3. Electroweak Standard Model

preliminaries

<u>Dirac matrices:</u> $\gamma^\mu \ (\mu = 0,1,2,3), \quad \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2\, g^{\mu\nu}$

$\overline{\Gamma} = \gamma^0 (\Gamma)^\dagger\, \gamma^0, \quad \Gamma$ *any Dirac matrix oder product of matrices*

further Dirac matrix: $\gamma_5 = i\,\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & \mathbf{1} \\ \mathbf{1} & 0 \end{pmatrix}$

$$\gamma_5\gamma^\mu + \gamma^\mu\gamma_5 = 0, \qquad \overline{\gamma_5} = -\gamma_5, \qquad \gamma_5^2 = \mathbf{1}$$

<u>chiral fermions:</u>

$\psi^L = \frac{1-\gamma_5}{2}\,\psi$ *left-handed spinor, L-chiral spinor*

$\psi^R = \frac{1+\gamma_5}{2}\,\psi$ *right-handed spinor, R-chiral spinor*

projectors on right/left chirality: $\omega_\pm = \frac{1\pm\gamma_5}{2}, \quad (\omega_\pm)^2 = \omega_\pm$

chiral currents:

$$\overline{\psi^L}\,\gamma^\mu\psi^L = \overline{\psi}\,\gamma^\mu\tfrac{1-\gamma_5}{2}\,\psi \equiv J^\mu_L$$ left-handed current

$$\overline{\psi^R}\,\gamma^\mu\psi^R = \overline{\psi}\,\gamma^\mu\tfrac{1+\gamma_5}{2}\,\psi \equiv J^\mu_R$$ right-handed current

$$J^\mu_V \;=\; \overline{\psi}\gamma^\mu\psi \;=\; J^\mu_L + J^\mu_R$$ vector current

$$J^\mu_A \;=\; \overline{\psi}\gamma^\mu\gamma_5\psi \;=\; -J^\mu_L + J^\mu_R$$ axialvector current

mass term:

$$m\,\overline{\psi}\psi \;=\; m\,(\overline{\psi^L}\,\psi^R + \overline{\psi^R}\,\psi^L)$$

connects L and R !

$$\boxed{\text{symmetry group: } SU(2)_I \times U(1)_Y}$$

$SU(2)_I$: weak isospin, generators $T_{\rm I}^a = \frac{1}{2}\,\sigma_a$ *for* L, $= 0$ *for* R

$U(1)_Y$: weak hypercharge, generator Y $\boxed{T_{\rm I}^3 + Y/2 = Q}$

fermion content (ignoring possibe right-handed neutrinos)

$$
\begin{array}{lllllll}
 & & & & & T_{\rm I}^3 & Y \\
\text{leptons:} & \Psi_L^{\rm L} = & \begin{pmatrix} \nu_e^{\rm L} \\ e^{\rm L} \end{pmatrix} & \begin{pmatrix} \nu_\mu^{\rm L} \\ \mu^{\rm L} \end{pmatrix} & \begin{pmatrix} \nu_\tau^{\rm L} \\ \tau^{\rm L} \end{pmatrix} & \begin{matrix} +\frac{1}{2} \\ -\frac{1}{2} \end{matrix} & \begin{matrix} -1 \\ -1 \end{matrix} \\
 & \psi_l^{\rm R} = & e^{\rm R} & \mu^{\rm R} & \tau^{\rm R} & 0 & -2 \\
\text{quarks:} & \Psi_Q^{\rm L} = & \begin{pmatrix} u^{\rm L} \\ d^{\rm L} \end{pmatrix} & \begin{pmatrix} c^{\rm L} \\ s^{\rm L} \end{pmatrix} & \begin{pmatrix} t^{\rm L} \\ b^{\rm L} \end{pmatrix} & \begin{matrix} +\frac{1}{2} \\ -\frac{1}{2} \end{matrix} & \begin{matrix} +\frac{1}{3} \\ +\frac{1}{3} \end{matrix} \\
 & \psi_u^{\rm R} = & u^{\rm R} & c^{\rm R} & t^{\rm R} & 0 & +\frac{4}{3} \\
 & \psi_d^{\rm R} = & d^{\rm R} & s^{\rm R} & b^{\rm R} & 0 & -\frac{2}{3}
\end{array}
$$

gauge boson content

$SU(2)_I$: generators $T_{\mathrm{I}}^1,\ T_{\mathrm{I}}^2,\ T_{\mathrm{I}}^3$

gauge fields $W_\mu^1,\ W_\mu^2,\ W_\mu^3$

also: $W_\mu^\pm = \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp i\,W_\mu^2\right),\ W_\mu^3$

$U(1)_Y$: generator Y

gauge field B_μ

$$\boxed{[T_{\mathrm{I}}^a,\ T_{\mathrm{I}}^b] = i\epsilon_{abc}T_{\mathrm{I}}^c\,, \quad [T_{\mathrm{I}}^a, Y] = 0}$$

$\boxed{\text{gauge boson content}}$

$SU(2)_I$: generators $T_{\mathrm{I}}^1,\ T_{\mathrm{I}}^2,\ T_{\mathrm{I}}^3$

gauge fields $W_\mu^1,\ W_\mu^2,\ W_\mu^3$

also: $W_\mu^{\pm} = \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp i\,W_\mu^2\right),\ W_\mu^3$

$U(1)_Y$: generator Y

gauge field B_μ

notation: $\not{\partial} = \gamma^\mu \partial_\mu, \quad \not{a} = \gamma^\mu a_\mu$

Free Lagrangian of (still massless) fermions:

$$\mathcal{L}_{0,\mathrm{ferm}} = \mathrm{i}\overline{\psi_f}\partial\!\!\!/\psi_f = \mathrm{i}\overline{\Psi^{\mathrm{L}}_L}\partial\!\!\!/\Psi^{\mathrm{L}}_L + \mathrm{i}\overline{\Psi^{\mathrm{L}}_Q}\partial\!\!\!/\Psi^{\mathrm{L}}_Q + \mathrm{i}\overline{\psi^{\mathrm{R}}_l}\partial\!\!\!/\psi^{\mathrm{R}}_l + \mathrm{i}\overline{\psi^{\mathrm{R}}_u}\partial\!\!\!/\psi^{\mathrm{R}}_u + \mathrm{i}\overline{\psi^{\mathrm{R}}_d}\partial\!\!\!/\psi^{\mathrm{R}}_d$$

Minimal substitution:

$$\partial_\mu \;\to\; D_\mu = \partial_\mu - \mathrm{i}g_2 T^a_{\mathrm{I}} W^a_\mu + \mathrm{i}g_1 \tfrac{1}{2} Y B_\mu \;=\; D^{\mathrm{L}}_\mu \omega_- + D^{\mathrm{R}}_\mu \omega_+,$$

$$D^{\mathrm{L}}_\mu = \partial_\mu - \frac{\mathrm{i}g_2}{\sqrt{2}} \begin{pmatrix} 0 & W^+_\mu \\ W^-_\mu & 0 \end{pmatrix} - \frac{\mathrm{i}}{2} \begin{pmatrix} g_2 W^3_\mu - g_1 Y^{\mathrm{L}} B_\mu & 0 \\ 0 & -g_2 W^3_\mu - g_1 Y^{\mathrm{L}} B_\mu \end{pmatrix},$$

$$D^{\mathrm{R}}_\mu = \partial_\mu + \mathrm{i}g_1 \tfrac{1}{2} Y^{\mathrm{R}} B_\mu$$

Photon identification:

"rotation":

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} c_{\mathrm{W}} & s_{\mathrm{W}} \\ -s_{\mathrm{W}} & c_{\mathrm{W}} \end{pmatrix} \begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix}, \qquad c_{\mathrm{W}} = \cos\theta_{\mathrm{W}},\; s_{\mathrm{W}} = \sin\theta_{\mathrm{W}},$$

θ_{W} = mixing angle

$$D^{\mathrm{L}}_\mu\Big|_{A_\mu} = -\frac{\mathrm{i}}{2} A_\mu \begin{pmatrix} -g_2 s_{\mathrm{W}} - g_1 c_{\mathrm{W}} Y^{\mathrm{L}} & 0 \\ 0 & g_2 s_{\mathrm{W}} - g_1 c_{\mathrm{W}} Y^{\mathrm{L}} \end{pmatrix} \stackrel{!}{=} \mathrm{i}eA_\mu \begin{pmatrix} Q_1 & 0 \\ 0 & Q_2 \end{pmatrix}$$

- charged difference in doublet $Q_1 - Q_2 = 1$ $\quad\to\quad$ $g_2 = \dfrac{e}{s_{\mathrm{W}}}$
- normalize $Y^{\mathrm{L/R}}$ such that $g_1 = \dfrac{e}{c_{\mathrm{W}}}$

$\hookrightarrow$ Y fixed by "Gell-Mann–Nishijima relation": $Q = T^3_{\mathrm{I}} + \dfrac{Y}{2}$

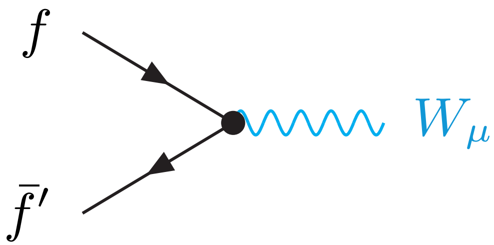
Fermion–gauge-boson interaction:

$$\mathcal{L}_{\mathrm{ferm,YM}} = \frac{e}{\sqrt{2}s_{\mathrm{W}}}\overline{\Psi_F^{\mathrm{L}}}\begin{pmatrix} 0 & \not{W}^{+} \\ \not{W}^{-} & 0 \end{pmatrix}\Psi_F^{\mathrm{L}} + \frac{e}{2c_{\mathrm{W}}s_{\mathrm{W}}}\overline{\Psi_F^{\mathrm{L}}}\sigma^3 \not{Z}\Psi_F^{\mathrm{L}}$$

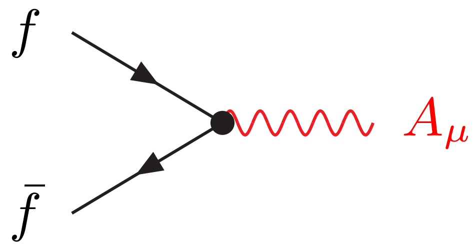
$$- e\frac{s_{\mathrm{W}}}{c_{\mathrm{W}}}Q_f\overline{\psi_f}\not{Z}\psi_f - eQ_f\overline{\psi_f}\not{A}\psi_f$$

(f=all fermions, F= all doublets)

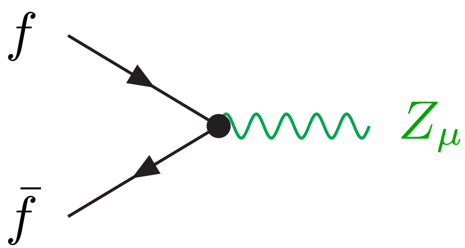
Feynman rules:



$$\frac{\mathrm{i}e}{\sqrt{2}s_{\mathrm{W}}}\gamma_\mu\omega_-$$



$$-\mathrm{i}Q_f e\gamma_\mu$$



$$\mathrm{i}e\gamma_\mu(g_f^+\omega_+ + g_f^-\omega_-) = \mathrm{i}e\gamma_\mu(v_f - a_f\gamma_5)$$

with $g_f^+ = -\frac{s_{\mathrm{W}}}{c_{\mathrm{W}}}Q_f, \quad g_f^- = -\frac{s_{\mathrm{W}}}{c_{\mathrm{W}}}Q_f + \frac{T^3_{\mathrm{I},f}}{c_{\mathrm{W}}s_{\mathrm{W}}},$

$$v_f = -\frac{s_{\mathrm{W}}}{c_{\mathrm{W}}}Q_f + \frac{T^3_{\mathrm{I},f}}{2c_{\mathrm{W}}s_{\mathrm{W}}}, \quad a_f = \frac{T^3_{\mathrm{I},f}}{2c_{\mathrm{W}}s_{\mathrm{W}}}$$

gauge field Lagrangian (Yang-Mills Lagrangian)

$$\mathcal{L}_{\mathrm{YM}} = -\frac{1}{4} W^a_{\mu\nu} W^{a,\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

Field-strength tensors:

$$W^a_{\mu\nu} \;=\; \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g_2 \epsilon^{abc} W^b_\mu W^c_\nu, \qquad B_{\mu\nu} \;=\; \partial_\mu B_\nu - \partial_\nu B_\mu$$

Lagrangian in terms of "physical" fields:

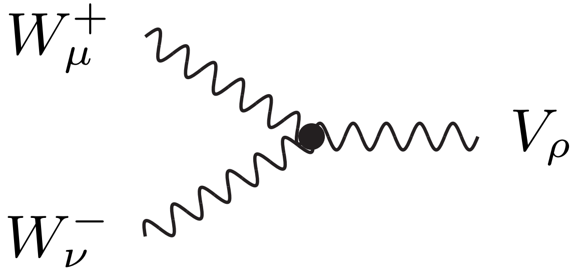
$$\begin{aligned}
\mathcal{L}_{\mathrm{YM}} = &-\frac{1}{2}(\partial_\mu W^+_\nu - \partial_\nu W^+_\mu)(\partial^\mu W^{-,\nu} - \partial^\nu W^{-,\mu}) \\
&- \frac{1}{4}(\partial_\mu Z_\nu - \partial_\nu Z_\mu)(\partial^\mu Z^\nu - \partial^\nu Z^\mu) - \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu)
\end{aligned}$$

$+$ (trilinear interaction terms involving AW^+W^-, ZW^+W^-)

$+$ (quadrilinear interaction terms involving AAW^+W^-, AZW^+W^-, ZZW^+W^-, $W^+W^-W^+W^-$)

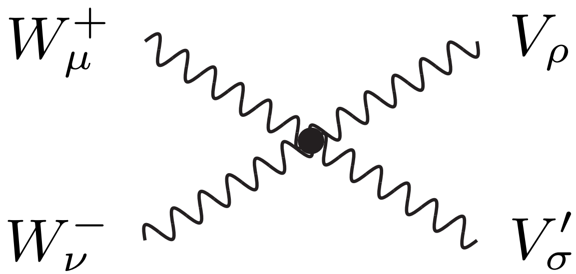
Feynman rules for gauge-boson self-interactions:

(fields and momenta incoming)



$$\mathrm{i}eC_{WWV}\Big[g_{\mu\nu}(k_+ - k_-)_\rho + g_{\nu\rho}(k_- - k_V)_\mu + g_{\rho\mu}(k_V - k_+)_\nu\Big]$$

with $C_{WW\gamma} = 1, \quad C_{WWZ} = -\dfrac{c_\mathrm{W}}{s_\mathrm{W}}$



$$\mathrm{i}e^2C_{WWVV'}\Big[2g_{\mu\nu}g_{\rho\sigma} - g_{\mu\rho}g_{\sigma\nu} - g_{\mu\sigma}g_{\nu\rho}\Big]$$

with $C_{WW\gamma\gamma} = -1, \qquad C_{WW\gamma Z} = \dfrac{c_\mathrm{W}}{s_\mathrm{W}},$

$$C_{WWZZ} = -\frac{c_\mathrm{W}^2}{s_\mathrm{W}^2}, \quad C_{WWWW} = \frac{1}{s_\mathrm{W}^2}$$

Higgs mechanism $\Rightarrow$ masses of W and Z bosons

spontaneous breaking $\quad SU(2)_I \times U(1)_Y \to U(1)_Q$

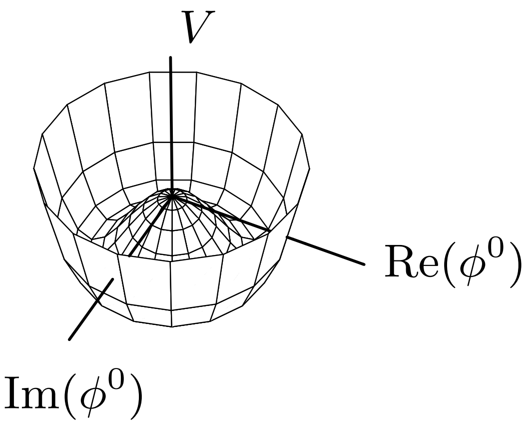
unbroken em. gauge symmetry, massless photon

Minimal scalar sector with complex scalar doublet $\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$, $Y_\Phi = 1$

Scalar self-interaction via Higgs potential:

$$V(\Phi) = -\mu^2 \Phi^\dagger \Phi + \frac{\lambda}{4}(\Phi^\dagger \Phi)^2, \quad \mu^2, \lambda > 0,$$

$$= \mathrm{SU(2)_I \times U(1)_Y} \text{ symmetric}$$



$$V(\Phi) = \text{minimal for} \quad |\Phi| = \sqrt{\frac{2\mu^2}{\lambda}} \equiv \frac{v}{\sqrt{2}} > 0$$

ground state Φ_0 (=vacuum expectation value of Φ) not unique

specific choice $\Phi_0 = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$ not gauge invariant $\Rightarrow$ spontaneous symmetry breaking

elmg. gauge invariance unbroken, since $\quad Q\Phi_0 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \Phi_0 = 0$

Field excitations in Φ:

$$\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \frac{1}{\sqrt{2}}\Big(v + H(x) + \mathrm{i}\chi(x)\Big) \end{pmatrix}$$

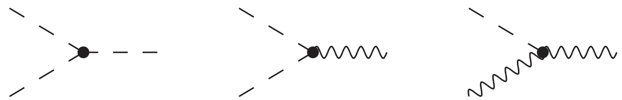
Gauge-invariant Lagrangian of Higgs sector: $\quad (\phi^- = (\phi^+)^\dagger)$

$$\mathcal{L}_\mathrm{H} = (D_\mu\Phi)^\dagger(D^\mu\Phi) - V(\Phi) \qquad \text{with } D_\mu = \partial_\mu - \mathrm{i}g_2\frac{\sigma^a}{2}W_\mu^a + \mathrm{i}\frac{g_1}{2}B_\mu$$

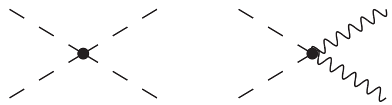
$$= (\partial_\mu\phi^+)(\partial^\mu\phi^-) - \frac{\mathrm{i}ev}{2s_\mathrm{W}}(W_\mu^+\partial^\mu\phi^- - W_\mu^-\partial^\mu\phi^+) + \frac{e^2v^2}{4s_\mathrm{W}^2}W_\mu^+W^{-,\mu}$$

$$+\frac{1}{2}(\partial\chi)^2 + \frac{ev}{2c_\mathrm{W}s_\mathrm{W}}Z_\mu\partial^\mu\chi + \frac{e^2v^2}{4c_\mathrm{W}^2s_\mathrm{W}^2}Z^2 + \frac{1}{2}(\partial H)^2 - \mu^2H^2$$

$+$ (trilinear SSS, SSV, SVV interactions)



$+$ (quadrilinear $SSSS$, $SSVV$ interactions)



Implications:

- gauge-boson masses: $\quad M_\mathrm{W} = \dfrac{ev}{2s_\mathrm{W}}, \quad M_\mathrm{Z} = \dfrac{ev}{2c_\mathrm{W}s_\mathrm{W}} = \dfrac{M_\mathrm{W}}{c_\mathrm{W}}, \quad M_\gamma = 0$
- physical Higgs boson H: $\quad M_\mathrm{H} = \sqrt{2\mu^2}$ = free parameter
- would-be Goldstone bosons $\phi^\pm$, χ: $\quad$ unphysical degrees of freedom

general gauge: Goldstone fields $\phi^{\pm}, \chi$ are present

required: gauge fixing term $\mathcal{L}_{fix}$

R_ξ gauge:

$$\mathcal{L}_{fix} = -\frac{1}{2\xi_\gamma}\left(F^\gamma\right)^2 - \frac{1}{2\xi_Z}\left(F^Z\right)^2 - \frac{1}{2\xi_W}\left(F^\pm\right)^2$$

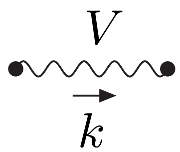
with the gauge-fixing functionals F^a: (ξ_V = arbitrary gauge-fixing parameters)

$$F^\pm = \partial W^\pm \mp \mathrm{i}\xi_\mathrm{W} M_\mathrm{W} \phi^\pm, \qquad F^\mathrm{Z} = \partial Z - \xi_\mathrm{Z} M_\mathrm{Z} \chi, \qquad F^\gamma = \partial A$$

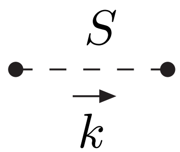
(*notation:* $\partial A = \partial_\mu A^\mu\,, \cdots$)

- elimination of mixing terms $(W^{\pm}_{\mu}\partial^{\mu}\phi^{\mp})$, $(Z_{\mu}\partial^{\mu}\chi)$ in Lagrangian
  $\hookrightarrow$ decoupling of gauge and would-be Goldstone fields (no mix propagators)

- boson propagators:



$$D^{VV}_{\mu\nu}(k) = -\mathrm{i}\left[\frac{g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}}{k^2 - M_V^2} + \frac{k_\mu k_\nu}{k^2}\frac{\xi_V}{k^2 - \xi_V M_V^2}\right], \qquad V = W, Z, \gamma$$



$$D^{SS}(k) = \frac{\mathrm{i}}{k^2 - \xi_V M_V^2}, \qquad S = \phi, \chi$$

- important special cases:
  - $\xi_V = 1$: 't Hooft–Feynman gauge
    $\hookrightarrow$ convenient gauge-boson propagators $\dfrac{-\mathrm{i}g_{\mu\nu}}{k^2 - M_V^2}$
  - $\xi_{\mathrm{W}}, \xi_{\mathrm{Z}} \to \infty$: "unitary gauge"
    $\hookrightarrow$ elimination of would-be Goldstone bosons

Fermion masses

fermions in chiral representations of gauge symmetry

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L , \quad e_R \quad \Rightarrow \text{mass term } m_e(\bar{e}_L e_R + \bar{e}_R e_L) = m_e \, \bar{e} e$$

not gauge invariant

solution of the SM: introduce <u>Yukawa interaction</u>

= new interaction of fermions with the Higgs field

gauge invariant interaction, g_e = Yukawa coupling constant

$$\mathcal{L}_{\text{Yuk}} = g_e \, [\, \overline{\psi^L} \, \Phi \, e_R \; + \; \overline{e_R} \, \Phi^\dagger \psi^L \,]$$

most transparent in unitary gauge

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H \end{pmatrix}$$

apply to the first lepton generation $\psi^L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \;\; e_R$:

$$\frac{g_e}{\sqrt{2}} \left[(\overline{\nu_L}, \overline{e_L}) \begin{pmatrix} 0 \\ v + H \end{pmatrix} e_R + \overline{e_R}\,(0, v + H) \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \right]$$

$$= \underbrace{\frac{g_e}{\sqrt{2}}v}_{m_e} [\overline{e_L} e_R + \overline{e_R} e_L] + \underbrace{\frac{g_e}{\sqrt{2}}}_{m_e/v} H [\overline{e_L} e_R + \overline{e_R} e_L]$$

$$= m_e\, \overline{e} e + \frac{m_e}{v} H\, \overline{e} e$$

3 generations of leptons and quarks

- for massless neutrinos: no generation mixing for leptons

  repeat construction for μ and τ with g_μ and g_τ
  $\Rightarrow \quad m_\mu,\ m_\tau$

- quark sector: generation mixing

  Yukawa couplings not generation-diagonal

$$U = \begin{pmatrix} u \\ c \\ t \end{pmatrix}, \quad D = \begin{pmatrix} d \\ s \\ b \end{pmatrix}, \qquad \overline{U} = (\overline{u},\, \overline{c},\, \overline{t})\,, \quad \overline{D} = (\overline{d},\, \overline{s},\, \overline{b})$$

unitary gauge:

$$\Phi = \tfrac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}, \quad \tilde{\Phi} = \tfrac{1}{\sqrt{2}} \begin{pmatrix} -v - H(x) \\ 0 \end{pmatrix},$$

$$\mathcal{L}_{\mathrm{Y}} = -\left(\overline{U}_L, \overline{D}_L\right) G_d\, \Phi\, D_R + \left(\overline{U}_L, \overline{D}_L\right) G_u\, \tilde{\Phi}\, U_R + \text{h.c.}$$

mass term for $H(x) = 0$:

$$-\overline{D}_L \underbrace{G_d \frac{v}{\sqrt{2}}}_{M_d} D_R - \overline{U}_L \underbrace{G_u \frac{v}{\sqrt{2}}}_{M_u} U_R + \text{h.c.}$$

mass matrices, non-diagonal

diagonalization by unitary matrices $V^u_{L,R},\, V^d_{L,R}$:

$$U_{L,R} = V^u_{L,R}\,\hat{U}_{L,R}\,, \qquad \overline{U}_{L,R} = \overline{\hat{U}}_{L,R}\,(V^u_{L,R})^\dagger$$
$$D_{L,R} = V^d_{L,R}\,\hat{D}_{L,R}\,, \qquad \overline{D}_{L,R} = \overline{\hat{D}}_{L,R}\,(V^d_{L,R})^\dagger$$

$\Rightarrow$ mass eigenstates $\hat{U},\, \hat{D}$

$$\overline{D}_L M_d D_R + \overline{U}_L M_u U_R \;=$$

$$\overline{\hat{D}}_L\; \underbrace{(V^d_L)^\dagger M_d V^d_R}_{M^{\rm diag}_d}\; \hat{D}_R + \overline{\hat{U}}_L\; \underbrace{(V^u_L)^\dagger M_u V^u_R}_{M^{\rm diag}_u}\; \hat{U}_R$$

diagonalization by unitary matrices $V^u_{L,R},\, V^d_{L,R}$:

$$U_{L,R} = V^u_{L,R}\,\hat{U}_{L,R} \qquad \overline{U}_{L,R} = \overline{\hat{U}}_{L,R}\,(V^u_{L,R})^\dagger$$

$$D_{L,R} = V^d_{L,R}\,\hat{D}_{L,R} \qquad \overline{D}_{L,R} = \overline{\hat{D}}_{L,R}\,(V^d_{L,R})^\dagger$$

$\Rightarrow$ mass eigenstates $\hat{U},\, \hat{D}$

$$\overline{D}_L M_d D_R + \overline{U}_L M_u U_R \; =$$

$$\overline{\hat{D}}_L \underbrace{(V^d_L)^\dagger M_d V^d_R}_{M_d^{\mathrm{diag}}} \hat{D}_R + \overline{\hat{U}}_L \underbrace{(V^u_L)^\dagger M_u V^u_R}_{M_u^{\mathrm{diag}}} \hat{U}_R$$

$$\begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix} \qquad \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix}$$

• Yukawa interactions in terms of mass eigenstates:

$$\begin{aligned}\mathcal{L}_{\mathrm{Y}} &= -\left(\overline{U}_L, \overline{D}_L\right) G_d\, \Phi\, D_R + \left(\overline{U}_L, \overline{D}_L\right) G_u\, \tilde{\Phi}\, U_R \;\; + \mathrm{h.c.} \\ &= -\overline{\hat{D}}_L\, M_d^{\mathrm{diag}}\, \hat{D}_R \left(1 + \frac{H}{v}\right) - \overline{\hat{U}}_L\, M_u^{\mathrm{diag}}\, \hat{U}_R \left(1 + \frac{H}{v}\right) \;\; + \mathrm{h.c.}\end{aligned}$$

flavor-diagonal interactions with H field

• neutral weak and electromagnetic currents:

$$\overline{U}_{L(R)}\gamma^\mu U_{L(R)} = \overline{\hat{U}}_{L(R)}\gamma^\mu \hat{U}_{L(R)}\,, \quad \overline{D}_{L(R)}\gamma^\mu D_{L(R)} = \overline{\hat{D}}_{L(R)}\gamma^\mu \hat{D}_{L(R)}$$

flavor-diagonal because $V_{L,R}^{u,d}$ are unitary matrices

• charged current:

$$\overline{U}_L\gamma^\mu D_L + \overline{D}_L\gamma^\mu U_L = \overline{\hat{U}}_L V \gamma^\mu \hat{D}_L + \overline{\hat{D}}_L V^\dagger \gamma^\mu \hat{U}_L$$

remnant: $V \equiv V_{\mathrm{CKM}} = (V_L^u)^\dagger V_L^d$

Features of the CKM mixing:

- V = 3-dim. generalization of Cabibbo matrix U_{C}
- V is parametrized by 4 free parameters: 3 real angles, 1 complex phase

  $\hookrightarrow$ complex phase is the only source of CP violation in SM

  counting:

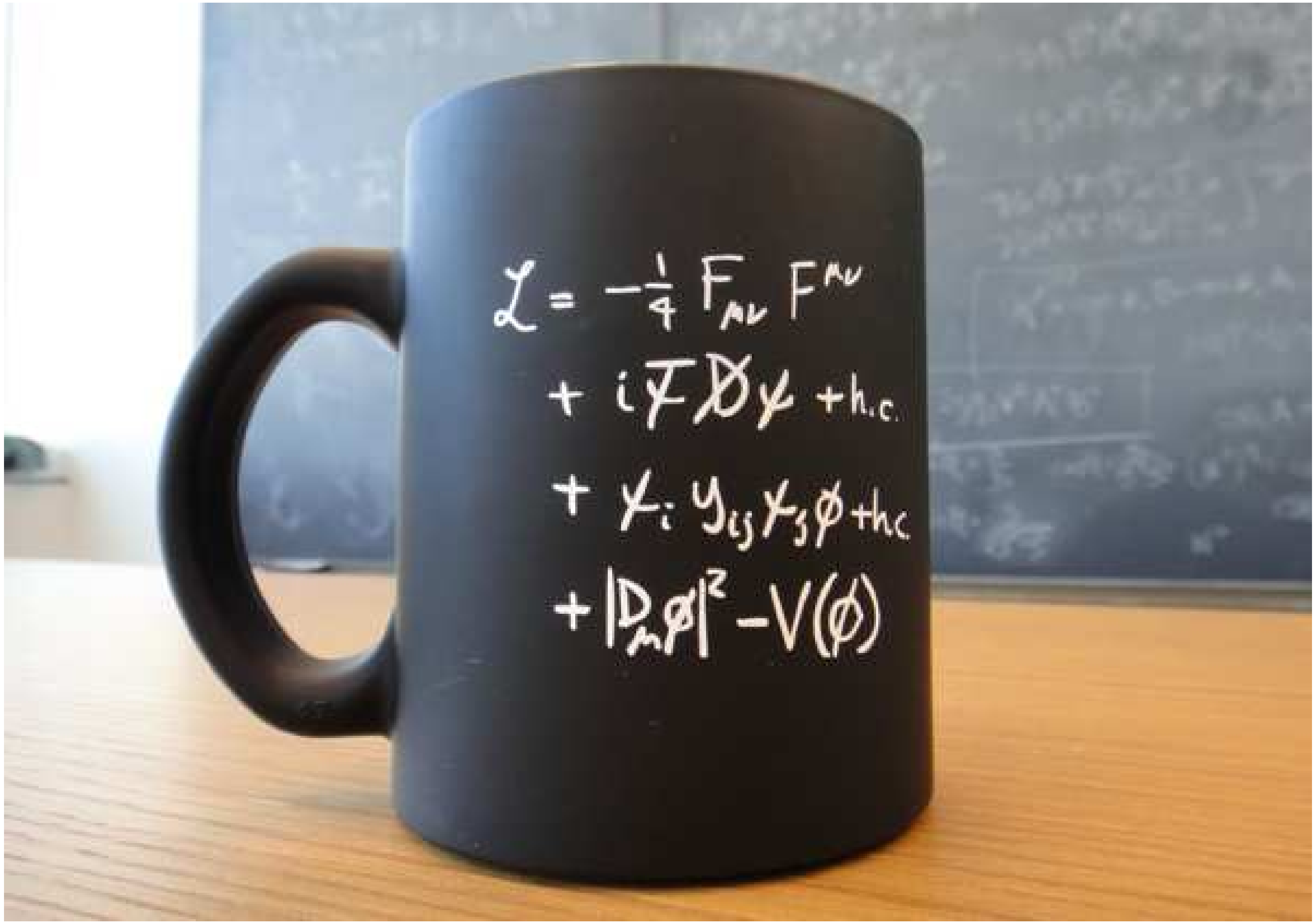
  $$\begin{pmatrix}\text{\#real d.o.f.}\\ \text{in } V\end{pmatrix} - \begin{pmatrix}\text{\#unitarity}\\ \text{relations}\end{pmatrix} - \begin{pmatrix}\text{\#phase diffs. of}\\ u\text{-type quarks}\end{pmatrix} - \begin{pmatrix}\text{\#phase diffs. of}\\ d\text{-type quarks}\end{pmatrix} - \begin{pmatrix}\text{\#phase diff. between}\\ u\text{- and } d\text{-type quarks}\end{pmatrix}$$

  $$= 18 - 9 - 2 - 2 - 1 = 4$$
- no flavour-changing neutral currents in lowest order,

  flavour-changing suppressed by factors $G_\mu(m_{q_1}^2 - m_{q_2}^2)$ in higher orders

  ("Glashow–Iliopoulos–Maiani mechanism")

The Standard Model Lagrangian



- renormalizable $\Rightarrow$ precision calculations
- quantum effects in precision observables detectable
- involve Higgs mass dependence

4. Phenomenology of W and Z bosons and precision tests

Cross sections and decay widths

scattering process: $a + b \quad \rightarrow \quad b_1 + b_2 + \cdots + b_n$

$$|a(p_a), b(p_b)> = |i>, \quad |b_1(p_1), \cdots b_n(p_n)> = |f>$$

matrix element = probability amplitude for $i \rightarrow f$:

$$S_{fi} =< f|S|i>$$

for $i \neq f$: $\quad S_{fi} = (2\pi)^4 \, \delta^4(P_i - P_f) \, \mathcal{M}_{fi} \, \left[\frac{1}{(2\pi)^{3/2}}\right]^{n+2}$

$P_i = p_a + p_b \; = \; P_f = p_1 + \cdots + p_n$ momentum conservation

factors $(2\pi)^{-3/2}$ from wave function normalization (plane waves)

$\mathcal{M}_{fi}$ from Feynman graphs and rules probability for scattering into phase space element $\mathrm{d}\Phi$:

$$\mathrm{d}W_{fi} = |S_{fi}|^2 \, \mathrm{d}\Phi, \qquad \mathrm{d}\Phi = \frac{\mathrm{d}^3 p_1}{2p_1^0} \cdots \frac{\mathrm{d}^3 p_n}{2p_n^0}$$

$$\frac{\mathrm{d}^3 p_i}{2p_i^0} = \mathrm{d}^4 p_i \, \delta(p_i^2 - m_i^2)$$ Lorentz invariant phase space

differential cross section:

$$\boxed{\mathrm{d}\sigma = \frac{(2\pi)^4}{4\sqrt{(p_a \cdot p_b)^2 - m_a^2 m_b^2}} \; |\mathcal{M}_{fi}|^2 \, (2\pi)^{-3n} \, \delta^4(P_i - P_f) \, \frac{\mathrm{d}^3 p_1}{2p_1^0} \cdots \frac{\mathrm{d}^3 p_n}{2p_n^0}}$$

decay process: $a \quad \rightarrow \quad b_1 + b_2 + \cdots + b_n$

$$|a(p_a)> = |i>, \quad |b_1(p_1), \cdots b_n(p_n)> = |f>$$

$$S_{fi} = (2\pi)^4 \, \delta^4(p_a - P_f) \, \mathcal{M}_{fi} \, \left[\tfrac{1}{(2\pi)^{3/2}}\right]^{n+1}$$

decay width (differential):

$$\boxed{\mathrm{d}\Gamma = \frac{(2\pi)^4}{2\,m_a} \; |\mathcal{M}_{fi}|^2 \, (2\pi)^{-3n} \, \delta^4(p_a - P_f) \, \frac{\mathrm{d}^3 p_1}{2p_1^0} \cdots \frac{\mathrm{d}^3 p_n}{2p_n^0}}$$

special case: 2-particle phase space

$$a + b \to b_1 + b_2, \qquad a \to b_1 + b_2$$

- cross section

in the CMS, $\vec{p}_a + \vec{p}_b = 0 = \vec{p}_1 + \vec{p}_2$

$$\boxed{\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{1}{64\pi^2 s} \frac{|\vec{p}_1|}{|\vec{p}_a|} \, |\mathcal{M}_{fi}|^2}$$

$$\mathrm{d}\Omega = \mathrm{d}\cos\theta \, \mathrm{d}\phi, \qquad \theta = \sphericalangle(\vec{p}_a, \vec{p}_1)$$

$$s = (p_a + p_b)^2 = E^2_{\mathrm{CMS}}$$

- decay rate

for final state masses $m_1 = m_2 = m$

$$\boxed{\frac{\mathrm{d}\Gamma}{\mathrm{d}\Omega} = \frac{1}{64\pi^2 \, m_a} \sqrt{1 - \frac{4m^2}{m_a^2}} \, |\mathcal{M}_{fi}|^2}$$

features of the ew Standard Model

- Higgs boson probably found, all other particles confirmed

- consistent quantum field theory
  - in accordance with unitarity
  - renormalizable $\Rightarrow$ predictions at higher orders

- formal parameters: $g_2,\ g_1,\ v,\ \lambda,\ g_f,\ V_{\mathrm{CKM}}$
  physical parameters: $\alpha,\ M_W,\ M_Z,\ M_H,\ m_f,\ V_{\mathrm{CKM}}$

Basic parameters and relations

ew mixing angle: $s_\mathrm{W} \equiv \sin\theta_\mathrm{W}, \quad c_\mathrm{W} \equiv \cos\theta_\mathrm{W}$

gauge coupling constants: $g_2 = \frac{e}{s_\mathrm{W}}, \quad g_1 = \frac{e}{c_\mathrm{W}}$

vector boson masses:

$$M_W = \frac{1}{2} g_2 v = \frac{ev}{2 s_\mathrm{W}}$$

$$M_Z = \frac{ev}{2 s_\mathrm{W} c_\mathrm{W}} = \frac{M_W}{c_\mathrm{W}}$$

$$s_\mathrm{W}^2 = 1 - \frac{M_W^2}{M_Z^2}$$

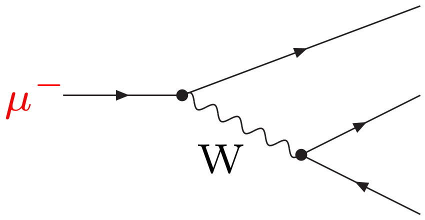
neutral current (NC) couplings:

$$a_f = \frac{g_2}{2 c_\mathrm{W}} T_3^f$$

$$v_f = \frac{g_2}{2 c_\mathrm{W}} (T_3^f - 2 Q_f s_\mathrm{W})$$

observables and experiments

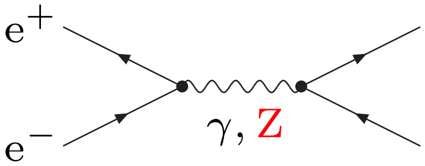
- Muon decay: $\mu^- \to \nu_\mu e^- \bar{\nu}_e$



determination of the Fermi constant

$$G_\mu = \frac{\pi \alpha M_Z^2}{\sqrt{2} M_W^2 (M_Z^2 - M_W^2)} + \dots$$

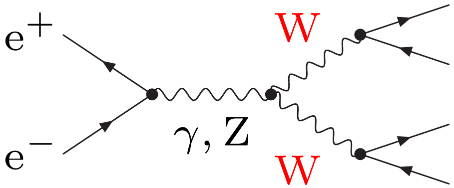
- Z production (LEP1/SLC): $e^+e^- \to Z \to f\bar{f}$

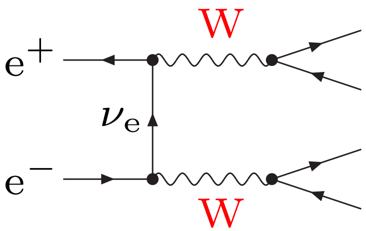


various precision measurements at the Z resonance: $M_Z, \Gamma_Z, \sigma_{had}, A_{FB}, A_{LR}$, etc.

$\Rightarrow$ good knowledge of the $Zf\bar{f}$ sector

- W-pair production (LEP2/ILC): $e^+e^- \to WW \to 4f(+\gamma)$





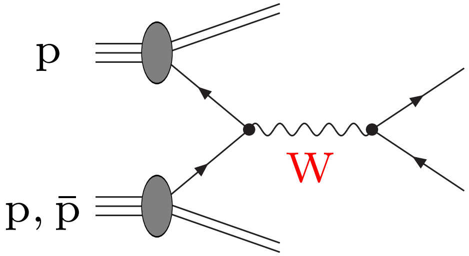
– measurement of M_W

– $\gamma WW/ZWW$ couplings

– quartic couplings: $\gamma\gamma WW$, γZWW

experiments at hadron colliders

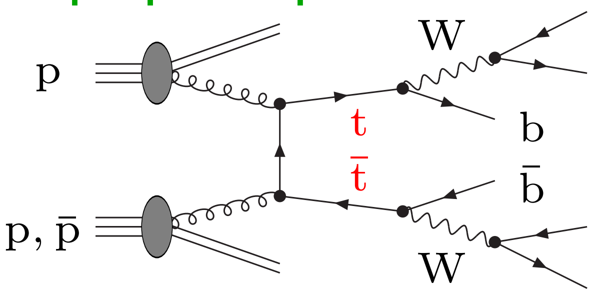
- W production (Tevatron/LHC): $\mathrm{pp}, \mathrm{p\bar{p}} \to \mathrm{W} \to l\nu_l(+\gamma)$



  – measurement of M_W

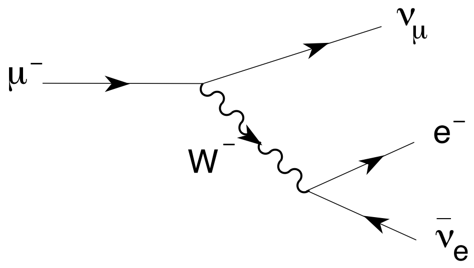
  – bounds on $\gamma\mathrm{WW}$ coupling

- top-quark production (Tevatron/LHC): $\mathrm{pp}, \mathrm{p\bar{p}} \to \mathrm{t\bar{t}} \to 6f$



  – measurement of m_t

μ decay



$$\mathcal{M} = \left(\frac{ig_2}{2\sqrt{2}}\right)^2 J_\rho^{(\mu)} \frac{-ig^{\rho\sigma}}{q^2 - M_W^2} J_\sigma^{(e)}$$

$|q|^2 \simeq m_\mu^2 \ll M_W^2$:
$$\mathcal{M} = -\frac{g_2^2}{8M_W^2} J_\rho^{(\mu)} J^{\rho\,(e)}$$

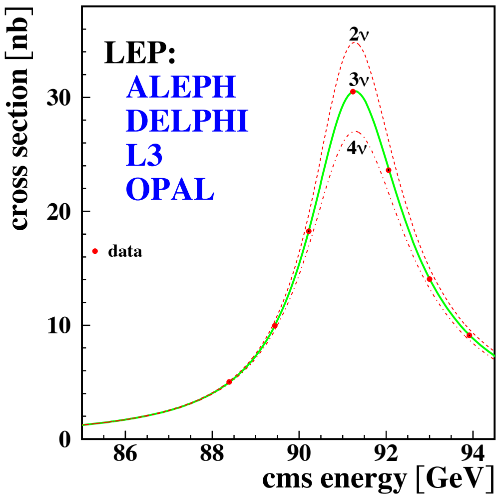
Fermi model *with point-like 4-fermion interaction:*

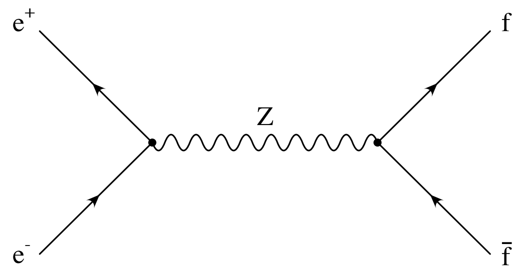
$$\mathcal{M} = \frac{G_F}{\sqrt{2}} J_\rho^{(\mu)} J^{\rho\,(e)}$$ *low-energy limit of SM*

$$\Rightarrow \quad \boxed{\frac{G_F}{\sqrt{2}} = \frac{g_2^2}{8M_W^2} = \frac{e^2}{8s_\mathrm{W}^2 c_\mathrm{W}^2 M_Z^2} = \frac{\pi\alpha}{2s_\mathrm{W}^2 c_\mathrm{W}^2 M_Z^2} = \frac{\pi\alpha}{2(1 - M_W^2/M_Z^2)M_W^2}}$$

$$G_F = 1.16637(1) \cdot 10^{-5}\,\mathrm{GeV}^{-2}$$

Z resonance





$$\mathcal{M} = J_\mu^{(e)} \frac{-ig^{\mu\nu}}{s - M_Z^2 + iM_Z\Gamma_Z} J_\nu^{(f)}$$

propagator with finite width Γ_Z *(unstable particle)*

$$\Gamma_Z = \sum_f \Gamma(Z \to f\bar{f}), \quad \Gamma(Z \to f\bar{f}) = \frac{M_Z}{12\pi}\,(v_f^2 + a_f^2)$$

differential cross section at $s = M_Z^2$:

$$\frac{d\sigma}{d\Omega} \sim (v_e^2 + a_e^2)(v_f^2 + a_f^2)\,(1 + \cos^2\theta)\; + \;(2v_e a_e)(2v_f a_f) \cdot 2\cos\theta$$

$\Rightarrow$ *forward-backward asymmetry* $A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = \frac{3}{4} A_e A_f$

$$A_f = \frac{2v_f a_f}{v_f^2 + a_f^2}$$

polarized cross section for $e^-_{L,R}$:

$\Rightarrow$ *left-right asymmetry* $A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = A_e$

$\boxed{\textit{asymmetries determine } \sin^2\theta_{\mathrm{W}}}$

input from experiments

- LEP1/SLC: $\quad e^+e^- \to Z \to f\bar{f}$
  LEP1: $\sim 4 \times 10^6$ events/experiment
  4 experiments (1989 – 1995)

- LEP2: $\quad e^+e^- \to W^+W^-$
  $\mathcal{O}(10^4)$ W pairs (1996 – 2000)

- Tevatron: $\quad q\bar{q}' \to W \to l\nu, q\bar{q}'$
  $(p\bar{p})$ $\quad q\bar{q}' \to t\bar{t},\ t \to W^+b \to \ldots$

- low-energy experiments (μ decay, νN scattering, νe scattering, atomic parity violation, ...)

experimental results (selection)

$$
\begin{array}{lll}
M_Z \ [\mathsf{GeV}] & = 91.1875 \pm 0.0021 & 0.002\% \\
\Gamma_Z [\mathsf{GeV}] & = \ \ 2.4952 \pm 0.0023 & 0.09\% \\
\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}} & = 0.23148 \pm 0.00017 & 0.07\% \\
M_W \ [\mathsf{GeV}] & = 80.385 \pm 0.015 & 0.02\% \\
m_t \ [\mathsf{GeV}] & = 173.2 \pm 0.9 & 0.52\% \\
G_F \ [\mathsf{GeV}^{-2}] & = 1.16637(1)10^{-5} & 0.001\%
\end{array}
$$

loop effects are at least one order of magnitude larger than experimental uncertainties