



# HEAVY IONS

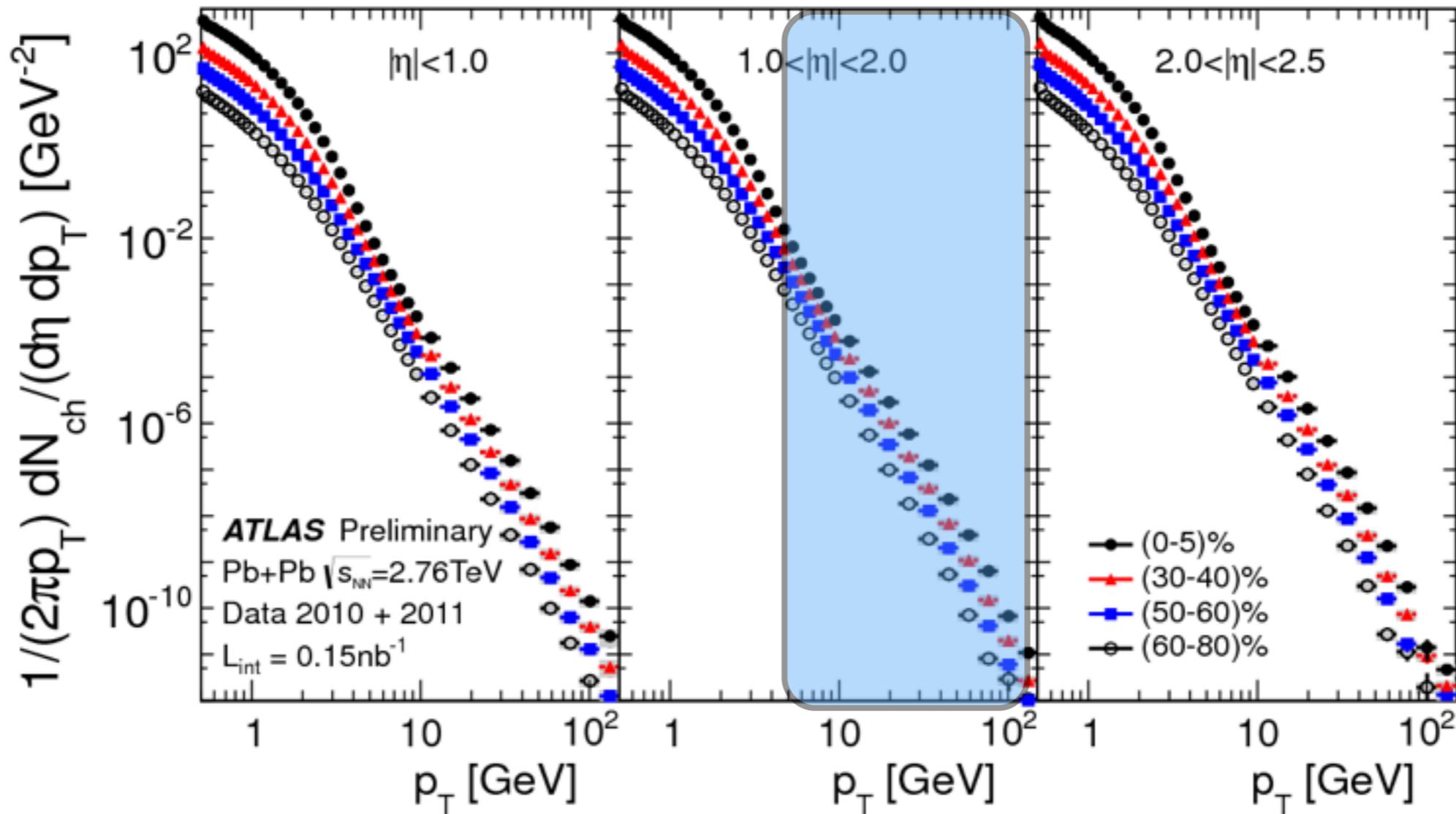
Konrad Tywoniuk

Taller de Altas Energias 2014, Sep 14 - 27, Benasque

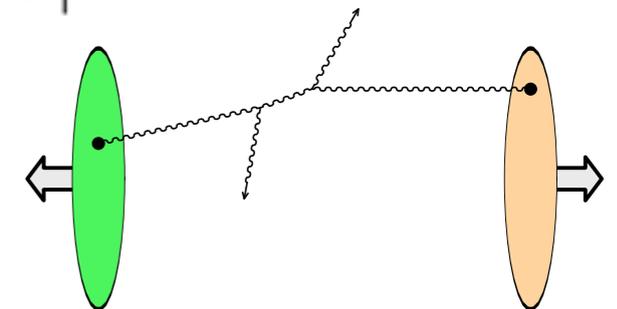
## Lecture II

- quick repetition about jets
- medium-induced radiation
- jet decoherence

# Charged particle spectrum

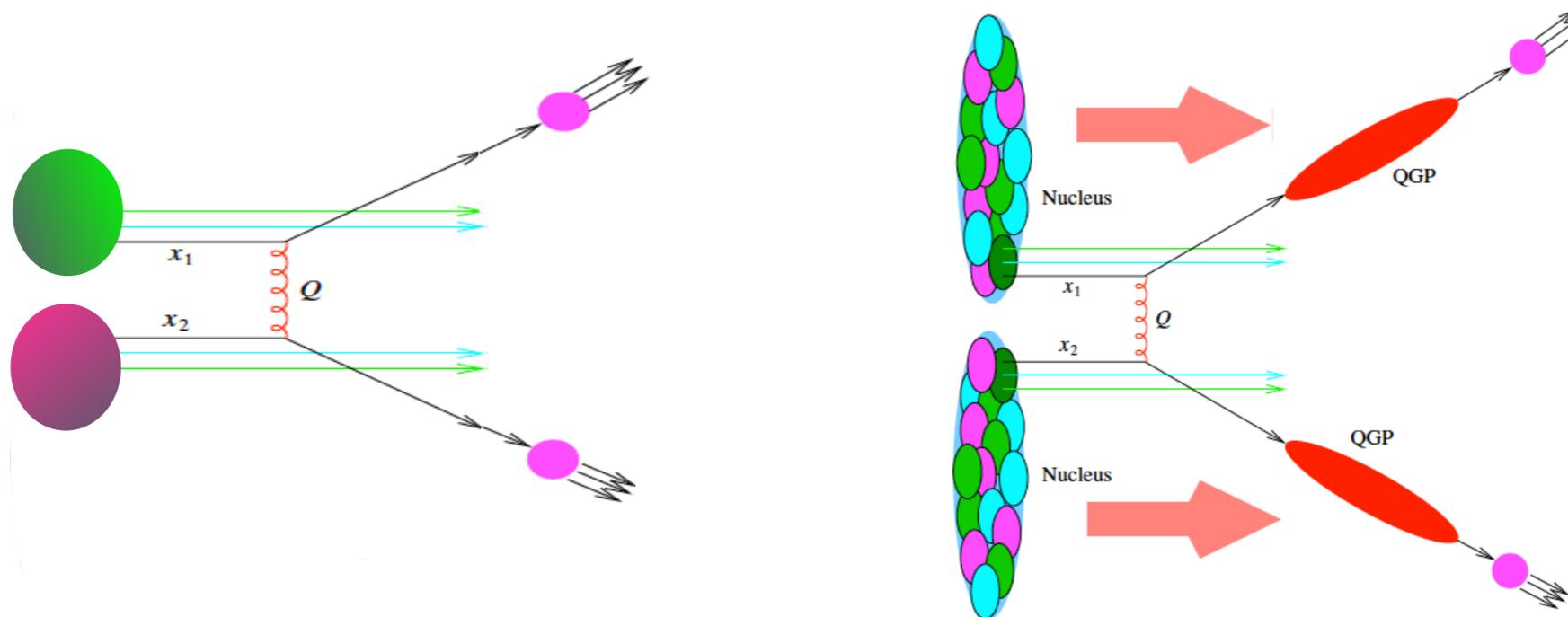


hard probes  
dilute regime  
power-like



# Collinear factorization

Separation of initial- and final-state effects.

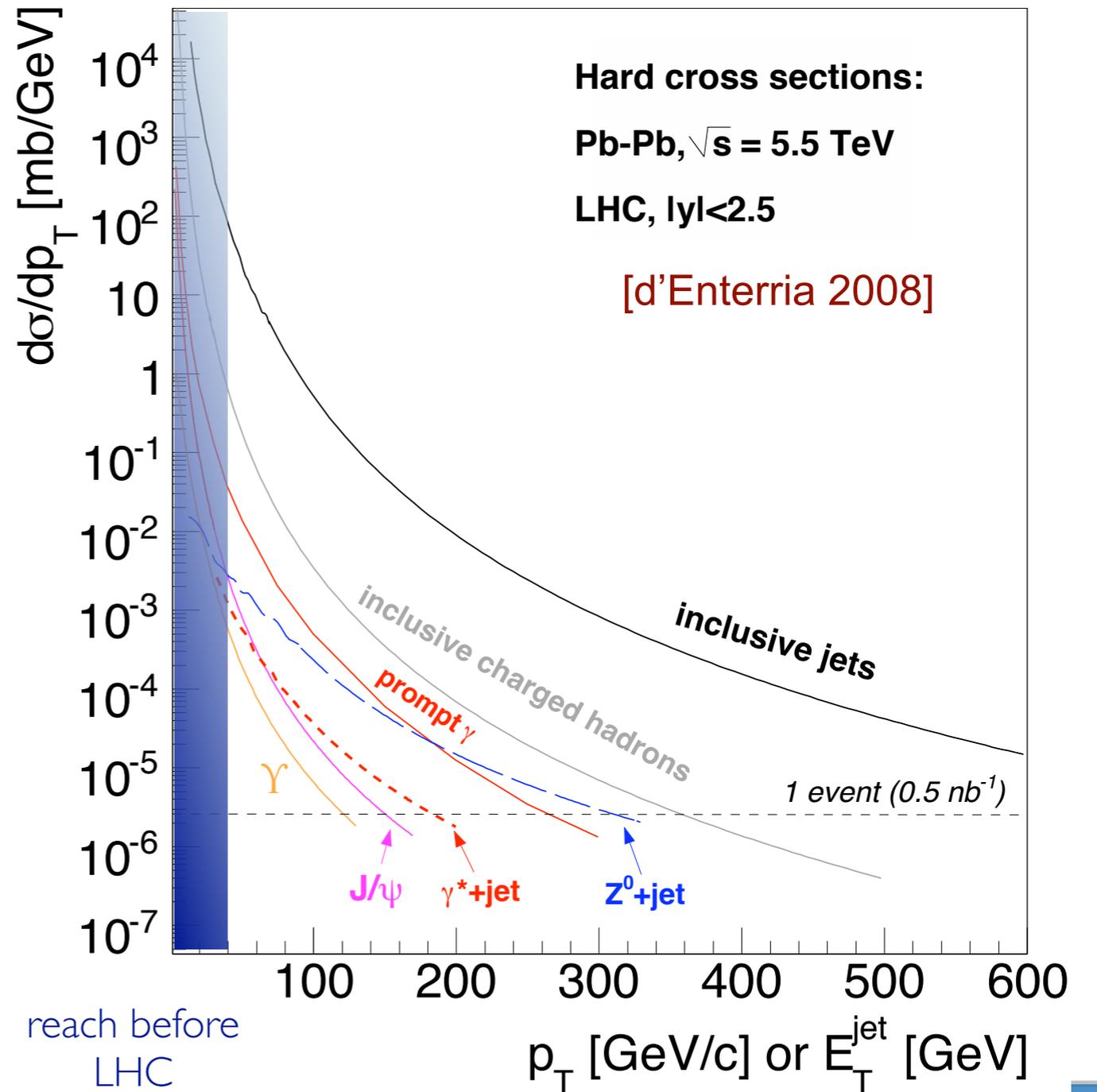
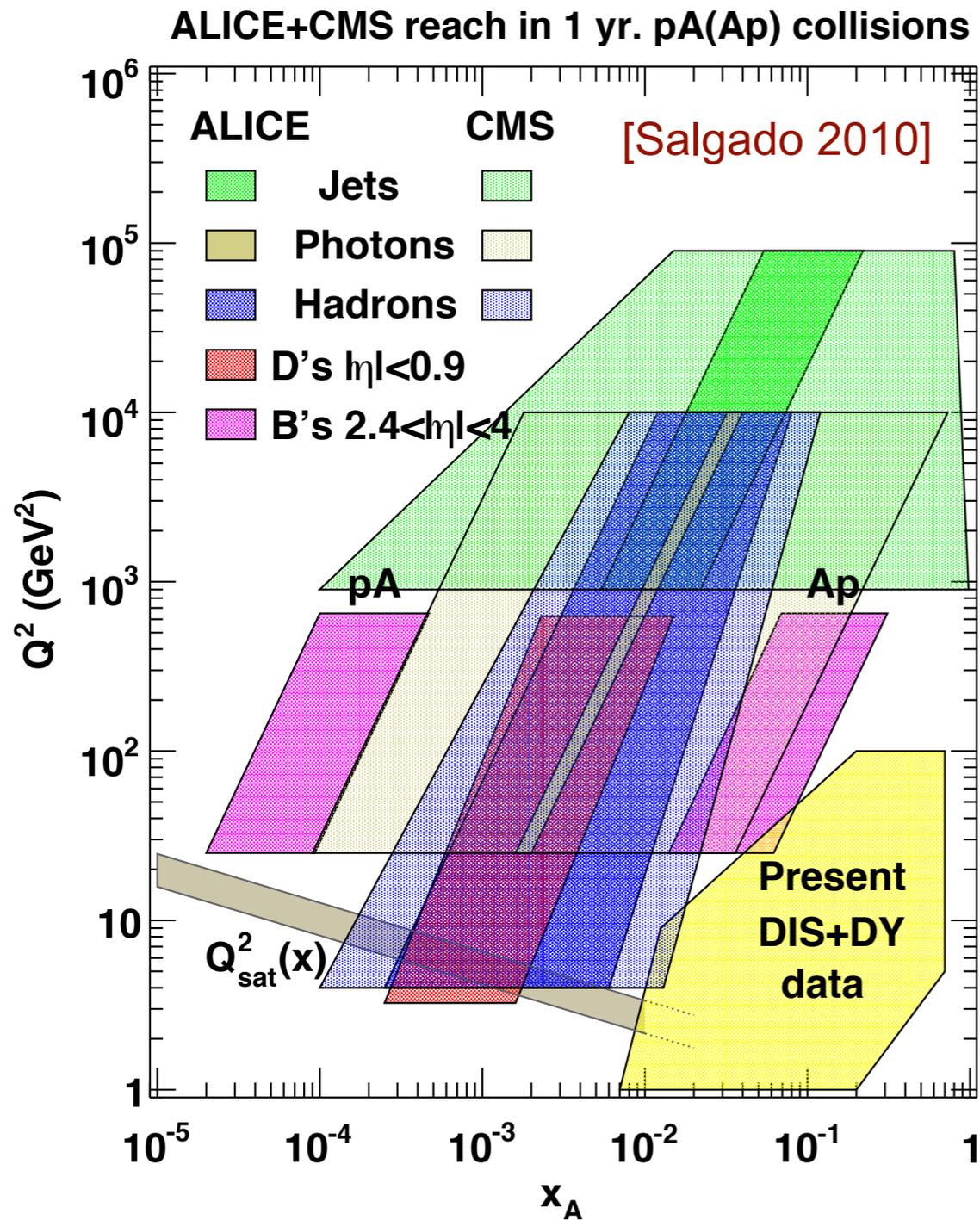


$$\sigma^{pp \rightarrow h} = f_p(x_1, Q^2) \otimes f_p(x_2, Q^2) \otimes \sigma(x_1, x_2, Q^2) \otimes D(z, Q^2)$$

Nuclear PDFs

Modified FFs

# Hard probes at LHC





CMS Experiment at LHC, CERN  
 Data recorded: Tue Nov 9 23:51:56 2010 CEST  
 Run/Event: 150590 / 776435  
 Lumi section: 183

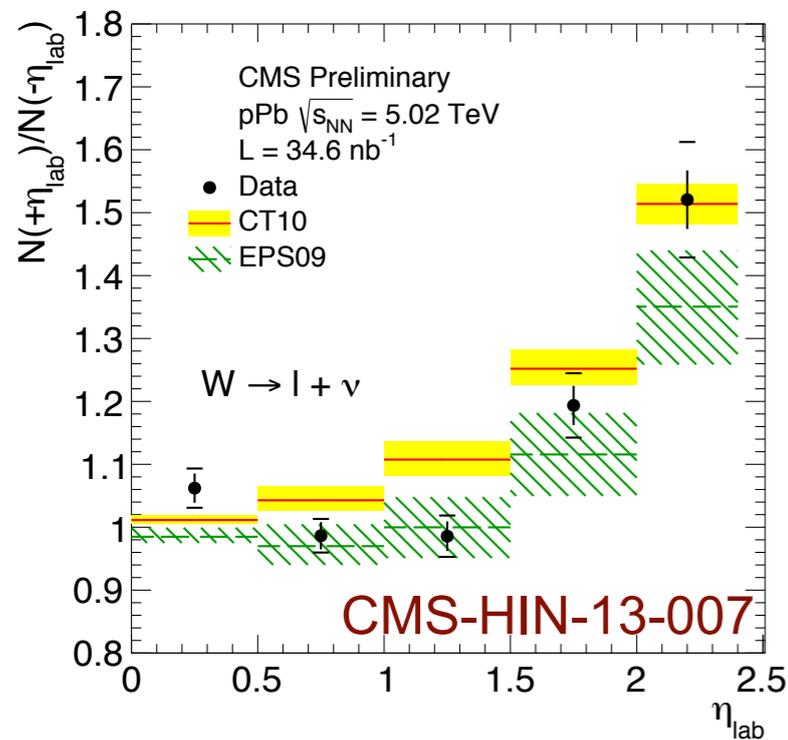
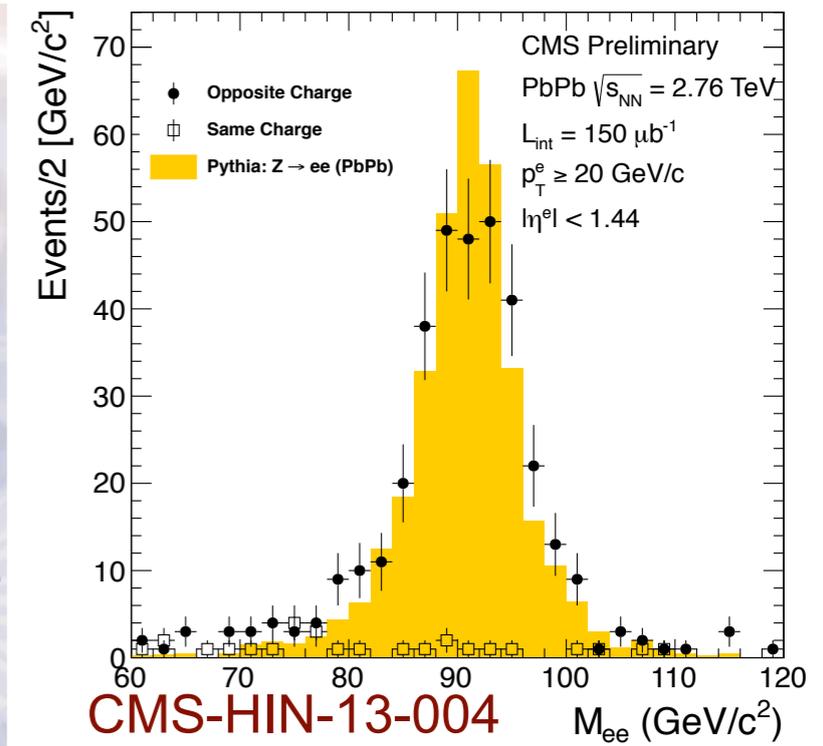
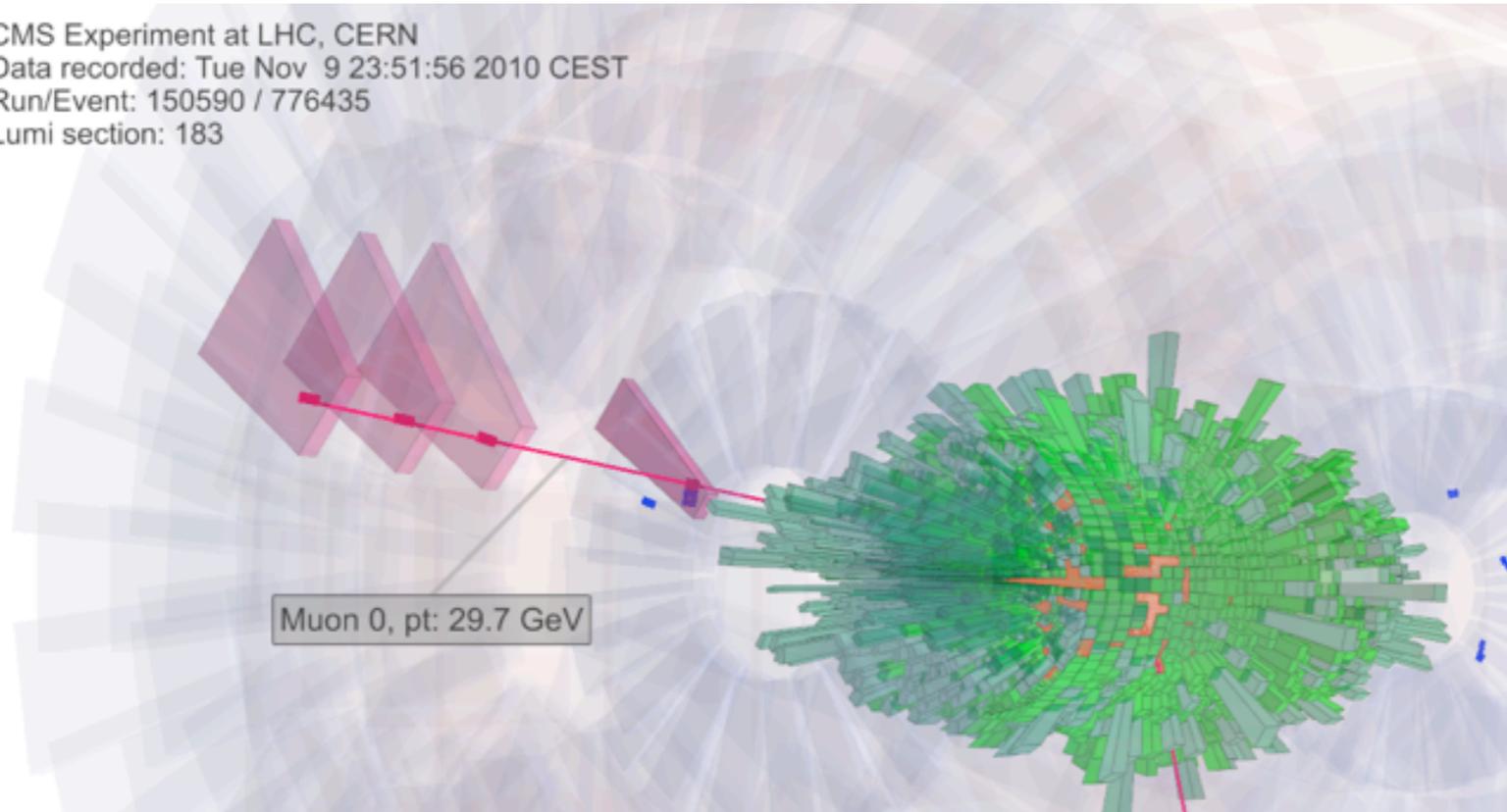
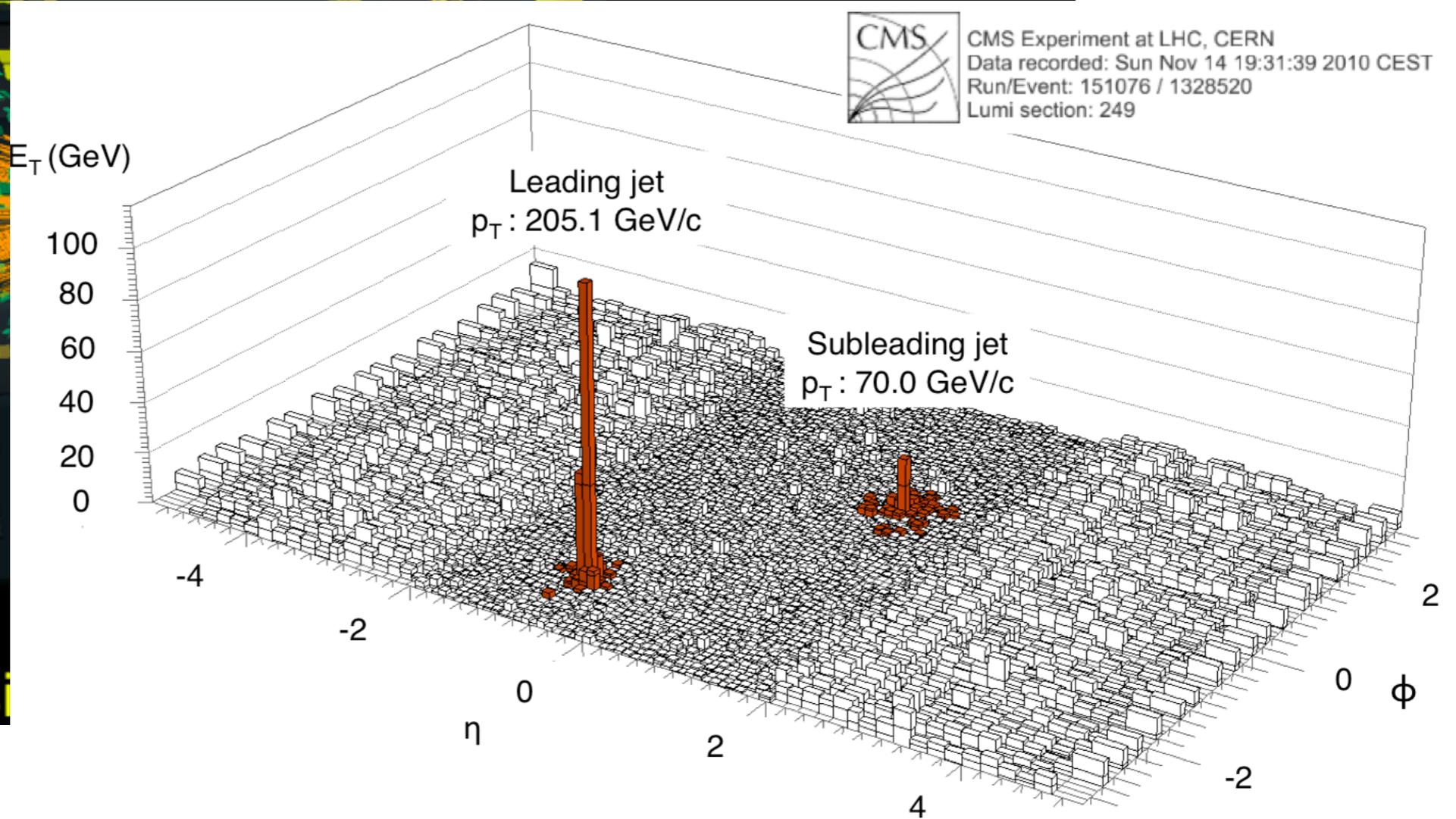
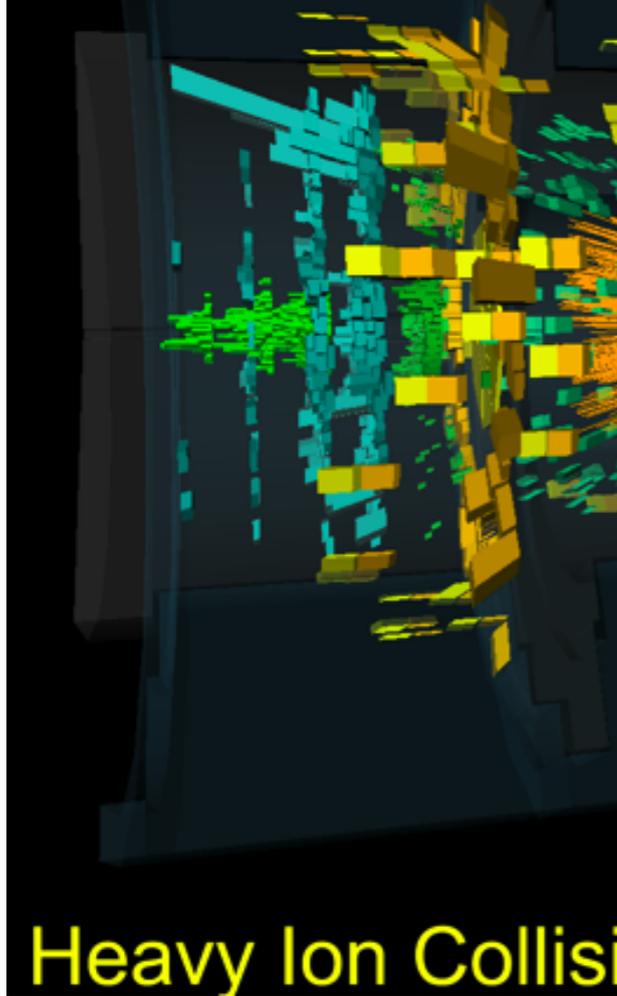


Figure 6: The  $N(+\eta_{lab})/N(-\eta_{lab})$  asymmetry.

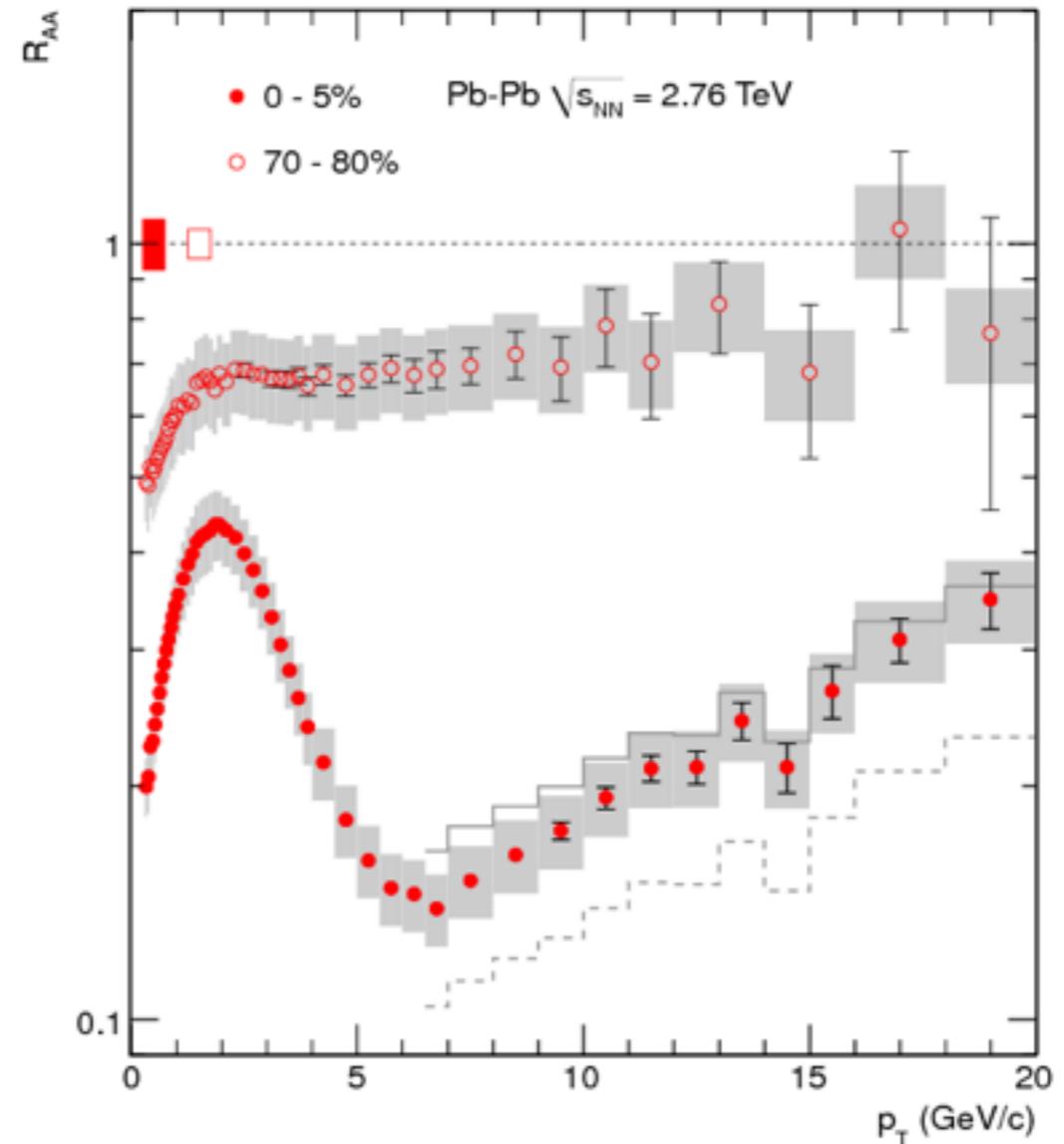
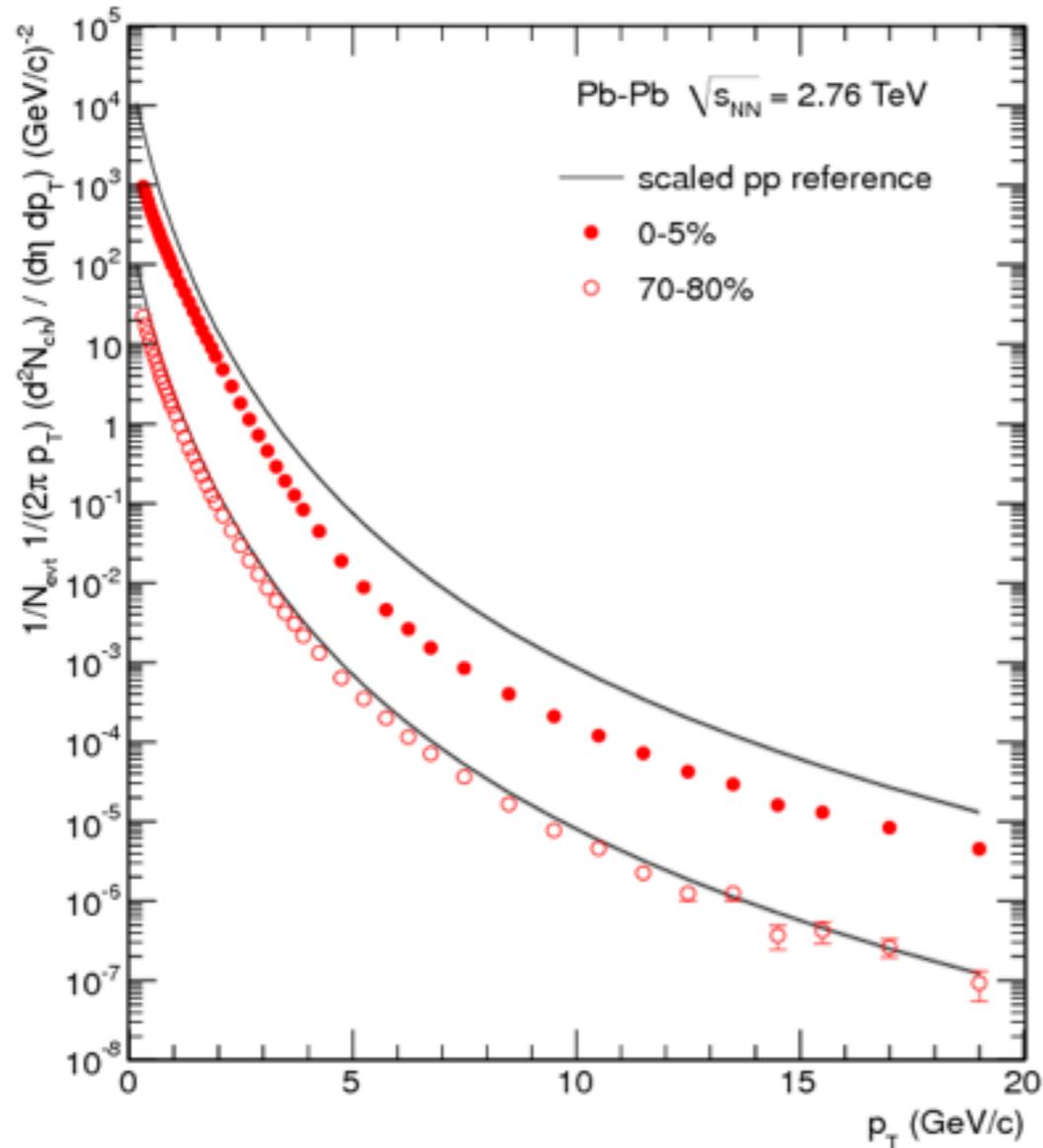
First observed Z-production event  
 in heavy-ion collisions!

Standard candles in pPb

Run 168875, Event 1577540  
Time 2010-11-10 01:27:38 CET

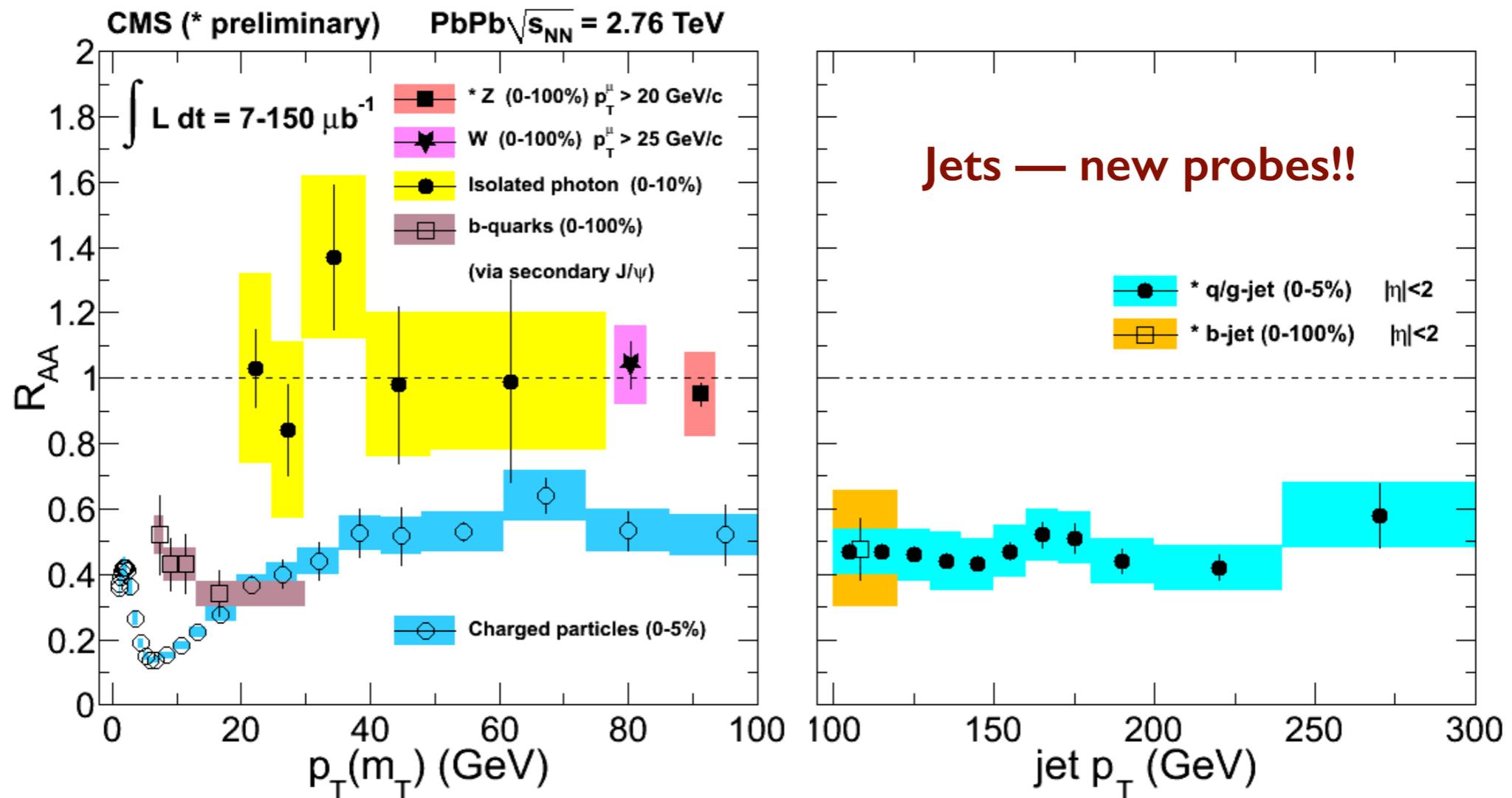


# The nuclear modification factor



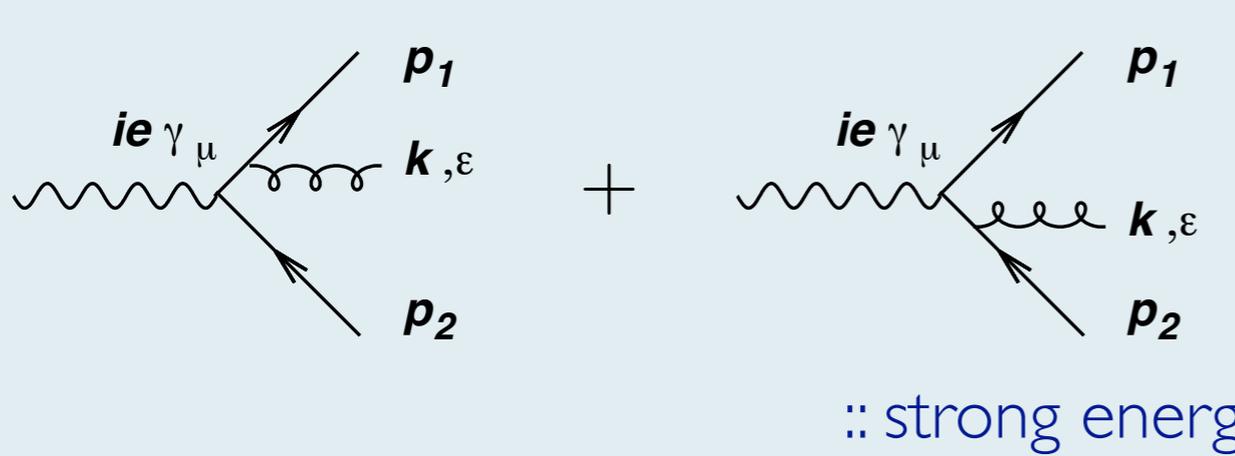
$$R_{AA}(p_T) = \frac{(1/N_{evt}^{AA}) d^2N_{ch}^{AA} / d\eta dp_T}{\langle N_{coll} \rangle (1/N_{evt}^{pp}) d^2N_{ch}^{pp} / d\eta dp_T}$$

# The T-shirt plot



- no modification for colorless probes :: baseline ok!
- light & heavy hadrons/jets suppressed by a factor 2-5

# Repetition:



The image shows two Feynman diagrams for the vertex of a quark-antiquark pair and a gluon. In the first diagram, a quark line with momentum  $p_1$  and an antiquark line with momentum  $p_2$  meet at a vertex. A gluon line with momentum  $k$  and polarization  $\epsilon$  is emitted from the quark line. The vertex factor is  $ie\gamma_\mu$ . The second diagram is similar, but the gluon is emitted from the antiquark line. The two diagrams are summed together.

$$= \bar{u}(p_1) (-ig_s t^a) \not{\epsilon} \frac{i(\not{p}_1 + \not{k})}{(p_1 + k)^2} (-ie_q \gamma_\mu) v(p_2) - \bar{u}(p_1) (-ie_q \gamma_\mu) \frac{i(\not{p}_2 + \not{k})}{(p_2 + k)^2} (-ig_s t^a) \not{\epsilon} v(p_2)$$

:: strong energy ordering!

$$i\mathcal{M}_{q\bar{q}g} = i\mathcal{M}_{q\bar{q}} \times g_s t^a \left( \frac{p_1 \cdot \epsilon}{p_1 \cdot k} - \frac{p_2 \cdot \epsilon}{p_2 \cdot k} \right) \Rightarrow J^\mu = Q_1 \frac{p_1^\mu}{p_1 \cdot k} + Q_2 \frac{p_2^\mu}{p_2 \cdot k}$$

:: classical current

$$\sum |\mathcal{M}_{q\bar{q}g}|^2 = \sum |\mathcal{M}_{q\bar{q}}|^2 \times g_s^2 C_F \frac{2p_1 \cdot p_2}{(p_1 \cdot k)(p_2 \cdot k)}$$

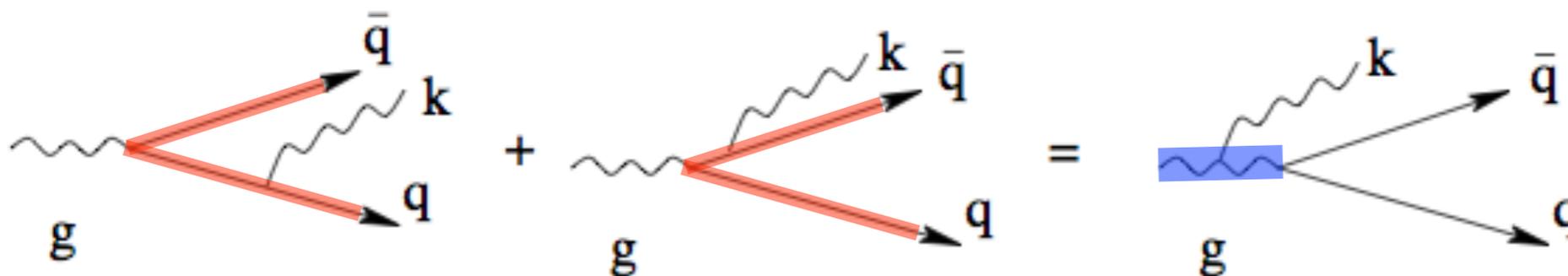
- factorization  $q\bar{q} \rightarrow q\bar{q} + g$  :: separation of time-scales!
- holds for higher order emissions too (leading logs)!

# Angular ordering

$$J^\mu = Q_1 \frac{p_1^\mu}{p_1 \cdot k} + Q_2 \frac{p_2^\mu}{p_2 \cdot k} \Rightarrow |J|^2 = Q_1^2 \mathcal{R}_1 + Q_2^2 \mathcal{R}_2 + 2Q_1 \cdot Q_2 \mathcal{J}$$

$$(Q_1 + Q_2)^2 = 0 \Rightarrow Q_1 \cdot Q_2 = -C_F \quad :: \text{singlet}$$

$$(Q_1 + Q_2)^2 = C_A \Rightarrow Q_1 \cdot Q_2 = \frac{C_A}{2} - C_F \quad :: \text{octet} \quad \text{etc...}$$



large-angle emissions are restored with the total charge!

$$\omega \frac{dN_g}{d\omega d^2k_\perp} \propto \frac{\alpha_s C_F}{k_\perp^2} + (q \rightarrow \bar{q})$$

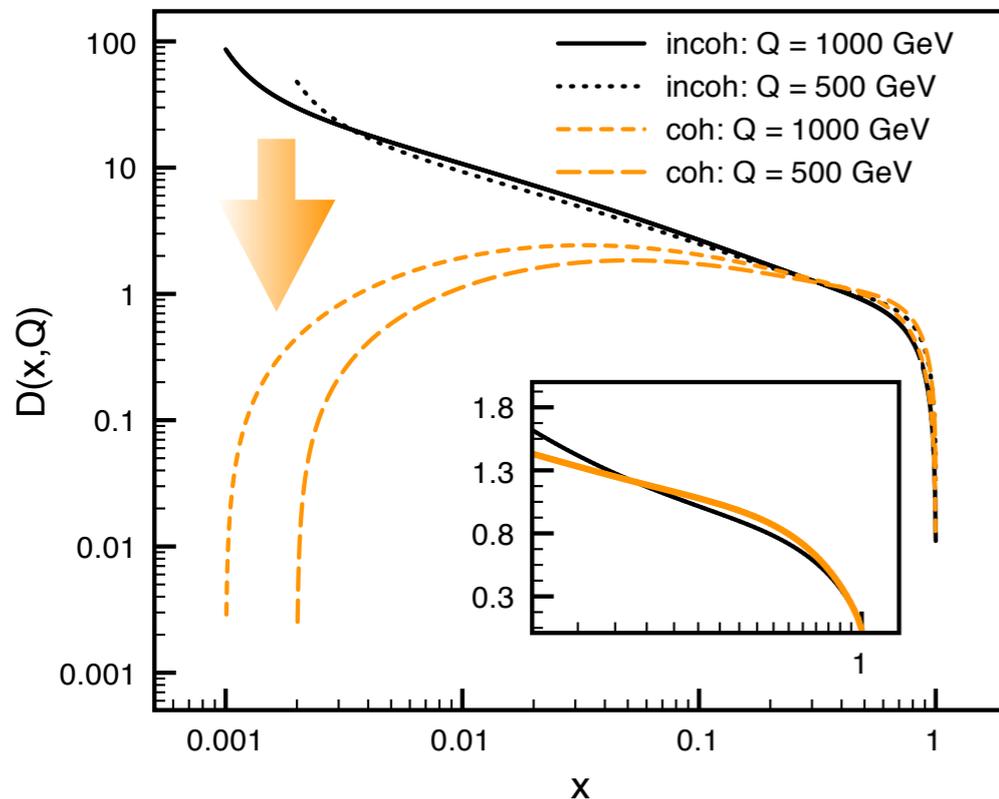
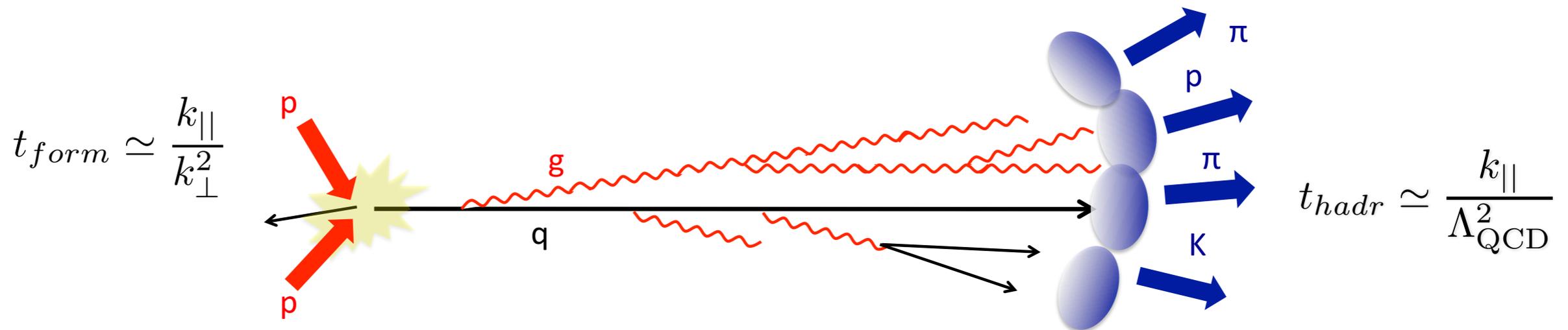
$$\theta \ll \theta_{q\bar{q}} \quad (k_\perp \ll \omega \theta_{q\bar{q}})$$

$$\omega \frac{dN_g}{d\omega d^2k_\perp} \propto \frac{\alpha_s C_A}{k_\perp^2}$$

$$\theta \gg \theta_{q\bar{q}} \quad (k_\perp \gg \omega \theta_{q\bar{q}})$$

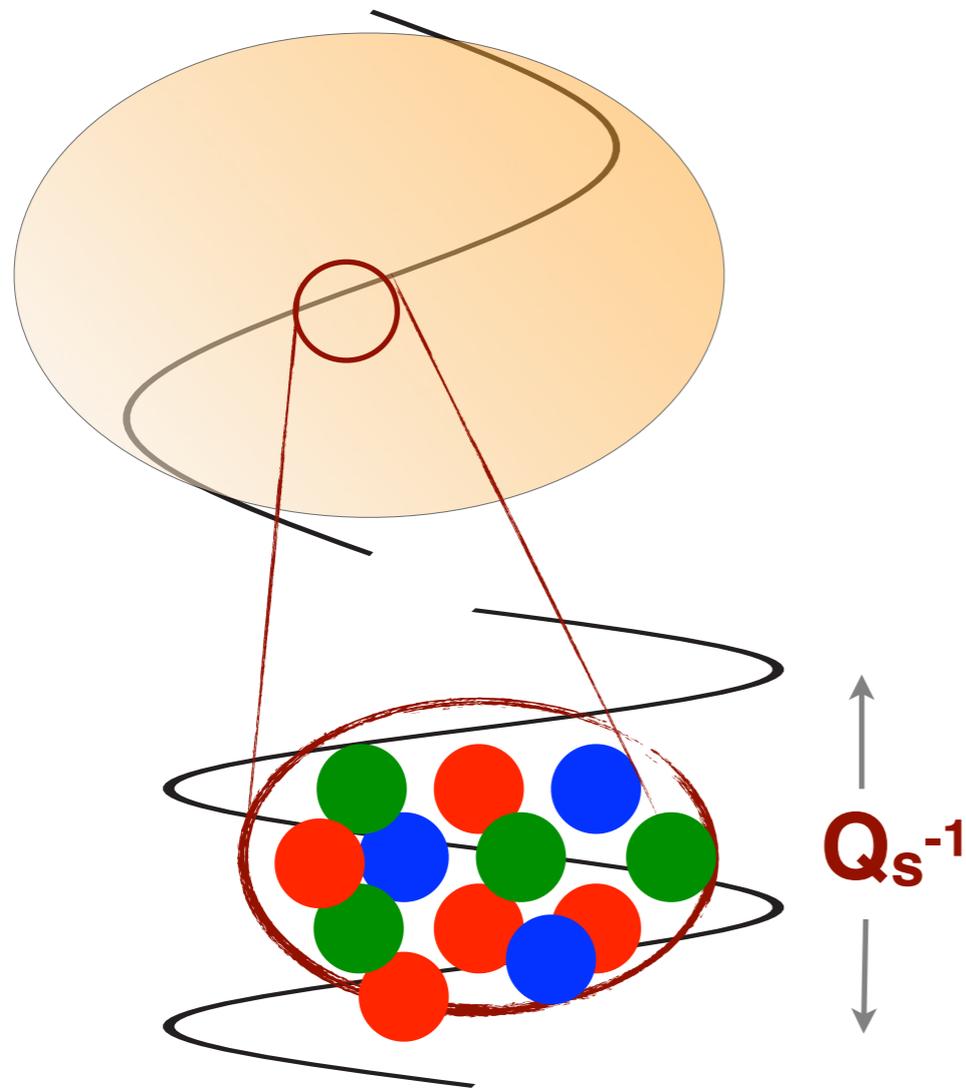
# Jets in vacuum

[Bassetto, Ciafaloni, Marchesini, Mueller, Dokshitzer, Fadin, Lipatov (80's)]



- separation of scales  $Q \gg \Lambda_{QCD}$  allows for resummation of branchings
- probabilistic picture (including quantum interference)
- delicate treatment of the soft sector
- basis for **precision pQCD**

# A new scale in the medium



- the jet scale  $Q = E\Theta_{jet}$
- the medium fluctuates with typical transverse wavelength  $Q_s^{-1}$
- medium “charge” is zero for  $\lambda > Q_s^{-1}$
- resolved by  $\lambda < Q_s^{-1}$

$$Q_s^2(t) = \hat{q}t$$

squared transverse momentum per unit length

# Eikonal propagation

$$u_\lambda(p) \xrightarrow{A^{b,\nu}(p'-p)} \bar{u}_{\lambda'}(p') \simeq -ig_s \overset{\text{spin}}{\delta_{\lambda\lambda'}} 2p^+ A^{a,-} t^a$$

$$\varepsilon_\mu^i(p) \xrightarrow{A^{b,\nu}(p'-p)} \varepsilon_\eta^{*,j}(p') \simeq -ig_s \overset{\text{polarization}}{\delta^{ij}} 2p^+ A^{a,-} (T^a)_{bc}$$

- conservation of energy during scattering
- no elastic energy loss
- no spin-flip, polarization
- color precession

S-matrix:

$$S(p', p) = \sum_{n=0}^{\infty} S_n(p', p)$$

$$\simeq 2\pi\delta(p'^+ - p^+) 2p^+ \int d^2\mathbf{x} e^{-i(\mathbf{p}' - \mathbf{p}) \cdot \mathbf{x}} U(x^+, x_0^+; [\mathbf{x}])$$

Wilson line:

$$U(x^+, 0; [\mathbf{r}]) \equiv \mathcal{P}_\xi \exp \left[ ig \int_0^{x^+} d\xi T \cdot A_{\text{med}}^-(\xi, \mathbf{r}(\xi)) \right]$$

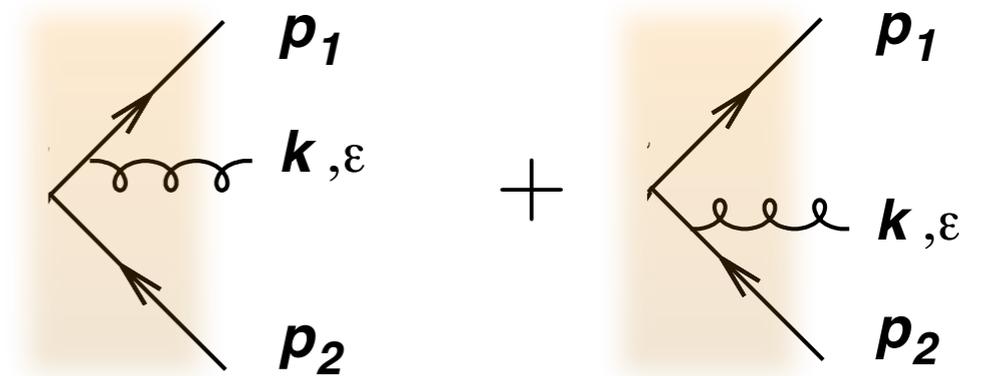
# Antenna in the medium

Classical current:

$$J^\mu = \sum_{i=1}^2 Q_i \frac{k_i^\mu}{k_i \cdot k} \quad \Rightarrow \quad J^\mu = \sum_{i=1}^2 Q_i U \left( x^+, 0; \left[ \mathbf{r} = \frac{\mathbf{k}_i}{k_i^+} \xi \right] \right) \frac{k_i^\mu}{k_i \cdot k}$$

Survival probability of the dipole:

$$\begin{aligned} \Delta_{\text{med}} &= 1 - \frac{1}{N_c} \langle \text{Tr} U_1(x_L^+, 0) U_2^\dagger(0, x_L^+) \rangle \\ &= 1 - \exp \left[ -\frac{1}{12} r_\perp^2(x_L^+) Q_s^2(x_L^+) \right] \end{aligned}$$



Only the interferences are modified!

$$\omega \frac{dN}{d^3k} = \frac{\alpha_s}{(2\pi)\omega^2} \left[ Q_1^2 \mathcal{R}_1 + Q_2^2 \mathcal{R}_2 + 2\mathbf{Q}_1 \cdot \mathbf{Q}_2 \left( 1 - \Delta_{\text{med}} \right) \mathcal{J} \right]$$

# In some detail...

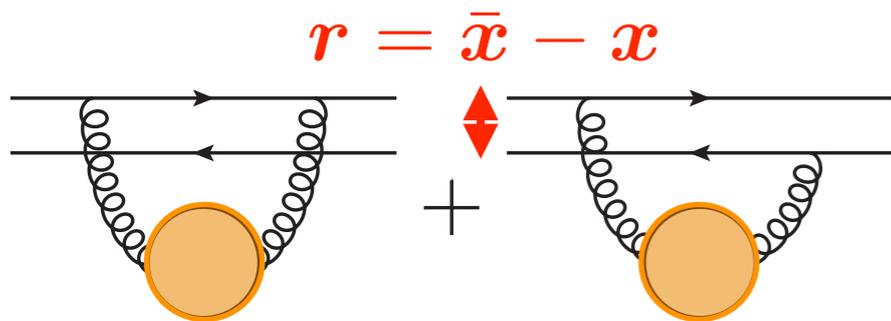
Two-point function:

$$C_q^{(2)}(x_L^+ - x_0^+; \mathbf{x} - \bar{\mathbf{x}}) = \frac{1}{N_c} \left\langle \text{Tr} U(x_L^+, x_0^+; \mathbf{x}) U^\dagger(x_0^+, x_L^+; \bar{\mathbf{x}}) \right\rangle$$

← comes from the number of quarks

Opacity expansion:

$$U(x_L^+, x_0^+; [\mathbf{r}]) = 1 + ig \int_{x_0^+}^{x_L^+} d\xi T \cdot A_{\text{med}}^-(\xi, \mathbf{r}) + (ig)^2 \int_{x_0^+}^{x_L^+} d\xi T \cdot A_{\text{med}}^-(\xi, \mathbf{r}) \int_{\xi}^{x_L^+} d\xi' T \cdot A_{\text{med}}^-(\xi', \mathbf{r})$$



$$\begin{aligned} \langle U^{(2)}(\mathbf{x}) \rangle &\sim \int_{\xi}^{x_L^+} d\xi' \int_{x_0^+}^{x_L^+} d\xi T^a T^b \langle A_{\text{med}}^{a,-}(\xi', \mathbf{x}) A_{\text{med}}^{b,-}(\xi, \mathbf{x}) \rangle \\ &\sim \gamma(\mathbf{r} = 0) \end{aligned}$$

$$\begin{aligned} \langle U^{(1)}(\mathbf{x}) U^{\dagger(1)}(\bar{\mathbf{x}}) \rangle &\sim \int_{x_0^+}^{x_L^+} d\xi' \int_{x_0^+}^{x_L^+} d\xi T^a T^b \langle A_{\text{med}}^{a,-}(\xi', \mathbf{x}) A_{\text{med}}^{\dagger b,-}(\xi, \bar{\mathbf{x}}) \rangle \\ &\sim \gamma(\mathbf{r}) \end{aligned}$$

# Medium averages

$$\langle A_{\text{med}}^{a,-}(x^+, \mathbf{q}) A_{\text{med}}^{b,+}(x'^+, \mathbf{q}') \rangle = \delta^{ab} n(x^+) \delta(x^+ - x'^+) (2\pi)^2 \delta(\mathbf{q} - \mathbf{q}') \gamma(\mathbf{q})$$

Medium average

:: instantaneous correlator  $\lambda_{\text{mfp}} > r_{\text{scr}}$

:: no cross-talk between scattering centers

Perturbatively:  $\lambda_{\text{mfp}} \sim 1/(g^2 T)$  and  $r_{\text{scr}} \sim 1/m_D \sim 1/gT$  :: ok!

Resumming all such contributions gives rise to exponentiation!

$$\begin{aligned} C_q^{(2)}(x_L^+ - x_0^+; \mathbf{r}) &= \frac{1}{N_c} \left\langle \text{Tr} U(x_L^+, x_0^+; \mathbf{x}) U^\dagger(x_0^+, x_L^+; \bar{\mathbf{x}}) \right\rangle \\ &= \exp \left\{ -g_s^2 N_c (x_L^+ - x_0^+) n_0 [\gamma(0) - \gamma(\mathbf{r})] \right\} \\ &= \exp \left\{ -\frac{1}{2} \int dx^+ \Gamma_2(\mathbf{r}, x^+) \right\} \end{aligned}$$

dipole scattering rate!

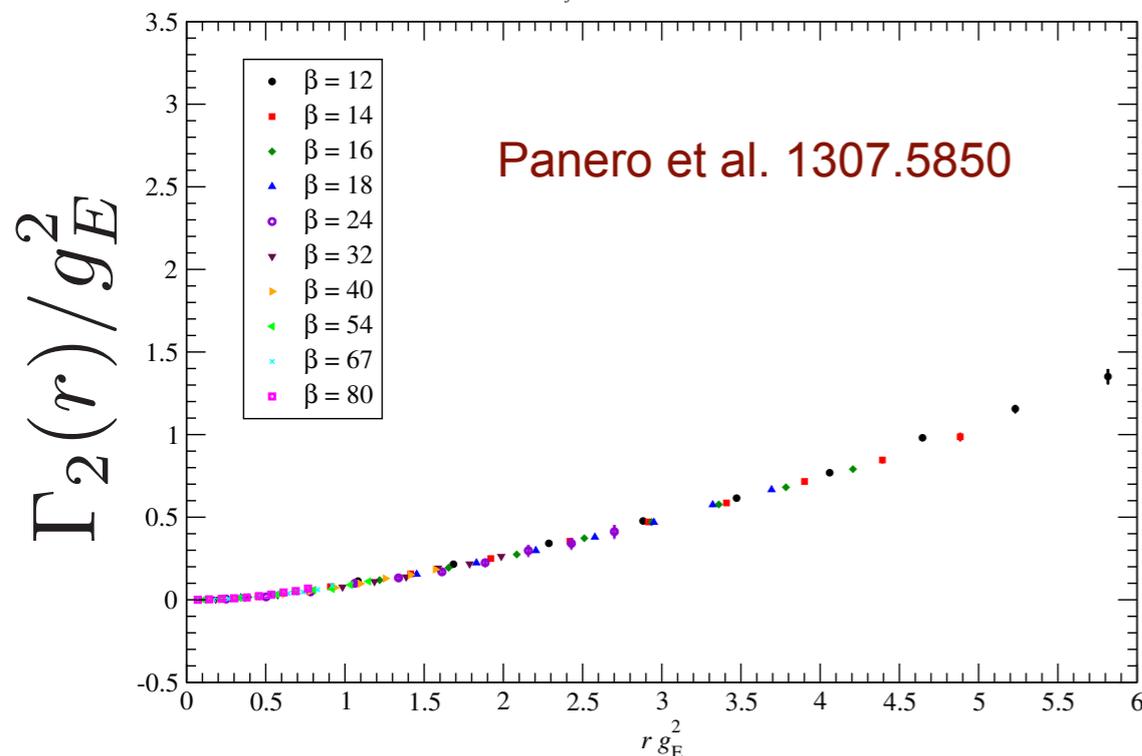
# Enter $\hat{q}$

$$\Gamma_2(\mathbf{r}, x^+) \simeq \frac{1}{2} \hat{q} r^2 \quad \Rightarrow \quad C_q^{(2)}(\Delta x^+; \mathbf{r}) = \exp\left(-\frac{\hat{q} \Delta x^+}{4} r^2\right)$$

“harmonic oscillator”/dipole approximation

In momentum space: 
$$\mathcal{P}(\mathbf{p} - \mathbf{p}_0, \Delta x^+) = \int d^2 \mathbf{r} e^{-i(\mathbf{p} - \mathbf{p}_0) \cdot \mathbf{r}} C_q^{(2)}(\Delta x^+; \mathbf{r})$$

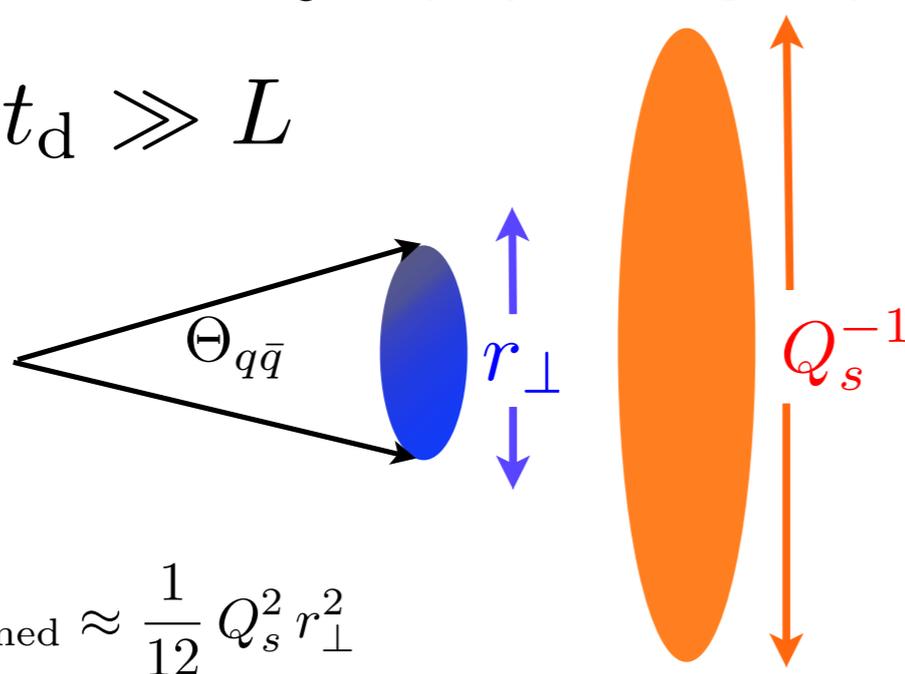
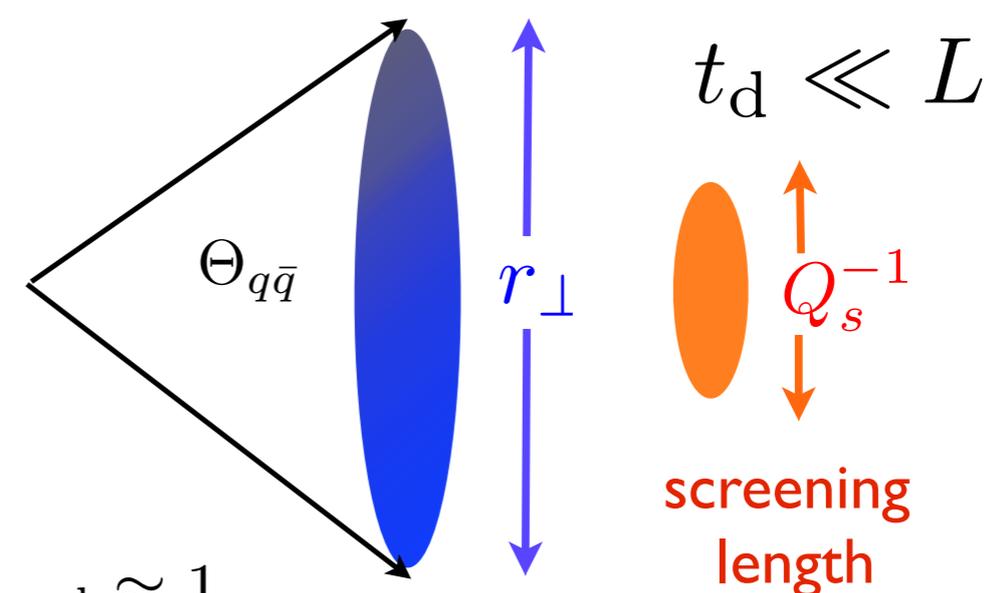
Coordinate-space collision kernel from EQCD  
( $n_f = 2, T \simeq 398$  MeV)



$$\mathcal{P}(\Delta \mathbf{p}, \Delta x^+) = \frac{4\pi}{\hat{q} \Delta x^+} e^{-\frac{\Delta \mathbf{p}^2}{\hat{q} \Delta x^+}}$$

- is a transport coefficient controlling Brownian motion/Gaussian broadening
- intuitively  $\hat{q} \sim 1/\eta$
- can be measured on the lattice in the high-energy limit

# Hard scale analysis

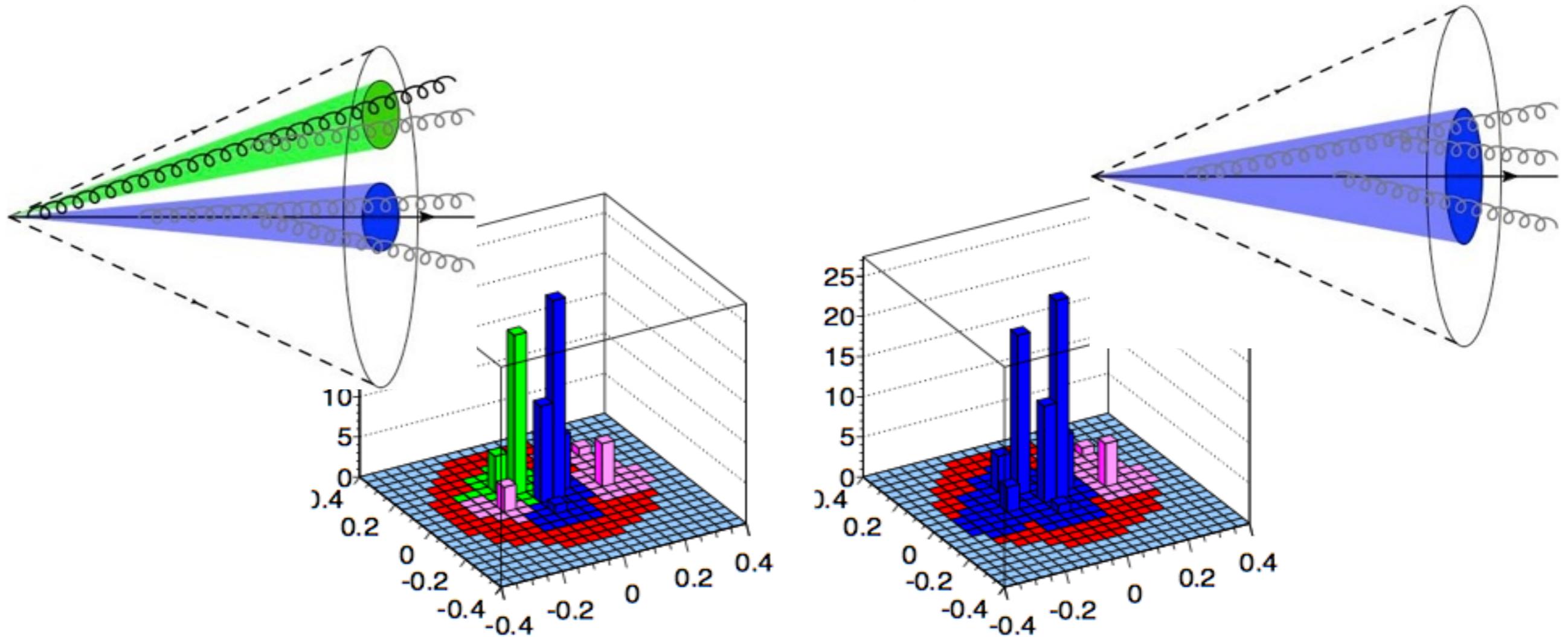
<ul style="list-style-type: none"> <li>• <math>r_{\perp} &lt; Q_s^{-1}</math> (Dipole regime)</li> </ul> <p><math>t_d \gg L</math></p>  <p><math>\Delta_{\text{med}} \approx \frac{1}{12} Q_s^2 r_{\perp}^2</math></p>	<ul style="list-style-type: none"> <li>• <math>r_{\perp} &gt; Q_s^{-1}</math> (Decoh. regime)</li> </ul> <p><math>t_d \ll L</math></p>  <p><math>\Delta_{\text{med}} \approx 1</math></p> <p>screening length</p>
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$$\Delta_{\text{med}} \approx 1 - \exp\left[-\frac{1}{12} Q_s^2 r_{\perp}^2\right]$$

$$r_{\perp} = \theta_{q\bar{q}} L$$

$Q_s$ : characteristic momentum scale of the medium

# Resolved jets

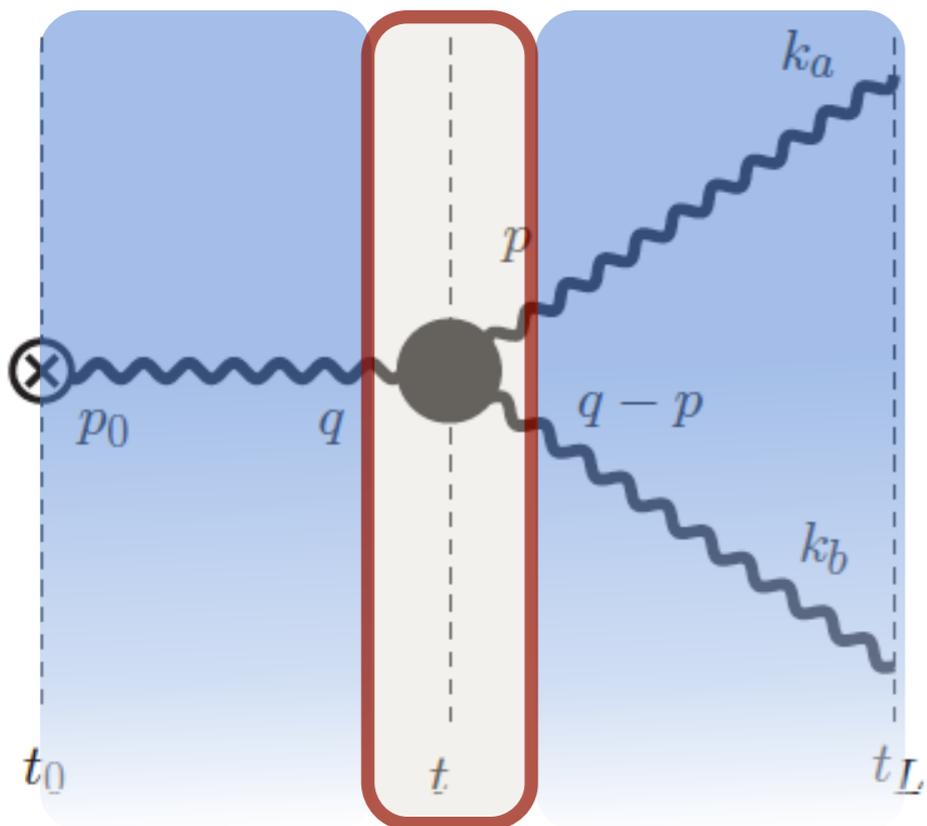


Dynamical process:

- medium resolves inner structure of the jet
- every resolved sub-jet = color current interacts incoherently with the medium

# Radiation in the medium

$$\frac{d^2\sigma}{d\Omega_{k_a} d\Omega_{k_b}} = 2g^2 z(1-z) \int_{t_0}^{t_L} dt \int_{\mathbf{p}_0, \mathbf{q}, \mathbf{p}} \mathcal{P}(\mathbf{k}_a - \mathbf{p}, t_L - t) \mathcal{P}(\mathbf{k}_b - \mathbf{q} + \mathbf{p}, t_L - t) \\ \times \mathcal{K}(\mathbf{p} - z\mathbf{q}, z, p_0^+, t) \mathcal{P}(\mathbf{q} - \mathbf{p}_0, t - t_0) \frac{d\sigma_{hard}}{d\Omega_{p_0}},$$

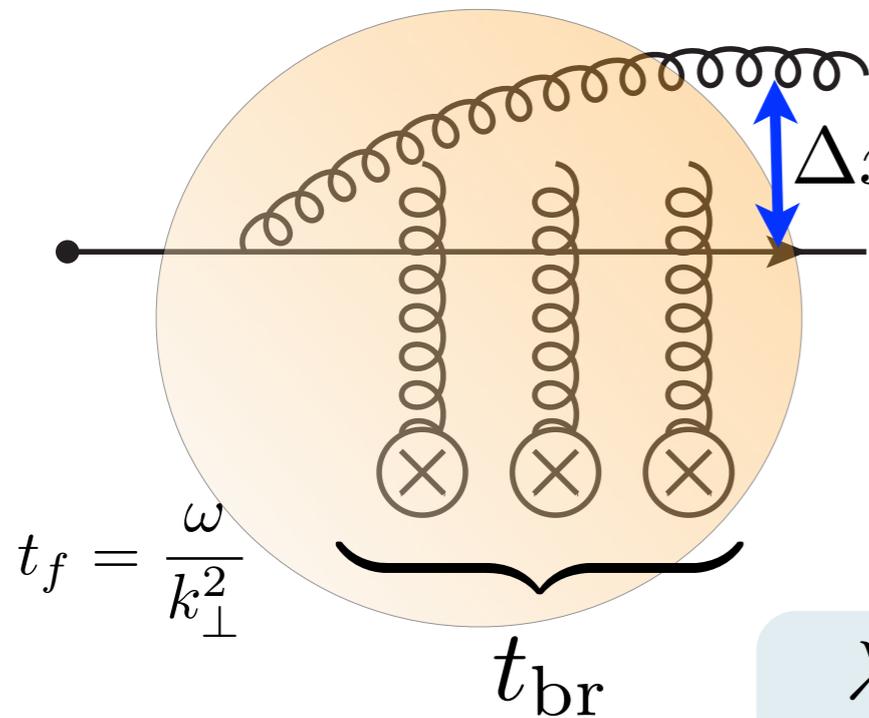


- factorization in the LPM regime
- $\mathcal{P}$  describes  $k_{\perp}$ -broadening
- $\mathcal{K}$  describes (quasi-instantaneous) emission  
→ splitting function

$$\mathcal{K}(\mathbf{p}, z, p_0^+) = \frac{2P_A^{gC}(z)}{z(1-z)p_0^+} \sin\left(\frac{\mathbf{p}^2}{2k_{br}^2}\right) \exp\left(-\frac{\mathbf{p}^2}{2k_{br}^2}\right)$$

$$k_{br}^2 = \sqrt{z(1-z)p_0^+ \hat{q}_{eff}} \quad \hat{q}_{eff} = \hat{q} \left[ (1-z)N_c - zC_R \right]$$

# Decoherence redux



Longitudinal coherence induces a characteristic formation time larger than mean free path

$$\left. \begin{aligned} t_{\text{br}} &= \lambda_{\text{mfp}} N_{\text{coh}} \\ k_{\text{br}}^2 &= \mu^2 N_{\text{coh}} \end{aligned} \right\} \begin{aligned} t_{\text{br}} &= \sqrt{\omega / \hat{q}} \\ k_{\text{br}}^2 &= \sqrt{\hat{q} \omega} \end{aligned}$$

$\lambda_{\text{mfp}} \rightarrow t_{\text{br}} \quad \therefore$  **Landau-Pomeranchuk-Migdal** effect

LPM spectrum:  $\omega \frac{dI^{\text{ind}}}{d\omega} \propto \alpha_s \frac{L}{t_{\text{br}}} = \alpha_s \sqrt{\frac{\hat{q} L^2}{\omega}} \quad \omega \ll \omega_c$

Mean energy loss:  $\Delta E = \int_0^\infty \omega \frac{dI^{\text{ind}}}{d\omega} \propto \alpha_s C_R \omega_c \quad \omega_c = \frac{1}{2} \hat{q} L^2$

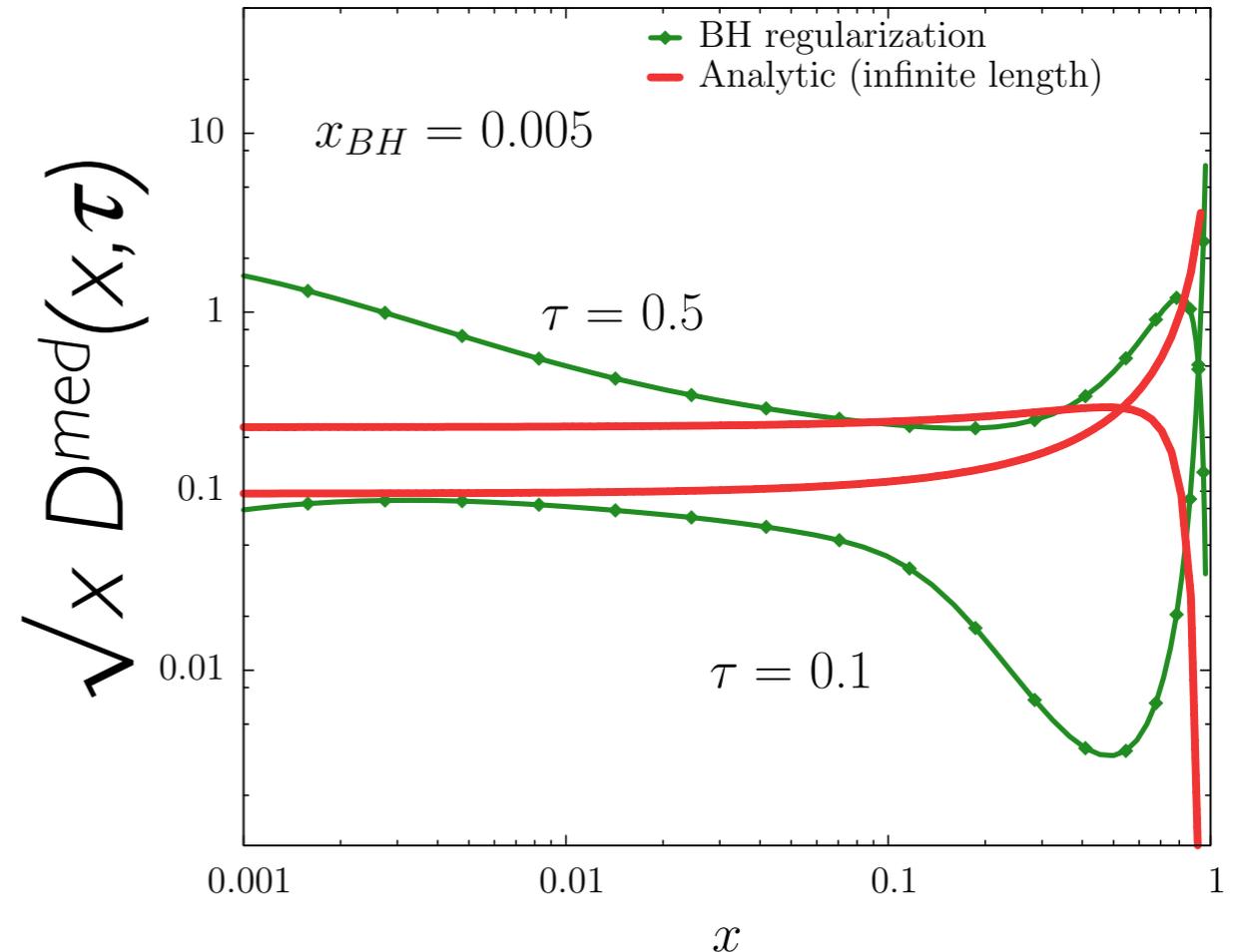
Baier, Dokshitzer, Mueller, Peigné, Schiff (1997-2000), Zakharov (1996),  
Wiedemann (2000), Gyulassy, Levai, Vitev (2000), Arnold, Moore, Yaffe (2001)

# Rate equation

$$\frac{\partial}{\partial L} D_i^{\text{med}}(x, p_{\perp}, L) = \int_0^1 dz \mathcal{K}_{ij} \left( z, \frac{x}{z} p_{\perp}; L \right) \times \left[ D_j^{\text{med}} \left( \frac{x}{z}, p_{\perp}, L \right) - z D_j^{\text{med}}(x, p_{\perp}, L) \right]$$

Distribution of energy in “time”:

$$\tau = \frac{\alpha_s N_c}{\pi} \sqrt{\frac{\hat{q} L^2}{E}}$$

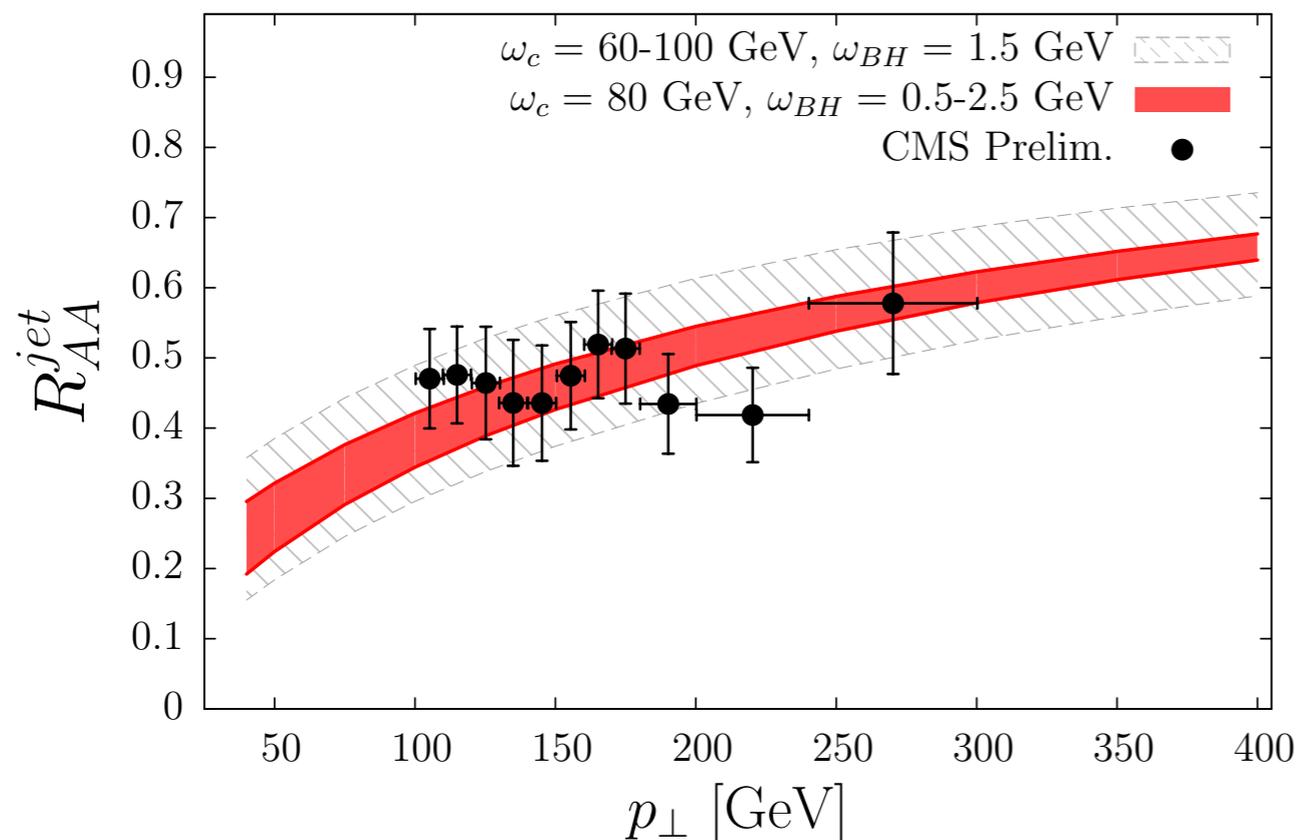


- $D_q(0) = \delta(1-x)$  and  $D_g(0) = 0$
- probabilistic interpretation
- turbulent flow: no intrinsic accumulation of energy
- effective in transporting sizable energy to large angles

Jeon, Moore hep-ph/0309332; Baier, Mueller, Schiff, Son hep-ph/0009237; Blaizot, Iancu, Mehtar-Tani 1301.6102

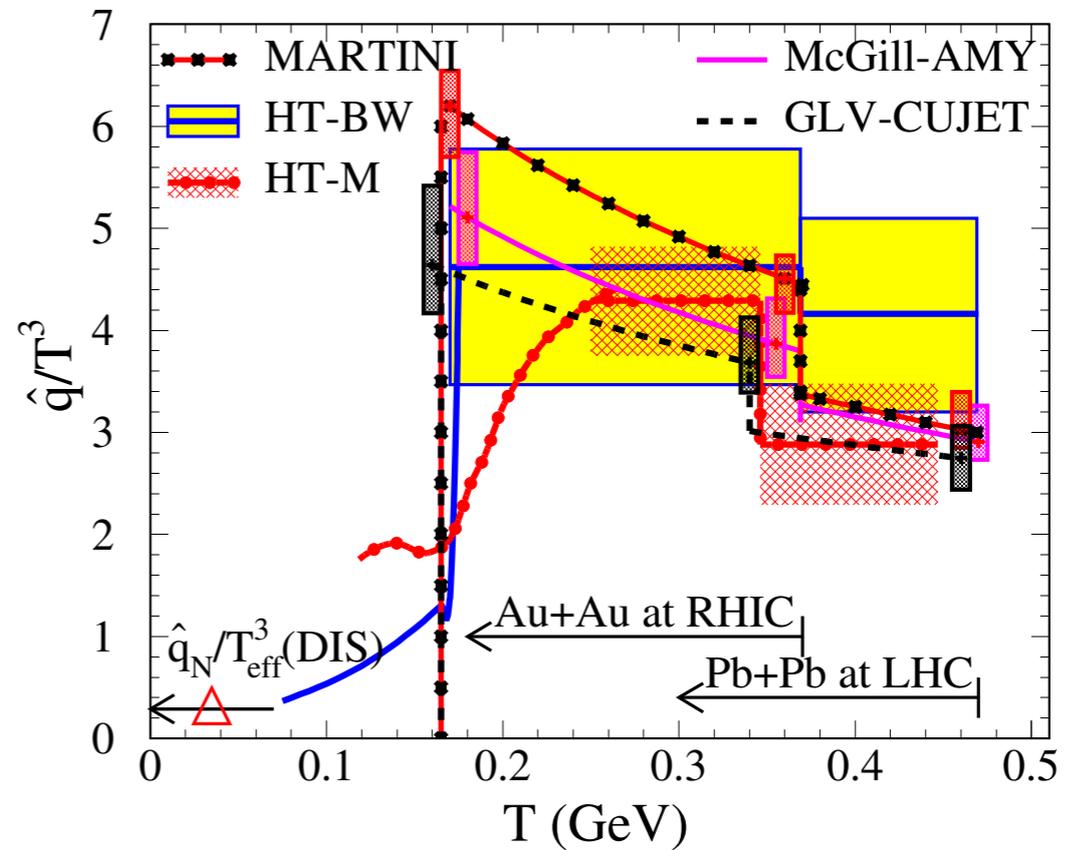
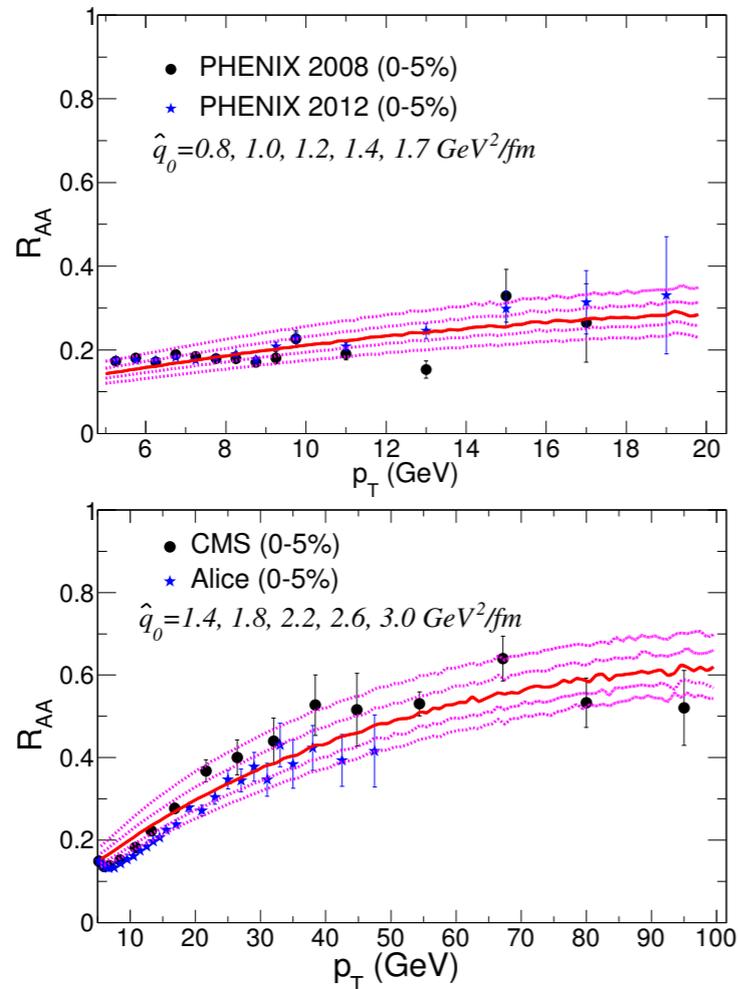
# Jet quenching

$$\frac{d^2 N_{\text{Pb-Pb}}^{\text{jet}}(p_{\perp})}{T_{\text{AA}} d^2 p_{\perp}} \simeq \int_0^1 \frac{dx}{x} D_q^{\text{med}} \left( x, \frac{p_{\perp}}{x}, L \right) \frac{d^2 \sigma_{\text{p-p}}^{\text{jet}} \left( \frac{p_{\perp}}{x} \right)}{d^2 p_{\perp}}$$



- assuming the jet is not resolved by the medium
- two medium parameters ( $\hat{q}$  and the mean free path) + geometry
- first step — improvements needed to study jet substructure

# Extraction of $\hat{q}$



Attempt at making a systematical comparison — still many caveats!

Within errors, a decreasing trend with  $T$ /collision energy — similar trend for  $1/\eta$ !

# Summary: quenched jets

- jets are excellent tools to study the QGP — vacuum baseline under control
- involves dynamical processes: fragmentation — resolution — radiation
- still many improvements to be made!