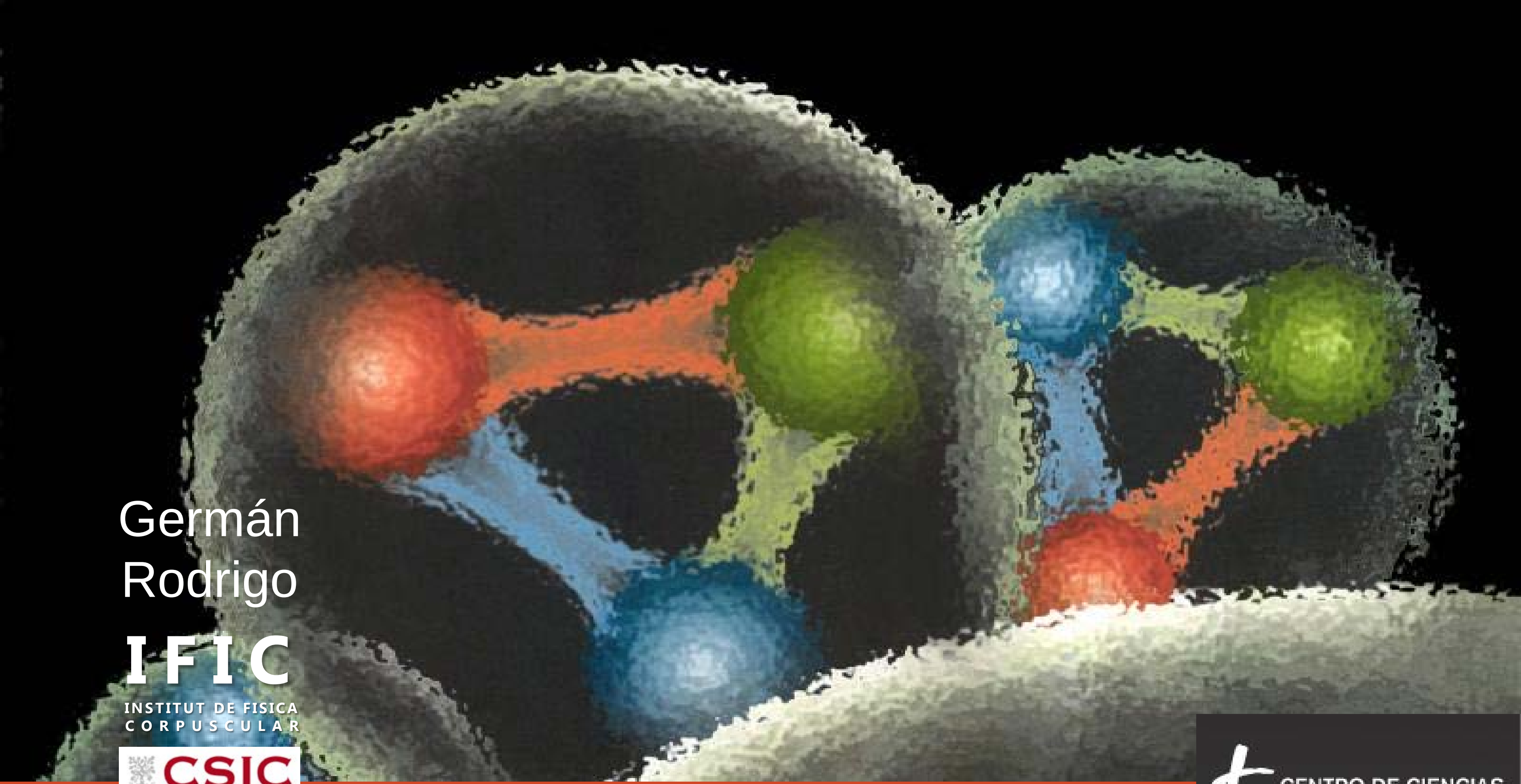

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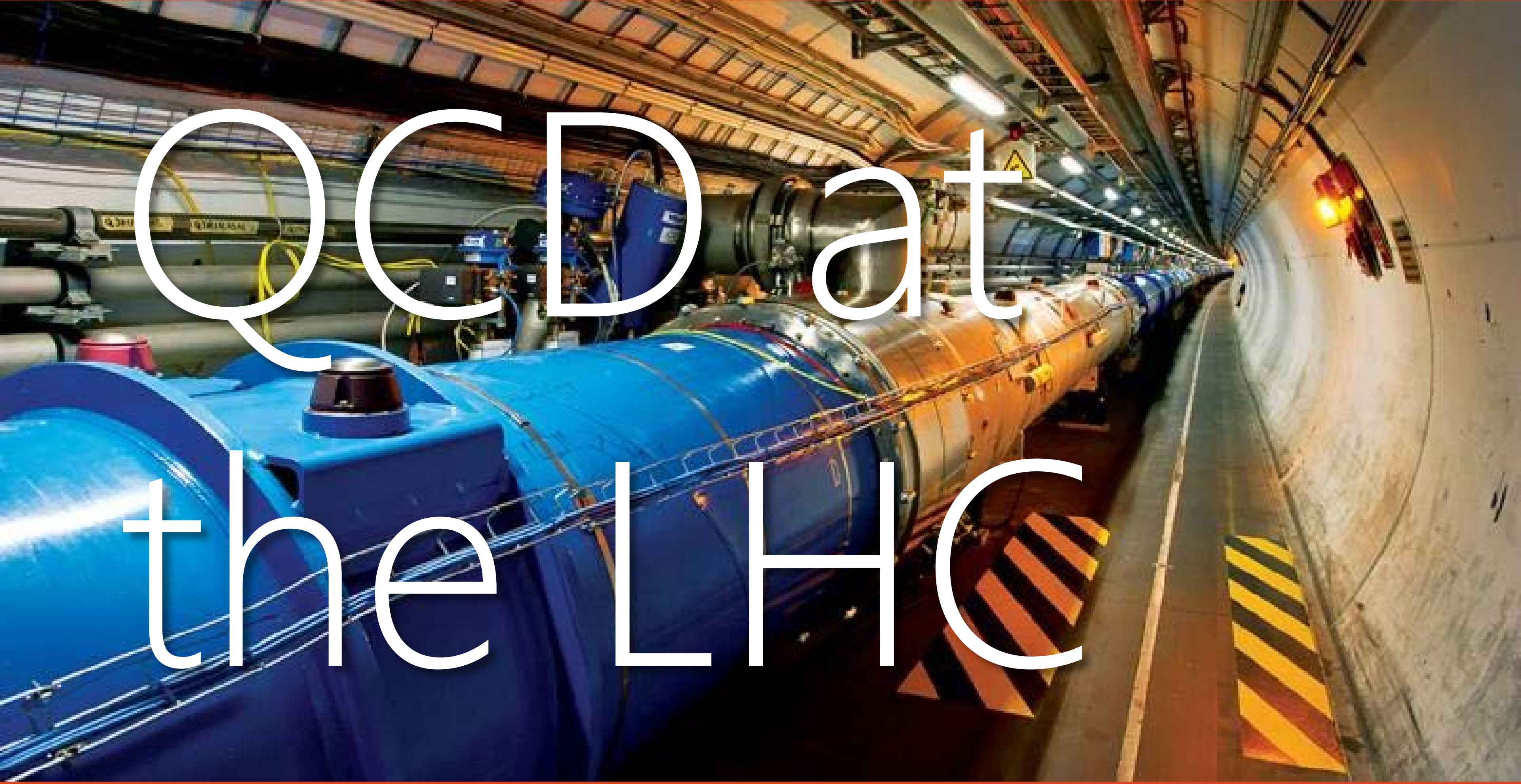
QCD, jets and Monte Carlo: 2nd lecture

Taller de Altas Energías TAE2014, September 2014

QCD at the LHC



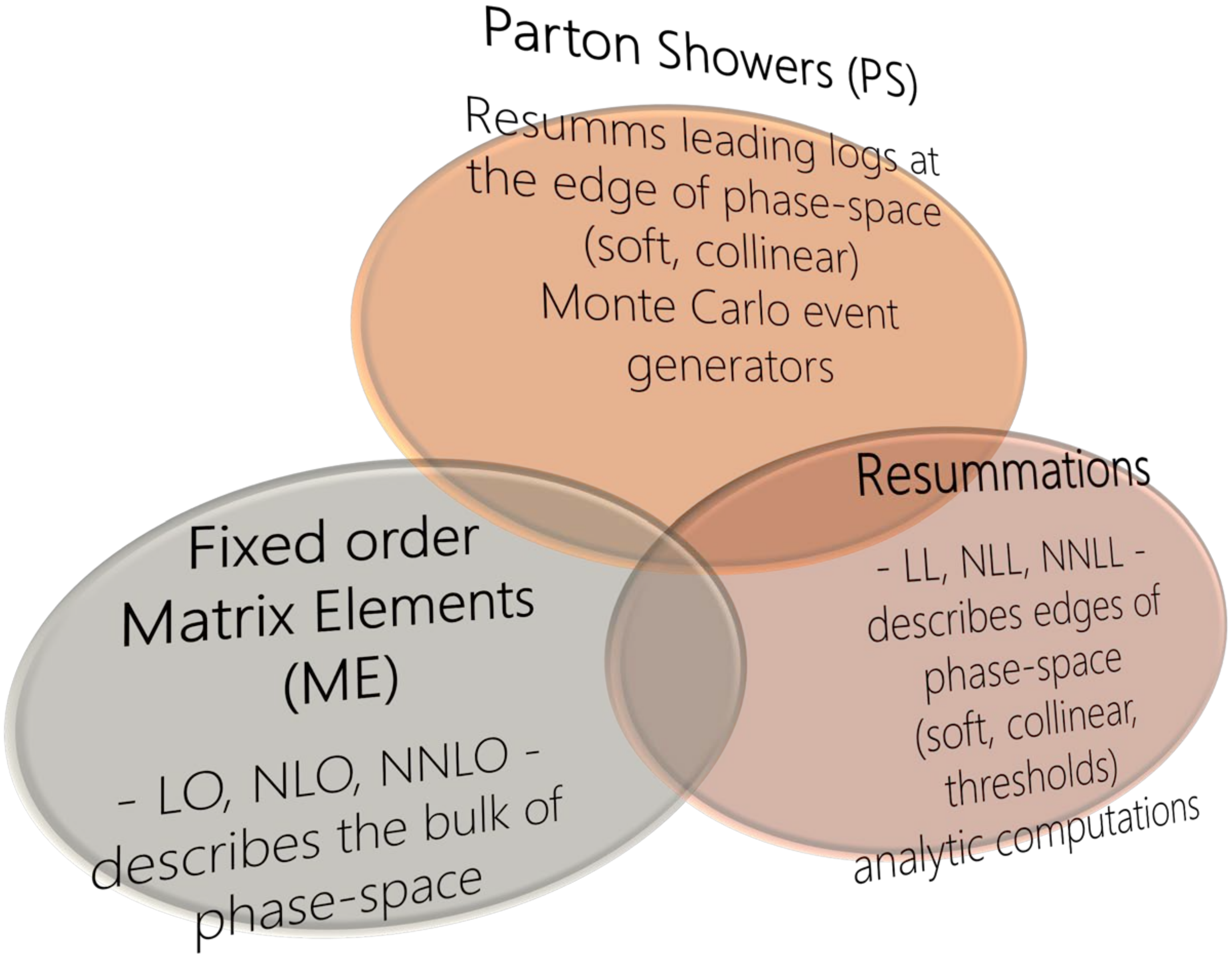
The LHC is a hadronic machine working at higher energies than ever before

- larger phase-space for hard radiation
- higher multiplicities (external legs)
  - more powers of α_S
  - multi-particle final states are the signal for new physics
  - multi-scale processes: logs of the ratio of very different scales
- proton is not elementary:
  - need to know PDF accurately
  - new channels might open at higher orders in pQCD

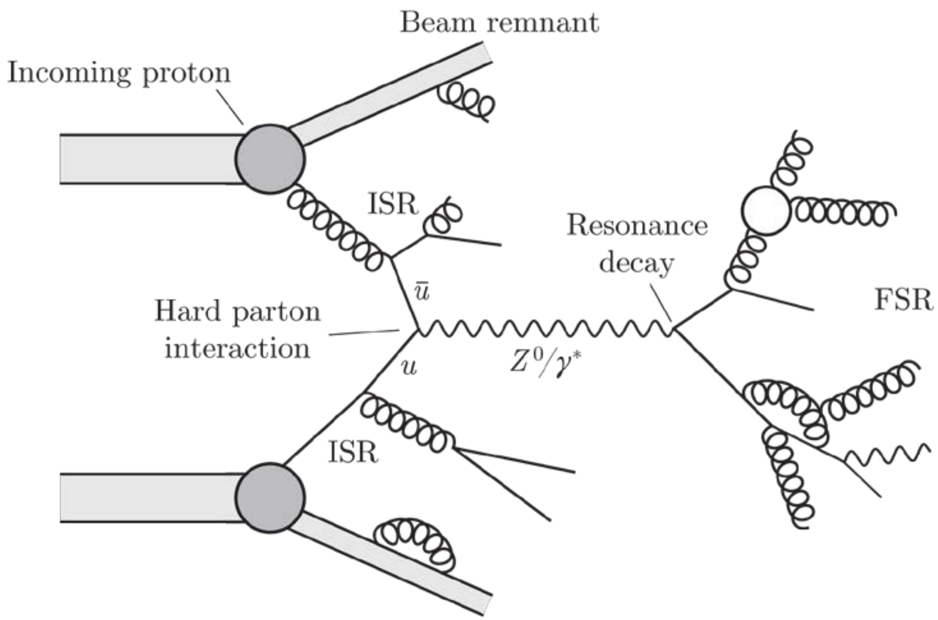
Huge radiative corrections

The absence so far of a clear signal BSM makes even more relevant the role of precision physics

The path to precision



Factorization in hadronic collisions



- Factorize physics into long distance (hadronic ~ M_{had}), and short distance (partonic $Q \gg M_{\mathrm{had}}$),
- factorization violation is power supressed ~ $\mathcal{O}\,(M_{\mathrm{had}}/Q)^q$

Parton densities
PDF

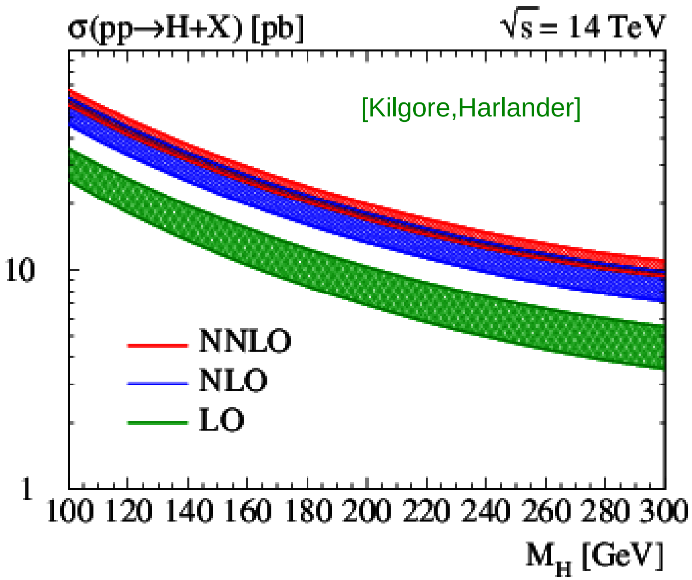
Hard scattering
cross-section

$$\sigma = \sum_{a,b} \int dx_1\, dx_2\, f_a(x_1, \mu_F)\, f_b(x_2, \mu)\, \hat{\sigma}_{ab}(x_1 p_A, x_2 p_B; \mu_F, \mu_R) + O\left(\frac{1}{Q}\right)$$

Factorization and renormalization scales

Higher
twist

Partonic cms energy $\hat{s} = x_1\, x_2\, s$



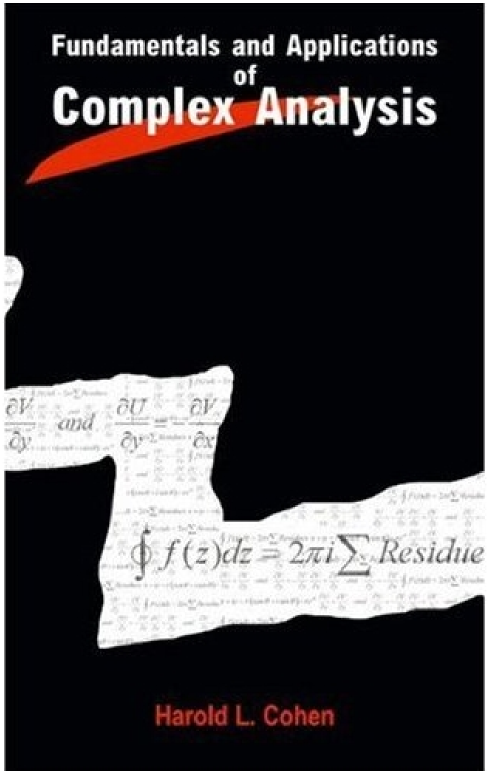
Perturbative view: higher orders improve systematically the precision of the theoretical predictions (estimated by varying the renormalization/factorization scales) for background and signal

- LO: fails to describe normalization (up to a factor 2). Monte Carlo event generators (LO + parton showers) : improves the shape of distributions, but normalization still underestimated
- NLO: first reliable estimate of central value
- NNLO: first serious estimate of the theoretical error

New perturb
metho

- To reach a new frontier in highe
- But also to better understand th

Recursion relations and unitarity methods



Properties of the S-Matrix

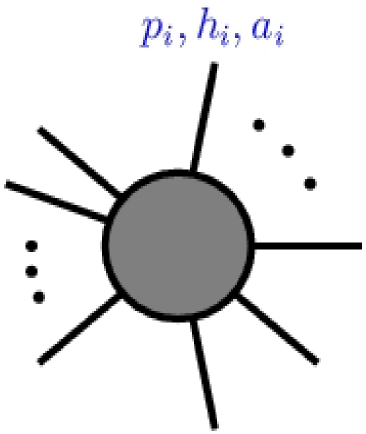
- Analyticity: scattering amplitudes are determined by their singularities
- Unitarity: the residues at singular points are products of scattering amplitudes with lower number of legs and/or less loops

Here are the words of some enthusiast: "One of the most remarkable discoveries in elementary particle physics has been that of the existence of the complex plane", "... the theory of functions of complex variables plays the role not of a mathematical tool, but of a fundamental description of nature inseparable from physics … ."

J. Schwinger, *Particles, Sources, and Fields*, Vol.1, p.36

- recycling: using scattering amplitudes to calculate other scattering amplitudes

Helicity basis + colour decomposition



Expressions simplify by using "right variables"

(1) for n-gluons at tree level

$$M_n(\{p_i, h_i, a_i\}) = \sum_{P(1,\ldots,n)} \mathrm{Tr}(\mathbf{T}^{a_1} \cdots \mathbf{T}^{a_n})\; A_n(\{p_i, h_i\})$$

SU(N_C) generators in the fundamental representation

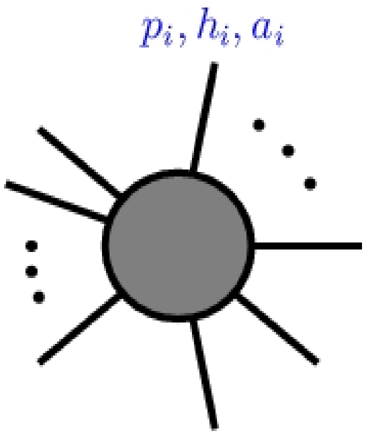
$$f^{abc} = \frac{-i}{\sqrt{2}} \left(\mathrm{Tr}(\mathbf{T^a T^b T^c}) - \mathrm{Tr}(\mathbf{T^a T^c T^c})\right)$$

colour subamplitude:
momenta and helicities
gauge-invariant
fixed cyclic order of external legs

n-gluon amplitude

n	# diagrams	# colour-ord diagrams
4	4	3
5	25	10
6	220	36
7	2485	133
8	34300	501
9	559405	1991
10	10525900	7225

[Cvitanovic, Lauwers, Scharbach, Berends, Giele, Mangano, Parke, Xu,Bern,Kosower, Lee, Nair]



(2) Four-dimensional spinors of definite helicity

$$|i^\pm\rangle = \frac{1}{2}(1 \pm \gamma_5)u(p_i) = v_\mp(p_i) \;, \qquad \langle i^\pm| = \bar{u}_\pm(p_i) = \bar{v}_\mp(p_i)$$

$$p_i^2 = 0 \;, \qquad p_i^{a\dot{a}} = k_i^\mu \sigma_\mu^{a\dot{a}} = \lambda_i^a \tilde{\lambda}_i^{\dot{a}}$$

- spinor inner products and other useful identities

$$\langle ij \rangle = \langle i^- | j^+ \rangle = \varepsilon_{ab} \lambda_i^a \lambda_j^b = \sqrt{|s_{ij}|} e^{i\phi_{ij}} = -\langle ji \rangle \qquad \text{holomorphic}$$

$$[ij] = [i^+ | j^-] = \varepsilon_{\dot{a}\dot{b}} \tilde{\lambda}_i^{\dot{a}} \tilde{\lambda}_j^{\dot{b}} = -\langle ij \rangle^* = -[ji] \qquad \text{antiholomorphic}$$

$$[i|\gamma^\mu|j\rangle = \langle j|\gamma^\mu|i]$$

$$s_{ij} = (p_i + p_j)^2 = \langle ij \rangle [ji]$$

$$\not{p}_i = |i\rangle[i| + |i]\langle i| \qquad \text{sum over polarizations}$$

$$\not{p}_i |i^\pm\rangle = 0 \qquad \text{equation of motion} \qquad \langle ij] = 0 = \langle ii \rangle$$

- polarization vector $\quad \epsilon^2 = 0 \;, \quad \epsilon^+ \cdot \epsilon^- = 0 \;, \quad k \cdot \epsilon^\pm(k) = 0$

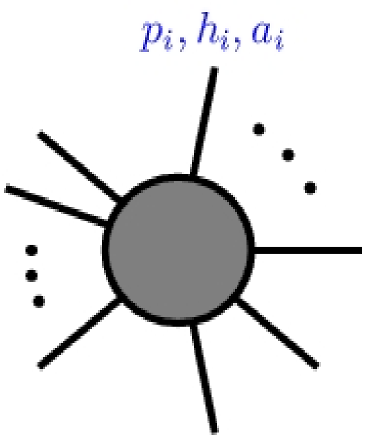
$$\epsilon_\mu^+(k,\xi) = \frac{\langle \xi | \gamma_\mu | k]}{\sqrt{2}\langle \xi k \rangle}$$

$$\epsilon_\mu^-(k,\xi) = \frac{[\xi | \gamma_\mu | k\rangle}{\sqrt{2}[k \xi]}$$

- equivalent to **axial gauge $\xi = n$**
- a clever choice of the gauge momentum can simplify calculations

Spinors

[Xu,Zhang,Chang, Berends, Kleiss, De Causmaeker, Gastmans, Wu,Gunion, Kunzst]



- spinor identities

$$\langle 1|\gamma^\mu|2][3|\gamma_\mu|4\rangle = 2\langle 14\rangle[32] \qquad \text{Fierz}$$

$$\langle 12\rangle\langle 34\rangle = \langle 14\rangle\langle 32\rangle + \langle 13\rangle\langle 24\rangle \qquad \text{Shouten}$$

Exercise: proof the Fierz and Shouten identities

Hint: divide and multiply by **⟨23⟩** and apply the Dirac identity

$$\gamma^\mu\gamma^\nu\gamma^\sigma\gamma_\mu = 4g^{\nu\sigma}$$

Exercises:

Calculate the scattering amplitudes and square amplitude for $e^+e^- \to q\bar{q}$ by using the helicity method, and compare with the traditional calculation

How many independent helicity amplitudes there are ?

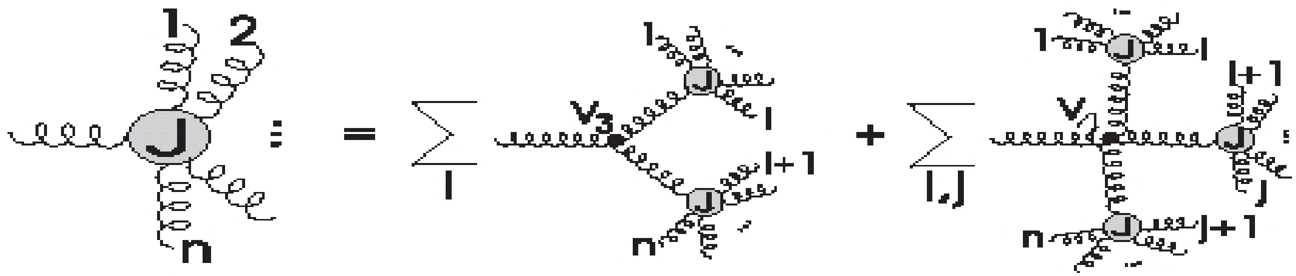
$$M_{e^+e^-\to q\bar{q}} \sim [\bar{u}(p_1)\gamma^\mu v(p_2)]\,[\bar{v}(p_3)\gamma^\nu u(p_4)]\, d_{\mu\nu}(p_{12},n)$$

$$|M|^2 \sim \mathrm{Tr}(\not{p}_1\gamma^\mu\not{p}_2\gamma^\sigma)\mathrm{Tr}(\not{p}_3\gamma^\nu\not{p}_4\gamma^\rho)d_{\mu\sigma}(p_{12},n)d_{\nu\rho}(p_{12},n)$$

Off-shell recursion relations

[Berends, Giele]

- Define Off-shell current: amplitude with one off-shell leg, building block for the off-shell current with higher multiplicity



the gluonic current particularly simple for some helicity configurations

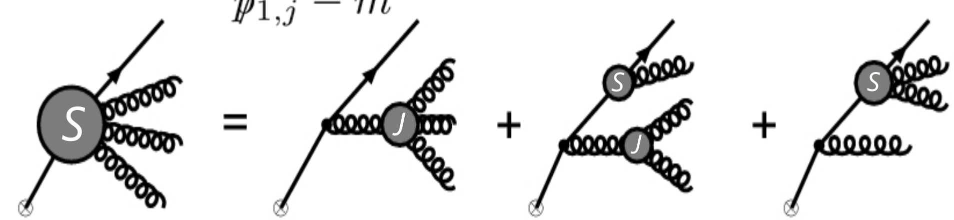
$$J^\mu(i^+,\ldots,j^+) = \frac{\langle \xi|\gamma^\mu \not{p}_{i,j}|\xi\rangle}{\sqrt{2}\langle \xi i\rangle\langle i(i+1)\rangle\cdots\langle j\xi\rangle}$$

on-shell amplitude by setting on-shell the off-shell leg

- Off-shell spinorial currents

$$M_n(1_q;2,\ldots,n-1;n_{\bar q}) = \sum_{P(2,\ldots,n-1)} (\mathbf{T}^{\mathbf{a}_2}\cdots\mathbf{T}^{\mathbf{a}_{n-1}})\, A_n(1_q;2,\ldots,n-1;n_{\bar q})$$

$$S(1_q;2,\ldots,j) = -\sum_{k=1}^{j-1} S(1_q;2,\ldots,k)\,\gamma\cdot J(k+1,\ldots,j)\frac{i}{\not{p}_{1,j}-m}$$



MHV amplitudes

Multi-gluonic amplitudes at tree level: Amplitude for all gluons of positive helicity or one single gluon of negative helicity vanishes

- two negative helicities (Maximal Helicity Violating Amplitude) rather simple [Parke-Taylor, 1986]

$$A_n(1^+,\ldots,i^\pm,\ldots,n) = 0$$

$$A_n^{\mathrm{MHV}}(1^+,\ldots,i^-,\ldots,j^-,\ldots,n^+) = i\frac{\langle ij\rangle^4}{\langle 12\rangle\langle 23\rangle\cdots\langle (n-1)n\rangle\langle n1\rangle}$$

proven via recursion relations [Berends-Giele, Mangano-Parke-Xu,1988]

next-to-MHV $A_n^{\mathrm{NMHV}}(1^+,\cdots,i^-,\cdots,j^-,\cdots,k^-,\cdots,n^+)$

does contain both $\langle ij\rangle$ and $[ij]$ [kosower,1990]

On-shell recursion relations at tree-level: BCFW

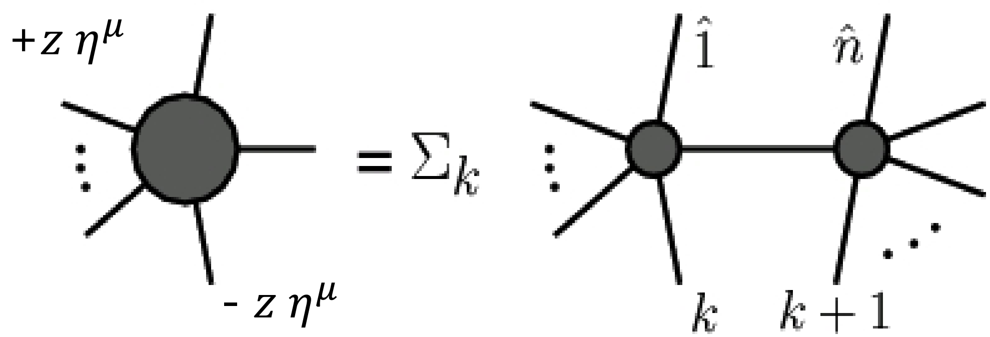
[Britto, Cachazo, Feng, Witten]

How to reconstruct scattering amplitude from its singularities

Add $z\,\eta^\mu$ (z complex) to the four-momentum of one external particle and subtract it on another such that the shift leaves them on-shell

$$0 = \frac{1}{2\pi i}\oint_{C\,\mathrm{at}\,\infty} \frac{A_n(z)}{z} = A_n(0) - \sum_{z_i} \frac{\mathrm{Res}_{z_i}(A_n(z))}{z_i}$$

has the correct residue at any multi-particle pole



$$A_n(1,2,\ldots,n) = \sum A_L(\hat{1},2,\ldots,-\hat{p}_{1,k})\,\frac{i}{s_{1,k}}\,A_R(\hat{p}_{1,k},k+1,\ldots,\hat{n})$$

- Diagrammatic proof [Draggiotis, Kleiss, Lazopoulos, Papadopoulos]
- Compact analytical results, although colour dressed Berends-Giele (off-shell recursion) might be more efficient numerically [Duhr, Höche, Maltoni]

in practice

holomorphic shift ((-,+) is not a safe shift)

$$\hat{p}_i^\mu = p_i^\mu + \frac{z}{2}[i|\gamma^\mu|j\rangle \qquad |\hat{i}\rangle = |i\rangle + z|j\rangle \qquad |\hat{i}] = |i]$$

$$\hat{p}_j^\mu = p_j^\mu - \frac{z}{2}[i|\gamma^\mu|j\rangle \qquad |\hat{j}\rangle = |j\rangle \qquad |\hat{j}] = |j] - z|i]$$

anti-holomorphic shift (i ↔ j)

z determined setting on-shell the intermediate momenta

$$\hat{p}_{1,k}^\mu = p_{1,k}^\mu + \frac{z}{2}[i|\gamma^\mu|j\rangle \ , \qquad \hat{p}_{1,k}^2 = 0 \ , \qquad z = -\frac{s_{1,k}}{[i|p_{1,k}|j\rangle}$$

☑ use only on-shell amplitudes

☑ rather compact expressions

☒ generates spurious poles at $[i|p_{1,k}|j\rangle$

while physical IR divergences at $s_{i,j} = (p_i + p_{i+1} + \ldots + p_j)^2$

Exercises:

Calculate by using BCFW the six-gluon amplitude

$$A_6(1^+,2^+,3^+,4^-,5^-,6^-) = \frac{i}{\langle 2|1+6|5]}\left(\frac{\langle 6|1+2|3]^3}{\langle 61\rangle\langle 12\rangle[34][45]s_{126}} + \frac{\langle 4|5+6|1]^3}{\langle 23\rangle\langle 34\rangle[56][61]s_{561}}\right)$$

One-loop amplitudes

The classical paradigm for the calculation of one-loop diagrams was established in **1979**

Calculation of one-loop scalar integrals

Reduction of tensor one-loop integrals to scalar integrals

Nuclear Physics B
Volume 153, 1979, Pages 365-401

Scalar one-loop integrals

G. 't Hooft, M. Veltman

Received 16 January 1979

Nuclear Physics B
Volume 160, 26 Nov 1979, Pages 151-207

One-loop corrections for e+e− annihilation into μ+μ− in the Weinberg model

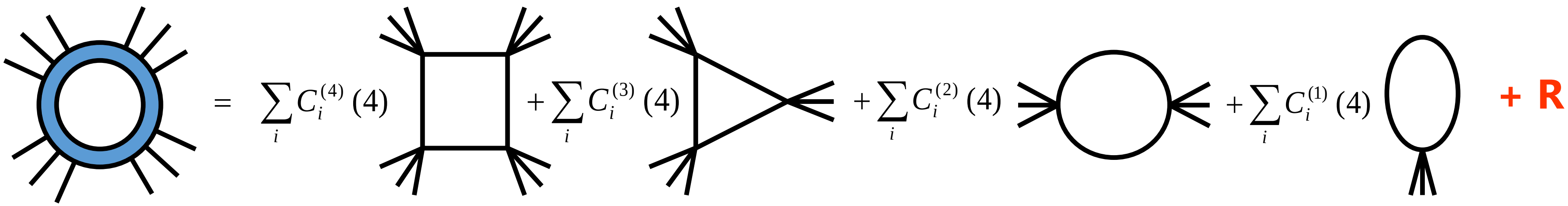
G. Passarino, M. Veltman

Received 22 March 1979

Not adequate for processes beyond 2→2
(Gramm determinants+large number of Feynman diagrams)

Generalized Unitarity: the one-loop basis

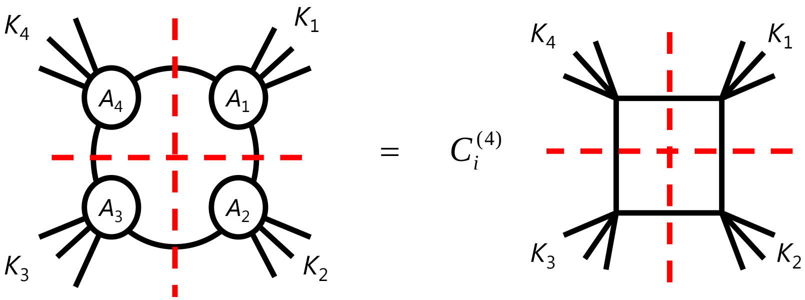
A dimensionally regulated n-point one-loop integral (scattering amplitude) is a linear combination of boxes, triangles, bubles and tadpoles with rational coefficients



- Pentagons and higher n-point functions can be reduced to lower point integrals and higher dimensional polygons that only contribute at O(ε) [Bern, Dixon, Kosower]
- The task is reduced to determining the coefficients: by applying multiple cuts at both sides of the equation [Brito, Cachazo, Feng]
- R is a finite piece that is entirely rational: can not be detected by four-dimensional cuts

Generalized Unitarity

Quadruple cut

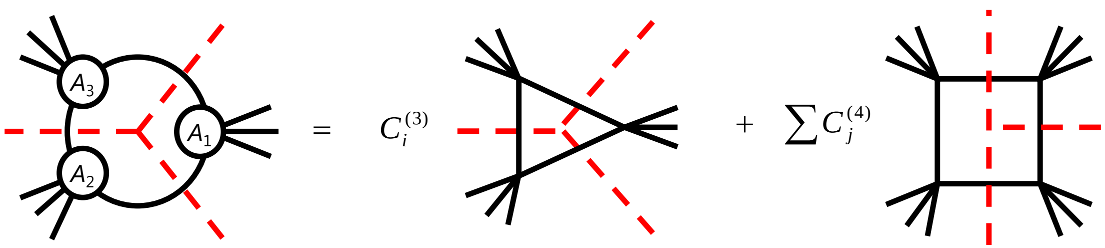


The discontinuity across the leading singularity is unique

$$C_i^{(4)} = A_1 \times A_2 \times A_3 \times A_4$$

Four on-shell constrains
➡ freeze the loop momenta

Triple cut



Only three on-shell constrains ➡ one free component of the loop momentum

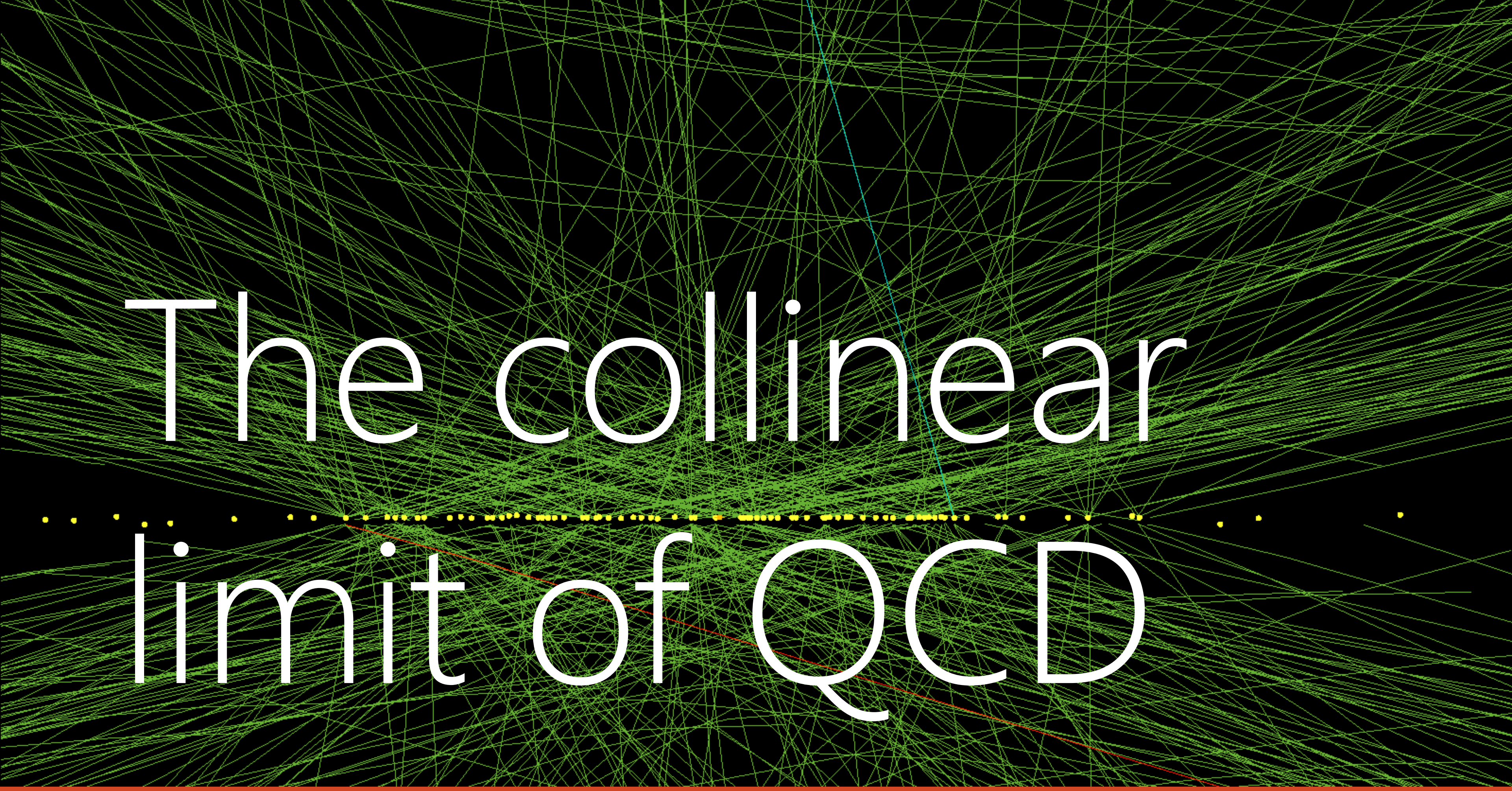
And so on for double and single cuts

- OPP [Ossola, Pittau, Papadopoulos]: a systematic way to extract the coefficients

Rational terms

d-dimensional cuts, recursion relations (BCFW), Feynman rules …

The collinear limit of QCD



Collinear limits in QCD

- evaluate IR finite cross-sections ▶ subtraction terms
- IR properties of amplitudes exploited to compute logarithmic enhanced perturbative terms ▶ resummations
- improve physics content of Monte Carlo event generators ▶ parton showers
- Evolution of PDF´s and fragmentation functions
- beyond QCD: hints on the structure of highly symmetric gauge theories (e.g. N=4 super-Yang-Mills)
- Factorization theorems: pQCD for hard processes

Altarelli, Parisi, Berends, Giele, Mangano, Parke …

Multiple (double) collinear limit

Momenta $p_1, \dots, p_m$ of m partons become parallel
Sub-energies $s_{ij} = (p_i + p_j)^2$ of the same order and vanish simultaneously

- Matrix element in perturbation theory (pQCD) $M = M^{(0)} + M^{(1)} + M^{(2)} + \dots$
- At tree-level ($s = s_{ij}, s_{ijk}$, or any sub-energy) $M^{(0)}(p_1, \dots, p_m; \dots, p_n) \simeq \left(\frac{1}{\sqrt{s}}\right)^{m-1}$
- At one-loop (scaling violation) $M^{(1)}(p_1, \dots, p_m; \dots, p_n) \simeq \left(\frac{1}{\sqrt{s}}\right)^{m-1} \left(\frac{s}{\mu^2}\right)^{-\epsilon}$

The momentum of the m partons in terms of two back-to-back light-like momenta $\tilde{P}^2 = 0, n^2 = 0$:

$$\widetilde{P}^\mu = p_{1,m}^\mu - \frac{s_{1,m}\, n^\mu}{2n \cdot \widetilde{P}}$$

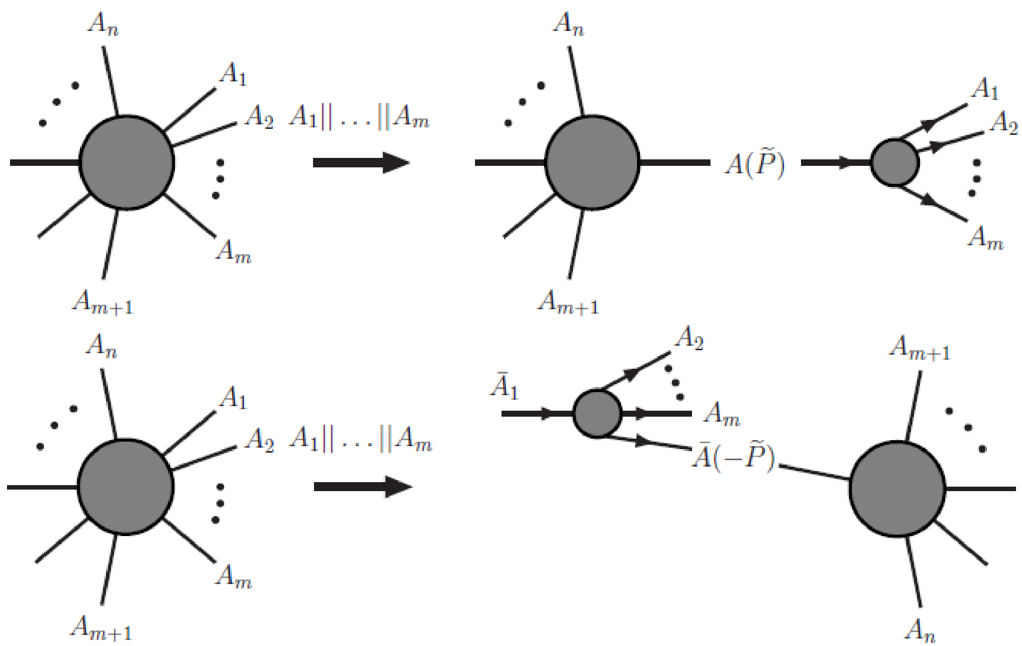
$\tilde{P}^\mu$: collinear direction
n^μ: describes how the collinear limit is approached
$z_i = \frac{n \cdot p_i}{n \cdot \tilde{P}}$: longitudinal momentum fraction, $\sum z_i = 1$

Collinear factorization at tree-level

- External legs on-shell with physical polarisations
- factorization in colour-space [Catani, de Florian, GR]
- Also colour stripped (Split function of colour-subamplitudes) [Bern, Chalmers, Dixon, Kosower, Catani, Grazzini, Glover, Campbell, del Duca, …]

time - like

space - like



Collinear limit

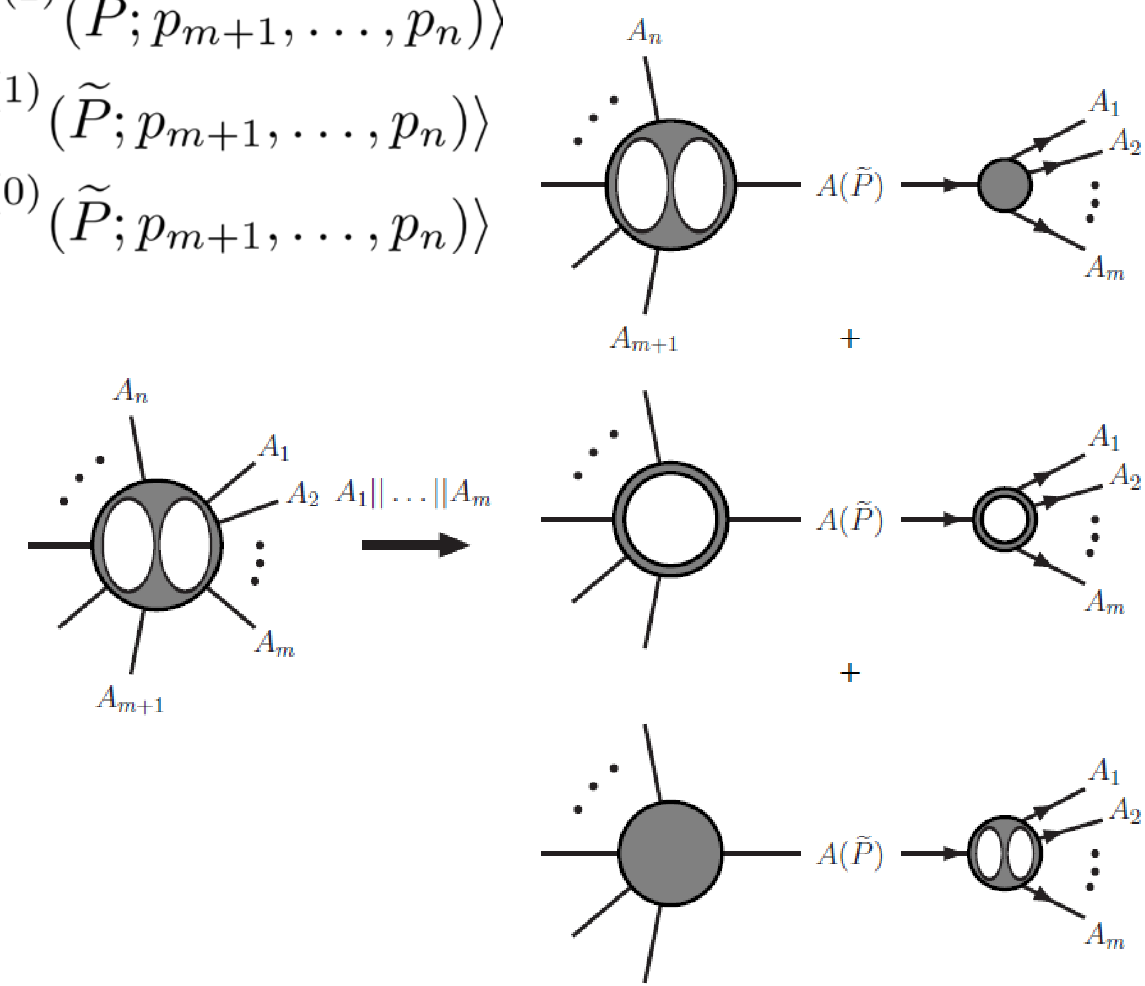
- Most singular behaviour captured by **universal** (process independent) factorisation properties
- **Splitting matrix** depends on the collinear partons only.
- Space-like and time-like related by crossing

$$|M^{(0)}(p_1, \ldots, p_n)\rangle$$

$$= \mathbf{Sp}^{(0)}(p_1, \ldots, p_m; \widetilde{P})\, |\overline{M}^{(0)}(\widetilde{P}; p_{m+1}, \ldots, p_n)\rangle + \mathcal{O}\left((\sqrt{s})^{3-m}\right)$$

At two loops

$$
\begin{aligned}
&|M^{(2)}(p_1,\ldots,p_n)\rangle \\
&\simeq \mathbf{Sp}^{(0)}(p_1,\ldots,p_m;\widetilde{P})\,|\overline{M}^{(2)}(\widetilde{P};p_{m+1},\ldots,p_n)\rangle \\
&+\mathbf{Sp}^{(1)}(p_1,\ldots,p_m;\widetilde{P})\,|\overline{M}^{(1)}(\widetilde{P};p_{m+1},\ldots,p_n)\rangle \\
&+\mathbf{Sp}^{(2)}(p_1,\ldots,p_m;\widetilde{P})\,|\overline{M}^{(0)}(\widetilde{P};p_{m+1},\ldots,p_n)\rangle
\end{aligned}
$$



The collinear projection

Work in the axial gauge (physical polarizations): only diagrams where the parent parton emitted and absorbed collinear radiation

$$\frac{1}{\not p_{12}} = \frac{1}{s_{12}} \not p_{12} = \frac{1}{s_{12}} \left(\not \widetilde{P} + \frac{s_{12}}{2n \cdot \widetilde{P}} \not n \right) \simeq \frac{1}{s_{12}} u(\widetilde{P}) \bar{u}(\widetilde{P}) + \ldots$$

$$d_{\mu\nu}(p_{12}, n) = d_{\mu\nu}(\widetilde{P}, n) + \cdots = \epsilon_\mu(\widetilde{P}) \epsilon^*_\nu(\widetilde{P}) + \cdots$$

The projection over the collinear limit is obtained by setting the parent parton at on-shell momenta $\tilde{P}$

Splitting functions

The square of the splitting amplitude, summed over final-state colours and spins, and averaged over colours and spins of the parent parton, defines the m-parton splitting function

$$\langle P^{(0)}_{a_1\cdots a_m}\rangle = \left(\frac{s_{1,m}}{2\mu^{2\epsilon}}\right)^{m-1} \overline{|\mathbf{Sp}^{(0)}_{a_1\ldots a_m}|}$$

Which is a generalization of the customary (i.e. with $m = 2$) Altarelli-Parisi splitting function

- Probability to emit further radiation with given longitudinal momenta, from the leading singular behavior
- Universal (process independent): e+e-, DIS or hadron collisions

Exercise:

Calculate the splitting functions for the collinear processes $q \to qg$, $g \to q\bar{q}$ and $g \to gg$ by using the helicity method

Hint:

$$\mathbf{Sp}^{(0)}_{q\to q_1 g_2} = \mathbf{T}^a \frac{1}{s_{12}} \bar{u}(p_1) \not{\varepsilon}(p_2)\, v(\widetilde{P})$$

$$P^{(0)}_{q\to q_1 g_2} = C_F \frac{1+z^2}{1-z} \qquad z = z_1 = \frac{n\cdot p_1}{n\cdot \widetilde{P}} \qquad z_2 = 1 - z$$