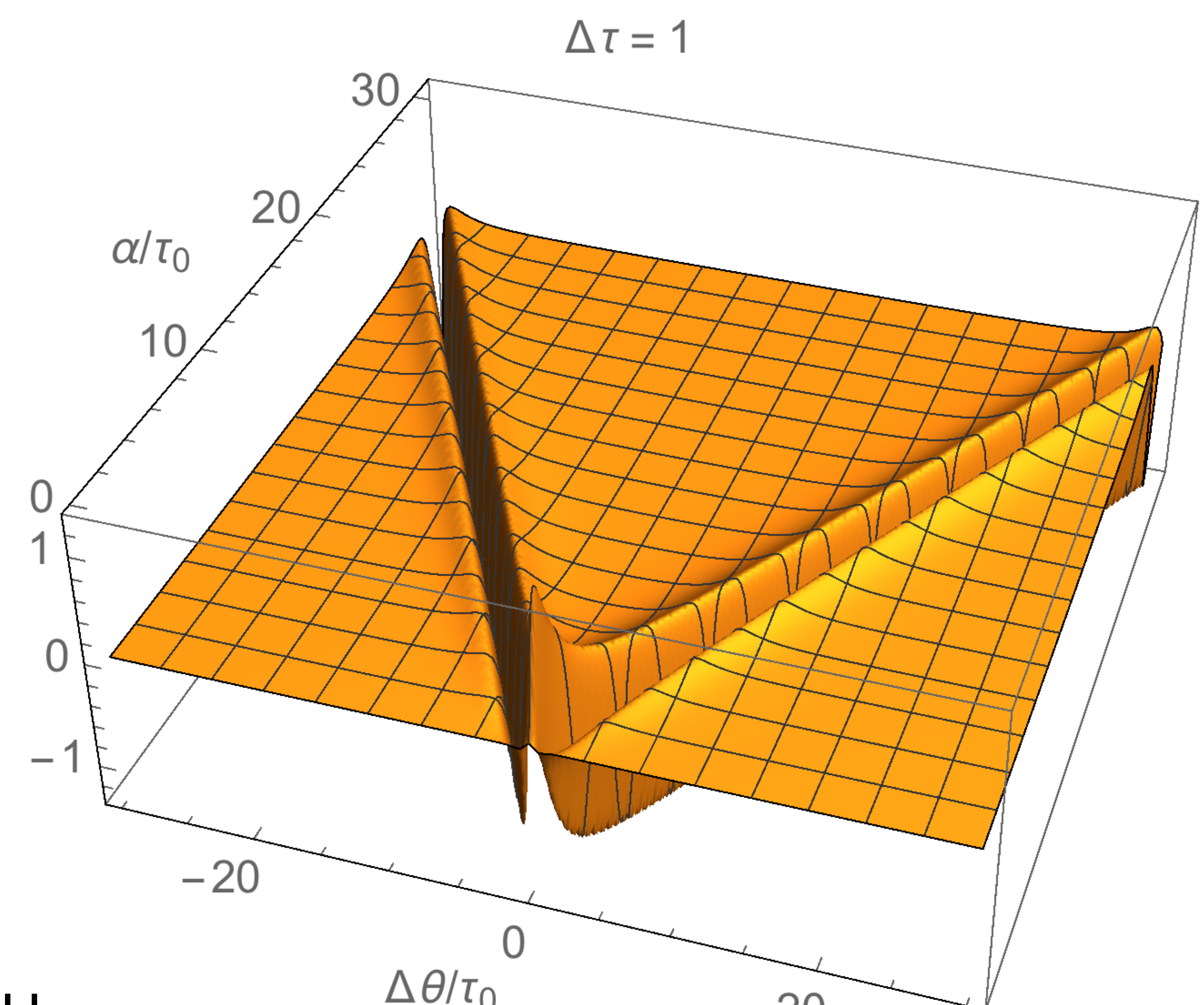
cf. Nozaki, Numasawa & Takayanagi 1401.0539

$$\langle T_{\tau\tau}\rangle = \frac{\langle 0|\, T_{\tau\tau}(\tau_0+\epsilon,\theta_0)\; T_{\tau\tau}(\tau,x)\; T_{\tau\tau}(\tau_0-\epsilon,\theta_0)\,|0\rangle}{\langle 0|\, T_{\tau\tau}(\tau_0+\epsilon,\theta_0)\; T_{\tau\tau}(\tau_0-\epsilon,\theta_0)\,|0\rangle} \approx -\frac{4\epsilon^2}{\pi}\,\frac{\Delta\tau^4 - 6\,\Delta\tau^2\Delta\theta^2 + \Delta\theta^4}{(\Delta\tau^2+\Delta\theta^2)^4}$$

Entanglement first law:

$$\delta S(\theta,\alpha) = -2\pi \int_{\theta-\alpha}^{\theta+\alpha} dx\, \boxed{\frac{\alpha^2-(\theta-x)^2}{2\alpha}}\, \langle T_{\tau\tau}\rangle(x)$$

dS$_2$ analogue of the bulk-boundary smearing function by Kabat et al.



# Generalization to higher spins in CFT$_2$

The exploratory idea of Hijano & Kraus 1406.1804 is the following:

$$\delta H_{mod} = -\int_{z_1}^{z_2} dz \left( \frac{(z - z_2)(z - z_1)}{z_2 - z_1} \right) T(z) + \text{antiholomorphic}$$

so in the presence of a higher spin current (here spin-3 current $W$) one can try

$$\delta H_{mod,3} = -3 \int_{z_1}^{z_2} dz \left( \frac{(z - z_2)(z - z_1)}{z_2 - z_1} \right)^2 W(z) + \text{antiholomorphic}$$

Any quantity behaving as $\sim \int_{\theta-\alpha}^{\theta+\alpha} dx \left( \frac{\alpha^2 - (\theta - x)^2}{2\alpha} \right)^\gamma f(x)$ leads to a dynamical fields in de Sitter with

$$m^2 L^2 = -\gamma - \gamma^2$$

perturbation in entanglement entropy

$$m^2 L^2 = -2$$

Hijano & Kraus construction

$$m^2 L^2 = -6$$

# Generalization to higher dimensions

The first law for any CFT in D spacetime dimensions

$$\delta S = -2\pi \int_{|\vec{x}|^2<\alpha^2} d^{(D-1)}x \, \frac{\alpha^2 - |\vec{x}|^2}{2\alpha} \, \delta\langle T_{\tau\tau}\rangle$$

implies propagation in D-dimensional de Sitter

$$\frac{1}{\sqrt{-g}}\partial_a \left(\sqrt{-g}\, g^{ab}\, \partial_b\, \delta S\right) - \frac{D}{L^2}\, \delta S = 0$$

with $g_{ab}\, dx^a dx^b = \frac{L^2}{\alpha^2}(-d\alpha^2 + d\vec{x}^2)$ being the inflationary patch of de Sitter$_D$ .

Note that free field theories have $\infty$ many higher spin currents.

see, e.g., Maldacena & Zhiboedov 1112.1016

By extrapolating 1406.1804 we might expect then $\infty$ many fields in de Sitter$_D$

# Relation to Einstein's equations in AdS

In 2013 several groups were able to derive linearized Einstein's equations from the Ryu-Takayanagi proposal and the first law of entanglement entropy.

In 1304.7100 Takayanagi et al. showed that linearized Einstein's equations in AdS$_3$ give

$$(\partial_t^2 - \partial_\xi^2)\Delta S_A(\xi, l, t) = 0,$$
$$\left[\partial_l^2 - \frac{1}{4}\partial_t^2 - \frac{2}{l^2}\right]\Delta S_A(\xi, l, t) = 0$$

Linear combination gives our wave equation on dS$_2$: $\left[-\partial_{l/2}^2 + \partial_\xi^2 - \frac{-2}{(l/2)^2}\right]\Delta S_A(\xi, l, t) = 0.$

In 1308.3792 Bhattacharya and Takayanagi obtained from Einstein's equations in AdS$_4$

$$\left[\frac{\partial^2}{\partial l^2} - \frac{1}{l}\frac{\partial}{\partial l}\boxed{-\frac{3}{l^2}} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2}\right]\Delta S_A(t, x, y, l) = 0.$$

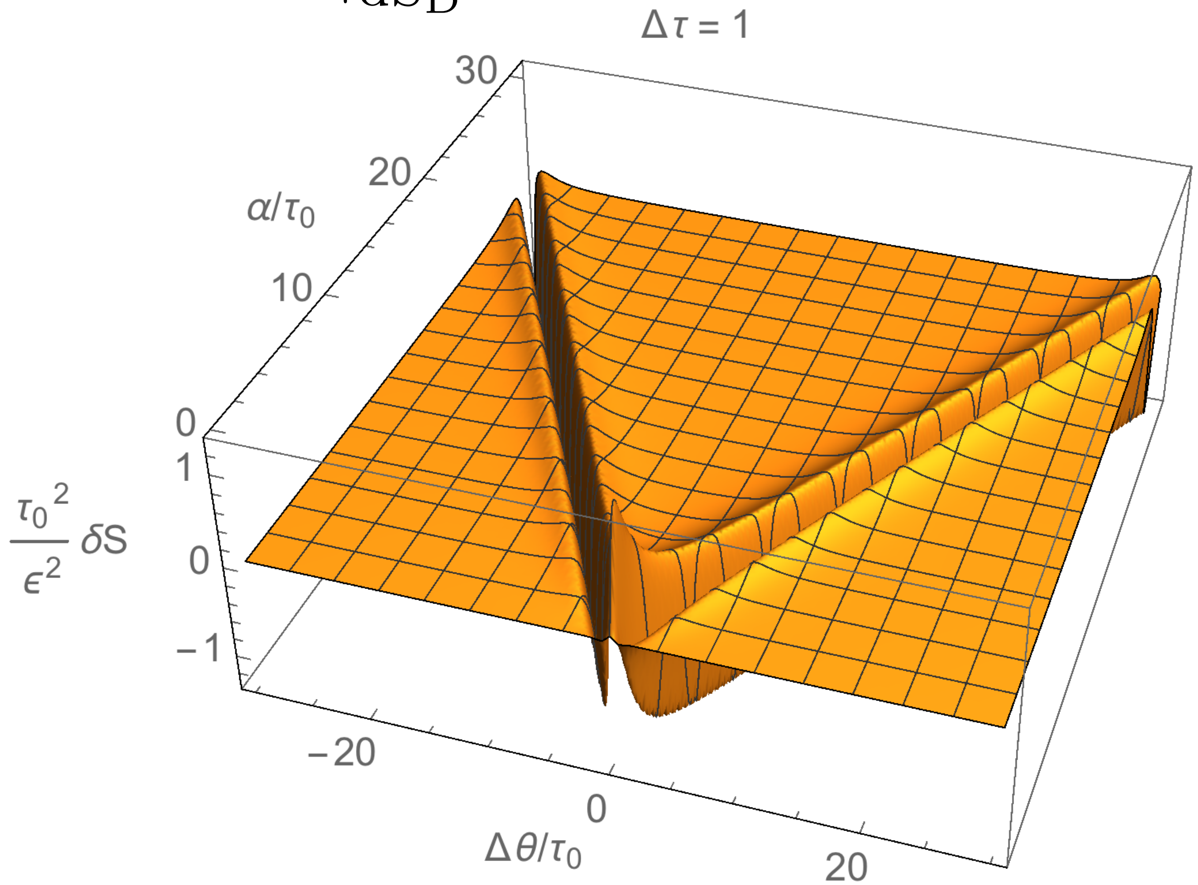
We see here that this equation in any D has nothing to do with a 2-derivative geometry and follows purely from the underlying conformal symmetry.

# Summary and open problems

# Summary ongoing work with Rob Myers and Guifre Vidal

Entanglement in excited states organizes itself in a Lorentzian way:

$$\nabla_a \nabla^a \big|_{\mathrm{dS_D}} \delta S - m^2 \delta S = 0 \qquad \text{with} \qquad m^2 L^2_{dS_D} = -D, \qquad \text{e.g.}$$




for in $T_{\tau\tau}(\tau_0, 0)\,|0\rangle$ ECFT$_2$

and $ds^2_{dS_2} = \frac{L^2}{\alpha^2}(-d\alpha^2 + d\theta^2)$.

This statement applies to <u>any CFT</u> in <u>any D</u> provided the first law holds!

The statement concerns constant time slices in a CFT.

For theories with higher spin charges: one dynamical field in dS$_D$ for each charge.

# Some open problems

Is recent explosion of emergent "geometries" an artifact of considering CFTs ?

Does auxiliary de Sitter = kinematic space ?

Higher dimensional generalization of differential entropy and hole-ography ?

Other fields in de Sitter, in particular gravitons? Nonlinear eoms? New holography ?

Local operators in de Sitter (generalization of Kabat et al.) ?

see Xiao Xiao 1402.7080

Link with MERA / cMERA ?