Neutrino Physics

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Problem set

Problem 0.1: *Let us define*

$$\psi_L = \frac{1}{2}(1-\gamma_5)\psi$$

the left-handed projection of a Dirac field. Under Lorentz transformations ψ *transforms like* $\psi(x) \to \psi'(x') = S(\Lambda)\psi(x)$ *with*

$$S(\Lambda) = \exp\left(-\frac{i}{4}\omega_{\mu\nu}\sigma^{\mu\nu}\right)$$

i) Check that both ψ_L *and* $\psi_L^c \equiv C\overline{\psi_L}^T$*, with* C *the charge conjugation matrix, transform as (*$\psi_L \to S(\Lambda)\psi_L$*,* $\psi_L^c \to S(\Lambda)\psi_L^c$*)*
ii) Check that $\gamma_5\psi_L^c = +\psi_L^c$*, that means that* ψ_L^c *is a right-handed spinor.*
iii) This allows us to write the following Lorentz invariant mass term (recall that $\bar{\psi} \to \bar{\psi}S(\Lambda)^{-1}$*)*

$$\mathscr{L}_M = i\overline{\psi_L}\partial\!\!\!/\psi_L - m\frac{1}{2}(\overline{\psi_L^c}\psi_L + \overline{\psi_L}\psi_L^c)$$

iv) Check that this mass term only exists for anticommuting fields ψ_L*.*
v) Check that this Lagrangian can be diagonalized and written in terms of the Majorana field $\psi_M = \psi_L + \psi_L^c$*, which is selfconjugate (*$\psi_M = \psi_M^c$*)*

$$\mathscr{L}_M = \frac{i}{2}\overline{\psi_M}\partial\!\!\!/\psi_M - m\frac{1}{2}\overline{\psi_M}\psi_M$$

vi) Using the Weyl representation of the Dirac matrices, write the Majorana Lagrangian in twocomponent notation. Obtain the equation of movement.

Problem 0.2: *Check that the "see-saw" Lagrangian*

$$\mathscr{L}_M = i\overline{\psi_L}\partial\!\!\!/\psi_L + i\overline{\psi_R}\partial\!\!\!/\psi_R - d\overline{\psi_R}\psi_L - \frac{1}{2}m\overline{\psi_R}\psi_R^c + \text{h.c.}$$

can be written as

$$\mathscr{L}_M = i\overline{\Psi_L}\partial\!\!\!/\Psi_L - \frac{1}{2}\overline{\Psi_L^c}M\Psi_L + \text{h.c.}$$

with

$$\Psi_L = \begin{pmatrix} \psi_L \\ \psi_R^c \end{pmatrix}, \qquad M = \begin{pmatrix} 0 & d \\ d & m \end{pmatrix}$$

Check that, in this case, d and m can be taken real. Diagonalize this Lagrangian and obtain the eigenmass fields.

i) *Obtain the mass spectrum in the limit $m \gg d$ ("see-saw")*

ii) *Obtain the mass spectrum in the limit $m \ll d$ ("pseudo-Dirac")*

iii) *In the see "see-saw" case, try to guess the result for n_ν "L" and "R" fields.*

Problem 0.3: *Using the "see-saw" Lagrangian before SSB,*

$$\mathscr{L} = i\overline{L_L}\not{D}L_L + i\overline{\nu_R}\not{D}\nu_R - \bar{L}_L Y_e \Phi e_R - \bar{L}_L Y_\nu \tilde{\Phi}\nu_R - \frac{1}{2}\overline{\nu_R^c} M \nu_R + \text{h.c.}$$

obtain the Weinberg operator by integrating ν_R using the equations of motion and expanding in M^{-1}.

Problem 0.4: *Estimate the neutrino mass in the inert doublet model and check that it is finite a) by mass insertion diagrams b) by diagonalization of the scalar mass matrix and computing the relevant selfenergies.*