Generation and decay of persistent currents in a toroidal Bose-Einstein condensate

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6.05.2005, Workshop Atomtronics

This talk is a review of the results obtained in collaboration with

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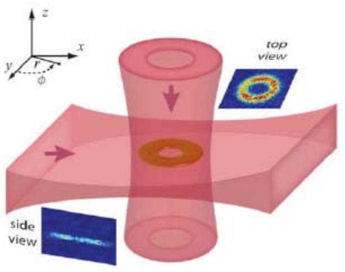
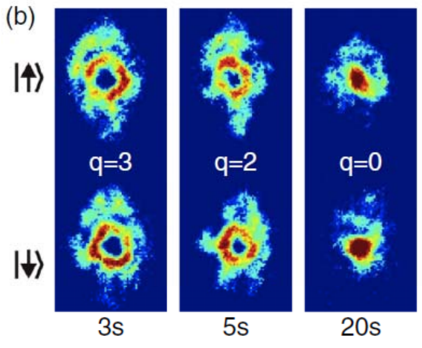
Elena Ostrovskaya
Nonlinear Physics Centre, Australian National University, Canberra, Australia

Outline





1. Stability of persistent currents in spinor Bose-Einstein condensates

Experiment: Beattie, Moulder, Fletcher, and Hadzibabic, PRL **110**, 025301 (2013)

Theory: Yakimenko, Isaieva, Vilchinskii, Weyrauch, PRA **88**, 051602(R) (2013)

2. Vortex excitations in toroidal BECs driven by

small stirrer

(diameter of the rotating barrier less than the width of the annulus)

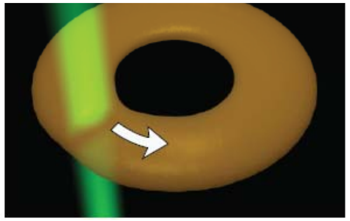
PHYSICAL REVIEW A **88**, 063633 (2013)

Threshold for creating excitations in a stirred superfluid ring

K. C. Wright,* R. B. Blakestad,† C. J. Lobb,‡ W. D. Phillips, and G. K. Campbell

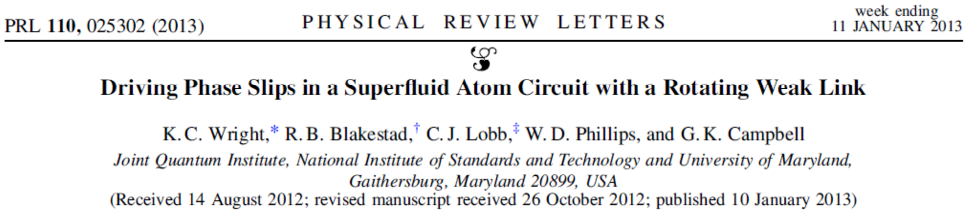
Joint Quantum Institute, National Institute of Standards and Technology and University of Maryland, Gaithersburg, Maryland 20899, USA

(Received 26 July 2013; published 20 December 2013)



rotating weak link

(wide barrier)



PRL **110**, 025302 (2013) PHYSICAL REVIEW LETTERS week ending 11 JANUARY 2013

Driving Phase Slips in a Superfluid Atom Circuit with a Rotating Weak Link

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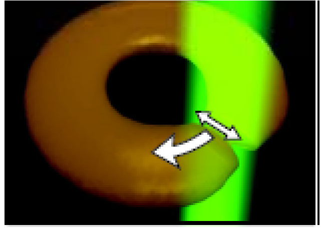
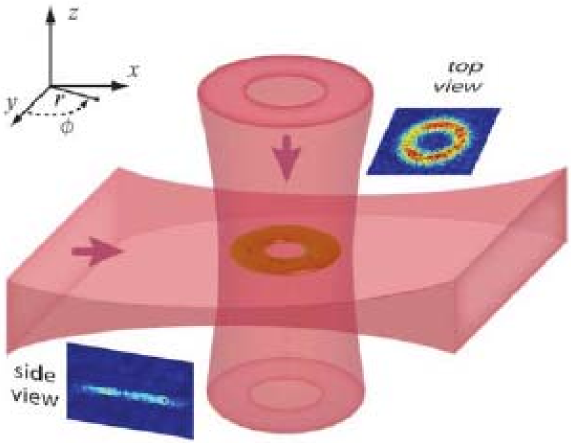
Joint Quantum Institute, National Institute of Standards and Technology and University of Maryland, Gaithersburg, Maryland 20899, USA

(Received 14 August 2012; revised manuscript received 26 October 2012; published 10 January 2013)



1. Stability of persistent currents in spinor Bose-Einstein condensates



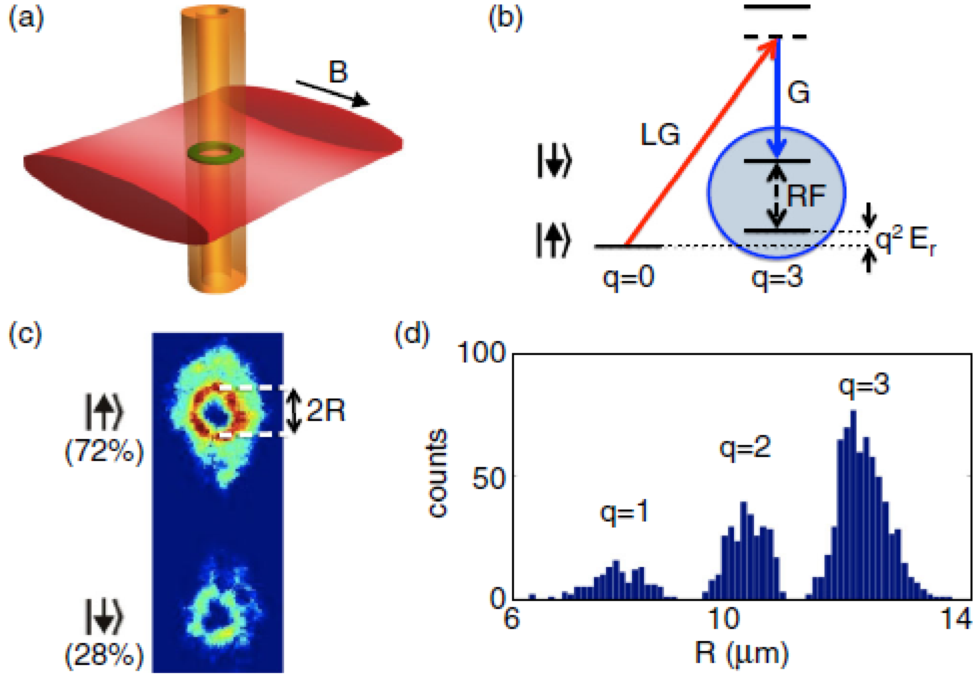
LG_{0q}

The persistent flow can be characterized by a q-charged vortex line pinned at the center of the ring-shaped condensate.

The external ring-shaped trap produces a huge central hole at the axis of the condensate cloud, where the vortex energy has a local minimum, thus the vortex core in toroidal traps is bounded by the potential barrier, which makes even the multi-charged vortices robust!

Persistent Currents in Spinor Condensates

Scott Beattie, Stuart Moulder, Richard J. Fletcher, and Zoran Hadzibabic
Cavendish Laboratory, University of Cambridge, J. J. Thomson Avenue, Cambridge CB3 0HE, United Kingdom
(Received 18 October 2012; published 9 January 2013)



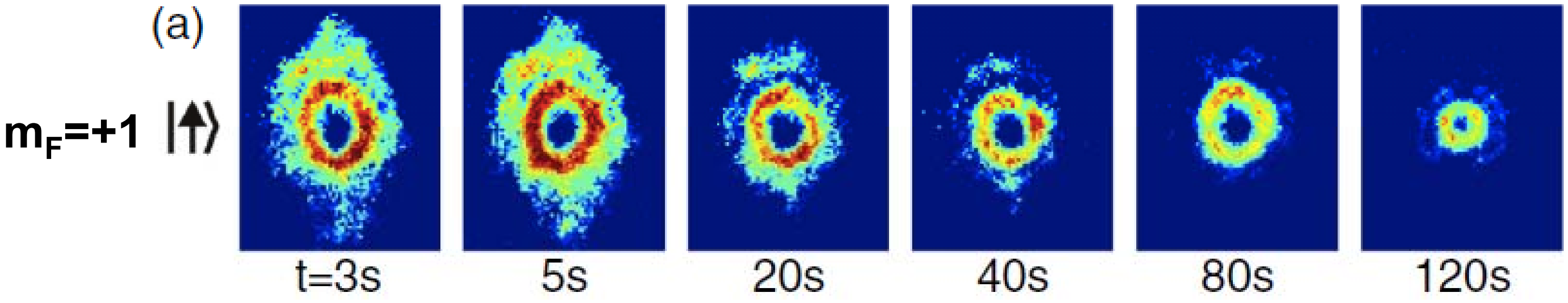
$$\boxed{M \oint_{\Gamma} \mathbf{v}_j d\mathbf{l} = 2\pi\hbar q_j}$$

$$\mathbf{v}_j = \frac{\hbar}{M}\nabla\Phi_j$$

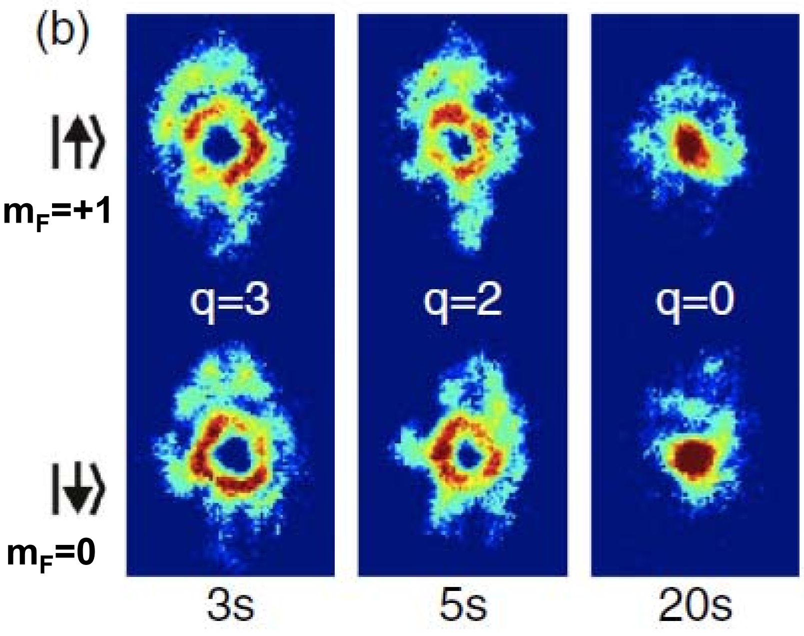
$$\Psi_j = |\Psi_j| e^{i\Phi_j}$$

- The system is two-component condensate of ^{87}Rb with $\boxed{m_F=+1,0}$
- Sheet beam and LG_0^3 ring beam

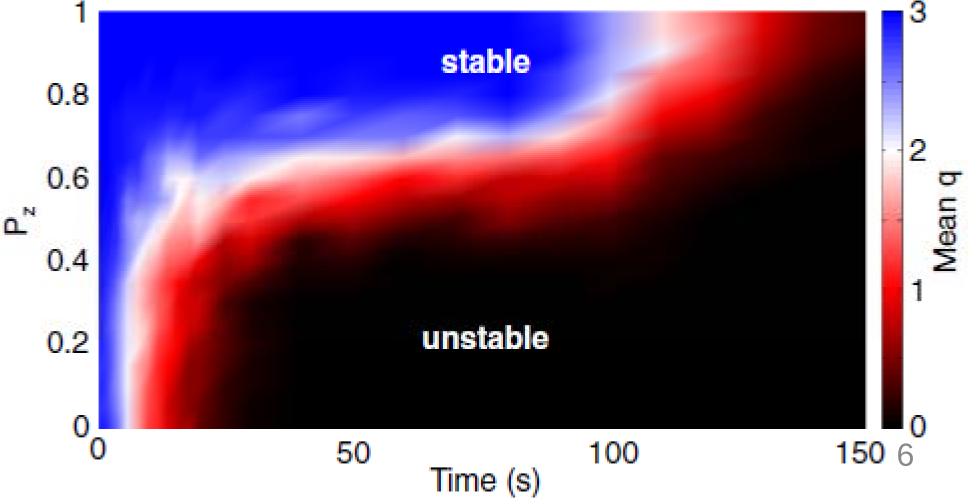
For a large spin-population imbalance supercurrents persisting for over two minutes.



However, supercurrent is unstable for spin polarization below a well-defined critical value.



$P_z=(N_+-N_0)/(N_++N_0)$



Mean-field theory for spinor BECs

$$H = \sum_{j=-,0,+} \int d\mathbf{r}\, \psi_j^* \left(-\frac{\hbar^2}{2m}\nabla^2 + \frac{c_0}{2} n + V(\mathbf{r}) \right) \psi_j + H_A,$$

$$n = \sum n_j = \sum |\psi_j|^2$$

$$H_{\mathrm{A}} = \int d\mathbf{r} \left(\sum_{j=-,0,+} E_j n_j + \frac{c_2}{2} |\mathbf{F}|^2 \right)$$

$$c_0 = 4\pi\hbar^2(2a_2 + a_0)/3m$$
$$c_2 = 4\pi\hbar^2(a_2 - a_0)/3m$$

$$\mathbf{F} = (F_x, F_y, F_z) = (\psi^\dagger \hat{F}_x \psi, \psi^\dagger \hat{F}_y \psi, \psi^\dagger \hat{F}_z \psi),$$

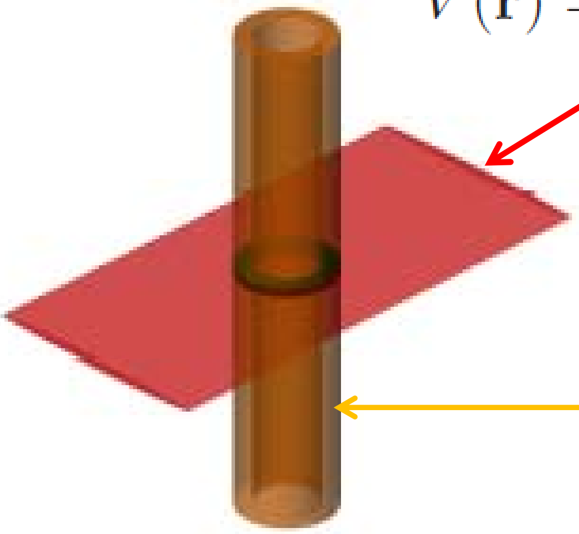
$$\hat{F}_x = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \hat{F}_y = \frac{i}{\sqrt{2}} \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \quad \hat{F}_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

In experiments the all-optical toroidal trap is formed by two red-detuned laser beams.

Harmonic potential creates a tight binding in z-direction (which models the elliptic highly anisotropic `sheet' beam)

Radial confinement is provided by Laguerre-Gauss LG_{03} potential (which models the `tube beam')

$$V(\mathbf{r}) = \frac{M\omega_z^2 z^2}{2} - V_0\left(\frac{r}{R}\right)^{2q} e^{-q\left(r^2/R^2-1\right)}$$



Disk-shaped spinor BEC can be described by set of 2D DGPEs:

$$\boxed{\begin{aligned} (i-\gamma)\frac{\partial \psi_\pm}{\partial t} &= \hat{\mathcal{H}}_\pm \psi_\pm + \nu_a n_0 \psi_\mp^*, \\ (i-\gamma)\frac{\partial \psi_0}{\partial t} &= \hat{\mathcal{H}}_0 \psi_0 + 2\nu_a \psi_+ \psi_- \psi_0^*, \end{aligned}}$$

$$\begin{aligned} \hat{\mathcal{H}}_\pm &= -\frac{1}{2}\Delta_\perp - \mu_\pm + V(r) + \nu_s n + \nu_a (n_0 + n_\pm - n_\mp), \\ \hat{\mathcal{H}}_0 &= -\frac{1}{2}\Delta_\perp - \mu_0 + V(r) - \epsilon + \nu_s n + \nu_a (n_+ + n_-). \end{aligned}$$

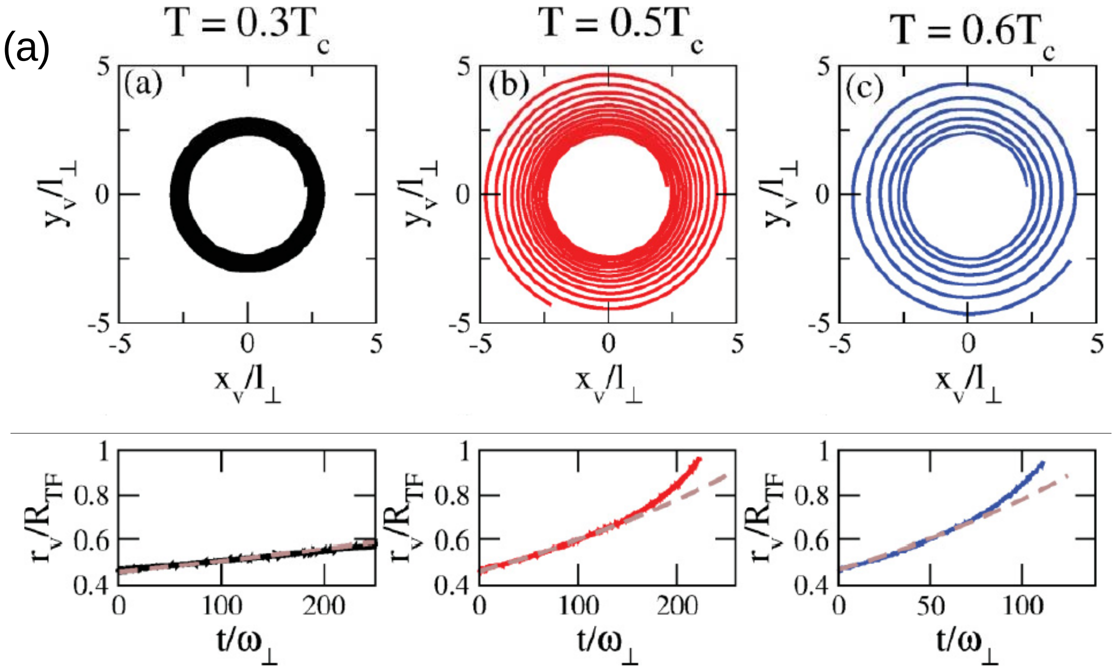
$$\nu_s = \mathrm{sgn}(c_0) = +1, \text{ and } \nu_a = c_2/c_0 = -4.66 \cdot 10^{-3}$$

Phenomenological dissipation provides qualitatively correct description of the vortex line dynamics

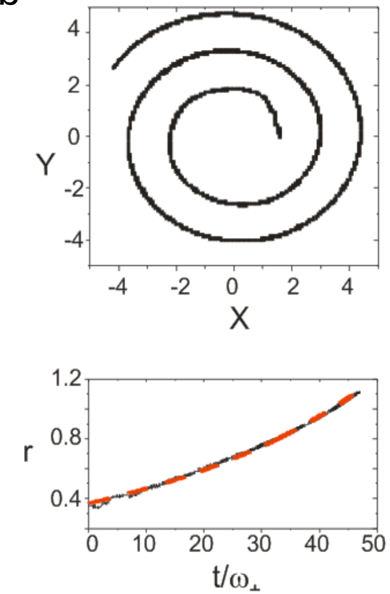
ZNG model [PRA 87, 013630 (2013)]

DGPE
[our 2D simulations]

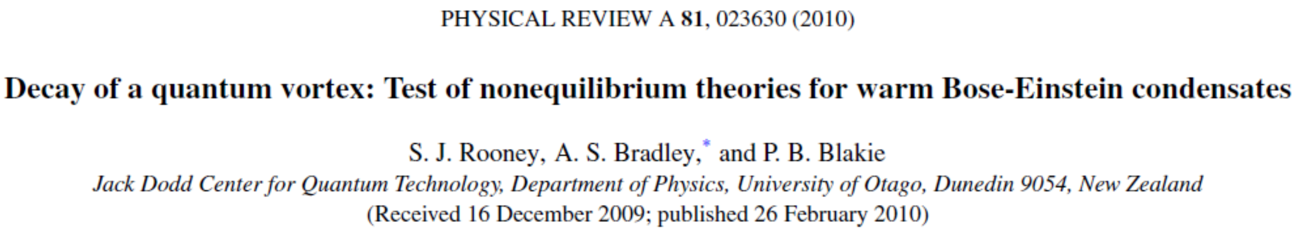
(a)



(b`



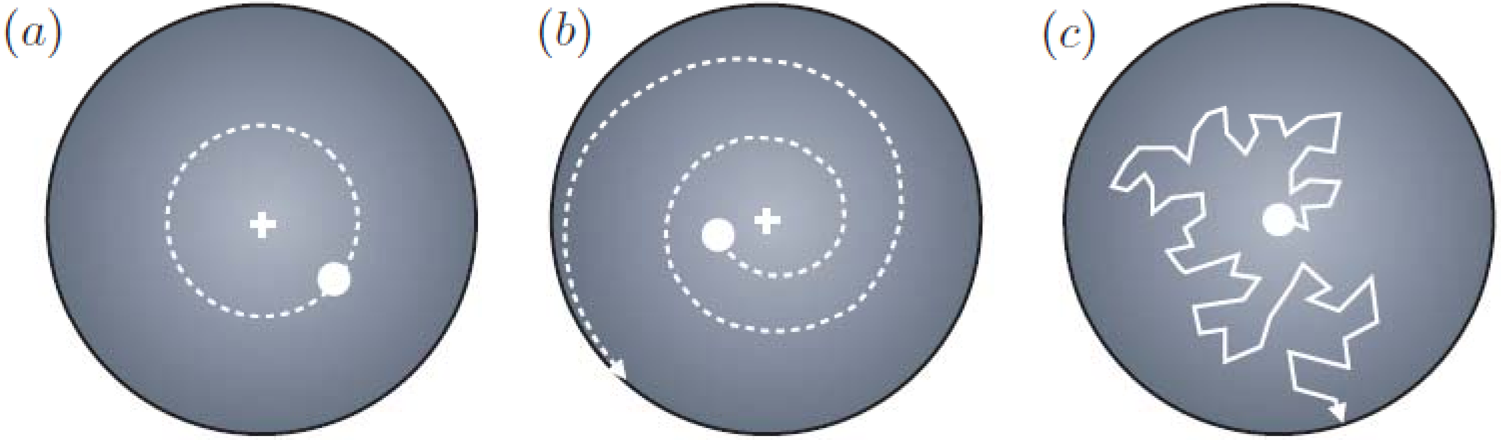
The noise produces a drift motion of collective excitations, such as dark solitons and vortices, and adds stochastic jitter to their trajectories



PHYSICAL REVIEW A **81**, 023630 (2010)

Decay of a quantum vortex: Test of nonequilibrium theories for warm Bose-Einstein condensates

S. J. Rooney, A. S. Bradley,* and P. B. Blakie
Jack Dodd Center for Quantum Technology, Department of Physics, University of Otago, Dunedin 9054, New Zealand
(Received 16 December 2009; published 26 February 2010)



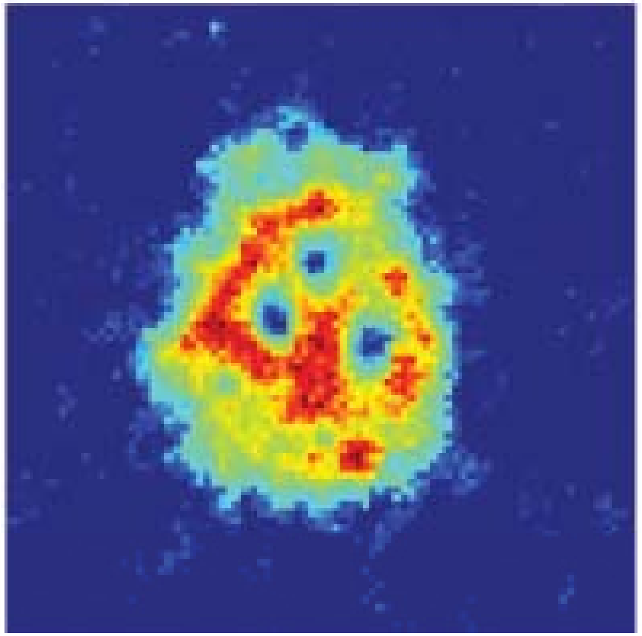
No dissipation **With dissipation** **With thermal fluctuations**

Note, the averaged direction of the vortex ring drift towards the lower density region remains the same!

How to estimate the phenomenological dissipative parameter γ?

Parameter γ is a constant, which can be estimated from quantum kinetic theory.

We take γ = 0.08 and verify by simulations in single-connected trap that 3-charged vortex line



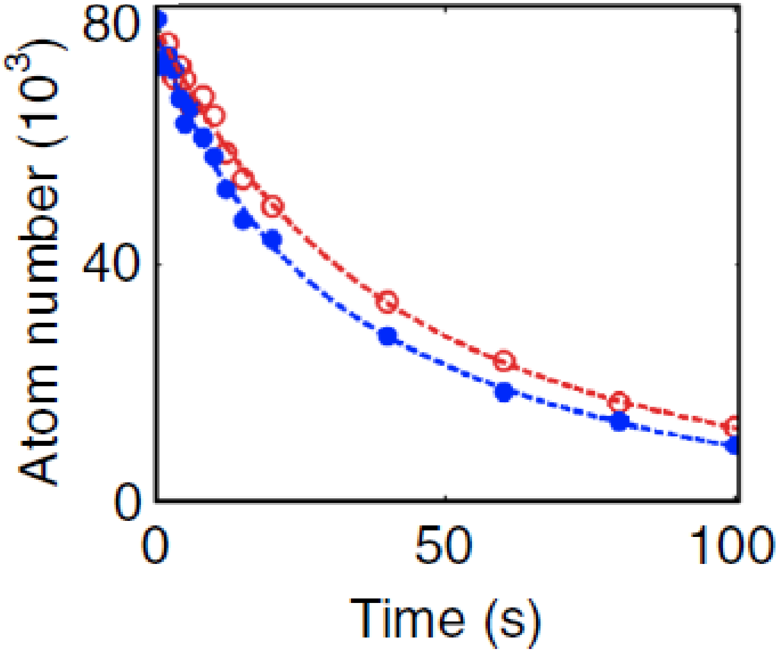
- splits into 3 vortices (1 sec)
- first vortex leaves the BEC cloud (10 sec)
- none vortex survives after 15 sec

PHYSICAL REVIEW A **86**, 013629 (2012)
Quantized supercurrent decay in an annular Bose-Einstein condensate
Stuart Moulder, Scott Beattie, Robert P. Smith,
Naaman Tammuz, and Zoran Hadzibabic
Cavendish Laboratory, University of Cambridge

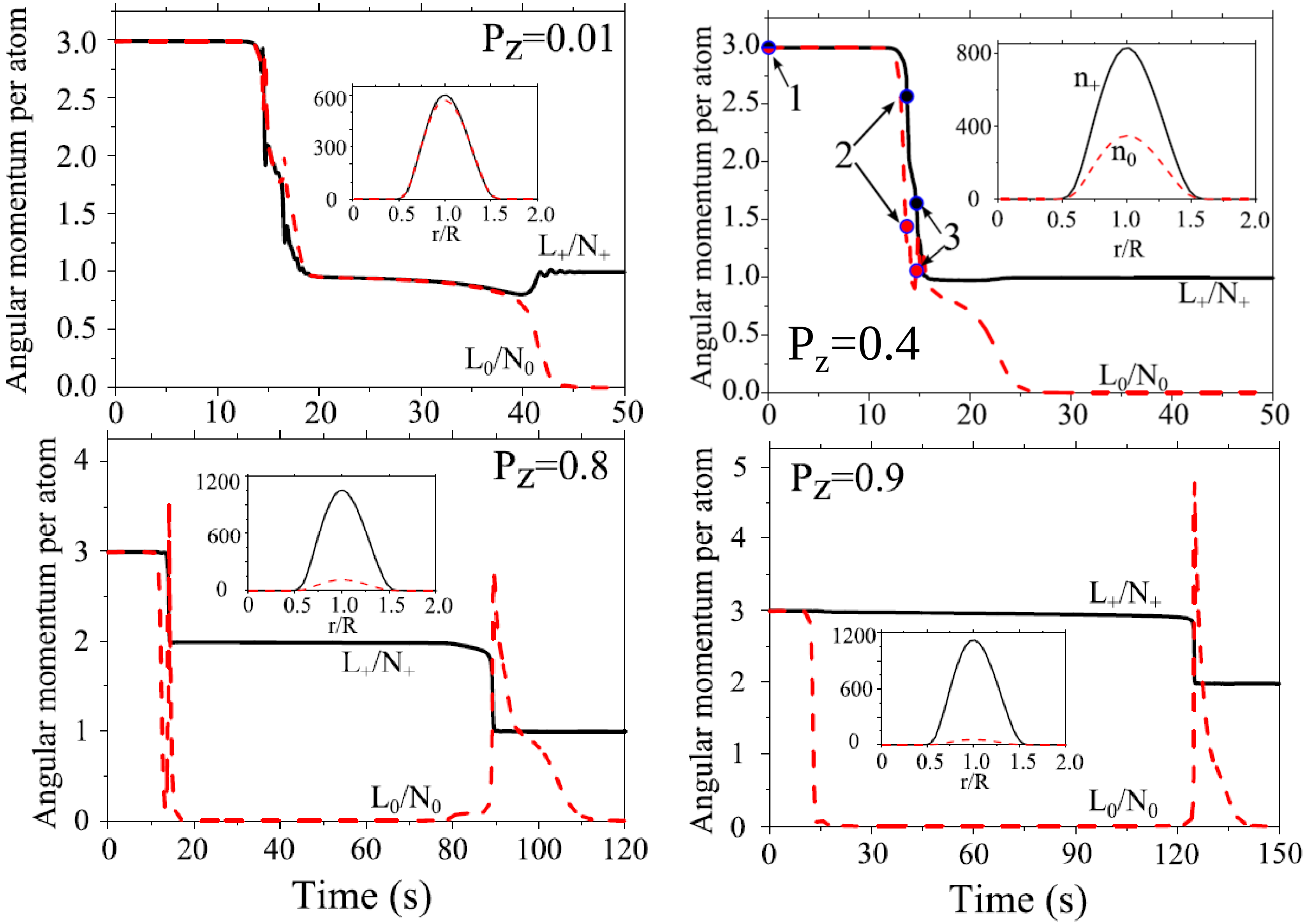
We fitted the condensate decay rate to the experimental data using time-dependent chemical potential μ(t)

$$N(t) = N(0)e^{-t/\tau_0}$$

$$\tau_0 = 39$$

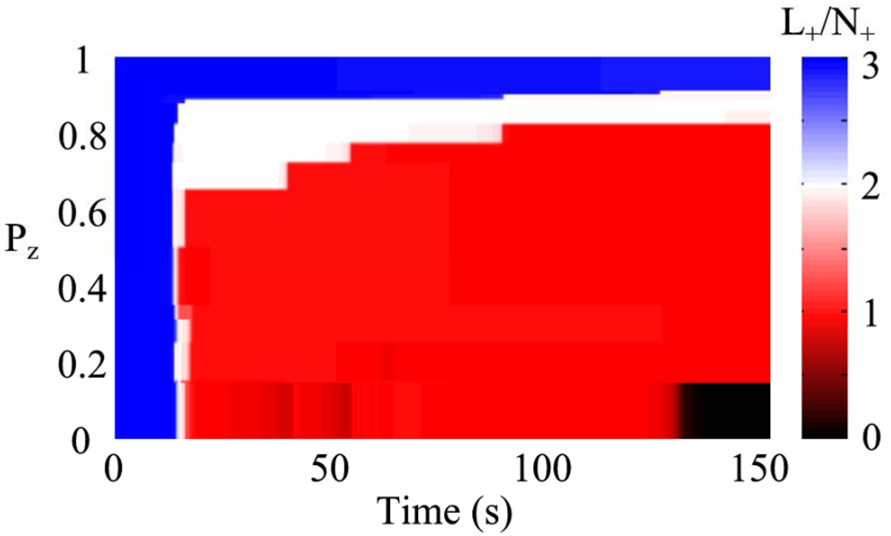


Angular momentum per particle is not integer close to the phase-slip

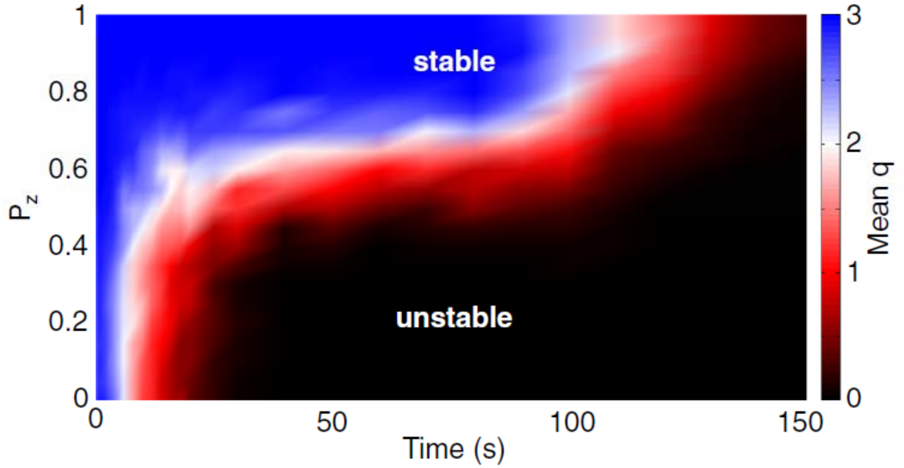


Our theoretical results agree with the experimental findings

Theory



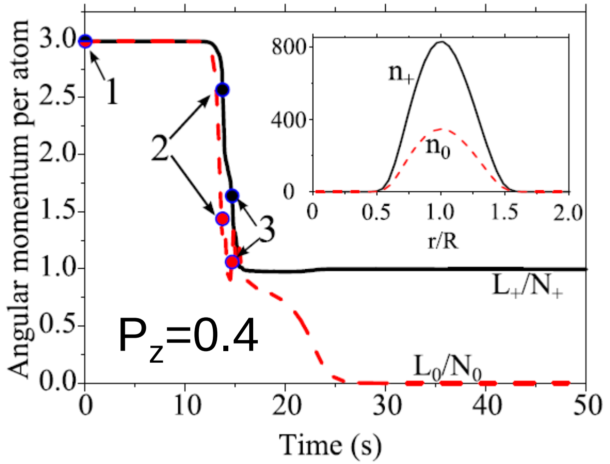
Experiment

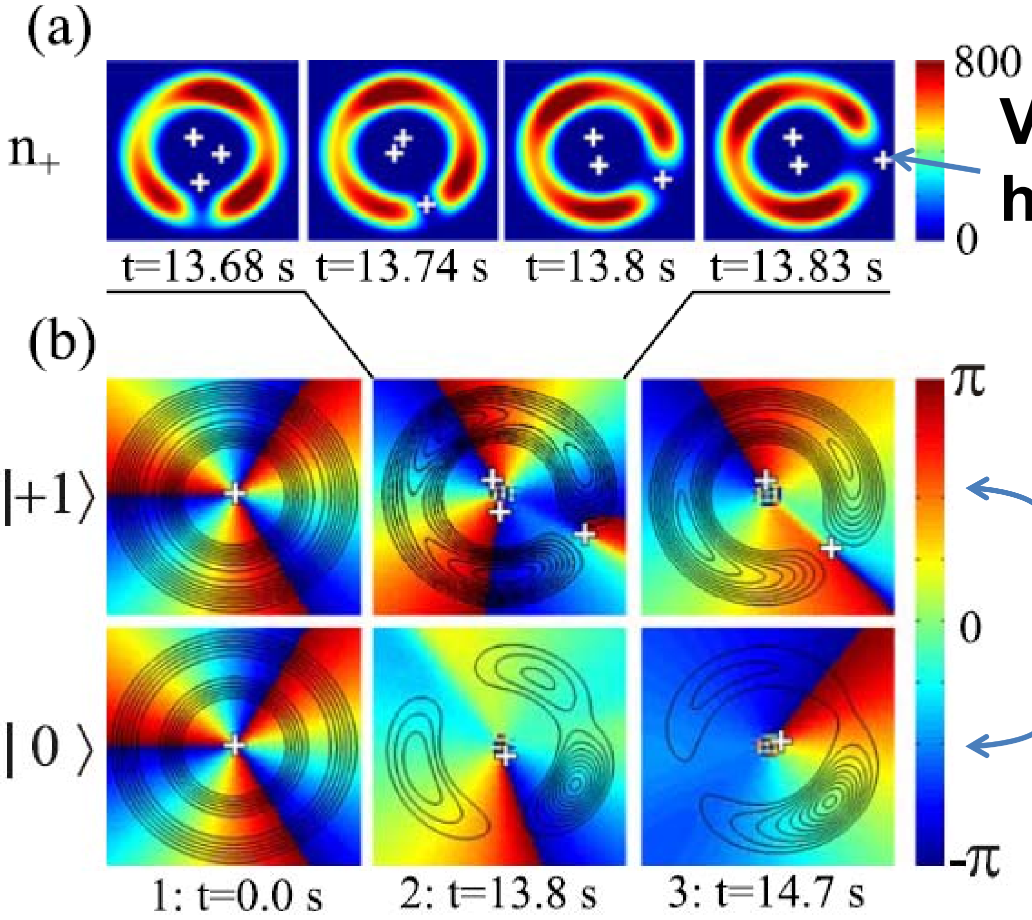


Why the superflow decays when admixture of second spin component is significant?

What is the microscopic mechanism of the instability, which destroys the persistent currents?

Dynamical simulations reveal the mechanism of the decay





Vortex comes through the hole (dynamical weak link)

Note the asymmetry in density distributions

Azimuthal symmetry-breaking instability of vector vortices is well known in BEC and nonliner optics

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PHYSICS LETTERS A
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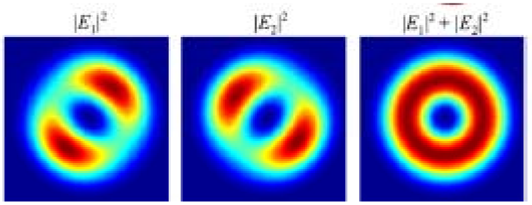
Stable counter-rotating vortex pairs in saturable media

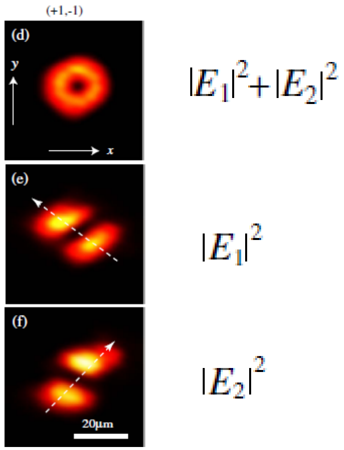
Anton S. Desyatnikov[a,*], Dumitru Mihalache[b], Dumitru Mazilu[b], Boris A. Malomed[c],
Falk Lederer[d]

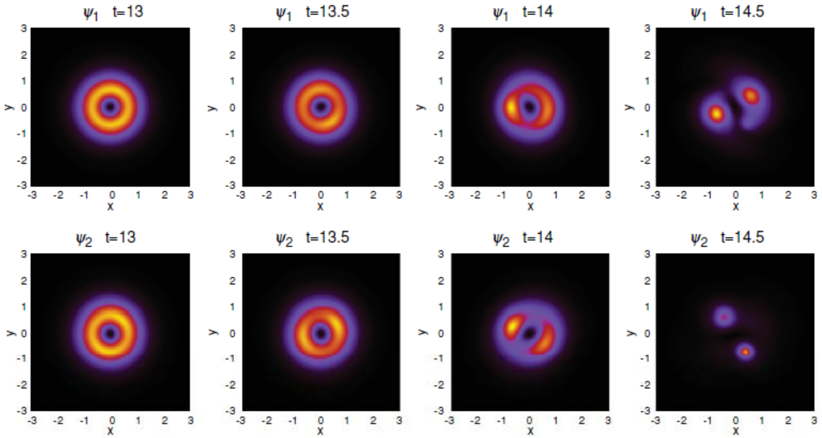


March 1, 2012 / Vol. 37, No. 5 / OPTICS LETTERS 767

Observation of vector solitons with hidden vorticity

Yana V. Izdebskaya,[1,*] Johannes Rebling,[1,2] Anton S. Desyatnikov,[1] and Yuri S. Kivshar[1]



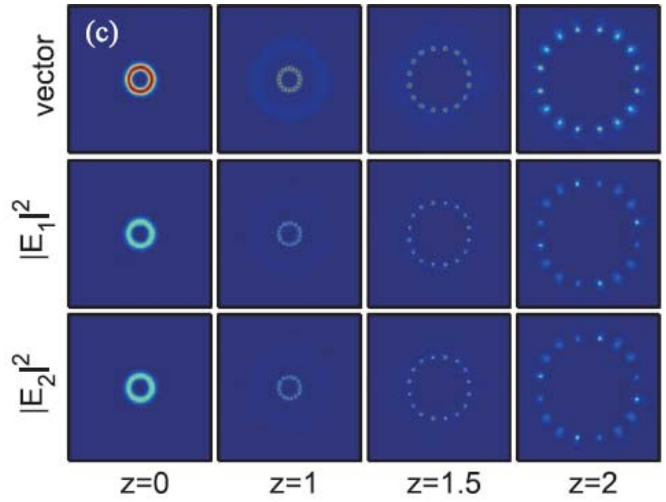


Brtka et.al. PRA **82, 053610 (2010)**

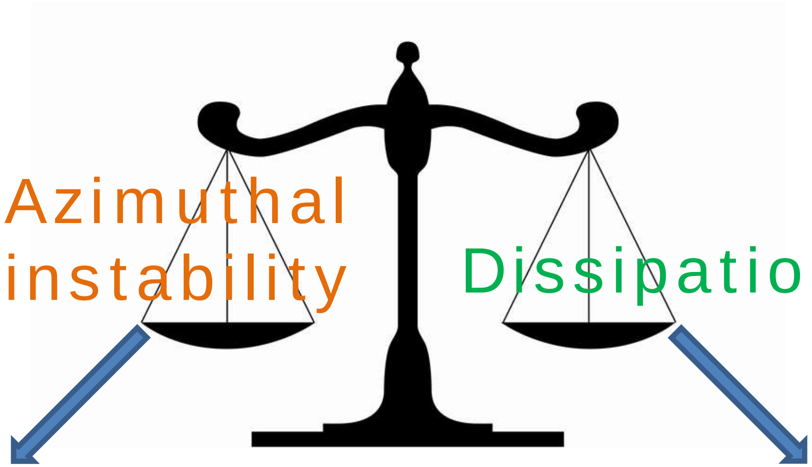
PHYSICAL REVIEW A **86**, 013827 (2012)

Stabilization of counter-rotating vortex pairs in nonlocal media

M. Shen,[1] J.-J. Zheng,[1] Q. Kong,[1] Y.-Y. Lin,[2] C.-C. Jeng,[3] R.-K. Lee,[2] and W. Krolikowski[4]



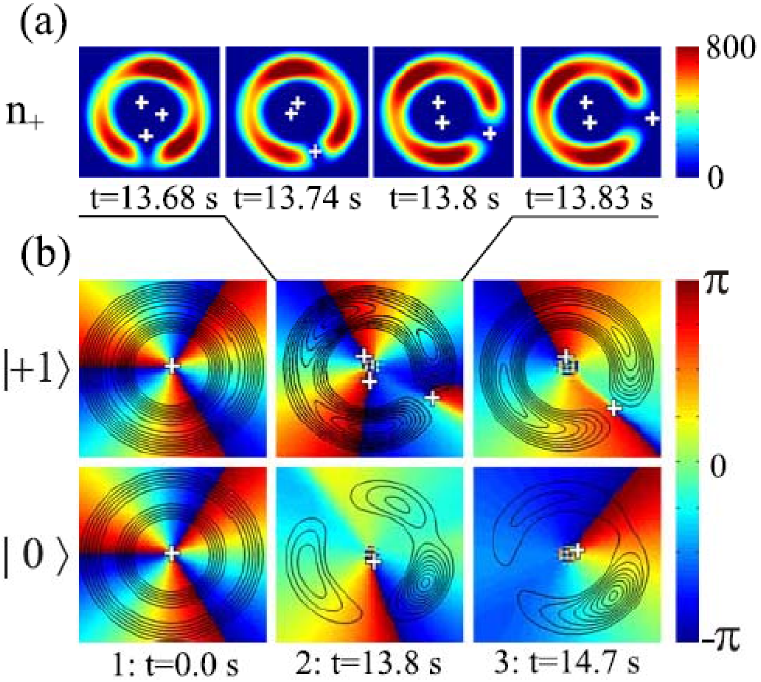
Phase-slip of superflow in two-component toroidal BECs is the result of two factors:



Azimuthal instability

Dissipation

Opens a ‘gate’ for vortices

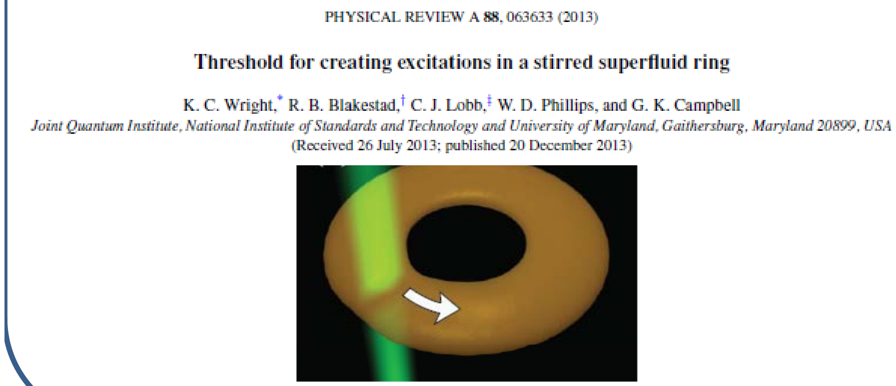


Vortex traverses the self-induced weak links

2. Vortex excitations in a stirred toroidal Bose-Einstein condensate

small stirrer

(diameter of the rotating barrier less than the width of the annulus)



PHYSICAL REVIEW A **88**, 063633 (2013)

Threshold for creating excitations in a stirred superfluid ring

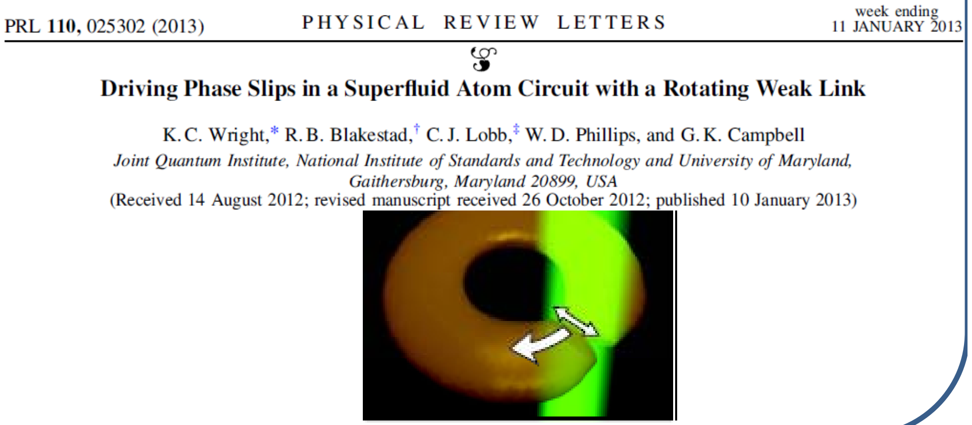
K. C. Wright,* R. B. Blakestad,† C. J. Lobb,‡ W. D. Phillips, and G. K. Campbell

Joint Quantum Institute, National Institute of Standards and Technology and University of Maryland, Gaithersburg, Maryland 20899, USA

(Received 26 July 2013; published 20 December 2013)

rotating weak link

(wide barrier)



PRL **110**, 025302 (2013) PHYSICAL REVIEW LETTERS week ending 11 JANUARY 2013

Driving Phase Slips in a Superfluid Atom Circuit with a Rotating Weak Link

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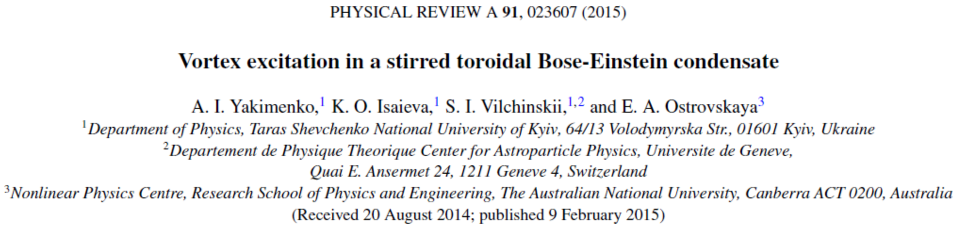

PHYSICAL REVIEW A **91**, 023607 (2015)

Vortex excitation in a stirred toroidal Bose-Einstein condensate

A. I. Yakimenko,[1] K. O. Isaieva,[1] S. I. Vilchinskii,[1,2] and E. A. Ostrovskaya[3]

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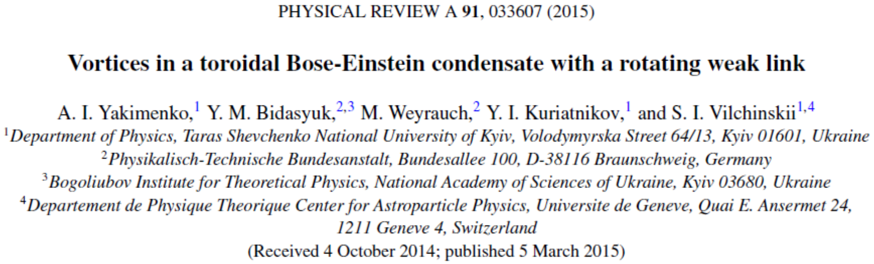
[2]*Departement de Physique Theorique Center for Astroparticle Physics, Universite de Geneve, Quai E. Ansermet 24, 1211 Geneve 4, Switzerland*

[3]*Nonlinear Physics Centre, Research School of Physics and Engineering, The Australian National University, Canberra ACT 0200, Australia*

(Received 20 August 2014; published 9 February 2015)



PHYSICAL REVIEW A **91**, 033607 (2015)

Vortices in a toroidal Bose-Einstein condensate with a rotating weak link

A. I. Yakimenko,[1] Y. M. Bidasyuk,[2,3] M. Weyrauch,[2] Y. I. Kuriatnikov,[1] and S. I. Vilchinskii[1,4]

[1]*Department of Physics, Taras Shevchenko National University of Kyiv, Volodymyrska Street 64/13, Kyiv 01601, Ukraine*

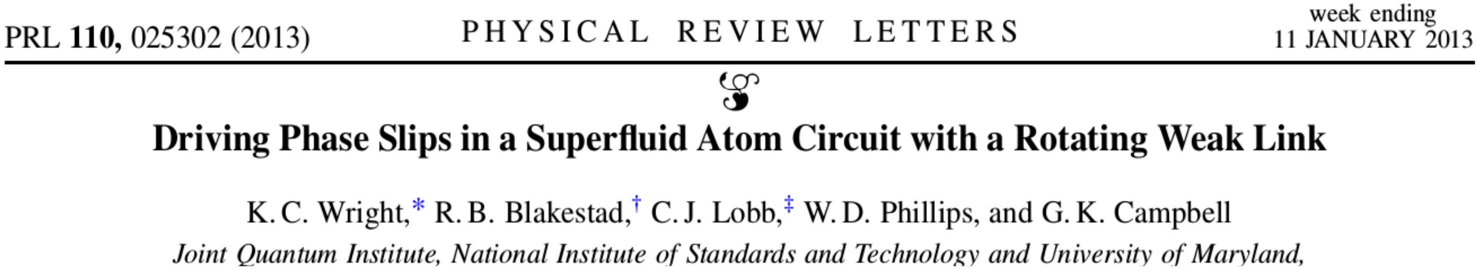
[2]*Physikalisch-Technische Bundesanstalt, Bundesallee 100, D-38116 Braunschweig, Germany*

[3]*Bogoliubov Institute for Theoretical Physics, National Academy of Sciences of Ukraine, Kyiv 03680, Ukraine*

[4]*Departement de Physique Theorique Center for Astroparticle Physics, Universite de Geneve, Quai E. Ansermet 24, 1211 Geneve 4, Switzerland*

(Received 4 October 2014; published 5 March 2015)

The experiment with a wide stirrer



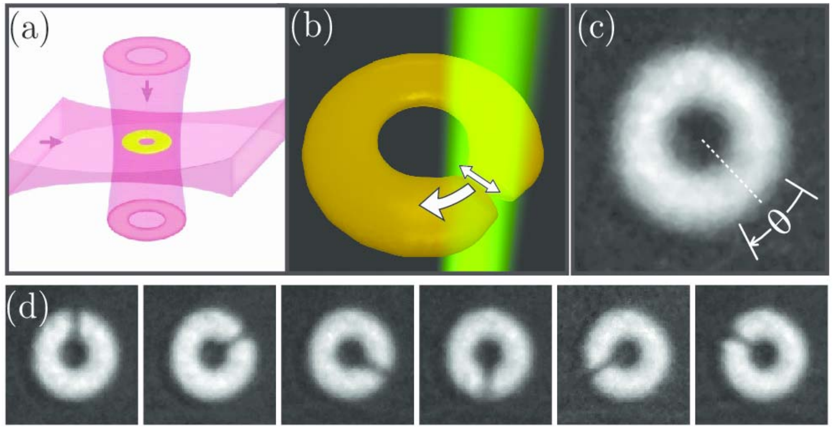
PRL **110,** 025302 (2013) PHYSICAL REVIEW LETTERS week ending 11 JANUARY 2013

Driving Phase Slips in a Superfluid Atom Circuit with a Rotating Weak Link

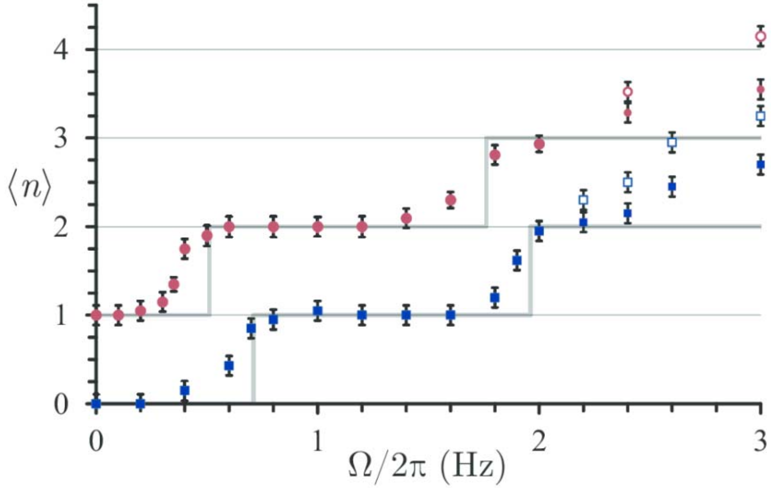
K. C. Wright,* R. B. Blakestad,† C. J. Lobb,‡ W. D. Phillips, and G. K. Campbell

Joint Quantum Institute, National Institute of Standards and Technology and University of Maryland,

The condensate of $6\cdot10^5$ ^{23}Na atoms is created in an all-optical dipole trap formed by two red-detuned laser beams. The rotating weak-link is created by a moving blue-detuned beam.

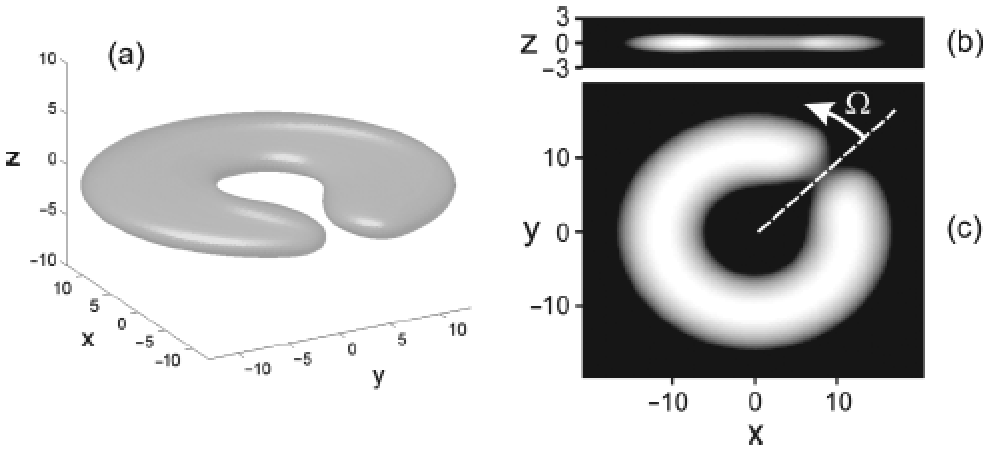


Trap setup with a rotating barrier beam



Vorticity changes sharply with barrier rotation speed (phase slips)

Model: 3D dissipative GPE

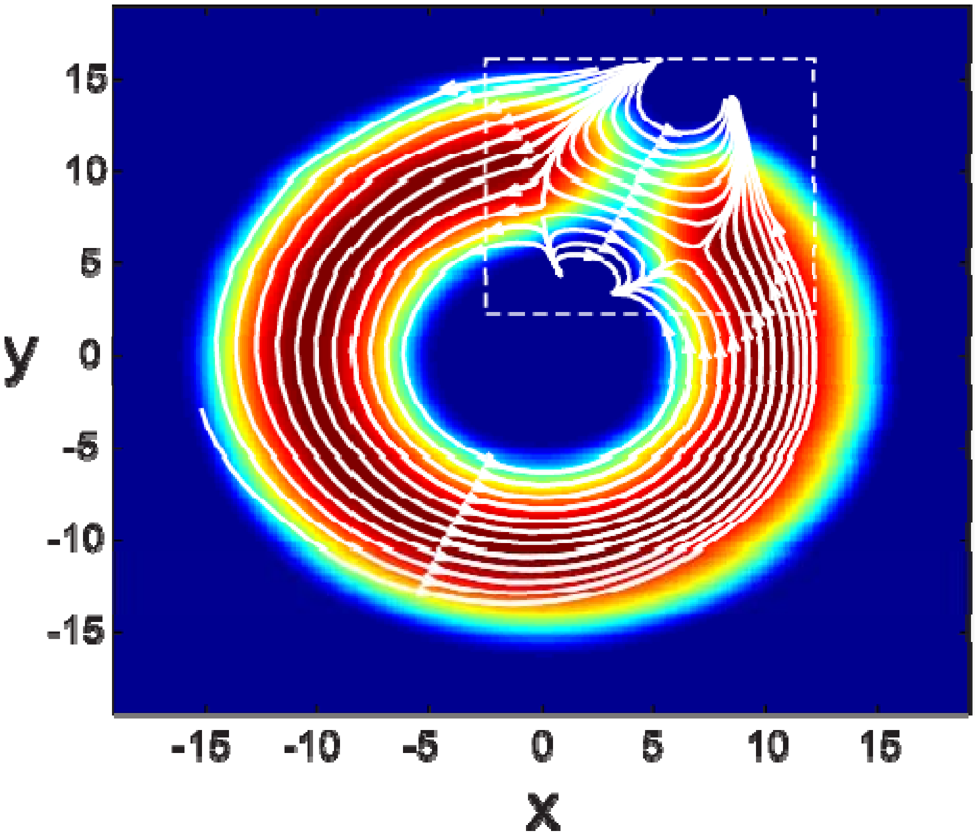


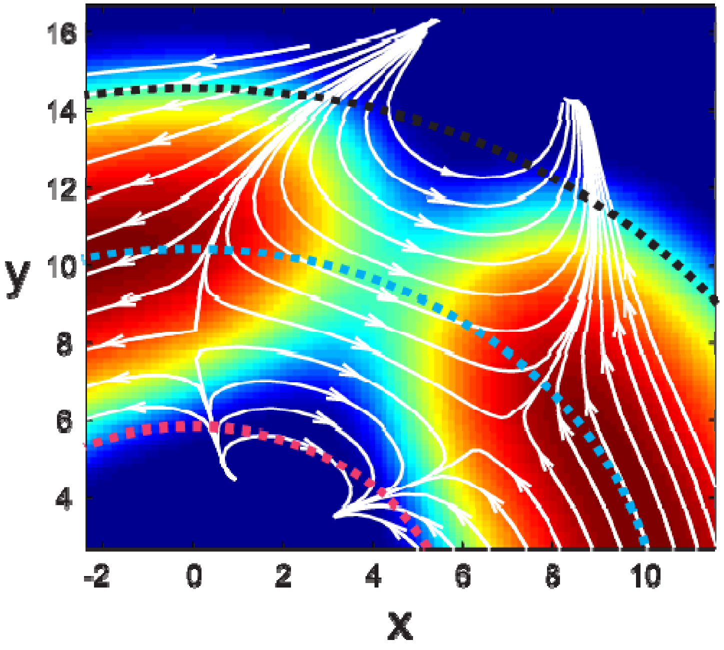
$$(i-\gamma)\frac{\partial \psi}{\partial t} = \left[-\frac{1}{2}\Delta + V(\mathbf{r},t) + g|\psi|^2 - \mu\right]\psi$$

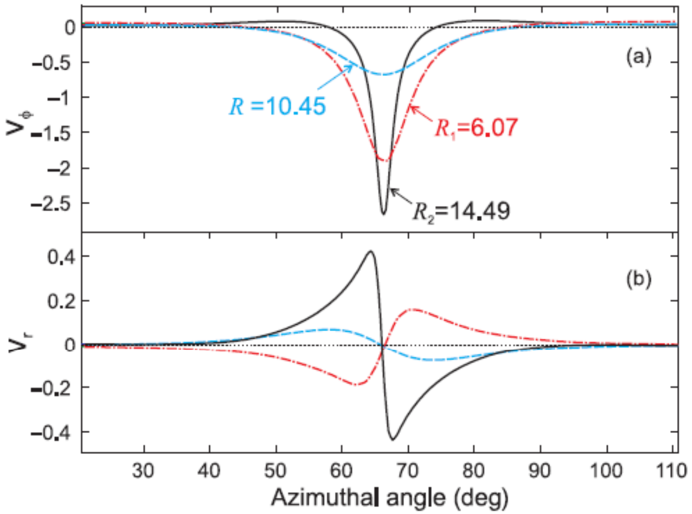
$$V_t(r,z) = \frac{1}{2}M\omega_r^2(r-R)^2 + \frac{1}{2}M\omega_z^2 z^2$$

$$V_b(\mathbf{r}_\perp,t) = U(t)\Theta(\mathbf{r}_\perp \cdot \mathbf{n})e^{-\frac{1}{2c^2}[\mathbf{r}_\perp \times \mathbf{n}]^2}$$

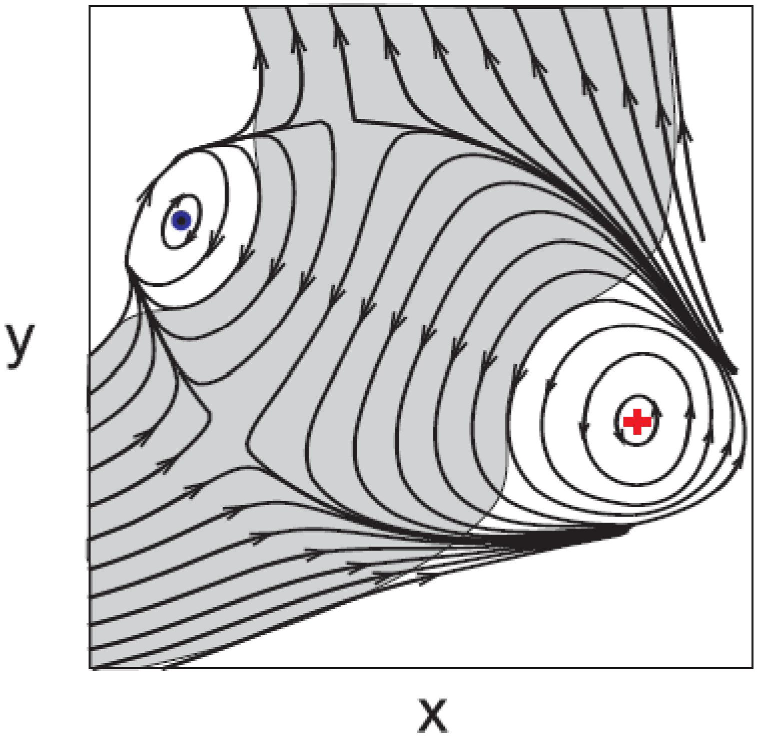
The superflow structure favors formation of a vortex-antivortex dipole inside the weak link



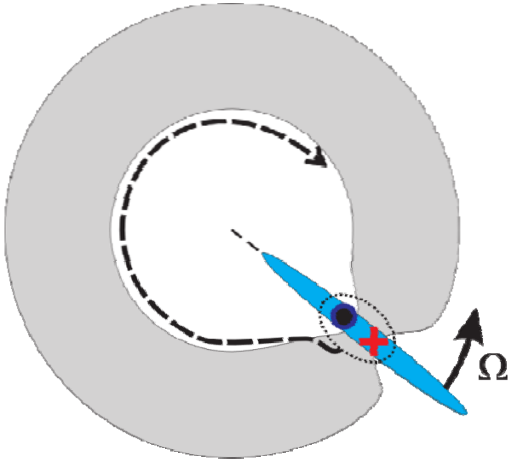




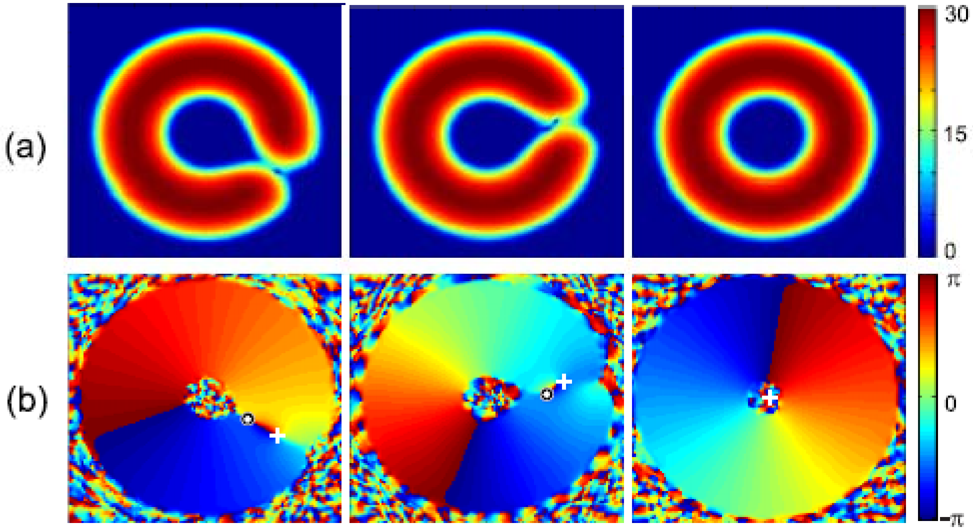
The phase slips observed in our numerical simulations are generally accompanied by formation of a <u>moving</u> vortex dipole.

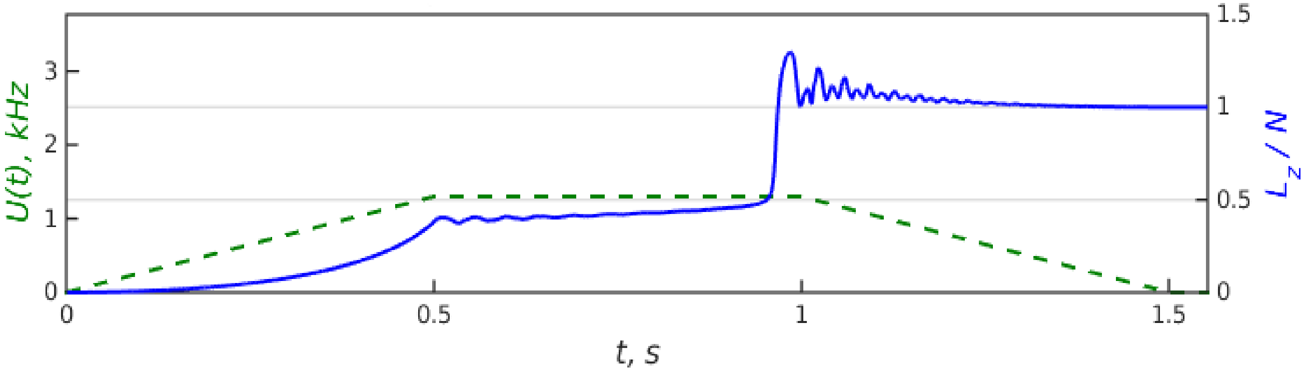


The phase slip occurs when the dipole is formed by a vortex from the outer region and anti-vortex from the inner region



$$v_d \sim \frac{\hbar}{MD}\ln(D/\xi)$$

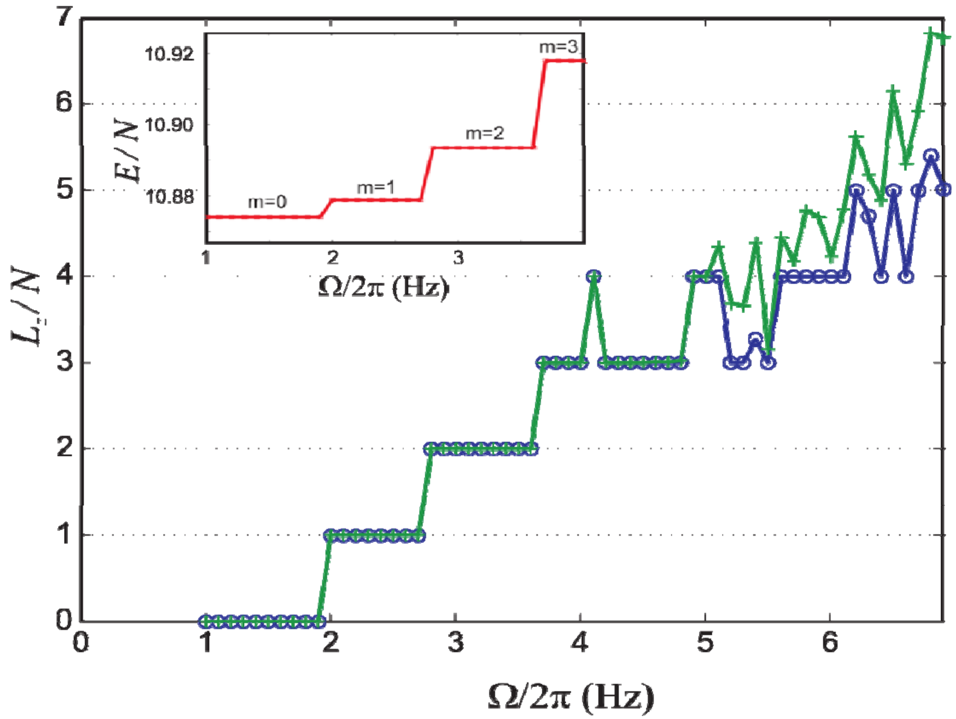




0 → 1 phase slip

$$\Omega/2\pi = 2\ \mathrm{Hz}$$

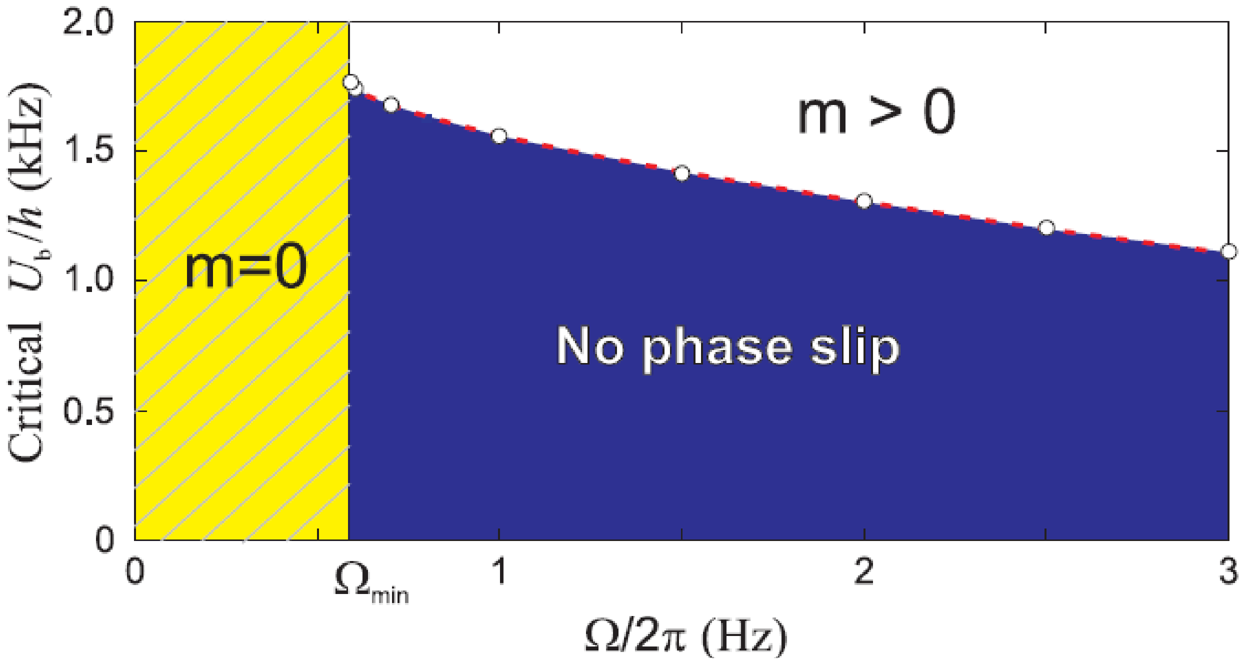
$$U_b = 1.3\ \mathrm{kHz}$$



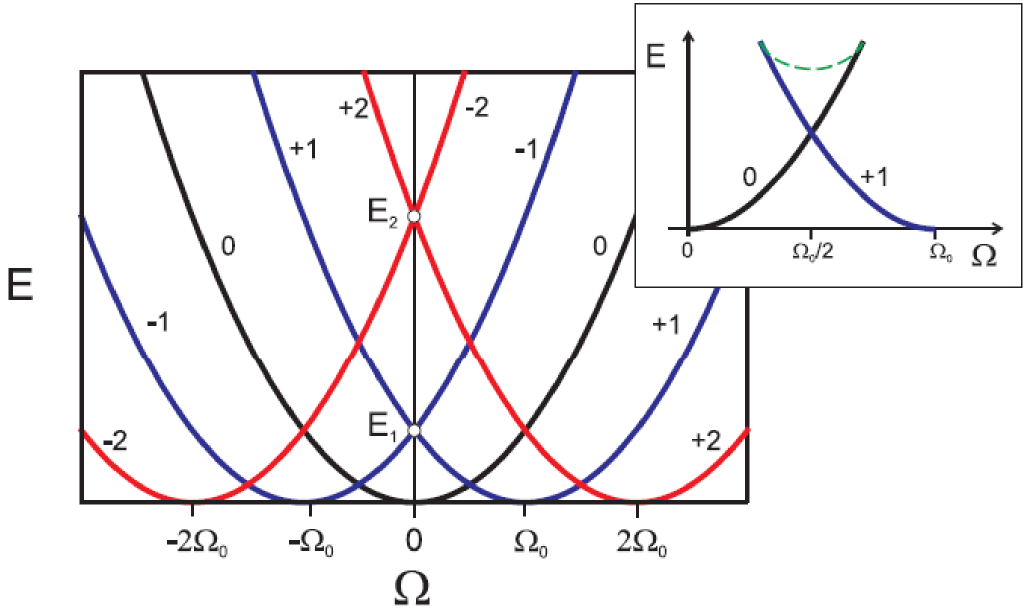
Obtained phase slips are in qualitative agreement with the experiment

The threshold for the phase slips in (U_b, Ω) plane correlates with analytical results

$$(\Omega - \Omega_\Lambda)^2 = \alpha^6 (U_\Lambda - U_b)^3$$



The phase slip corresponds to the classical cusp catastrophe.



$$(\Omega-\Omega_\Lambda)^2=\alpha^6(U_\Lambda-U_b)^3$$

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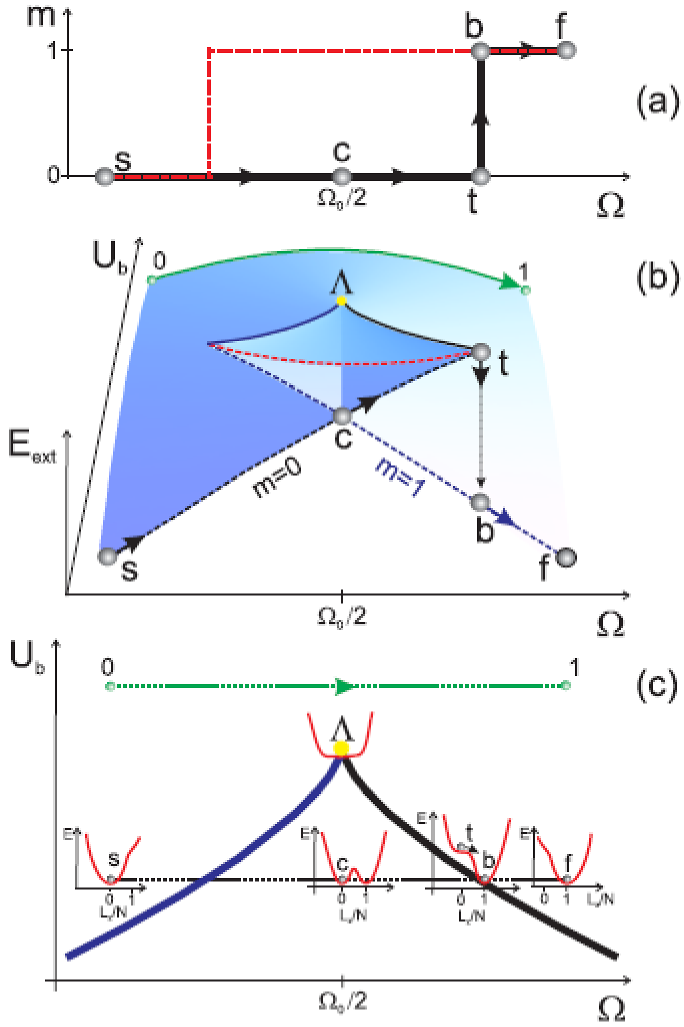
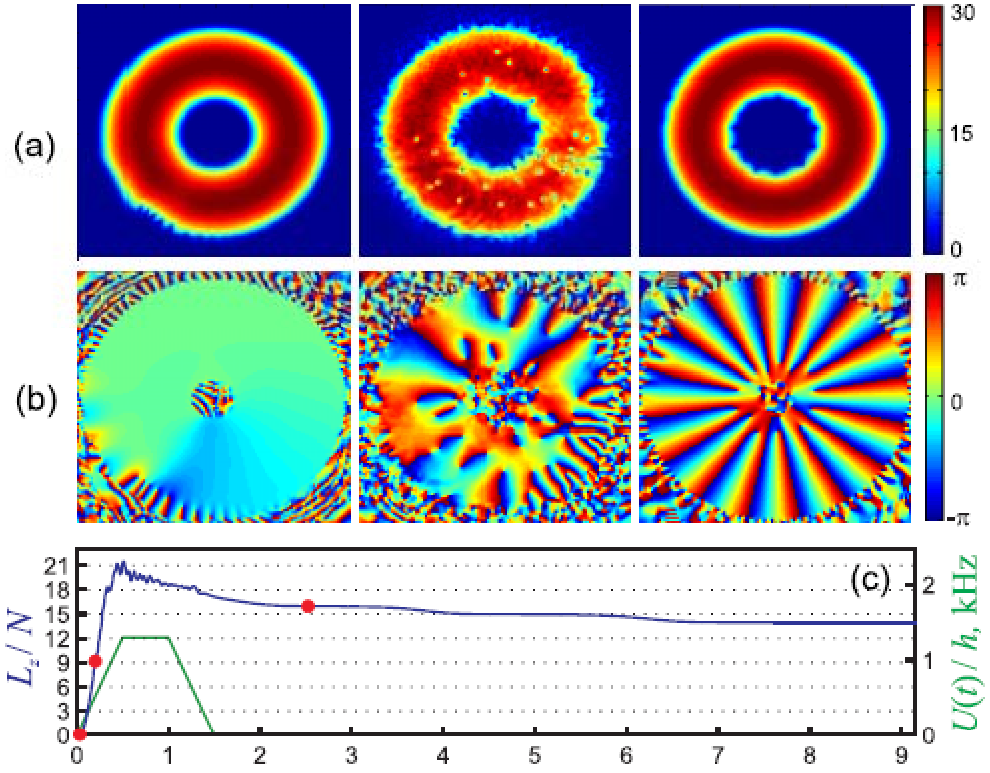
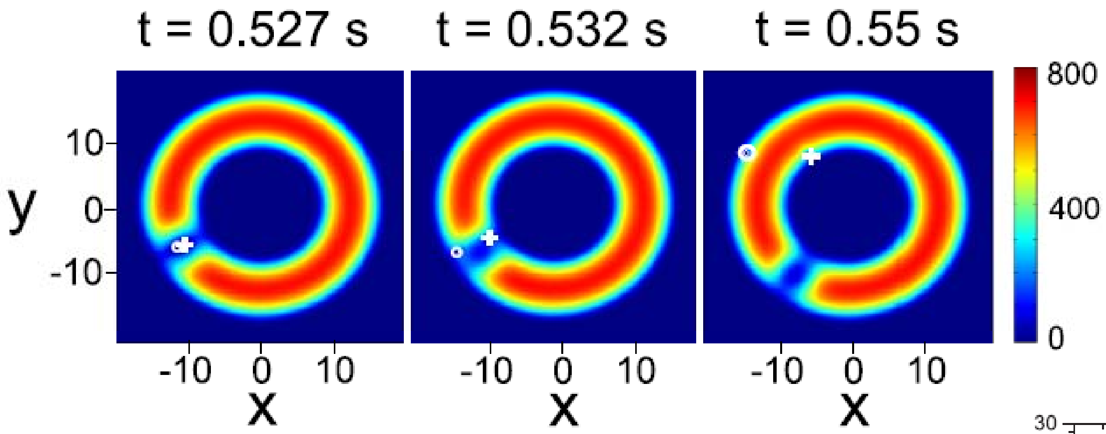
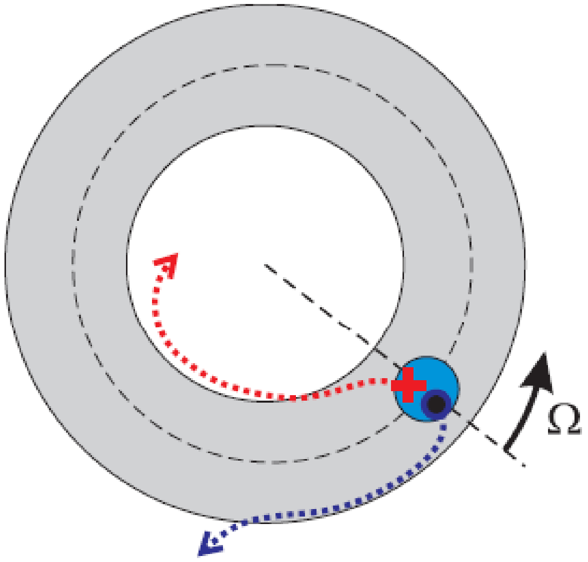
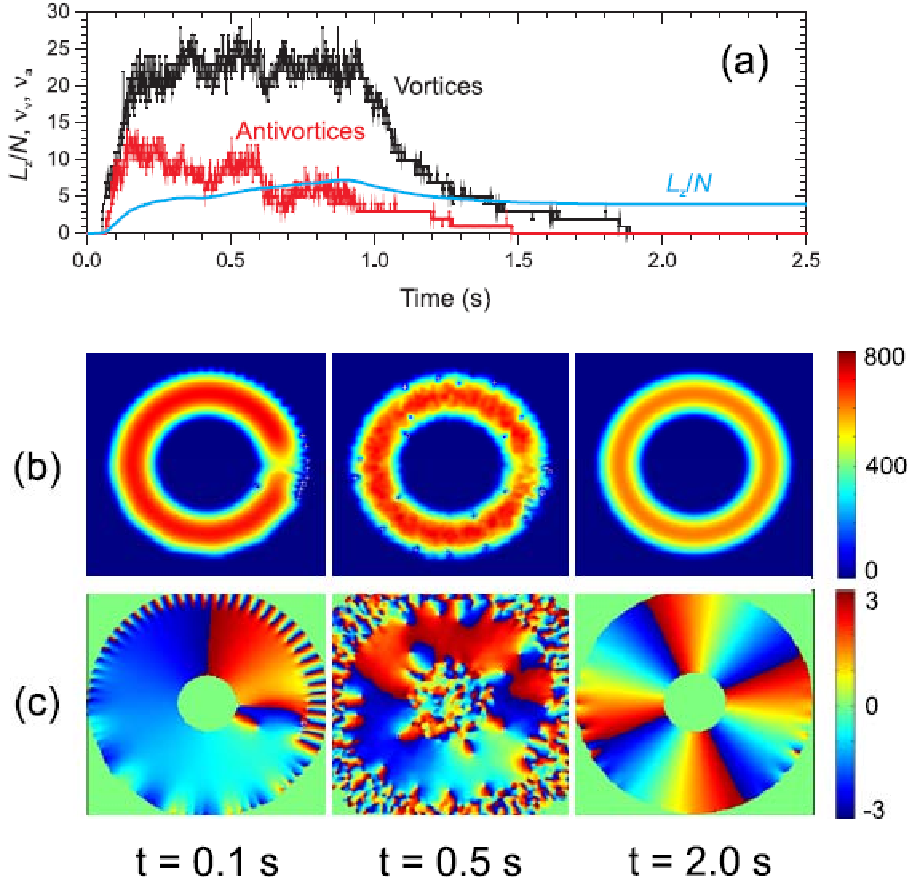


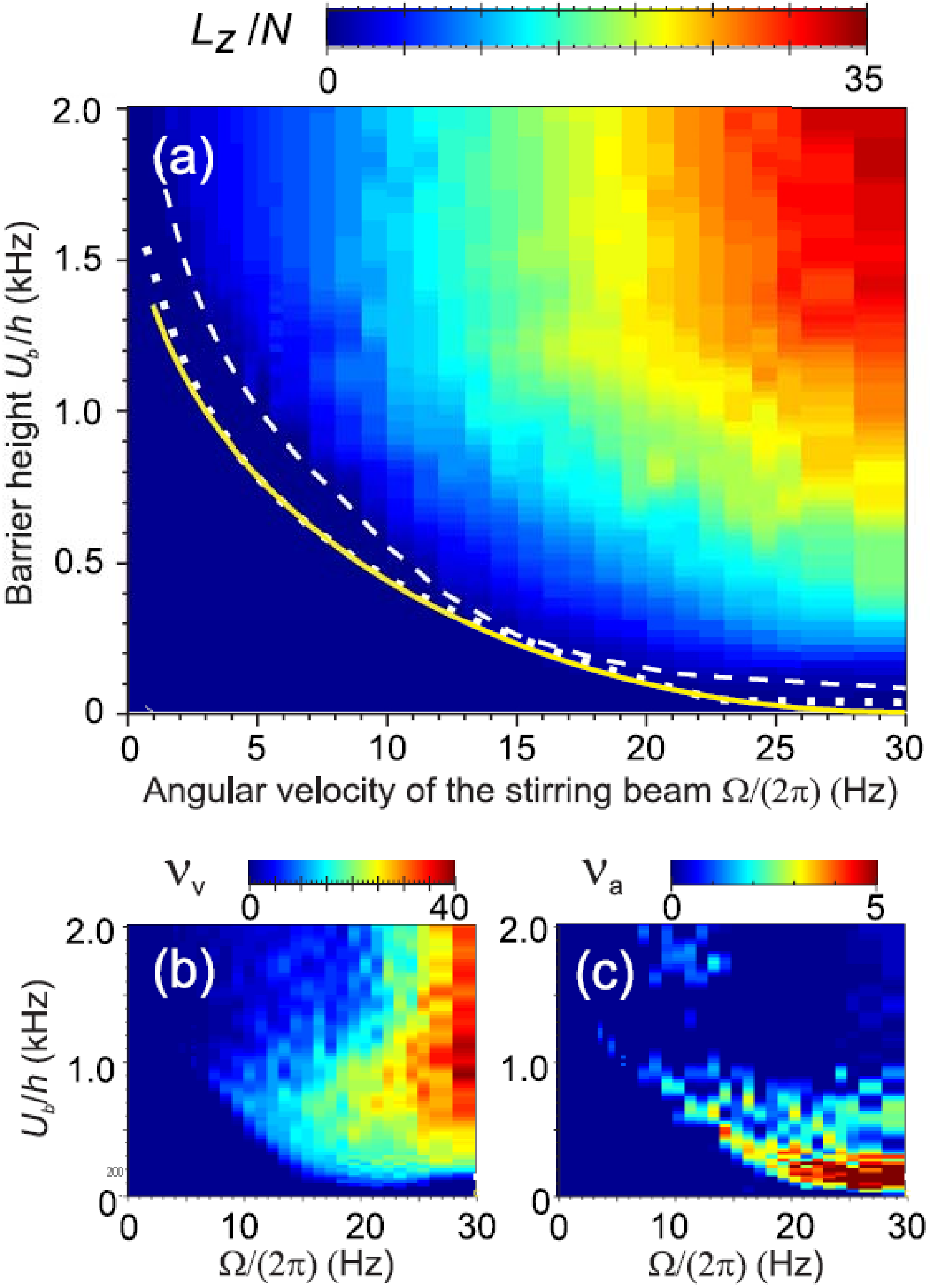
Rapidly rotating weak link produces 2D quantum turbulence, which relaxes to the large-scale multiply-charged persistent current



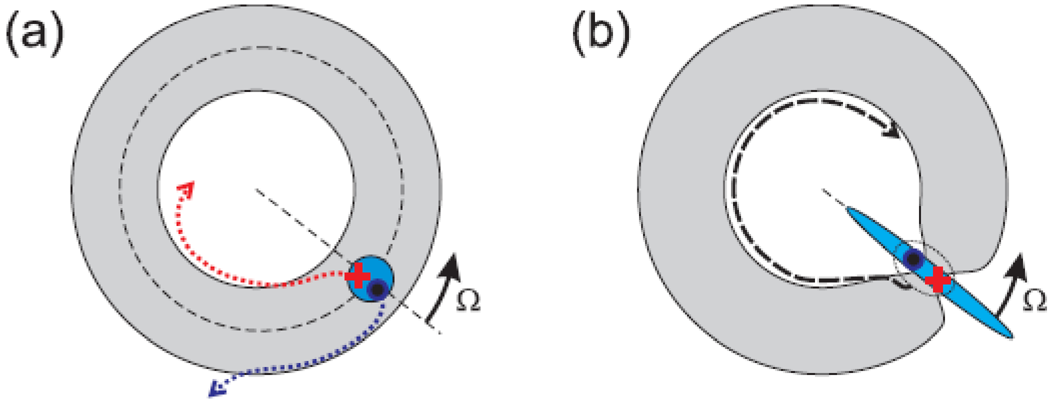

t = 0.527 s
t = 0.532 s
t = 0.55 s
800
400
0
y
10
0
-10
-10
0
10
x
x
x


Ω


(a)
L_z/N, ν_v, ν_a
30
25
20
15
10
5
0
Vortices
Antivortices
L_z/N
0.0
0.5
1.0
1.5
2.0
2.5
Time (s)
(b)
(c)
800
400
0
3
0
-3
t = 0.1 s
t = 0.5 s
t = 2.0 s


L_Z / N
0
35
(a)
Barrier height U_b/h (kHz)
2.0
1.5
1.0
0.5
0
0
5
10
15
20
25
30
Angular velocity of the stirring beam $\Omega/(2\pi)$ (Hz)
ν_v
0
40
(b)
U_b/h (kHz)
2.0
1.0
200
0
0
10
20
30
$\Omega/(2\pi)$ (Hz)
ν_a
0
5
(c)
2.0
1.0
0
0
10
20
30
$\Omega/(2\pi)$ (Hz)

There are two different scenarios of the persistent current generation in toroidal BEC by rotating blue-detuned laser beams.



Decay of the persistent current and hysteresis in a toroidal BEC

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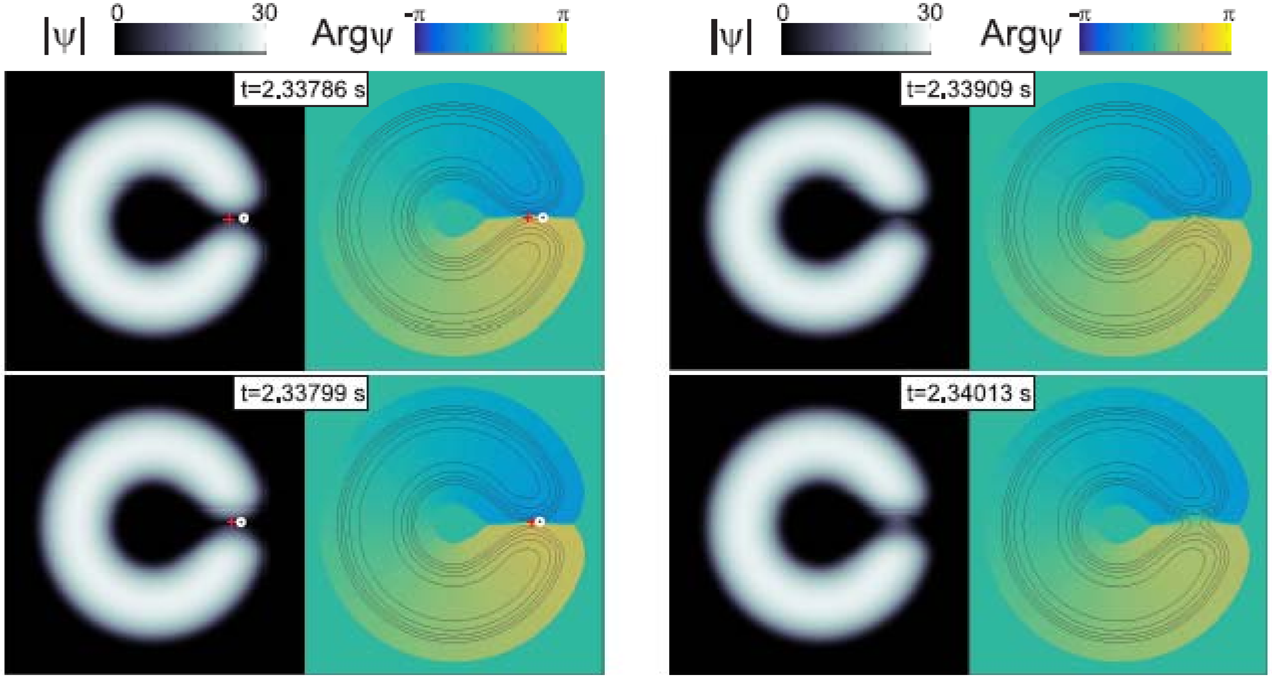
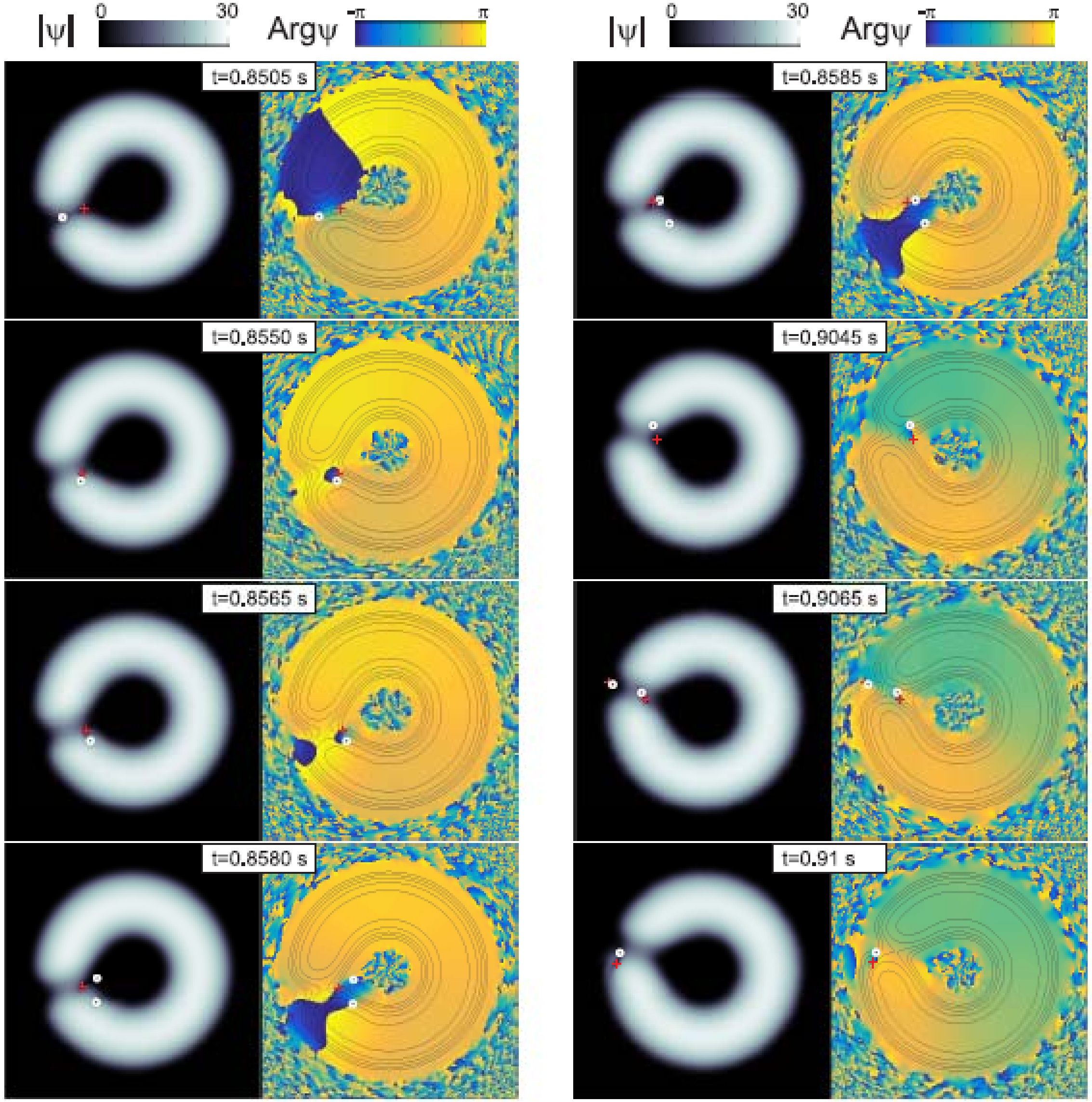
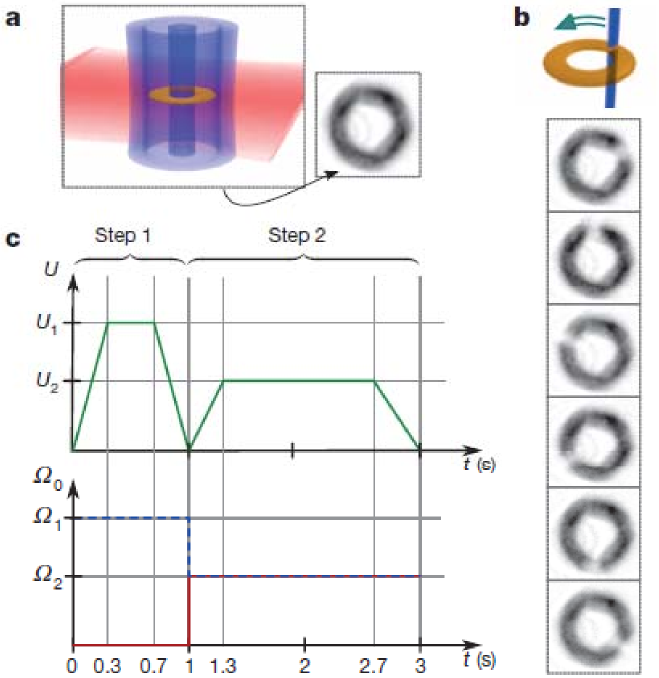
As known [F. Piazza (2009)], the non-rotating barrier leads to decay of the persistent current via annihilation of the vortex-antivortex pair inside the weak link.

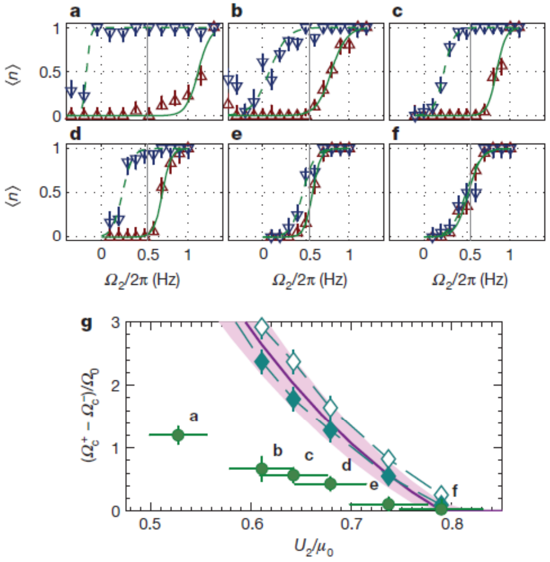


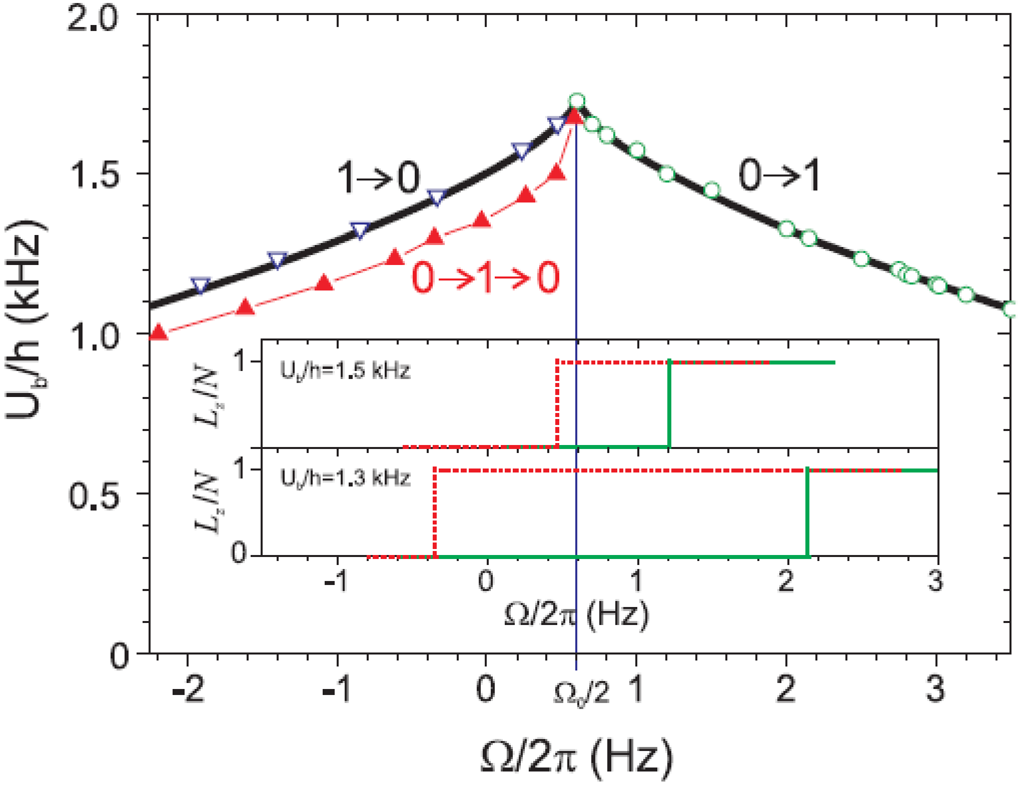
Rotating barrier destroys the persistent current by producing a moving dipole





Nature, **506**, (2014)





Conclusion

To obtain a quantative correspondence with the experiment the 1/e lifetime τ of the atomic cloud should be about 3 s. In real experiments τ ≈10-20 s.

Possible solutions of the hysteresis puzzle:
theory – stochastic fluctuations
experiment – using the improved trapping potential without significant azimuthal impurities.