## Imaging the collective excitations of a quantum gas using statistical correlations



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### Introduction

- Following the dynamics of a quantum gas is relevant for many problems: superfluid behaviour (superfluid fraction [Grimm,Stringari], symmetry breaking (e.g. in 2D) ...), atomtronics, Kibble-Zurek mechanisms, transport, etc.
- Superfluidity in particular is a dynamical property, characterised by collective modes, vortices, critical velocity, persistent flow ...
- Tracking the excitations is also a relevant diagnostic, for atomtronics (performance of waveguides, critical velocity through a rotating barrier...) or to study quantum turbulence [Tsubota, B. Anderson, Gasenzer, D. Hall, Bagnato...]
- This requires new analysis capabilities (e.g.: analysing movies)



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- Impressive improvement in imaging techniques [Greiner,Bloch]
- Time-resolved multiple detection of the same sample possible in some cases (phase contrast imaging)

 $\Rightarrow$  image analysis also must be improved when many pictures are taken, noise must be eliminated

Statistical tools are relevant to extract the maximum information from a complex system or a noisy environment.



- Statistical data analysis is widely used in astronomy or biology (low signal, noisy environment)
- Recently applied to cold atoms to perform noise filtering (e.g. in interferometry [Dana Anderson2010,Kasevich2011]) or noise analysis, when taking many pictures in the same conditions [Farkas et al. 2015]
- Principal component analysis is a powerful tool, more robust than FFT

In this talk, application of principal component analysis to a series of absorption images to identify the amplitudes of the true collective excitations of a real (not idealised) 2D gas in a harmonic trap.



## Outline

#### Collective modes in two dimensions

- The 2D Bose gas
- Excitation spectrum and critical velocity
- A very smooth trap for the study of collective modes
- The monopole mode
- The scissors mode

#### Imaging the collective modes with PCA

- Excitation of the 2D gas
- Applying PCA
- Identification of the collective modes
- Comparison with numerical simulations

### 3 Outlook



## The two-dimensional Bose gas 2D: A marginal dimension

2D is a very special case! In contrast with 3D, superfluidity doesn't appear together with BEC [bosons: ENS, Chicago, Palaiseau, Seoul...]

 homogeneous case: Berezinslii-Kosterlitz-Thouless transition: a superfluid transition relying on vortex-antivortex pairing, but not BEC fraction.



• BEC recovered in a trap. Summary:

lueal	interacting	
no BEC, no SF	BKT SF	
BEC, no SF	BKT+BEC	
	no BEC, no SF BEC, no SF	



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#### The two-dimensional Bose gas Scaling symmetry

2D is a very special case!

- scaling invariance  $r \to \lambda r$ :  $E_K \to \frac{1}{\lambda^2} E_K$ ,  $E_{int} \to \frac{1}{\lambda^2} E_{int}$
- no length scale: dimensionless interaction strength  $\tilde{g}$

$$g=rac{\hbar^2}{M} ilde{g}$$

cf EOS( $\mu/k_BT$ ) work at ENS / Chicago

• Pitaevskii-Rosch monopole mode in an isotropic trap:



- no damping [ENS 2002, in a cigar]
- $\Omega_{M}=2\omega$  for all amplitudes
- linked to scaling symmetry



Low energy modes of a trapped 2D superfluid

Excitation spectrum in a trap:





Low energy modes of a **trapped 2D superfluid** Excitation spectrum in a trap:



- dipole mode (m = 1), both superfluid and thermal: centre of mass oscillation



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- monopole (m = 0): superfluid and thermal signature of the EOS









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Low energy modes of a **trapped 2D superfluid** Excitation spectrum in a trap:



- monopole (m = 0): superfluid and thermal signature of the EOS
- quadrupole (m = 2)superfluid only





- dipole mode (m = 1), both superfluid and thermal: centre of mass oscillation

- scissors for  $\omega_x \neq \omega_y$ superfluid only

[N.B.: thermal gas always in the collisionless regime]

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Superfluid two-dimensional quantum gas Measuring the critical superfluid velocity

Low energy modes of a trapped 2D superfluid

Excitation spectrum in a trap:



red line: gives the critical velocity, related to surface modes [Anglin2001]. Can be measured (in principle) with a rotating defect, see [Desbuquois2012].





Superfluid two-dimensional quantum gas Measuring the critical superfluid velocity

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Global measurement: A heating is measured as the laser is stirred. But the (first) excited modes have not been identified.  $\Rightarrow$  a way to identify multiple excited modes is missing!



#### Two examples of collective modes Monopole vs scissors

We study experimentally two low energy collective modes:

- The monopole mode probes the equation of states.
- The scissors mode probes superfluidity.

Experimental constraints:

- The gas is confined to 2D ...
- in a very smooth harmonic potential.

First approach: 'top-down', or fit with an expected mode shape.



#### Confining a gas to two dimensions A very anisotropic trap

- very anisotropic harmonic trap  $\omega_z \gg \omega_x, \omega_y$
- z degree of freedom is frozen if  $\mu$ ,  $k_BT < \hbar\omega_z$
- confinement size  $a_z = \sqrt{\frac{\hbar}{M\omega_z}}$



Dimensionless coupling constant depends on confinement:

$$\tilde{g} = \sqrt{8\pi} \frac{a}{a_z}$$

Study of collective modes: need a smooth horizontal potential.



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#### rf-induced adiabatic potentials The dressed guadrupole trap

Adiabatic potentials for rf-dressed atoms: the dressed quadrupole trap

- smooth potentials (magnetic fields with large coils)
- naturally very anistropic
- geometry can be modified dynamically





Atoms are confined to the isopotentials of a quadrupole field.



## rf-induced adiabatic potentials

isomagnetic surfaces: ellipsoids with  $r_0 \propto \frac{\omega_{\rm rf}}{b'}$ 



$$\omega_z \propto rac{b'}{\sqrt{\Omega}} \sim 1\text{-}2 \ \mathrm{kHz} \qquad \omega_x, \omega_y \propto \sqrt{rac{g}{r_0}} \sim 20\text{-}50 \ \mathrm{Hz}$$

anisotropy  $\eta = \frac{\omega_{\chi}}{\omega_{y}}$  controlled through rf polarisation NB:  $\eta = 1$  with a circular rf polarisation



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isomagnetic surfaces: ellipsoids with  $r_0 \propto \frac{\omega_{\rm rf}}{h'}$ 



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temperature T controlled with a rf knife at  $\omega_{\rm rf} + \Omega + \nu_{\rm cut}$ 



isotropic harmonic 2D trap, frequency  $\omega$ 

• monopole probes the compressibility  $\Rightarrow \Omega_M$  is related to the 2D EOS  $\mu(n)$ :

$$\Omega_{M}=\sqrt{2(2+\epsilon)}\,\omega$$
 with  $\epsilon=rac{n\mu^{\prime\prime}(n)}{\mu^{\prime}(n)}$ 

cf Rudi Grimm's expt with fermions [Altmeyer 2006]

• Ex: 2D weakly interacting gas:  $\mu(n) = gn \Rightarrow \Omega_M = 2\omega$ 



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- Ex: flat, but 3D gas:  $\mu(n) \propto n^{2/3} \Rightarrow \Omega_M = \sqrt{10/3} \, \omega$



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- Ex: 2D weakly interacting gas:  $\mu(n) = gn \Rightarrow \Omega_M = 2\omega$
- Ex: quantum anomaly:  $\tilde{g}/(16\pi)$  positive shift [Olshanii 2010]
- Ex: flat, but 3D gas:  $\mu(n) \propto n^{2/3} \Rightarrow \Omega_M = \sqrt{10/3} \, \omega$
- we probe the intermediate case: for non negligible interactions is there a shift as a function of  $\alpha = \frac{\mu}{2\hbar\omega_z}$ ? [Merloti 2013]



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# Observation of the monopole mode Isotropic trap

Circular rf polarisation  $\Rightarrow$  isotropic 2D trap Excitation through a sudden change in  $\omega$ Very low T (no thermal fraction)

• experimental data: fit of TF radius after a tof \_\_\_\_\_\_ sinusoidal fit [Merloti NJP2013]



typical data:  $\Omega_M$  close to  $2\omega$ ; no measurable damping



## Results: shift of the monopole mode A modified EOS

We observe a small negative shift as a function of  $\alpha$  [Merloti PRA2013]:





## Results: shift of the monopole mode A modified EOS

#### Comparison with a **3D** GPE simulation:





## Results: shift of the monopole mode A modified EOS

Comparison with a perturbation theory:





Scissors mode: anisotropic harmonic trap, frequencies  $\omega_x \neq \omega_y$ , anisotropy  $\eta = \omega_y / \omega_x \sim 1.3$ , mean frequency  $\omega_0 = \sqrt{\omega_x \omega_y}$ 

• no scissors mode in the thermal phase, only harmonic modes  $\omega_{\rm X}\pm\omega_{\rm Y}$ 



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- scissors mode expected at  $\omega_{\rm sc} = \sqrt{\omega_{\rm x}^2 + \omega_{\rm y}^2}$  for a superfluid



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- scissors mode expected at  $\omega_{sc} = \sqrt{\omega_x^2 + \omega_y^2}$  for a superfluid
- damping when T increases
- different behaviour in 3D and 2D expected
- $\Rightarrow$  Use the scissors mode as a signature of superfluidity across the BKT transition!



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2D: positive shift expected

#### The scissors mode 3D vs 2D as a function of temperature





Excitation method:

• Prepare a 2D gas in an anisotropic trap





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- Prepare a 2D gas in an anisotropic trap
- Sudden change of the eigenaxes with rf polarisation





Excitation method:

- Prepare a 2D gas in an anisotropic trap
- Sudden change of the eigenaxes with rf polarisation
- Record the cloud axis angle with an *in situ* image





#### The scissors mode Time evolution of the fitted axis angle



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#### Results with a model fit Frequency vs temperature



superfluid / thermal

Thermal gas: the slow oscillation expected at  $\omega_v - \omega_x$ (10 Hz) is not observed: is it really there, hidden by damping or noise?



#### Results with a model fit Frequency vs temperature



Thermal gas: the slow oscillation expected at  $\omega_y - \omega_x$  (10 Hz) is not observed: is it really there, hidden by damping or noise?

#### How can we extract more information with less assumptions?



### Statistical analysis applied to quantum gases

What are the experimental imaging conditions?

- Dynamical system, evolving with time
- Snapshots taken after various times, not necessarily evenly spaced
- Technical noise (fringes, etc.)
- Trap imperfections lead to errors in the predicted shape of collective modes
- Fits made difficult when many parameters are to be fitted
- Excitation procedure, though well controlled, can excited several modes
- $\Rightarrow$  we need a method for extracting data with a minimum input.



### Principal component analysis

• Main idea: extract correlations between images from a given series.



Example: if the left-right dipole mode is excited, there will be a positive correlation between the left pixels in the series, even for irregular sampling. The right pixels will also be correlated, with an opposite phase.



## Principal component analysis

• Main idea: extract correlations between images from a given series.



Method:

Example: if the left-right dipole mode is excited, there will be a positive correlation between the left pixels in the series, even for irregular sampling. The right pixels will also be correlated, with an opposite phase.

- prepare an excited 2D gas
- 2 take successive in situ images
- Sompute and diagonalize the covariance matrix (quite fast!)
- identify the excited modes and filter noise.

See R. Dubessy et al., Fast Track Comm. of New J. Phys. 16, 122001 (2014) + video abstract.



## Expected collective modes

From Bogolubov diagonalisation of an idealised case

Bogolubov modes computed numerically for the 2D gas in a harmonic anisotropic trap  $\omega_x, \omega_y$ : n (0.998) o [1.332] p [1.552] m 2 dipoles  $(\omega_x, \omega_y)$ , quadrupole-like ( $\omega_{\Omega}$ ), q [1.674] r [1.988] s [2.024] t [2.356] scissors  $(\omega_{S}=\sqrt{\omega_{x}^{2}+\omega_{y}^{2}})$ , 4 more modes of higher order symmetry and then x [2.701] u [2.366] v [2.438] w [2.697] monopole-like ( $\omega_M$ )



4 3 b

### Exciting the atomic cloud

A BEC prepared in a plugged quadrupole trap is transferred too fast into the 2D rf-dressed quadrupole trap, whose axes are also suddenly rotated. Several modes are excited during this process.



excited cloud

2D trap frequencies:  $\omega_x = 2\pi \times 33$  Hz,  $\omega_y = 2\pi \times 44$  Hz

133 images taken during 100 ms, after various holding times.



N images,  $N_p$  pixels labeled by an index i: pix<sub>1</sub>, pix<sub>2</sub>,... pix<sub>i</sub> ...



Element (i, j) of the covariance matrix:

$$S_{i,j} = \langle \mathsf{pix}_i \mathsf{pix}_j \rangle - \langle \mathsf{pix}_i \rangle \langle \mathsf{pix}_j \rangle$$

Diagonalization: S has a huge size  $N_p \times N_p$  but its eigenvalues are the same than the covariance of the image series, with a size  $N \times N$  (much smaller).



Diagonalization gives the eigenvariances and the eigenimages or principal components.





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#### Decomposition onto the components



Each image from the original series can be decomposed onto the various PC's:



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# Recovering the modes Dipole mode

Weight onto PC1 as a function of time: oscillation at 44 Hz ( $\omega_y$ ).





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## Recovering the modes Dipole mode

Weight onto PC1 as a function of time: oscillation at 44 Hz ( $\omega_y$ ). We can identify the dipole mode along y.



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#### Recovering the modes Scissors mode

Back to the scissors mode: weight onto PC4.



Fit with 3 frequencies: 12 Hz, 55 Hz, 77 Hz. Now we can identify the 3 frequencies at which the scissors component  $\langle x^2 - y^2 \rangle$  oscillates:  $\omega_y - \omega_x$  becomes visible. The three frequencies can thus coexist also in 2D.



2D modes PCA Outlook Excitation PCA Modes Simulation

### Comparison with a Thomas-Fermi fit

A Thomas-Fermi fit of the density profile with the same data set is able to recover the dipole oscillation, but not the scissors mode:



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### Simulation of GPE vs experiment

PCA applied to a simulation of GPE in the experimental conditions vs PCA applied to the experiment:

а	b [0.410]	c [0.336]	d [0.116]	а	b [0.549]	c [0.243]	d [0.111]
	-		0	•			
e [0.032]	f [0.024]	g [0.011]	h [0.005]	e [0.070]	f [0.007]	g [0.005]	h [0.004]
<b>*</b>	۲	•			9		$\bigcirc$
i [0.004]	j [0.004]	k [0.003]	I [0.003]	i [0.002]	j [0.002]	k [0.001]	I [0.001]
							$\bigcirc$
PCA ap	plied to e	experimer	nt	PCA app	blied to a	simulatior	ı



## Simulation of GPE vs Bogoliubov diagonalization

PCA applied to simulation vs the Bogolubov diagonalisation: the mode amplitudes, with their sign, are clearly captured

m	n [0.998]	o [1.332]	p [1.552]	а	b [0.549]	c [0.243]	d [0.111]
•			•	•			
q [1.674]	r [1.988]	s [2.024]	t [2.356]	e [0.070]	f [0.007]	g [0.005]	h [0.004]
					0	$\bigcirc$	
u [2.366]	v [2.438]	w [2.697]	x [2.701]	i [0.002]	j [0.002]	k [0.001]	I [0.001]
		٢					$\bigcirc$
Bogolubov modes			PCA applied to simulation				



## Mode frequencies

 PCA correctly identifies the modes, with the expected frequency (in units of ω<sub>x</sub>):

mode	$\omega_{ m pca}$	$\omega_{ m diag}$	$\omega_{ m th}$
dipole (x)	0.999	0.998	1
dipole (y)	1.332	1.332	1.334
quadrupole-like	1.547	1.552	1.548
scissors	1.674	1.674	1.667
monopole-like	2.441	2.438	2.438

- PCA gives access to the true modes when the potential differs from the model harmonic potential.
- It is not sensitive to aliasing (like FFT), and also works for not evenly spaced sampling times.



• A 2D Bose gas in a very smooth trap with a tunable geometry





- A 2D Bose gas in a very smooth trap with a tunable geometry
- monopole (EOS) and scissors (superfluidity) modes observed







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- A 2D Bose gas in a very smooth trap with a tunable geometry
- monopole (EOS) and scissors (superfluidity) modes observed
- principal component analysis identifies the mode shapes and frequencies
- The low frequency ω<sub>y</sub> ω<sub>x</sub> component recovered, all 3 frequencies present in 2D







# Outlook: Application to an annular superfluid Superfluid in a ring trap

What do we expect in a ring?

Yet another ring trap, well adapted to 2D Bose gases [Morizot et al. PRA 2006, also Foot 2008]:



rf bubble + light potential



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Very anisotropic trap with independent frequencies (vertical frequency 1-10 kHz, radial 300 Hz - 1 kHz), with dynamically adjustable radius, anisotropy, frequency, etc  $\Rightarrow$  probe 2D superfluidity



#### Critical velocity of an annular superfluid The role of edge modes

#### [Dubessy et al., PRA 2012]

- 2D/3D rings: The Bologlubov spectrum gives access to the critical angular velocity and to the energy stability, evidencing the role of surface modes.
- Due to the way a superfluid rotates, the two edges don't play the same role, in contrast with a superfluid flowing in a straight tube.

inner surface mode



outer surface mode





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2D modes PCA Outlook

## Simulation of the dynamics of stirring

#### Excitation rotating at an increasing frequency $\boldsymbol{\Omega}$



 $\ell = 4$  excitation



rotating barrier

Simulations by Thomas Liennard in his PhD thesis (Paris 13, 2011). See also recent work by Yakimenko et al. Next step: use PCA to identify the excitations triggering the critical velocity or responsible for phase slips!



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