#### precise experiments need precise calculations



"I think you should be more explicit here in step two."

#### example: 1-loop diagrams for $\mu$ decay amplitude



Example of loop integral:



- $\Rightarrow$  integral diverges for large q
- $\Rightarrow$  theory in this form not physically meaningful
- needs (i) regularization
  - (ii) renormalization

# **Regularization:**

theory modified such that expressions become mathematically meaningful

 $\Rightarrow$  ''regulator'' introduced, removed at the end

e.g. cut-off in loop integral

$$\int_0^\infty d^4q \to \int_0^{\Lambda} d^4q; \quad \Lambda \to \infty \text{ at the end}$$

technically more convenient: dimensional regularization

$$\int d^4q \rightarrow \int d^Dq, \quad D = 4 - \varepsilon; \quad D \rightarrow 4 \text{ at the end}$$

# Renormalization:

- absorption of divergencies
- determination of physical meaning of parameters order by order in perturbation theory

add counterterms that absorb divergent parts

- parameters in  $\mathcal{L}$  are formal, "bare parameters"  $g_0 = g + \delta g$  for a coupling,  $m_0 = m + \delta m$  for a mass
- $\blacksquare$  g, m are "physical", *i.e.* measurable

mass renormalization,  $m_0^2 = m^2 + \delta m^2$ 

Physical mass: pole of propagator

inverse propagator up to 1-loop order:



on-shell renormalization:  $\delta m^2 = \operatorname{Re} \Sigma(m^2)$ 

# charge renormalization: $e_0 = e + \delta e$

 $\delta e$  cancels loop contributions to  $ee\gamma$  vertex in the Thomson limit

e 
$$k \rightarrow 0$$
  $k \rightarrow 0$   $ie\gamma_{\mu}$  for on-shell electrons

 $\Rightarrow e =$  elementary charge of classical electrodynamics

fine-structure constant  $\alpha(0) = \frac{e^2}{4\pi} = 1/137.03599976$ 

 $\delta e$  contains photon vacuum polarization  $\Pi^{\gamma}(k^2=0)$  :

$$\Pi^{\gamma}(0) = \underbrace{\Pi^{\gamma}(0) - \Pi^{\gamma}(M_Z^2)}_{non-perturbative} + \underbrace{\Pi^{\gamma}(M_Z^2)}_{perturbative}$$

## photon vacuum polarization

 $\gamma$  $\wedge$ 

$$\Pi^{\gamma}(M_Z^2) - \Pi^{\gamma}(0) \equiv \Delta \alpha \quad \rightarrow \quad \alpha(M_Z) = \frac{\alpha}{1 - \Delta \alpha} \simeq \frac{1}{129}$$

$$\Delta \alpha = \Delta \alpha_{\text{lept}} + \Delta \alpha_{\text{had}},$$

$$\Delta \alpha_{\text{lept}} = 0.031498 \quad (3 - \text{loop})$$

$$\Delta \alpha_{\rm had} = 0.02750 \pm 0.00033$$

$$= 0.02757 \pm 0.00010$$

arXiv:1010.4180

$$\Delta \alpha_{\rm had} = -\frac{\alpha}{3\pi} M_Z^2 \operatorname{Re} \int_{4m_\pi^2}^{\infty} ds' \, \frac{R_{\rm had}(s')}{s'(s' - M_Z^2 - i\epsilon)}$$



$$R_{\text{had}} = \frac{\sigma(e^+e^- \to \gamma^* \to \text{hadrons})}{\sigma(e^+e^- \to \gamma^* \to \mu^+\mu^-)}$$

 $M_W - M_Z$  correlation





 $M_W - M_Z$  correlation



$$\frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{2M_W^2 \left(1 - M_W^2 / M_Z^2\right)}$$

$$M_W = 80.939 \pm 0.002 \,\text{GeV}$$
 from  $G_F, \, lpha, \, M_Z$   
 $M_W = 79.965 \pm 0.005 \,\text{GeV}$  with  $lpha o lpha(M_Z)$   
 $M_W = 80.385 \pm 0.015 \,\text{GeV}$  exp.  $37\sigma / 28 \,\sigma$ 

#### with loop contributions

$$\frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{2M_W^2 \left(1 - M_W^2/M_Z^2\right)} \cdot \left(1 + \Delta r\right)$$

 $\Delta r$ : quantum correction  $\Delta r = \Delta r(m_t, M_H)$ 

$$\Delta r = \Delta \alpha - \frac{c_W^2}{s_W^2} \Delta \rho + \cdots$$
$$\Delta \rho \sim \frac{m_t^2}{M_W^2}$$

determines W mass

 $M_W = M_W(\alpha, G_F, M_Z, \boldsymbol{m_t}, \boldsymbol{M_H})$ 

complete at 2-loop order

#### 1-loop examples



full structure of SM











2-loop examples

## effects of higher-order terms on $\Delta r$



variation of  $\Delta r$  by 0.001  $\Rightarrow \delta M_W = 18 \,\mathrm{MeV}$ 

**3-loop** ( $\Delta \rho$ )  $\Rightarrow \delta M_W = 12 \,\mathrm{MeV}$ 

present exp. error:

 $\Delta M_W = 15 \,\mathrm{MeV}$  / theo:  $4 \,\mathrm{MeV}$ 

# LEP Electroweak Working Group



Z resonance



• effective Z boson couplings with higher-order  $\Delta g_{V,A}$ 

$$v_f \to g_V^f = v_f + \Delta g_V^f, \qquad a_f \to g_A^f = a_f + \Delta g_A^f$$

• effective ew mixing angle (for f = e):

$$\sin^2 \theta_{\text{eff}} = \frac{1}{4} \left( 1 - \operatorname{Re} \frac{g_V^e}{g_A^e} \right) = \kappa \cdot \left( 1 - \frac{M_W^2}{M_Z^2} \right)$$



2-loop examples for Z couplings

complete 2-loop calculation available for  $\sin^2 \theta_{\rm eff}$ 

# EW 2-loop calculations for $\Delta r$

Freitas, Hollik, Walter, Weiglein

Awramik, Czakon

Onishchenko, Veretin

# EW 2-loop calculations for $\sin^2 heta_{ m eff}$

Awramik, Czakon, Freitas, Weiglein

Awramik, Czakon, Freitas

Hollik, Meier, Uccirati

# universal terms at 3- and 4-loops (EW and QCD)

van der Bij, Chetyrkin, Faisst, Jikia, Seidensticker Faisst, Kühn Seidensticker, Veretin Boughezal, Tausk, van der Bij Schröder, Steinhauser Chetyrkin, Faisst, Kühn Chetyrkin, Faisst, Kühn, Maierhofer, Sturm Boughezal, Czakon

## importance of two-loop calculations



lowest order:  $\sin^2 \theta_W = 1 - \frac{M_W^2}{M_Z^2} = 0.22290 \pm 0.00029$ exp. value:  $\sin^2 \theta_{\text{eff}} = 0.23153 \pm 0.00016$  Global analysis within the SM



before the top quark was discovered (< 1995): indirect mass determination  $\Rightarrow$  m<sub>t</sub> = 178 ± 8  $^{+17}_{-20}$  GeV



before the top quark was discovered (< 1995): indirect mass determination  $\Rightarrow$  m<sub>t</sub> = 178 ± 8  $^{+17}_{-20}$  GeV



top discovery: Tevatron 1995  $m_t = 180 \pm 12 \, \mathrm{GeV}$ 

before the top quark was discovered (< 1995): indirect mass determination  $\Rightarrow$  m<sub>t</sub> = 178 ± 8  $^{+17}_{-20}$  GeV



# The way to the Higgs boson

development of bounds from direct and indirect searches



# Global fit to the Higgs boson mass



#### blueband: Theory uncertainty

"Precision Calculations at the Z Resonance" CERN 95-03 [Bardin, WH, Passarino (eds.)]

 $M_{\rm H} < 161 \; {\rm GeV} \quad ({\rm at} \; 95\% \; {\rm C.L.})$ 



after the 2011 results from the LHC on the Higgs boson mass

 $M_{\rm H} < 152 \,\,{\rm GeV} \quad (95\% {\rm C.L.})$  $M_{\rm H} = 94^{+29}_{-24} \,\,{\rm GeV}$ 

# 5. Higgs bosons

Higgs potential: 
$$V = -\mu^2 \left(\Phi^{\dagger}\Phi\right)^2 + \frac{\lambda}{4} \left(\Phi^{\dagger}\Phi\right)^4$$

Higgs field in unitary gauge:  $\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$ 

H(x) : real scalar field, describes neutral spin-0 bosons

minimum of V:  $v = \frac{2\mu}{\sqrt{\lambda}}, \quad M_H = \mu\sqrt{2}$ 

$$\Rightarrow \lambda = \frac{4\mu^2}{v^2} = \frac{2M_H^2}{v^2}$$

$$V = \frac{M_H^2}{2}H^2 + \frac{M_H^2}{2v}H^3 + \frac{M_H^2}{8v^2}H^4$$

 $M_H$  is the only free parameter

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general gauge: 
$$\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \frac{1}{\sqrt{2}} \left[ v + H(x) + i\chi(x) \right] \end{pmatrix}$$

#### gauge invariant Lagrangian of the Higgs sector

#### $\Rightarrow$ H-V-V gauge interactions, V=W and Z

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#### $\Rightarrow$ H-V-V gauge interactions, V=W and Z

$$\mathcal{L}_{\text{Yuk}} = -\sum_{f} \left( m_{f} + \frac{m_{f}}{v} H \right) \overline{\psi}_{f} \psi_{f} + \cdots \left( f f \chi, \phi^{\pm} \right)$$

#### $\Rightarrow$ H-f-f Yukawa interactions



$$\Gamma(H \to f\bar{f}) = N_c \frac{G_F M_H}{4\pi\sqrt{2}} m_f^2 \beta_f^3, \quad N_C = 3 \,(1) \text{ for quarks (leptons)}$$

$$\Gamma(H \to VV) = \frac{G_F M_H^3}{8\pi\sqrt{2}} F(r) \left(\frac{1}{2}\right)_Z, \quad r = \frac{M_V}{M_H}$$

#### **Higgs boson decay channels**

branching ratios  $BR(H \to X) = \frac{\Gamma(H \to X)}{\Gamma(H \to \text{all})}$ 



loop-induced (rare) decays



 $H \rightarrow \gamma \gamma$ 





#### **Higgs decays into 4 fermions**

also below VV threshold with one or two V off-shell



[Djouadi]

 $H \to VV \to 4f$ 

needs also background pocesses + h.o.



Bredenstein et al.  $\rightarrow$  **PROPHECY** 

 $H \to ZZ \to l^+ l^- \ l^+ l^-$ 



signal + background

## the Higgs – or not the Higgs?



CERN, July 2012



Oviedo, October 2013

#### A Standard Model Higgs boson at the LHC?



Theory:  $\sigma(pp \rightarrow H) \cdot BR(H \rightarrow X)$ 

#### Higgs production at the LHC



Handbook of Higgs Cross sections, arXiv:1101.0593, arXiv:1201.3084

#### cross section for Higgs-boson production – theory



Spira, Djouadi, Graudenz, Zerwas '91, '93 NLO: Dawson '91 ~80% NNLO: RH, Kilgore '02 ~30% Anastasiou, Melnikov '02 Ravindran, Smith, v. Neerven '03 **Resummation:** Catani, de Florian, Grazzini, Nason '02 Ahrens, Becher, Neubert, Zhang '08 ~10% **Electroweak:** Actis, Passarino, Sturm, Uccirati '08 Aglietti, Bonciani, Degrassi, Vicini '04 Degrassi, Maltoni '04 ~5% Djouadi, Gambino '94 Mixed EW/QCD: Anastasiou, Boughezal, Petriello '09 Fully differential NNLO: Anastasiou, Melnikov, Petriello '04 Catani, Grazzini '07

[courtesy R. Harlander]

Higgs production at LEP:





# Higgs production at the Tevatron:



# Theoretical bounds on Higgs boson mass

- unitarity  $\rightarrow$  upper bound
- Landau pole  $\rightarrow$  upper bound
- vacuum stability  $\rightarrow$  lower bound

# unitarity

# scattering of longitudinally polarized W bosons: $W_L W_L \rightarrow W_L W_L$



Extra contribution from scalar particle:



$$= g_{WWH}^2 \frac{E^2}{M_W^4} + \mathcal{O}(1) \qquad \text{for } E \to \infty$$

$$\mathcal{M} = \mathcal{M}_V + \mathcal{M}_S$$

 $\Rightarrow$  terms with bad high-energy behavior cancel for

 $g_{WWH} = g M_W$ 

for 
$$s >> M_W^2$$
, with  $t = -\frac{s}{2} (1 - \cos \theta)$ ,  
 $\mathcal{M} \approx \frac{M_H^2}{v^2} \left( 2 + \frac{M_H^2}{s - M_H^2} + \frac{M_H^2}{t - M_H^2} \right)$ 

partial wave expansion:

$$\mathcal{M}(s,t) = 8\pi \sum_{l=0}^{\infty} (2l+1) P_l(\cos\theta) \mathbf{a_l}$$

unitarity condition:  $|a_l| < 1$ 

project on l = 0 partial wave:

$$a_{0} = \frac{1}{16\pi} \int_{-1}^{1} d\cos\theta \,\mathcal{M}(s,t)$$

$$= \frac{M_{H}^{2}}{8\pi v^{2}} \left[ 2 + \frac{M_{H}^{2}}{s - M_{H}^{2}} - \frac{M_{H}^{2}}{s} \log\left(1 + \frac{s}{M_{H}^{2}}\right) \right]$$

$$\approx \frac{M_{H}^{2}}{8\pi v^{2}} \quad \text{for} \quad s >> M_{H}^{2}$$

 $a_0 < 1 \quad \Rightarrow \quad M_H < 872 \,\mathrm{GeV}$ 

# Landau pole

Higgs self coupling is scale dependent,  $\lambda(Q)$ 

variation with scale Q described by RGE

$$Q^2 \frac{d\lambda}{dQ^2} = \beta(\lambda) = \frac{3}{4\pi^2} \,\lambda^2$$

solution:

$$\lambda(Q) = \frac{\lambda(v)}{1 - \frac{3}{4\pi^2}\lambda(v)\log\frac{Q^2}{v^2}} \quad \text{with} \quad \lambda(v) = \frac{M_H^2}{2v^2}$$

diverges at scale  $Q = \Lambda_C$  (Landau pole)

$$\Lambda_C = v \, \exp\left(\frac{4\pi^2 v^2}{3M_H^2}\right)$$

self-coupling diverges at

$$\Lambda_C = v \, \exp\left(\frac{4\pi^2 v^2}{3M_H^2}\right)$$

maximum Higgs mass by condition  $\Lambda_C > M_H$ 

 $\Rightarrow M_H < 800 \,\mathrm{GeV}$ 

# vacuum stability

top-quark Yukawa coupling  $g_t \sim m_t$  contributes to the running Higgs self coupling  $\lambda(Q)$  through top loop  $\sim g_t^4$ 

H H

F

H

variation with scale Q described by RGE

$$Q^2 \frac{d\lambda}{dQ^2} = \frac{3}{4\pi^2} \left(\lambda^2 - \frac{m_t^4}{v^4}\right)$$

approximate solution:

$$\lambda(Q) = \lambda(v) - \frac{3m_t^4}{2\pi^2 v^4} \log \frac{Q}{v}$$

 $\lambda(Q) < 0$  for  $Q > \Lambda_C \rightarrow$  vacuum not stable

high value of  $\Lambda_C$  needs  $M_H$  large enough

combined effects:

$$\frac{d\lambda}{dt} = \frac{1}{16\pi^2} \left( 12\lambda^2 - 3g_t^4 + 6\lambda g_t^2 + \cdots \right)$$





<sup>[</sup>Degrassi et al. 2012]

#### **Status of the Standard Model**



SM input now completey determined  $\Rightarrow$  PO uniquely predicted

	theo	ехр
$\sin^2 heta_{ m eff}$	$0.23152 \pm 0.00005 \pm 0.00005$	$0.23153 \pm 0.00016$
$M_W ({ m GeV})$	$80.361 \pm 0.006 \pm 0.004$	$80.385 \pm 0.015$



# The success of the Standard Model

- impressive confirmation by a huge data sample from low to high energies, no significant deviations
- quantum effects have been established at many  $\sigma$
- perfect indirect and direct determination of the top quark
- now being repeated for the Higgs boson
- new particle around 126 GeV strong candidate for the Higgs boson
- if confirmed: Standard Model closed

Happy End of a successful story ?

# **Shortcomings of SM**

• no mass terms for neutrinos [introduce  $\nu_R$  ...]

- hierarchy problem  $v \ll M_{\rm Pl}$ ,  $M_H \ll M_{\rm Pl}$
- Iarge number of free parameters  $g_1, g_2, v, m_f, V_{\text{CKM}}$
- no further unification of forces
- missing link to gravity

- nature of dark matter?
- baryon asymmetry of the universe?

- next steps with upgraded LHC
  - confirm the Higgs boson properties
  - check versus electroweak precision measurements
  - or find deviations, new structures:
  - more Higgs bosons (doublets, singlet, ...)
  - supersymmetry (minimal or non-minimal)
  - new strong sector, substructure
  - **\_** ...



# extra slides

#### few observables with not-so-good agreement

- in general, SM is in overall agreement with data
- yet a few quantities prefer to stand a bit apart ( $\sim 3\sigma$ )
  - the forward-backward asymmetry for b quarks,  $A_{\rm FB}^{b\bar{b}}$  at the Z peak
  - the anomalous magnetic moment of the muon
  - ${\scriptstyle \bullet}~$  the forward-backward asymmetry for top quarks at the Tevatron,  $p\bar{p} \rightarrow t\bar{t}$

no conclusive situation

SM Higgs:

- $\lambda H^4$  term ad hoc
- Higgs boson mass: free parameter  $\sim \sqrt{\lambda}$
- no a-priori reason for a light Higgs boson

SM Higgs:

- $\checkmark \lambda H^4$  term ad hoc
- Higgs boson mass: free parameter  $\sim \sqrt{\lambda}$
- no a-priori reason for a light Higgs boson

SUSY Standard Model avoids these questions

$$H_{2} = \begin{pmatrix} H_{2}^{+} \\ v_{2} + H_{2}^{0} \end{pmatrix}, \qquad H_{1} = \begin{pmatrix} v_{1} + H_{1}^{0} \\ H_{1}^{-} \end{pmatrix}$$
  
couples to  $u$  couples to  $d$ 

- SUSY gauge interaction  $\rightarrow$   $H^4$  terms
- self coupling remains weak

spectrum of Higgs bosons in the MSSM:  $h^0$ ,  $H^0$ ,  $A^0$ ,  $H^{\pm}$ 



spectrum of Higgs bosons in the MSSM:  $h^0$ ,  $H^0$ ,  $A^0$ ,  $H^{\pm}$ 



 $m_{h}^{0}$  strongly influenced by quantum effects, e.g. t,  $\tilde{t}$ 



## sensitivity to mass/mixing parameters

 $m_{h^0}$  prediction at different levels of accuracy:



$$X_t = A_t - \mu \cot \beta$$

#### allowed region for top-squark mass and mixing



[Heinemeyer, Staal, Weiglein '12]

compatible with light top-squarks ongoing experimental search