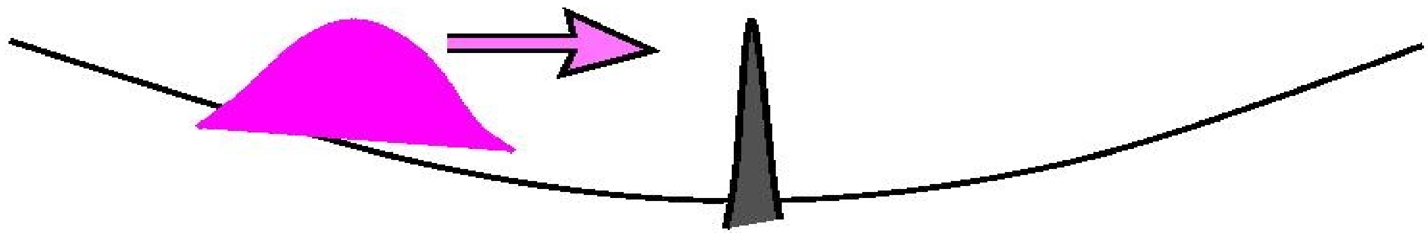

Dipole modes in a split trap

Marco Cominotti, Frank Hekking, Anna Minguzzi

LPMMC Theory and Modeling of Condensed Systems

Grenoble

LPMC laboratoire de physique et de modélisation des milieux condensés

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Plan

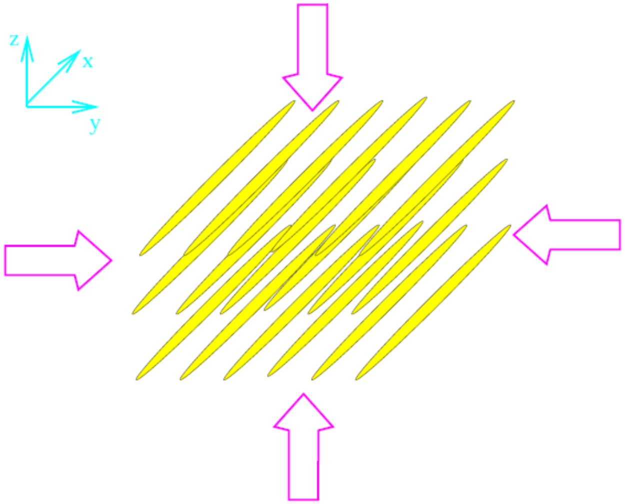
- *1D gases with a barrier* : effect of interactions and quantum fluctuations
- *Protocol : dipole mode quench to study transport across the barrier*
  - Exact solution at infinite interactions
  - Numerical Diagonalization
  - Gross-Pitaevskii solution
  - inhomogeneous Luttinger Liquid theory
- *Main result and some 'what if...'*

→ Two nontrivial effects of interactions ←

1D quantum gases

- Cylindrical geometry
- Very large transverse confinement

Realizations : 2D optical lattices, chip traps,...



- All energy scales smaller than transverse energy

$$\mu, k_B T \ll \hbar\omega_\perp$$

- Important quantum fluctuations at intermediate - large interactions

Interactions in 1D

- Interactions due to atom-atom collisions in a waveguide

  (3D s-wave scattering length) →

- Effective 1D interactions

$$v(x) = g\delta(x)$$

$$g = 2a_s\hbar\omega_\perp(1 - 0.4602\, a_s/a_\perp)^{-1}$$

- Hamiltonian (Lieb-Liniger)

$$\mathcal{H} = \sum_i -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_i^2} + V(x_i) + g\sum_{i<j}\delta(x_i - x_j)$$

Dimensionless interaction strength :

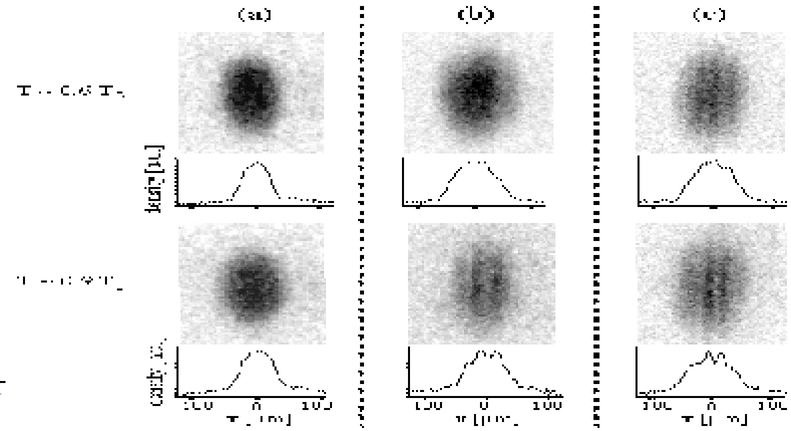
$$\gamma = gn/(\hbar^2 n^2/m)$$

interaction energy/kinetic energy

Important quantum fluctuations

- No Bose-Einstein condensation in uniform 1D system – phase fluctuations increase at increasing interactions

$$\rho_1(x,x') = \langle \Psi^\dagger(x)\Psi(x')\rangle \to \frac{1}{|x-x'|^{1/2K}}$$

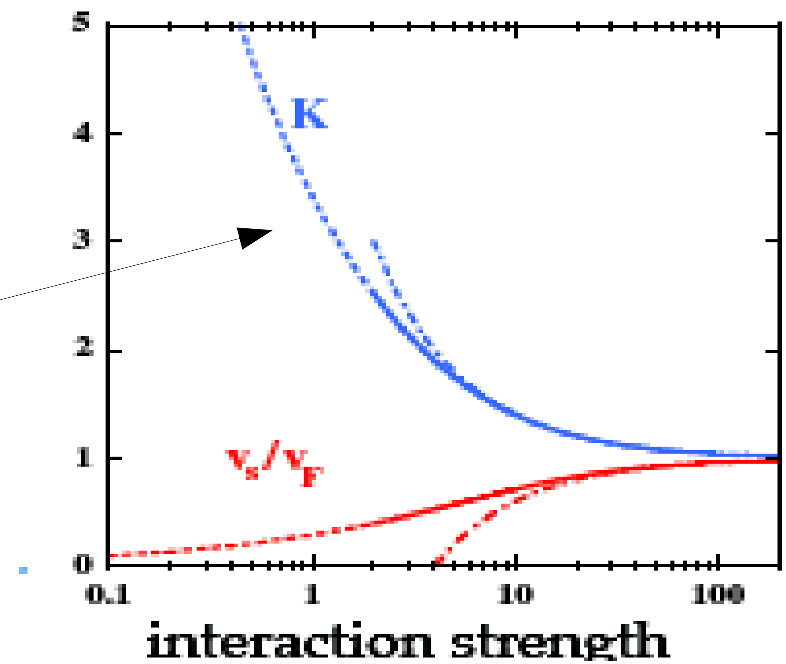


Hannover, 2001

- Duality : density fluctuations decrease at increasing interactions

$$\langle \rho(x)\rho(0)\rangle \sim |x|^{-2K}$$

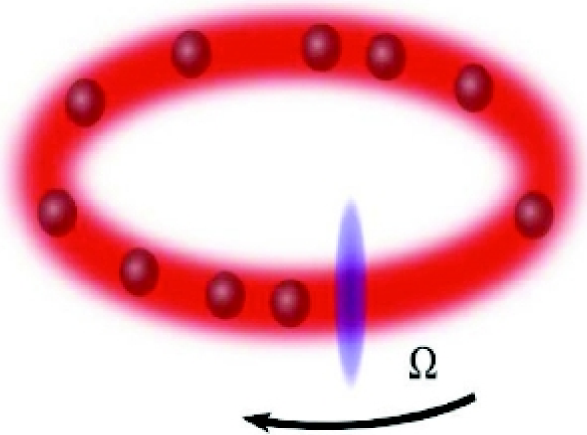
K = Luttinger parameter,
depending on interaction strength

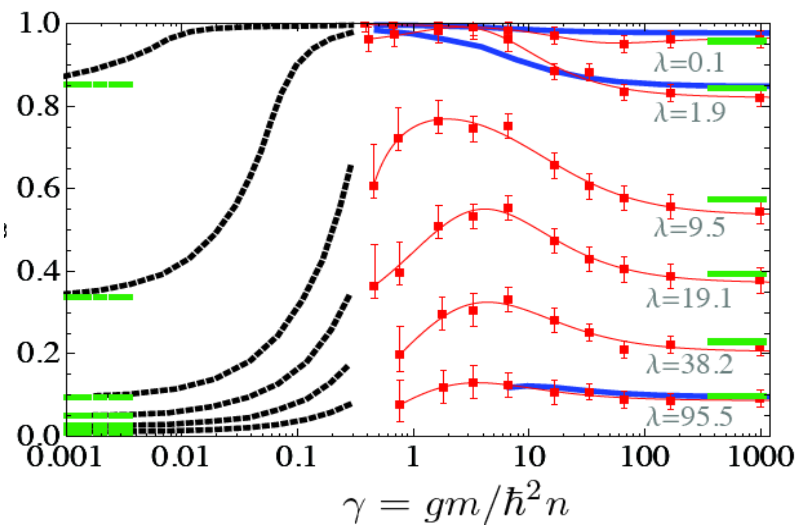


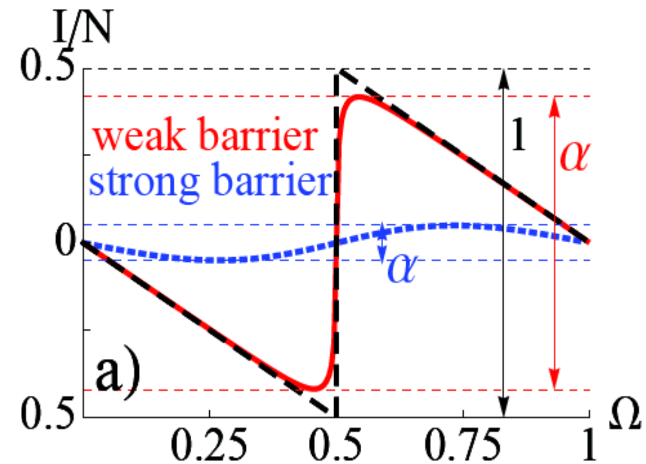
M. Cazalilla, JPhysB 2001

1D interacting gas *with a barrier*

- Our work on persistent currents : nonmonotonous behaviour of the current amplitude *vs*. interaction strength







Cominotti et al PRL 113, 025301 (2014)

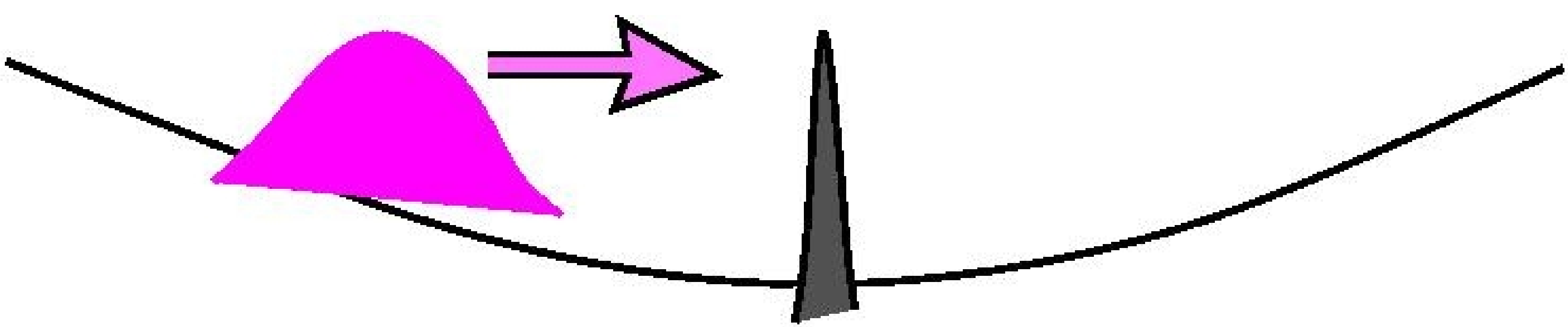
competition between barrier screening and renormalization due to quantum fluctuations

Barrier renormalization in transport of quantum gases ?

Barrier screening / renormalization also visible in other physical observables ?

→ **eg transport / dynamical phenomena ?**
→ eg analog of Hawking radiation emission ?
→ ….

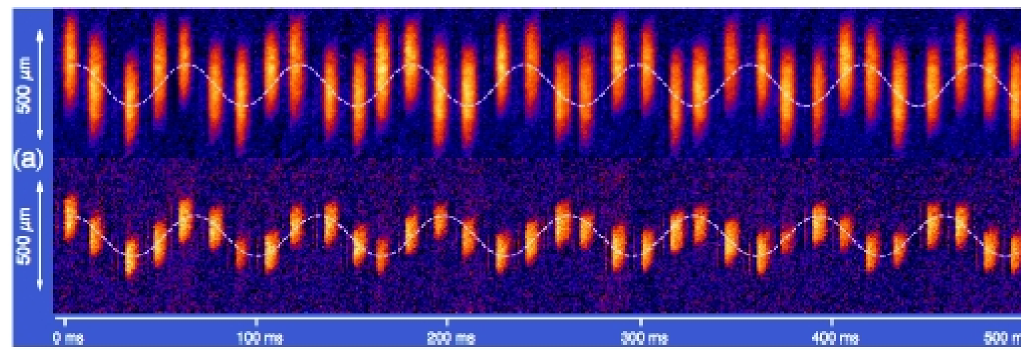
With ultracold gases : induce transport in a confined geometry : *sloshing dipole mode*



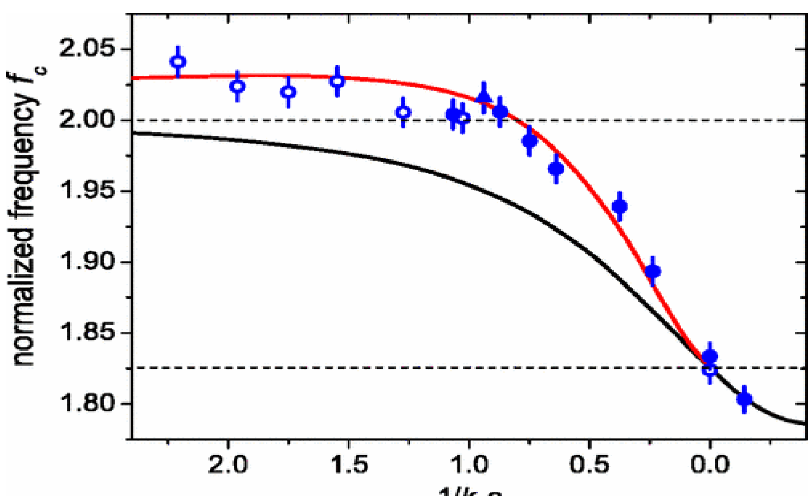
Collective excitations in quantum gases : high-precision tool

Collective-modes frequencies are measured with high precision → information on

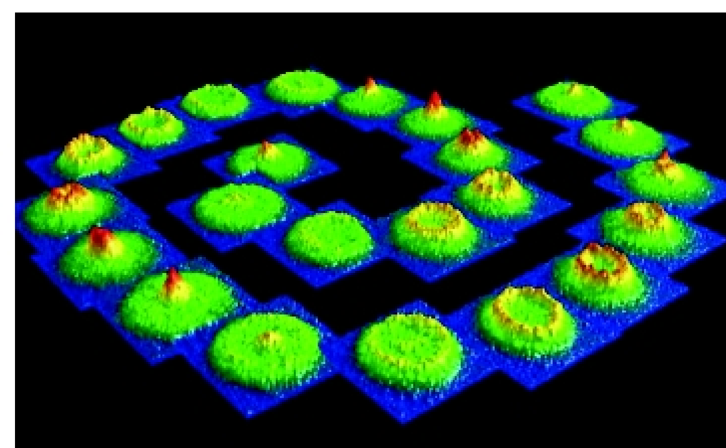
– equation of state
– superfluidity, vortices
– scale invariance
– beyond mean-field effects



Ferrier-Barbut et al Science 345, 1035 (2014)

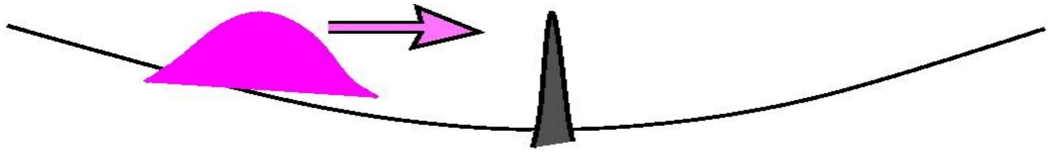


Altmeyer et al PRL 98, 040401 (2007)

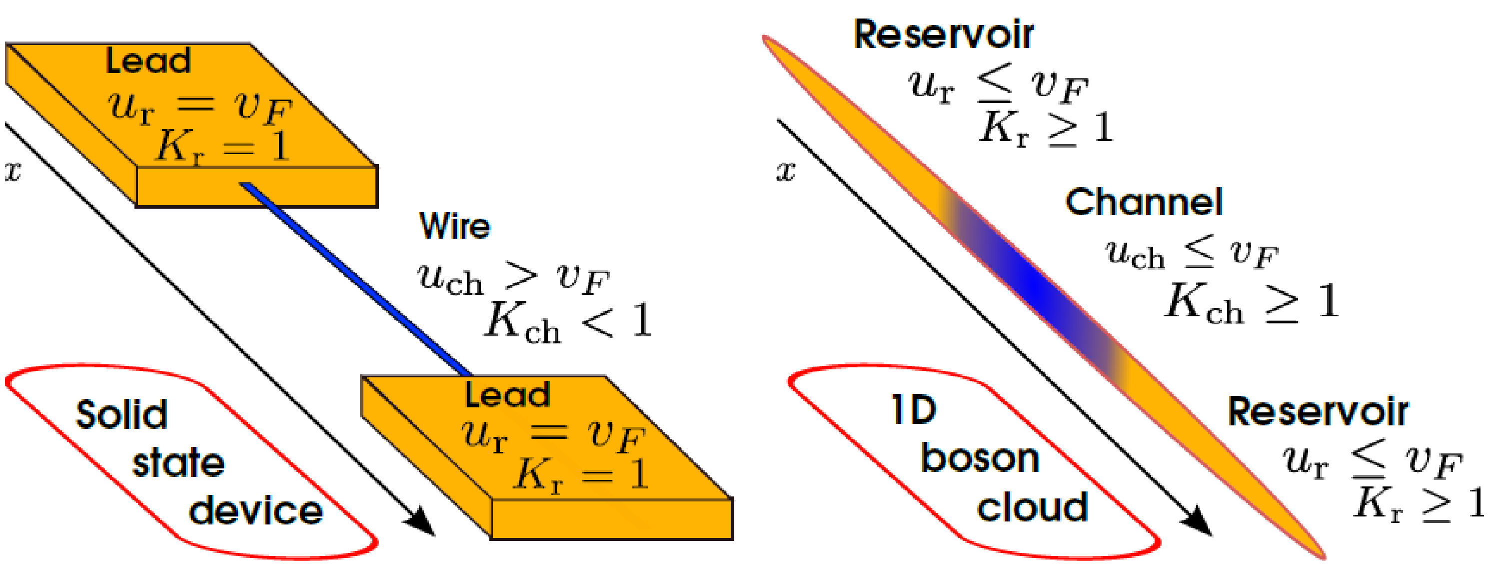


Chevy et al PRL 88, 250402 (2002)

Barrier renormalization in transport of quantum gases?



… Dipole mode in a split trap : a setup complementary to the 2-reservoir one mimicking mesoscopic physics devices



Filippone, Hekking, Minguzzi, arXiv:1410.5841

Kohn's theorem

- In a purely harmonic trap $V(x) = \frac{1}{2} m\omega_0^2 x^2$ the dipole sloshing mode has frequency ω_0
- holds for arbitrary interactions
- not a compressional mode :

$$n(x,t) = n_0(x - x_0(t))$$

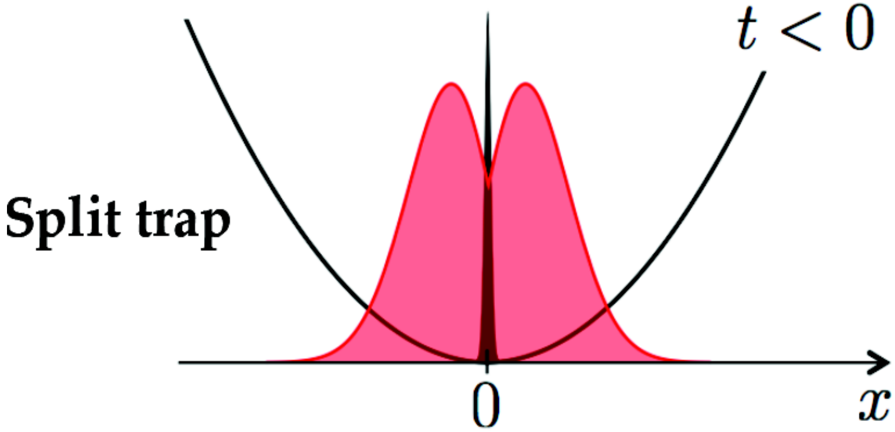
- simmetry preperty, consequence of the harmonic- trap geometry :

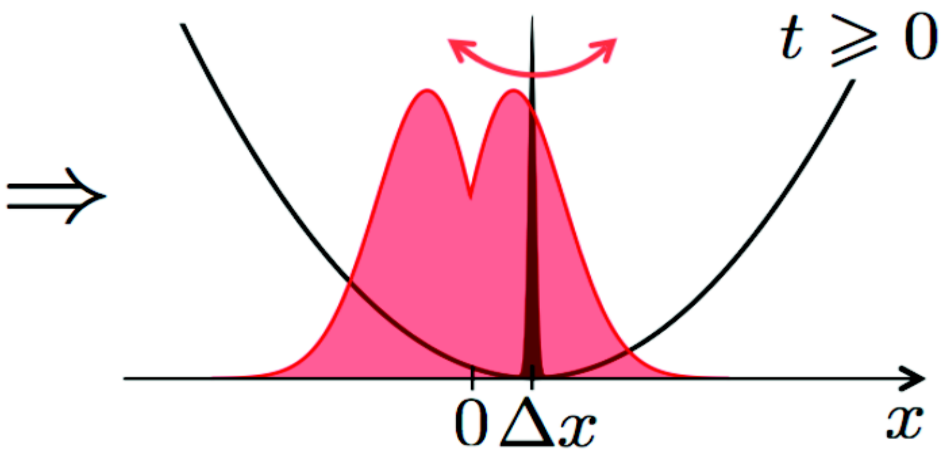
  equivalent to looking at the system from an oscillating accelerated frame

Minguzzi PRA 64, 033604 (2001)

The system and protocol

t<0 : 1D interacting bosons at equilibrium in a *split trap* (harmonic trap + thin barrier)





t=0 : sudden shift of the split-trap position

t>0 : time evolution ?

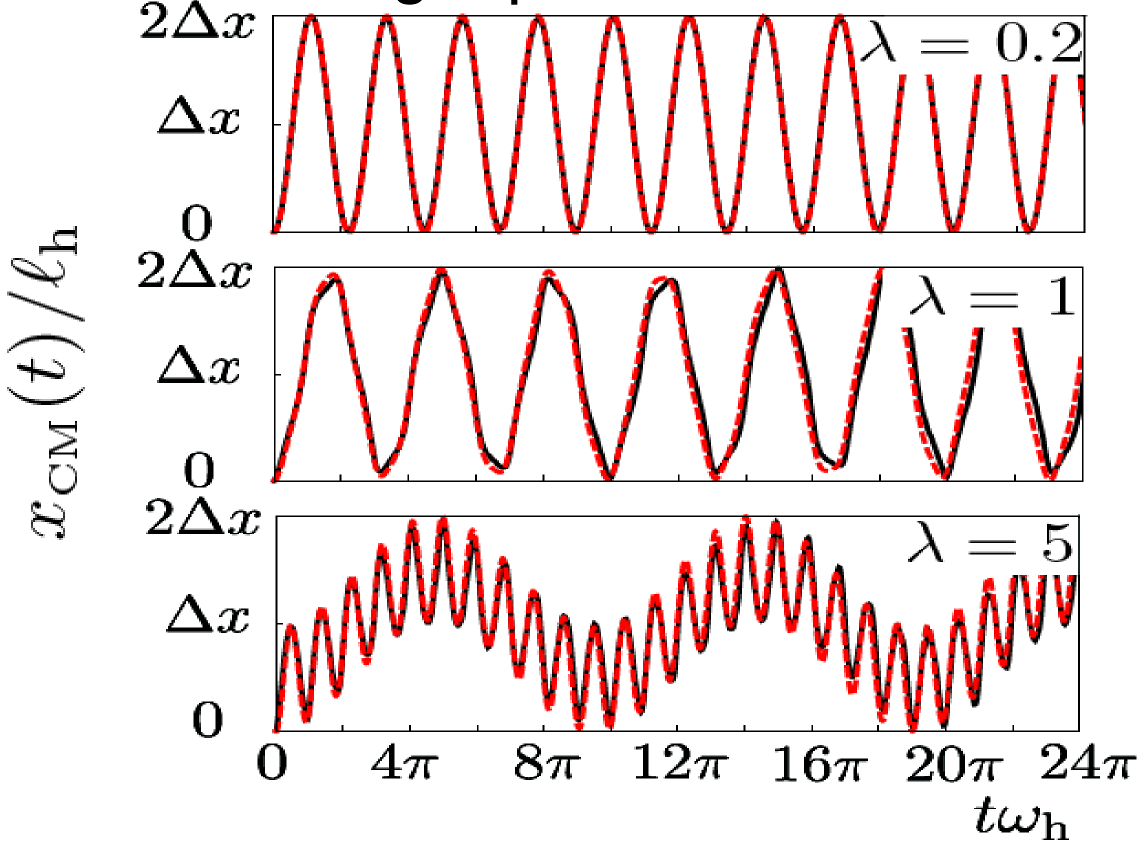
Quench dynamics

for small trap displacement

Follow the center-of mass position $x_{\rm CM}(t) = \int {\rm d}x\, x |n(x,t)|^2$

with $n(x,t) = N \int dx_2 ... dx_N |\Psi(x_1, x_2, ...x_N, t)|^2$

Single-particle solution



At increasing barrier strength :

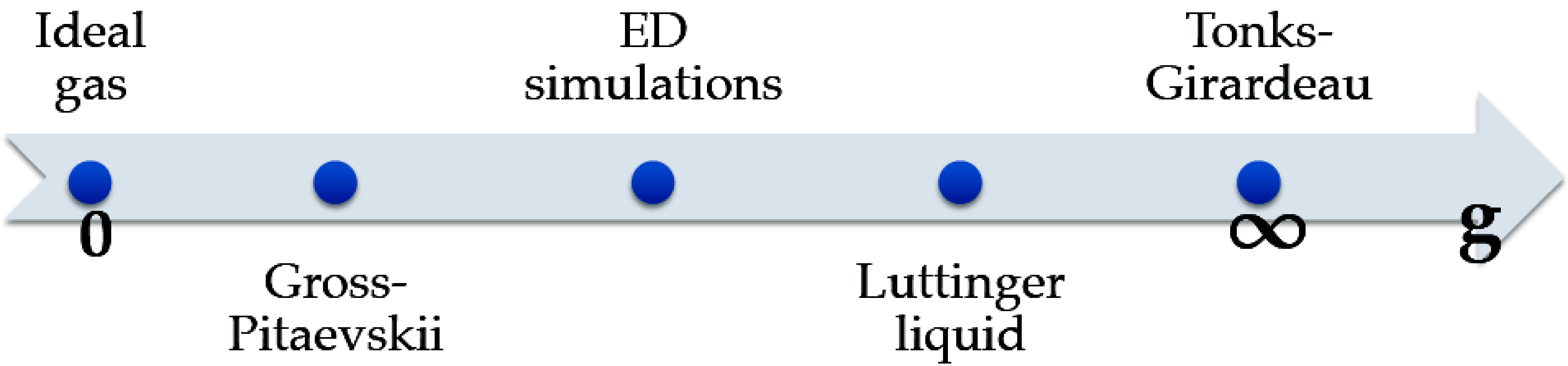
→ additional harmonics

→ frequency shift of the dipole mode – violation of the Kohn's theorem due to the presence of the barrier

Interaction regimes

1D bosons at zero temperature

To treat arbitrary interactions, we emply a combination of techniques:



** benchmark of inhomogeneous Luttinger liquid theory with exact results and numerical diagonalisation

Tonks-Girardeau regime

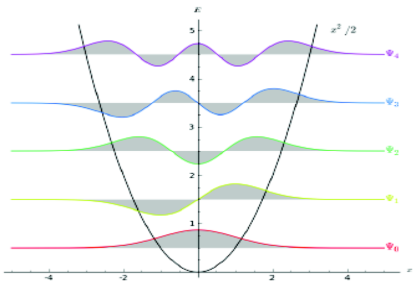
Time-dependent Bose-Fermi mapping *[Girardeau,Wright, PRL (2000)]:*

$$\Psi_B(x_1, ...x_N, t) = \Pi_{1\leq j\leq \ell}\mathrm{sgn}(x_j - x_\ell)\Psi_F(x_1, ...x_N, t)$$

→ the cusp condition is preserved in the dynamics

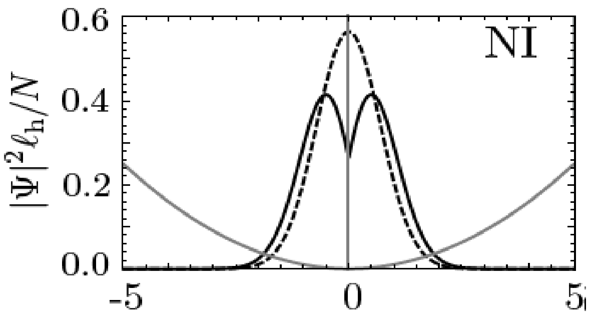
Note : exact solution of the quench dynamics for arbitrary
– barrier strength
– time evolution
– trap shift

Needs the solution of the time-dependent single-particle problem



Single-particle solution in a split trap

$$\left[-\frac{\hbar^2}{2m}\partial_x^2 + U_0\delta(x) + \frac{1}{2}m\omega_{\rm h}^2 x^2\right]\psi_n = \varepsilon_n\psi_n$$



- Initial equilibrium state: analytical solution
  [Busch et al, J. Phys. B 36, 2553 (2003)]

Cusp condition : $\partial_x\psi_n(0^+) - \partial_x\psi_n(0^-) = \lambda\psi_n(0)$

Wavefunctions (confluent hypergeometric functions) :

$$\psi_n(x) = \cos\left(\frac{\pi}{4} - \frac{\pi E_n}{2}\right)Y_1 - \sin\left(\frac{\pi}{4} - \frac{\pi E_n}{2}\right)Y_2$$

$$Y_1 = \frac{\Gamma(\frac{1}{4}+\frac{E_n}{2})}{\sqrt{\pi}2^{(\frac{1}{4}-\frac{E_n}{2})}}e^{-\frac{x^2}{2}}M\left(\frac{1}{4}-\frac{E_n}{2},\frac{1}{2},x^2\right)$$

$$Y_2 = \frac{\Gamma(\frac{3}{4}+\frac{E_n}{2})}{\sqrt{\pi}2^{(-\frac{1}{4}-\frac{E_n}{2})}}e^{-\frac{x^2}{2}}\sqrt{2}|x|M\left(\frac{3}{4}-\frac{E_n}{2},\frac{3}{2},x^2\right)$$

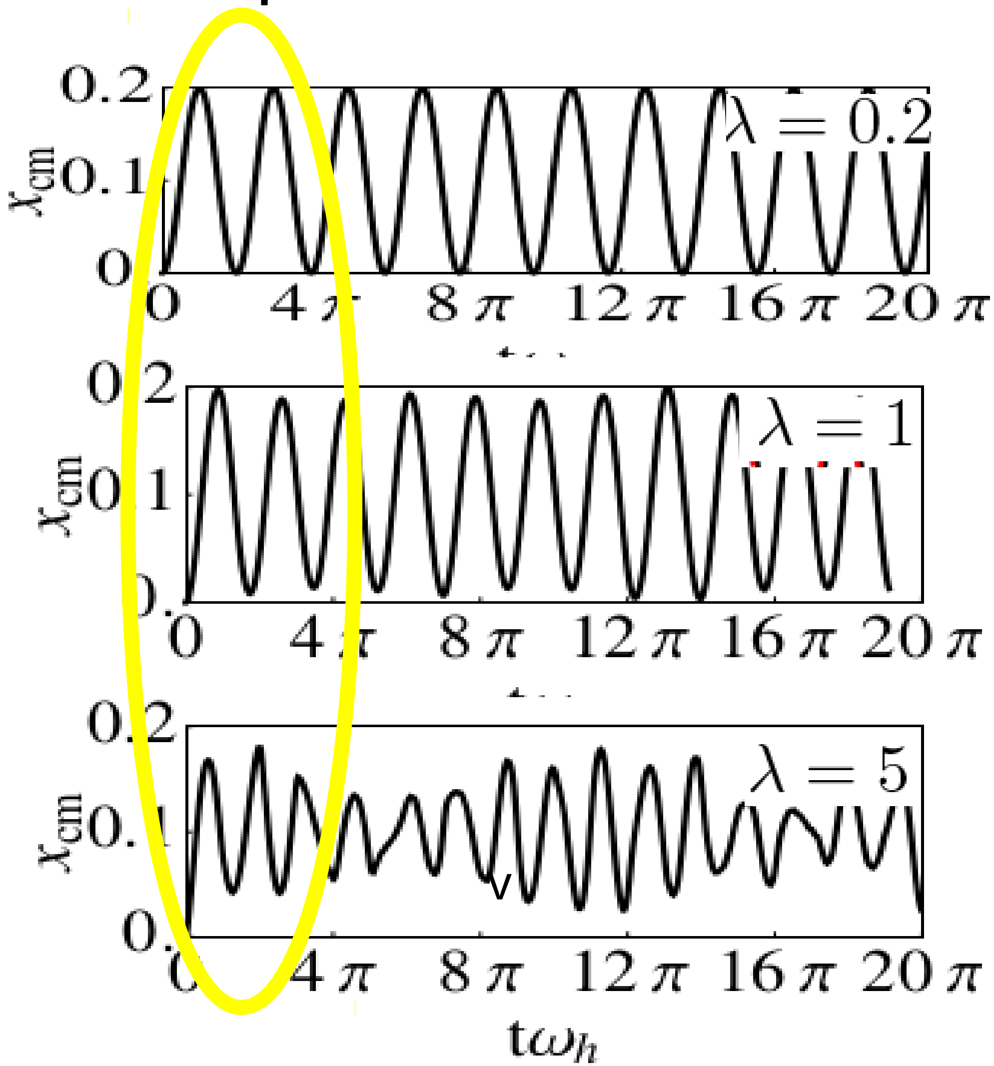
Eigenvalues:

$$\frac{\Gamma\left(\frac{3}{4}-\frac{E_n}{2}\right)}{\Gamma\left(\frac{1}{4}-\frac{E_n}{2}\right)} = -\lambda/2$$

- Real-time evolution by numerical integration of the Schroedinger equation

Center-of-mass evolution of a Tonks-Girardeau gas

Some results for the full time evolution for small displacement:



N=8

the dipolar frequency increases at increasing the barrier strength

focus on $\lambda = 1$

At arbitrary interactions
for small trap displacement

Focus on dipole-mode frequency
– use perturbation theory

$$|\Psi_0^{t\geqslant 0}(t)\rangle = \exp(-i\mathcal{H}^{t\geqslant 0}t/\hbar)|\Psi_0^{t<0}\rangle$$

$$\mathcal{H}^{t<0} \simeq \mathcal{H}^{t\geqslant 0} + \Delta x \partial_x V_{\rm ext}^{t\geqslant 0}$$

$$|\Psi_0^{t<0}\rangle = |\Psi_0^{t\geqslant 0}\rangle + \Delta x \sum_{k=0}^{\infty} \frac{\langle \Psi_k^{t\geqslant 0}|\partial_x V_{\rm ext}^{t\geqslant 0}|\Psi_0^{t\geqslant 0}\rangle}{(E_0^{t\geqslant 0} - E_k^{t\geqslant 0})}|\Psi_k^{t\geqslant 0}\rangle$$

$$\Rightarrow \quad \omega_{\rm d} = (E_1^{t\geqslant 0} - E_0^{t\geqslant 0})/\hbar$$

At arbitrary interactions : ground and first-excited state from exact diagonalization

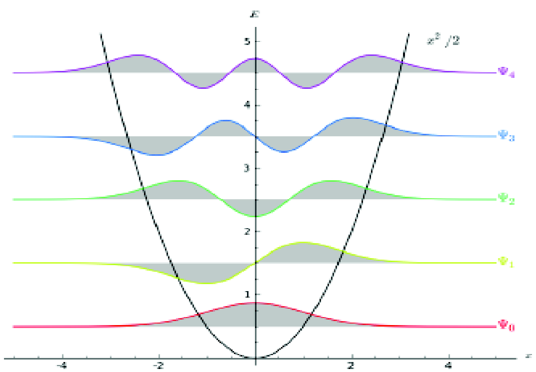
Tonks-Girardeau limit : an easier route than the real-time dynamics....

Tonks-Girardeau regime :
...prediction of parity effect

Dipole-mode frequency for a weak barrier $U_0\delta(x)$:

$$\hbar\omega_{\rm d} = E_1^{\rm TG} - E_0^{\rm TG} = \hbar\omega_{\rm h} + \langle\Psi_1^{\rm TG}|\mathcal{H}_b|\Psi_1^{\rm TG}\rangle - \langle\Psi_0^{\rm TG}|\mathcal{H}_b|\Psi_0^{\rm TG}\rangle$$

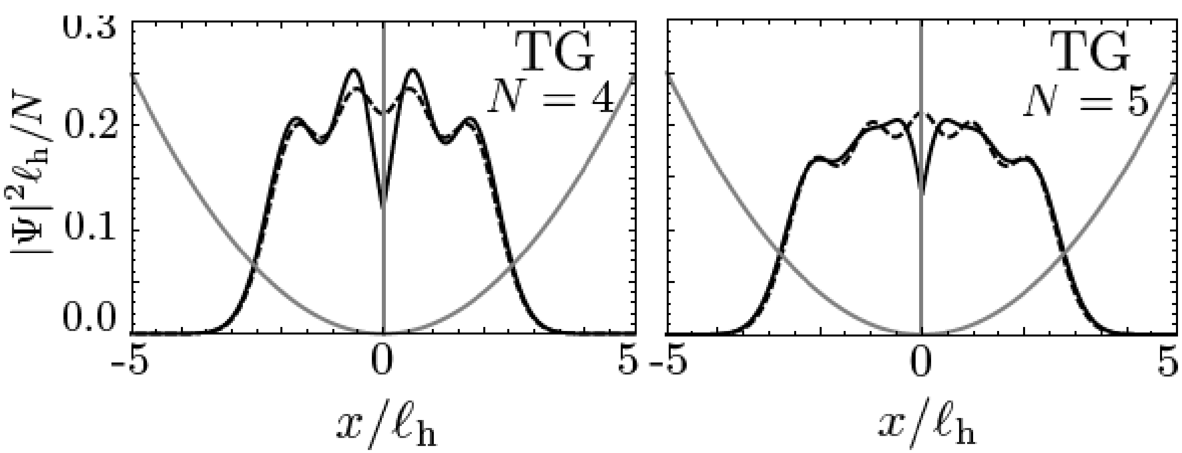
$$\Rightarrow \hbar\omega_d = \hbar\omega_{\rm h} + U_0(|\psi_{N+1}(0)|^2 - |\psi_N(0)|^2)$$



for HO confinement, one of the two last orbital vanishes in x=0 → parity effect :

– for N *odd :* $\omega_d < \omega_h$

– for N *even :* $\omega_d > \omega_h$



Transport occurs at the Fermi surface for a strongly correlated Bose gas !

Exact diagonalization

Determine to high accuracy the ground- and first- excited state of the many-body Hamiltonian

$$\mathcal{H} = \sum_{j=1}^{N} -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial^2 x_j} + U_0\delta(x_j) + \frac{1}{2}m\omega_{\mathrm{h}}^2 x_j^2 + \frac{g}{2}\sum_{j,l=1}^{N}\delta(x_l - x_j)$$

- represented on the basis of the single particle problem

- truncated Hilbert space :

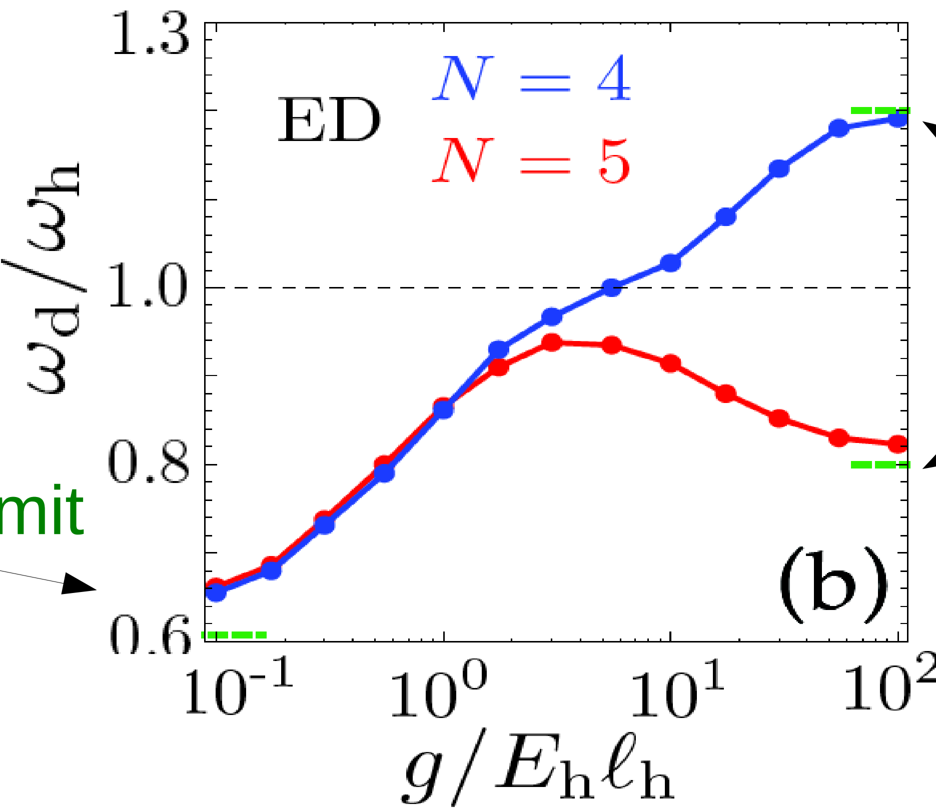
N # particles
S # states $\Longrightarrow$ Size Hilbert space $\binom{S+N-1}{N}$

Number of particles limited to N< 10

$\Longrightarrow$ Use Luttinger Liquid theory for larger N

Dipole mode frequency vs interaction strength

Exact diagonalization results for N=4 and 5



Barrier strength $\lambda = 1$

Tonks-Girardeau exact result

Ideal gas limit

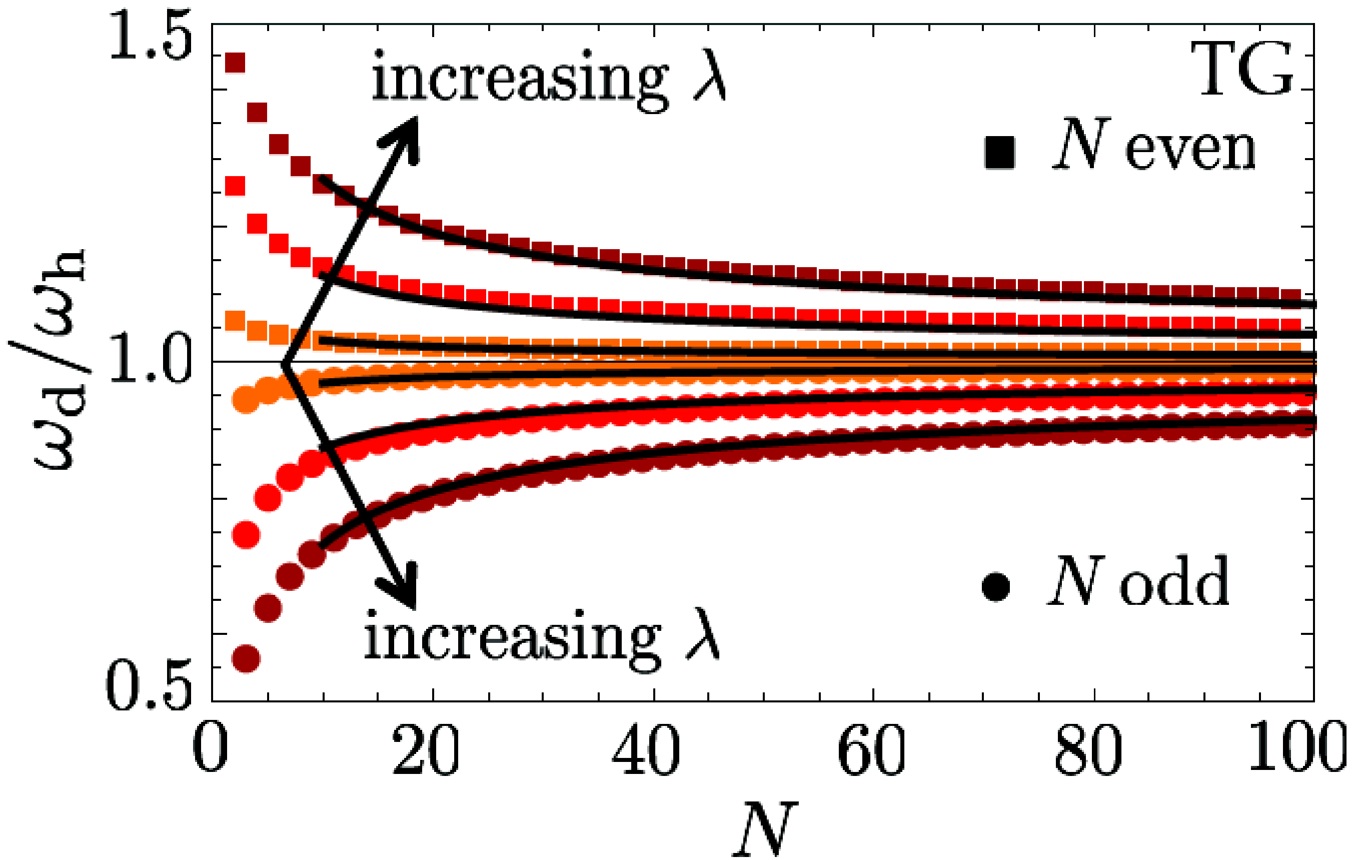
Cominotti, Hekking, Minguzzi, arXiv:1503.07776

- Parity effect at large interactions : a correlation effect
- Nontrivial frequency shift with interaction strength ...
  → barrier screening and renormalization

Scaling of parity gap with size

- The parity effect vanishes in the thermodynamic limit
- From the Tonks-Girardeau solution :

$$|\omega_\mathrm{d} - \omega_\mathrm{h}| \propto 1/\sqrt{N}$$



Effective barrier : similar results at larger N with a larger barrier

Larger particle numbers : Gross-Pitaevskii equation

- Neglect quantum fluctuations – *in 1D always an approximation*

$$\left[-\frac{\hbar^2}{2m}\partial_x^2 + \lambda\delta(x) + \frac{1}{2}m\omega_{\mathrm{h}}^2 x^2 + gN|\Phi|^2\right]\Phi = \mu\Phi$$

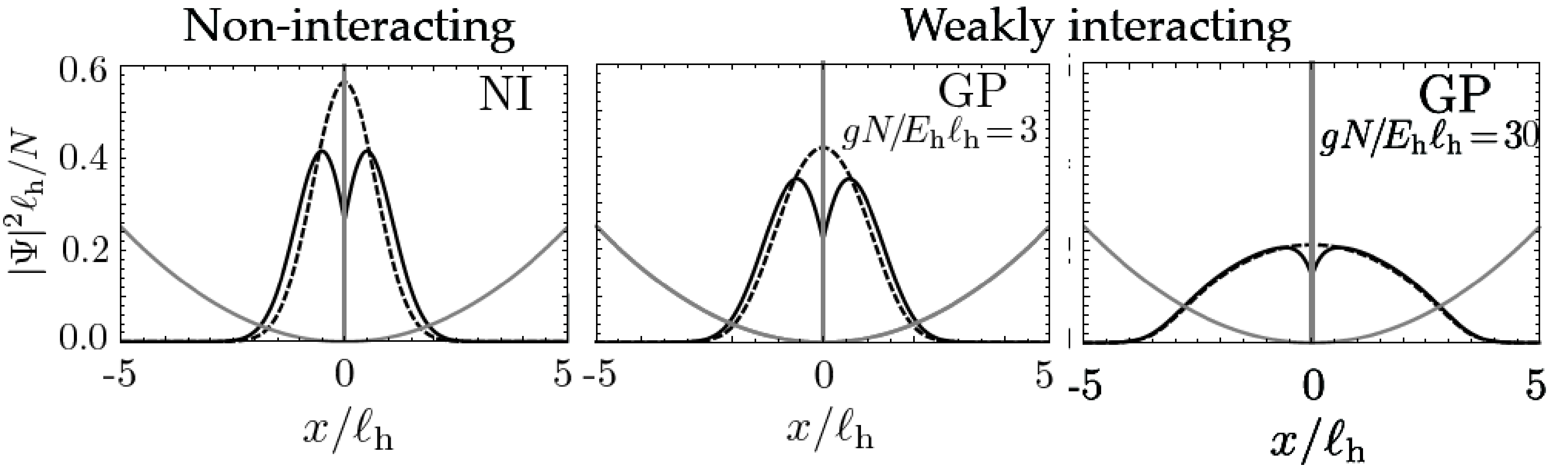
- Initial state : numerical evolution in imaginary times under the pre-quench Hamiltonian

- Dynamics : numerical evolution in real times under the post-quench Hamiltonian

$$x_{\mathrm{CM}}(t) = \int \mathrm{dx}|\Phi^{\mathrm{t}\geqslant 0}(\mathrm{t},\mathrm{x})|^2\mathrm{x}$$

Gross-Pitaevskii

- Equilibrium results at increasing interaction strength



Classical screening of the barrier : the effective barrier seen by the fluid is smaller at increasing interactions

healing length $\xi = \hbar/\sqrt{2mng}$

Luttinger-liquid theory

- Effective low-energy theory – quantum hydrodynamics
  → bosonic field operator

$$\psi(x) = \sqrt{\rho(x)}e^{i\phi(x)}$$

- Fields for phase $\phi(x)$ and density $\theta(x)$ fluctuations

- Non-linear dependence of the fields in the density operator

$$\rho(x) = [n(x) + \partial_x\theta(x)/\pi] \sum_{l=-\infty}^{+\infty} e^{2il\theta(x)+2il\pi\int_{-\infty}^{x} \mathrm{d}x' n(x')}$$

- Density and phase are canonically conjugate

$$[\partial_x\theta(x), \phi(x')] = i\pi\delta(x - x')$$

Inhomogeneous Luttinger

- Slowly varying inhomogeneity : use the local-density approximation *for the harmonic confinement*

$$\mathcal{H}_0^{\rm LL} = \frac{\hbar}{2\pi}\int_{-\infty}^{\infty} {\rm d}x \left[v_s(x) K(x) (\partial_x \phi(x))^2 + \frac{v_s(x)}{K(x)} (\partial_x \theta(x))^2 \right]$$

- position-dependent Luttinger parameters

$$v_s(x) K(x) = \hbar \pi n(x)/m \qquad \frac{v_s(x)}{K(x)} = \frac{1}{\hbar\pi} \partial_n \mu(n(x))$$

- to proceed analytically, Ansatz for the equation of state

$$\mu(n) = \eta n^\nu \quad \rightarrow \quad \mu(n) = gn \quad \text{GP}$$

$$\mu(n) = \eta n^\nu \quad \rightarrow \quad \mu(n) = \frac{\hbar^2\pi^2}{2m} n^2 \quad \text{TG}$$

η, ν ← Bethe ansatz

Normal modes for the inhomogeneous Luttinger liquid

- Mode expansion

$$-\frac{\theta(x,t)}{\pi} = \sum_{j=0}^{\infty} \sqrt{\frac{\hbar n(x)}{2m\omega_j}} \left(\varphi_j(x) e^{i\omega_j t} b_j^\dagger + \varphi_j^*(x) e^{-i\omega_j t} b_j \right)$$

$$\partial_x \phi(x,t) = \sum_{j=0}^{\infty} i \sqrt{\frac{m\omega_j}{2\hbar n(x)}} \left(\varphi_j(x) e^{i\omega_j t} b_j^\dagger - \varphi_j^*(x) e^{-i\omega_j t} b_j \right)$$

- Diagonal Hamiltonian : $\mathcal{H}_0^{\mathrm{LL}} = \sum_{j=0}^{\infty} \hbar\omega_j \left(b_j^\dagger b_j + \frac{1}{2} \right)$
- Mode amplitudes

$$-\omega_j^2 \sqrt{v_s(x) K(x)} \varphi_j(x) = v_s(x) K(x) \partial_x \left(\frac{v_s(x)}{K(x)} \partial_x (\sqrt{v_s(x) K(x)} \varphi_j(x)) \right)$$

- Solution : Gegenbauer polynomials; dispersion :

$$(\omega_j/\omega_{\mathrm{h}})^2 = (j+1)(1 + j\nu/2)$$

Barrier renormalization with Luttinger-Liquid theory

- The barrier is very localized → *cannot be treated with LDA*

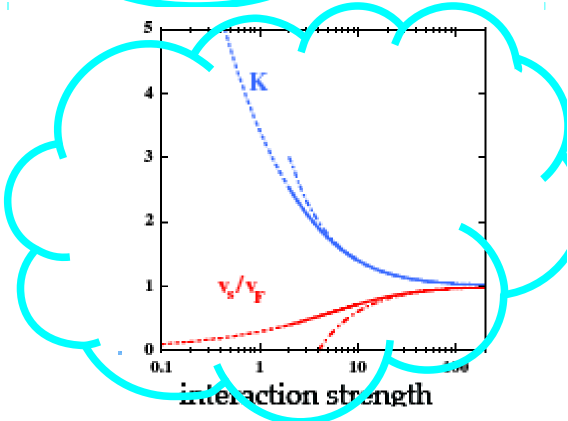
$$\mathcal{H}_{\mathrm{b}} = \int_{-\infty}^{\infty} \mathrm{d}x\, U_0 \delta(x) \rho(x)$$

- Integrating out the higher-energy density fluctuation modes :

$$\mathcal{H}_{\mathrm{b}}^{LL} \sim 2n(0) U^{\mathrm{eff}} \cos[2\theta_0(0) + 2\pi \int_{-\infty}^{0} \mathrm{d}x\; n(x)]$$

- Barrier renormalization by quantum fluctuations of the density:

$$\langle \rho(x) \rho(0) \rangle \sim |x|^{-2K}$$



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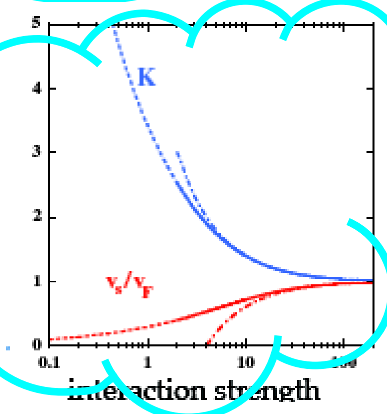
- Barrier renormalization by quantum fluctuations of the density:

$$U^{\rm eff} = U_0 \langle 0| \cos(2 \sum^{N/c} \theta_j(0))|0\rangle \sim U_0 \left(\frac{a}{N}\right)^{\kappa}$$

$$\langle \rho(x) \rho(0) \rangle \sim |x|^{-2K}$$

$$\kappa = K(0) v_s(0)/\omega_{\rm h} R = K_0 \sqrt{\frac{\nu}{2}}$$

- Ueff decreases at decreasing interactions
- The exponent is *different from the homogeneus case* !



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$$\kappa = K(0) v_s(0)/\omega_{\rm h} R = K_0 \sqrt{\frac{\nu}{2}}$$

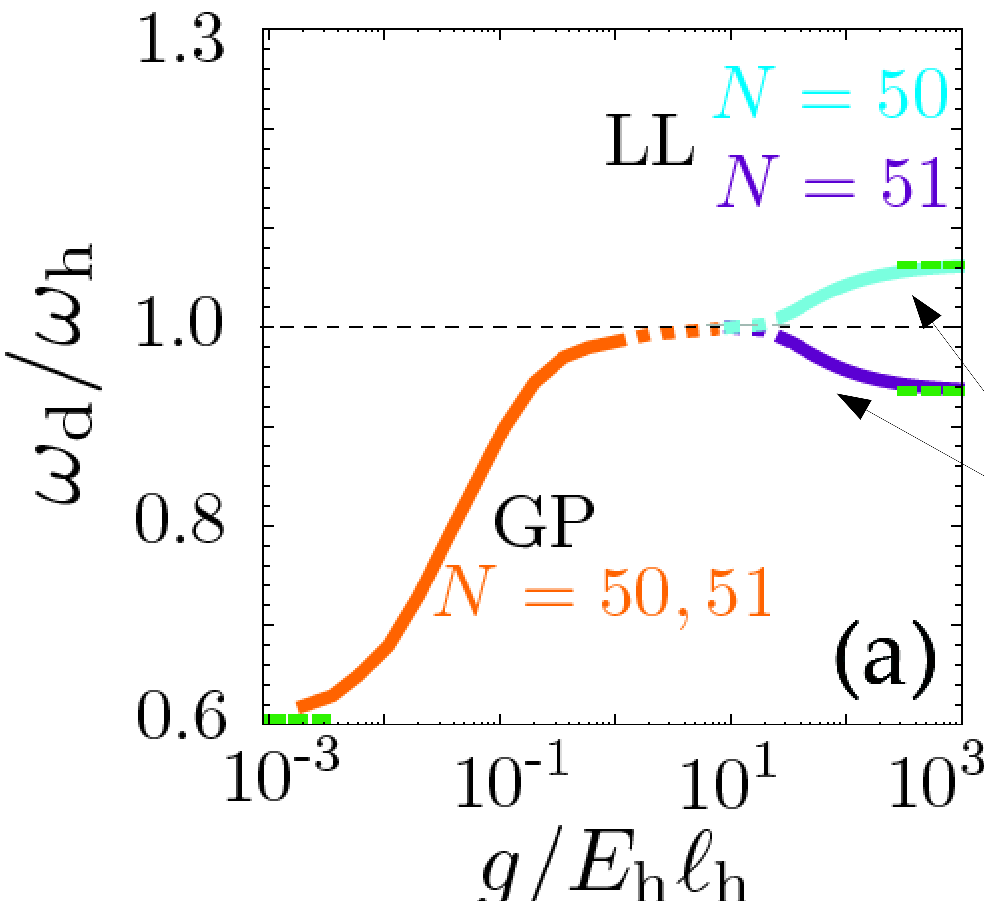
- Parity effect :

$$(-1)^N$$

- Ueff decreases at decreasing interactions
- The exponent is *different from the homogeneus case* !

Dipole mode frequency vs interaction strength

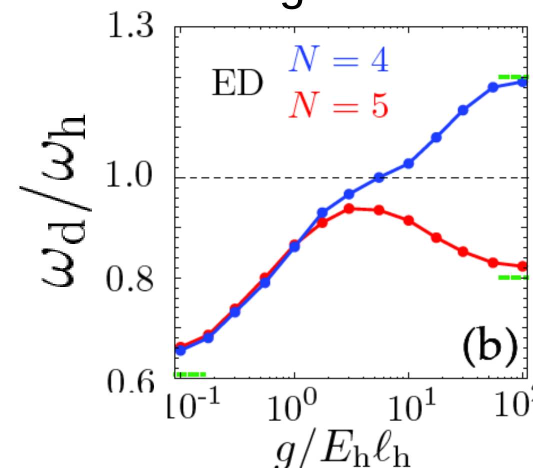
- From Gross-Pitaevskii and Luttinger liquid theory



Barrier strength

$\lambda = 1$

Exact diagonalization :



At decreasing interactions, the effective barrier is reduced

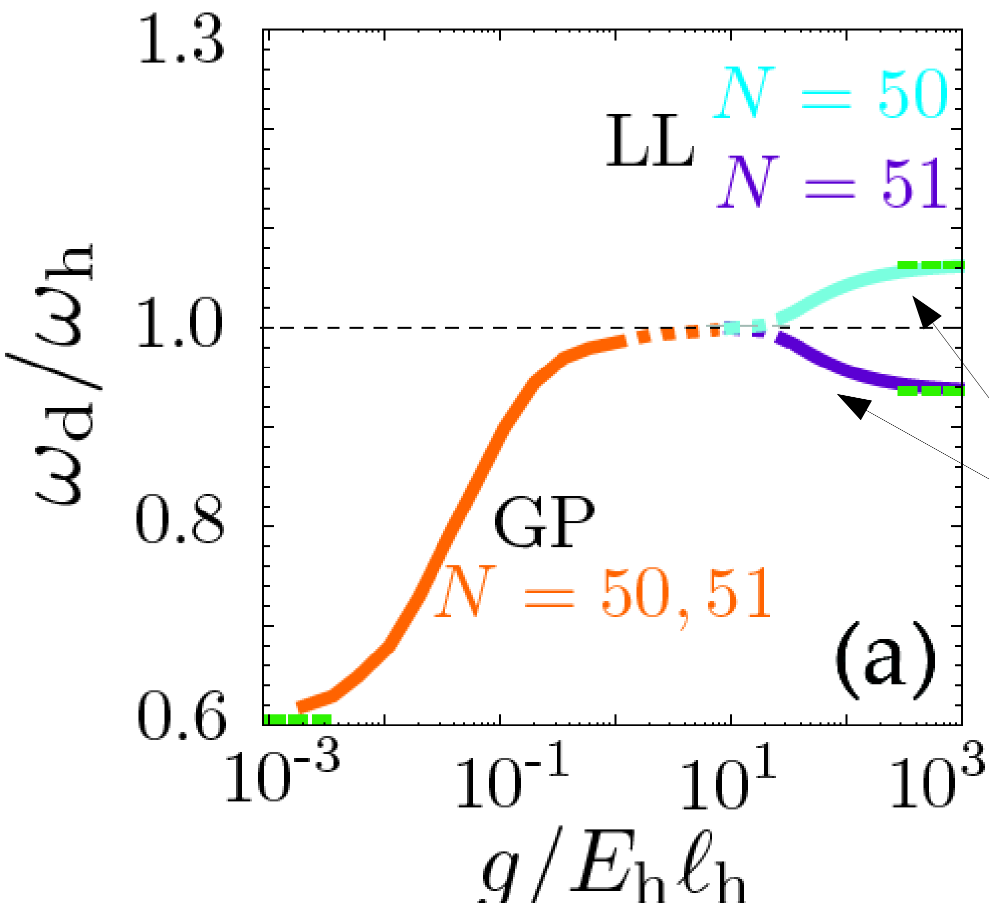
$$U^{\mathrm{eff}} = U_0 \left(\frac{a}{N}\right)^{\kappa}$$

Luttinger Liquid : $\omega_d - \omega_h = (-1)^N n(0) U^{\mathrm{eff}} K_0 \sqrt{\frac{\nu}{2}\frac{1}{\hbar}} \left(\frac{(\frac{1}{\nu}+\frac{1}{2})\Gamma^2(\frac{1}{\nu}+\frac{1}{2}) 2^{\frac{2}{\nu}+2}}{\Gamma(\frac{2}{\nu}+1)} \right)$

Cominotti, Hekking, Minguzzi, arXiv:1503.07776

Dipole mode frequency vs interaction strength

- From Gross-Pitaevskii and Luttinger liquid theory



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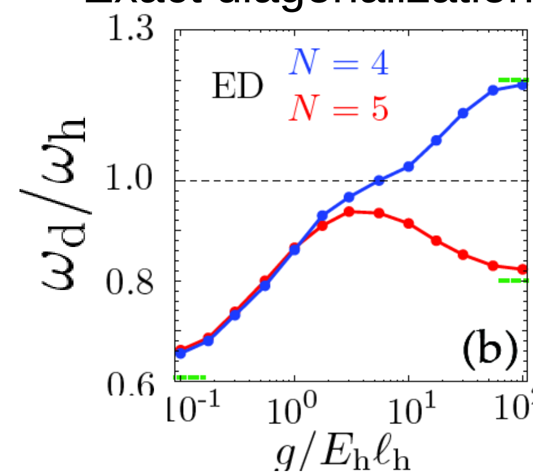
Exact diagonalization :



At decreasing interactions, the effective barrier is reduced

$$U^{\mathrm{eff}} = U_0 \left(\frac{a}{N}\right)^{\kappa}$$

The frequency shift is a direct measure of barrier screening or renormalization by quantum fluctuations

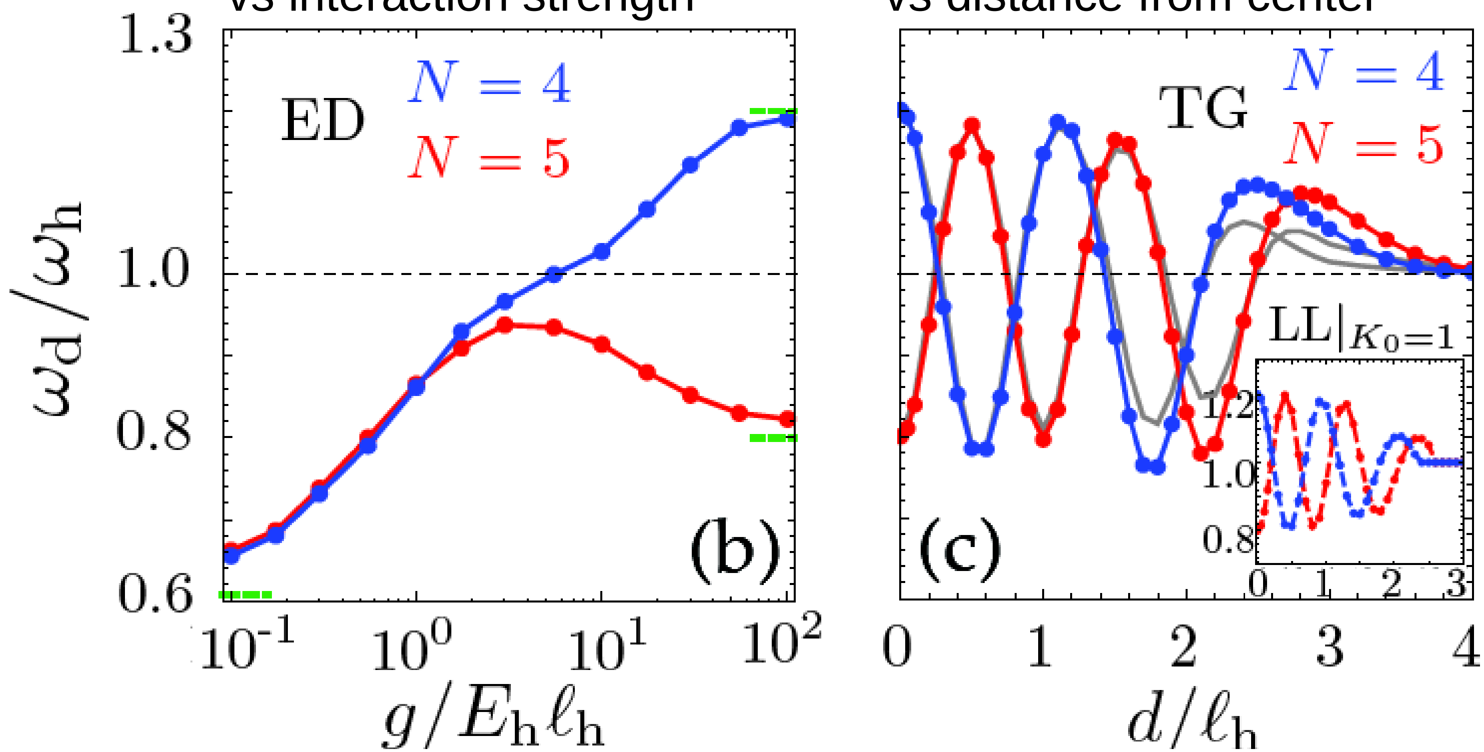
Cominotti, Hekking, Minguzzi, arXiv:1503.07776

Effect of non-centered barrier

- Dipole-mode frequency from Tonks-Girardeau and Luttinger liquid theory

centered barrier, freqency vs interaction strength

non-centered barrier, freqency vs distance from center



$$\omega_d - \omega_h = \frac{n(d)U^{\mathrm{eff}}(d)K(d)v_s(d)}{\hbar\omega_h R}\left(\frac{(\frac{1}{\nu}+\frac{1}{2})\Gamma^2(\frac{1}{\nu}+\frac{1}{2})2^{\frac{2}{\nu}+2}}{\Gamma(\frac{2}{\nu}+1)}\right)\cos\left(2\pi\int_{-\infty}^{d}\mathrm{d}x\, n(x)\right)$$

Cominotti, Hekking, Minguzzi, arXiv:1503.07776

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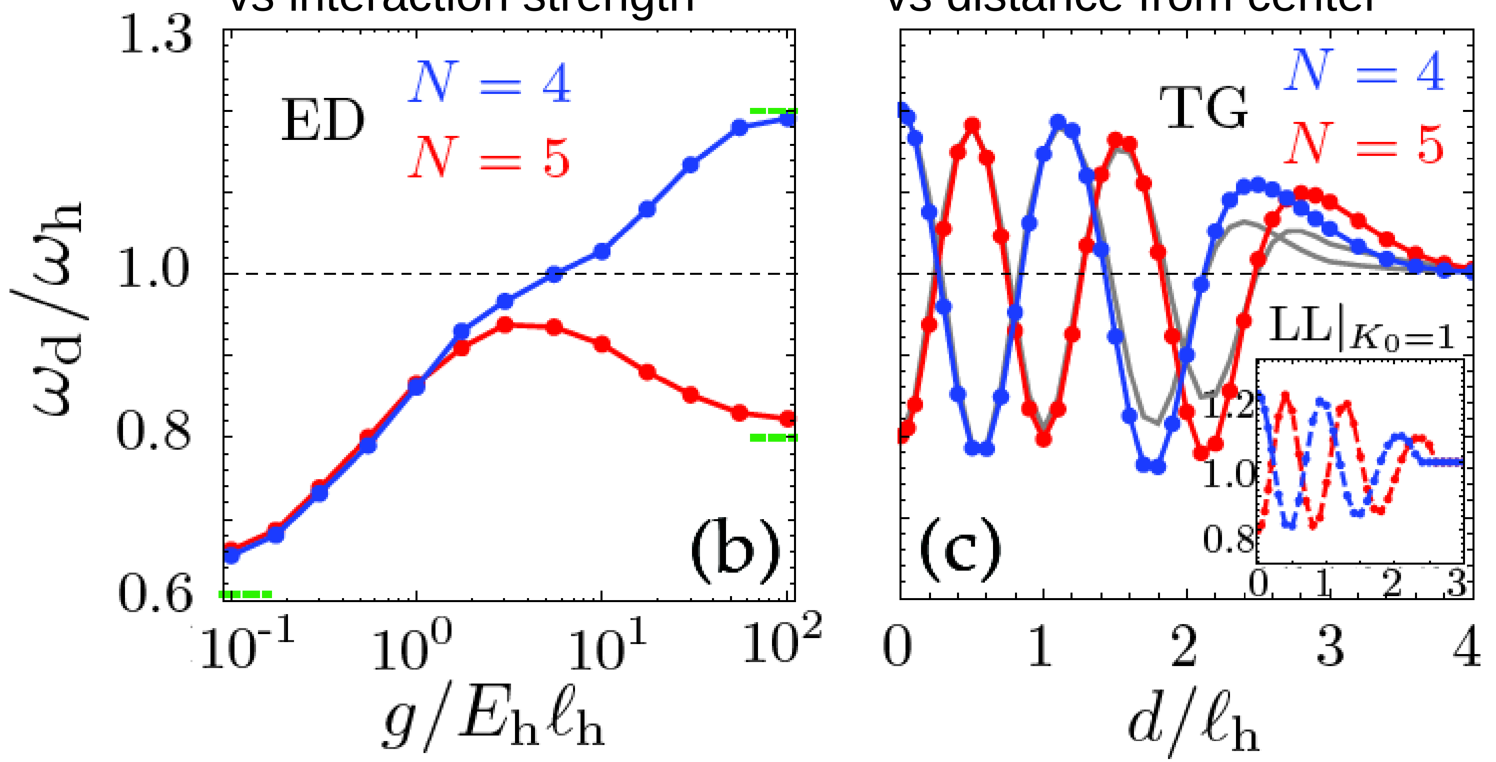
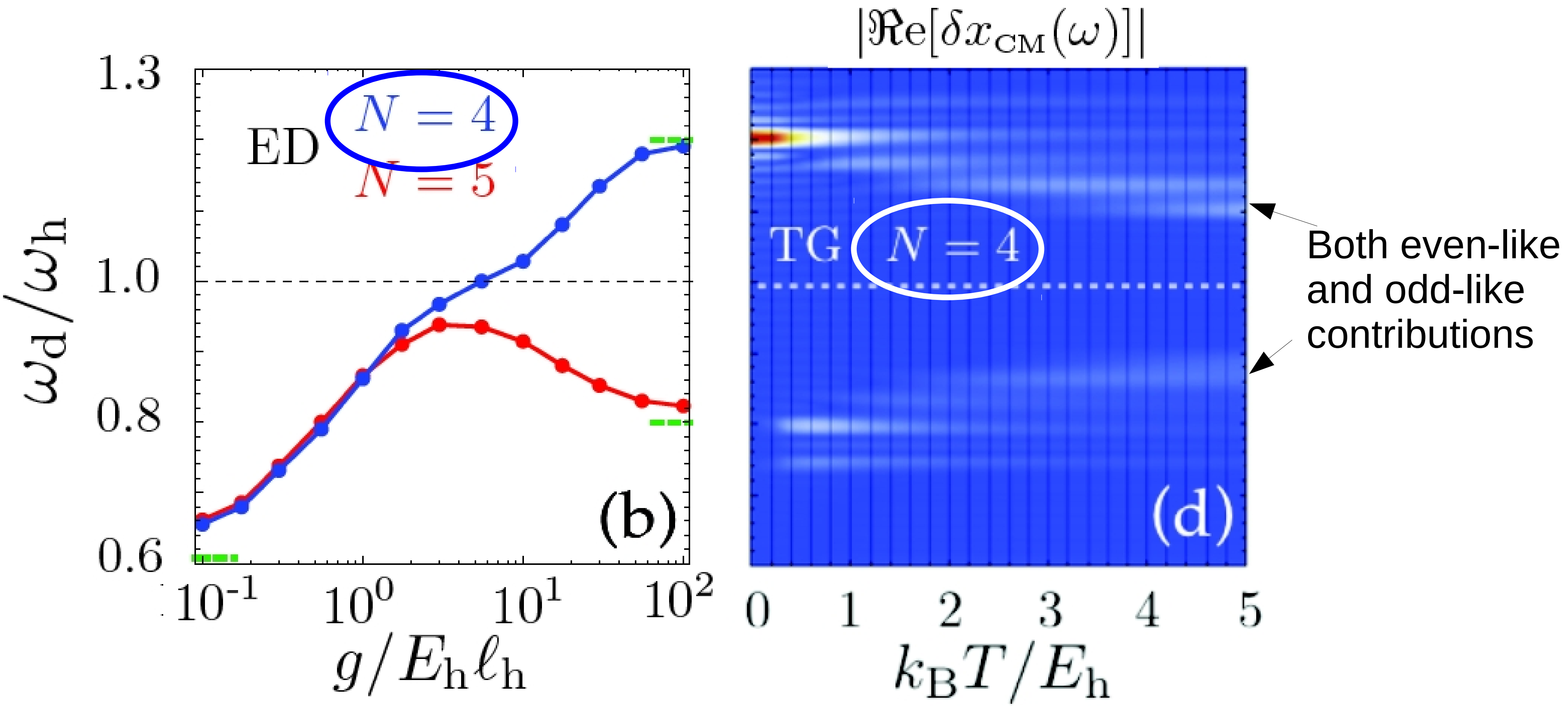


Oscillatory behaviour vs barrier distance, 'particle-counting effect' – well accounted for by the Luttinger-liquid theory

Cominotti, Hekking, Minguzzi, arXiv:1503.07776

...and at finite temperature ?

- Exact solution for the quench dynamics at finite temperature using the thermal Bose-Fermi mapping
- Dipole frequency from a Fourier analysis of $x_{\rm CM}(t)$

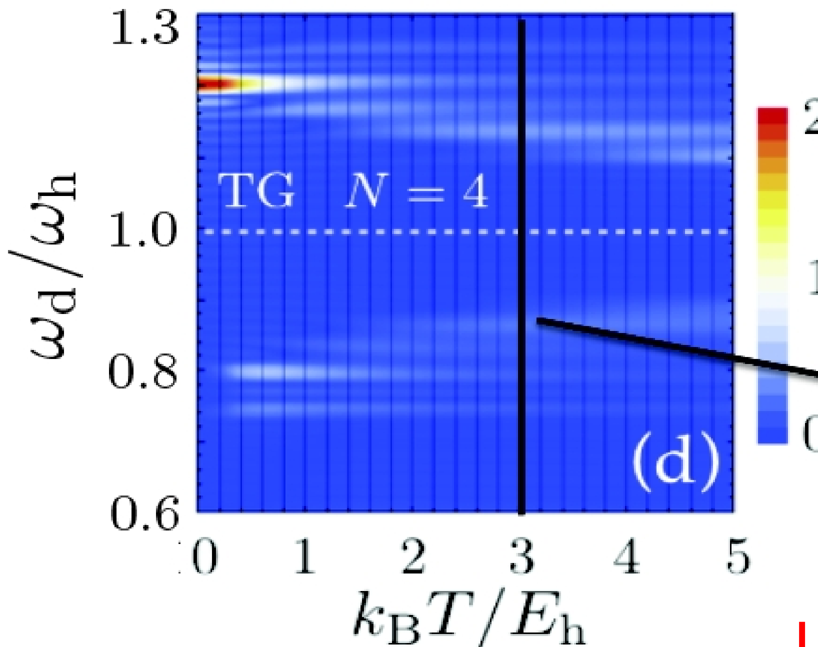


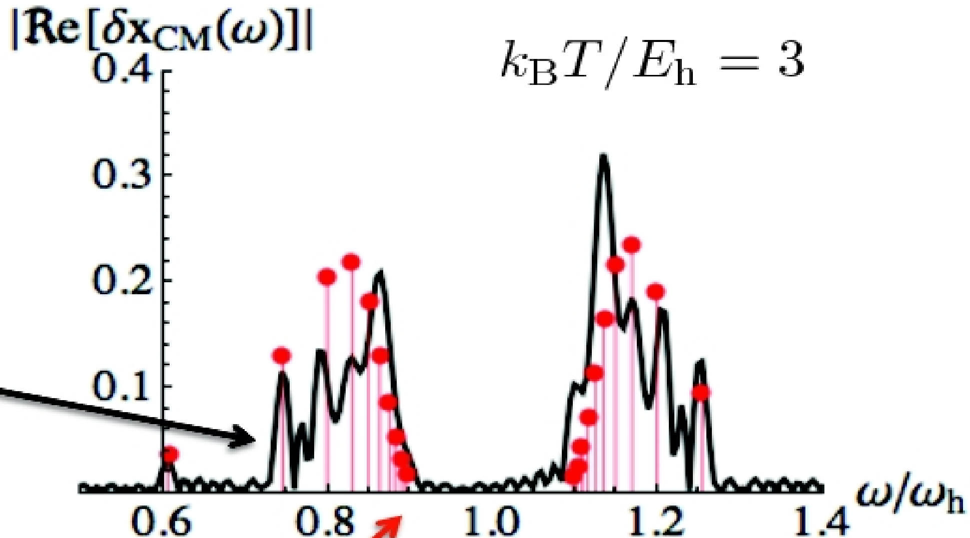
Cominotti, Hekking, Minguzzi, arXiv:1503.07776

Spectral function at finite T

- Understanding the frequency contributions to the center-of-mass motion at finite temperature

$|\Re\mathrm{e}[\delta x_{\mathrm{CM}}(\omega)]|$ from exact dynamical evolution (on a finite time)





Linear response theory:

$$V_p(x,t) = \theta(t)\Delta x(m\omega_h^2 x + U_0\delta'(x))$$

$$\delta x_{\mathrm{CM}}(\omega) = \int \mathrm{d}x\, x \int \mathrm{d}x'\, \chi(x,x';\omega)V_p(x',\omega)$$

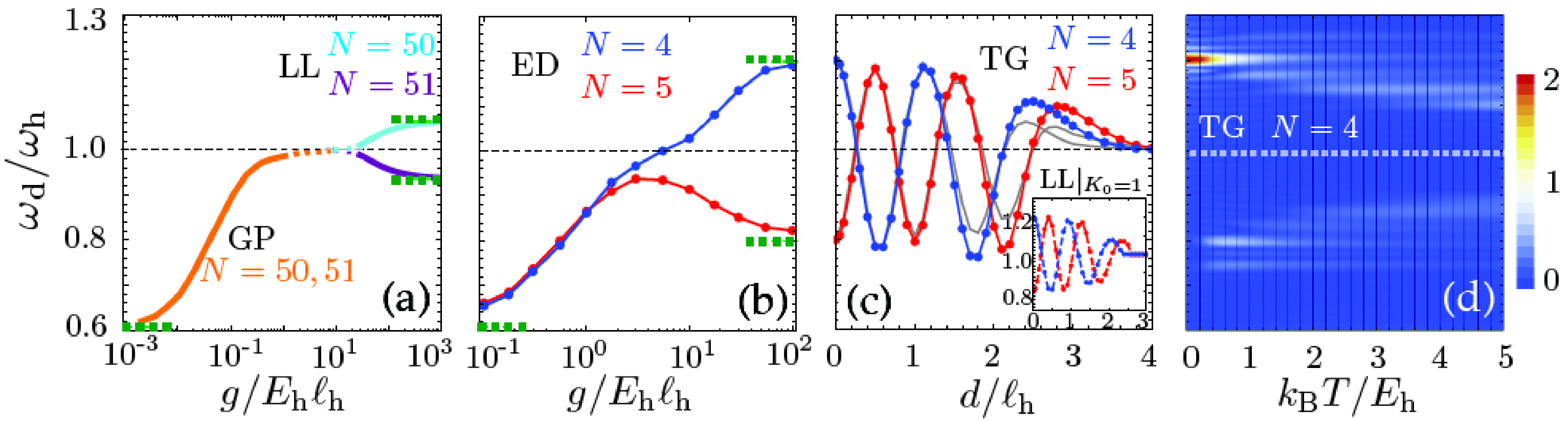
$$\chi(x,x';\omega) = (1/\hbar)\sum_{j\neq k}\psi_j^*(x)\psi_k(x)\psi_k^*(x')\psi_j(x')f(\varepsilon_j)[1-f(\varepsilon_k)]$$

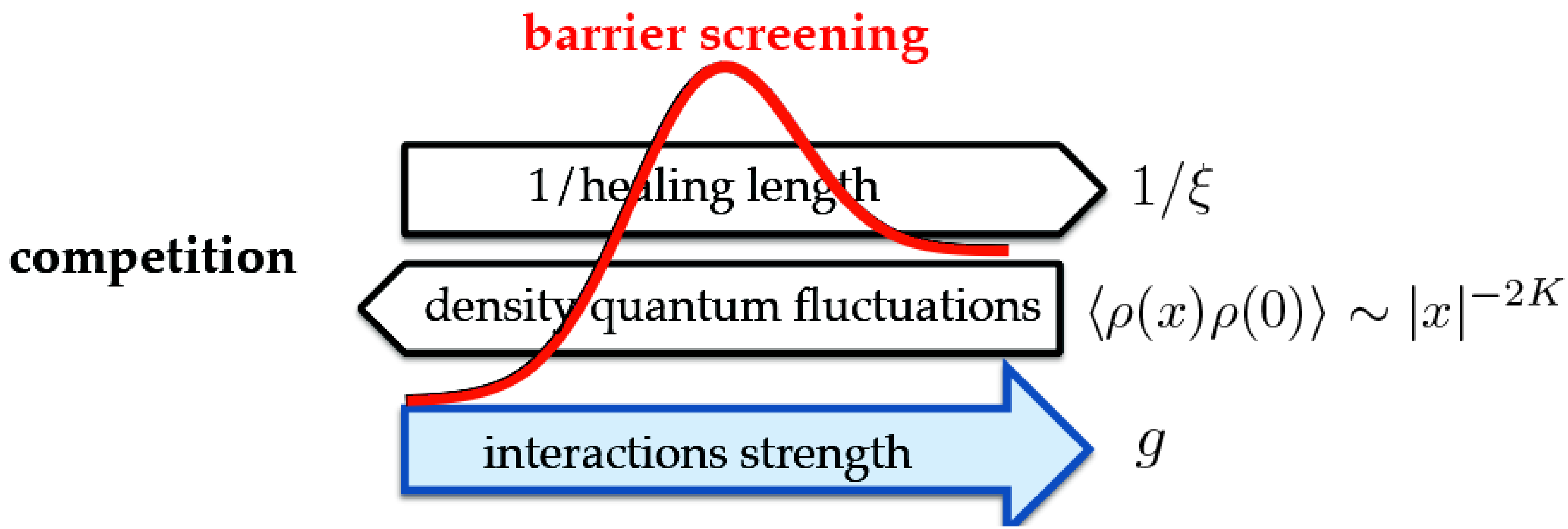
$$\times\left(\frac{1}{(\omega-(\varepsilon_k-\varepsilon_j)/\hbar))+i\epsilon} - \frac{1}{(\omega+(\varepsilon_k-\varepsilon_j)/\hbar))+i\epsilon}\right)$$

Parity effect still visible at finite temperature !

Conclusions

- Dipole frequency as a powerful tool to explore barrier screening and renormalization





Outlook

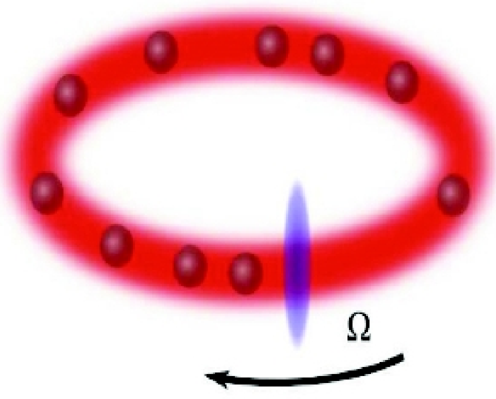
- Beyond small shift / weak barrier regime :

  – damping ?

  – thermalisation ? *[depinning : Cartarius, Kawasaki, Minguzzi, arXiv : 1505.01009]*

  – phase slips ? *[lattice model : I. Danshita, PRL 2013]*

- Fermions, multimode,...
- Back to ring geometry !



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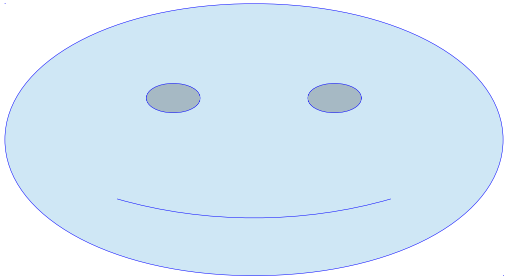

Thank you !