Heavy ions — exercises

(Dated: September 19, 2014)

THE BJORKEN MODEL

1) Write t and z expressed in terms of the proper time $\tau = \sqrt{t^2 - z^2}$ and the space-time rapidity $\eta = \frac{1}{2}\ln\frac{t+z}{t-z}$. Write the expression for the local flow velocity $u^\mu \equiv \mathrm{d}x^m u/\mathrm{d}\tau$.

2) Given the stress-energy tensor for a perfect fluid $T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu - P g^{\mu\nu}$, where ϵ is the energy density and P the pressure, write an evolution equation in propert time for ϵ. Solve the equation for a typical value of the speed of the sound $c_s^2 = \mathrm{d}P/\mathrm{d}\epsilon$. **Hint:** Use the conservation equation $\partial_\mu T^{\mu\nu} = 0$.

3) Using the fundamental thermodynamical relation (at fixed volume), $\mathrm{d}\epsilon = T\mathrm{d}s$, write and solve the evolution equation for the entropy density s. Furthermore, using the relations $\mathrm{d}P = s\mathrm{d}T$ (at fixed volume) write and solve the evolution equation for the temperature.

THE EIKONAL APPROXIMATION AND THE PATH-ORDERED WILSON LINE

Definitions: Light-cone coordinates $x^\pm \equiv \frac{1}{\sqrt{2}}(x^0 \pm x^3)$, and $p\cdot x = p^+x^- + p^-x^+ - \boldsymbol{p}\cdot\boldsymbol{x}$.

1) Assuming the dominance of the +-momentum, calculate the S-matrix of an on-shell quark scattering off *one* color potential $A_\mu(x)$ (in the fundamental representation), $S_1(p', p)$. Show that $A_\mu(x) = A_\mu(x^+, \boldsymbol{x})$ results in the conservation of +-momentum, $2\pi\delta(p'^+ - p^+)$, and discuss why. **Hint:** Make use of the fact that $\frac{1}{2}\sum_\lambda \bar{u}^\lambda(p')\gamma^\mu u^\lambda(p) = 2p^\mu$ in the eikonal approximation.

2) Using the path ordering property,

$$\int \mathrm{d}x_1 \ldots \mathrm{d}x_n \Theta(x_2 - x_1)\ldots\Theta(x_n - x_{n-1})A(x_1)\ldots A(x_n) = \frac{1}{n!}\mathcal{P}\left[\int \mathrm{d}x\, A(x)\right]^n \tag{1}$$

calculate the S-matrix of 2 and n scatterings in the medium. Resum the S-matrices and identify the path-ordered Wilson line

$$\mathcal{U}(\boldsymbol{x}) = \mathcal{P}\exp\left[ig\int \mathrm{d}x^+ A^-(x^+, \boldsymbol{x})\right]. \tag{2}$$

ENERGY LOSS

1) Assuming that every particle looses a *constant* amount of energy in the plasma, calculate the high-$p_\perp$ behavior of the nuclear modification factor

$$R_{AA} = \frac{\mathrm{d}N_{AA}/\mathrm{d}\eta\mathrm{d}p_\perp}{N_{\mathrm{coll}}\,\mathrm{d}N_{pp}/\mathrm{d}\eta\mathrm{d}p_\perp} \tag{3}$$

when the underlying (pp) spectrum was **a)** a power-like or **b)** exponential.

2) How is this behavior modified if interaction in the plasma simply absorb particles. Discuss both cases.