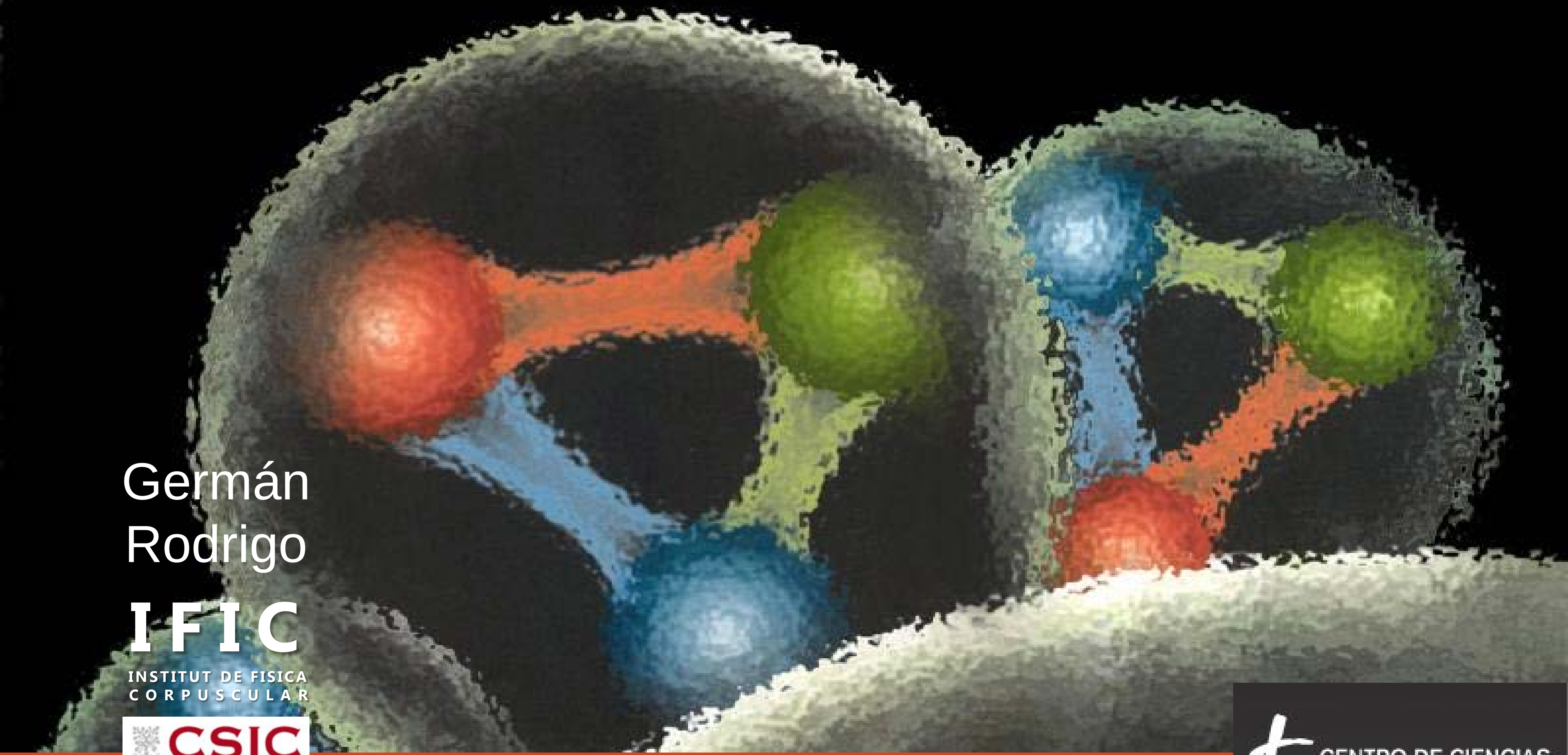

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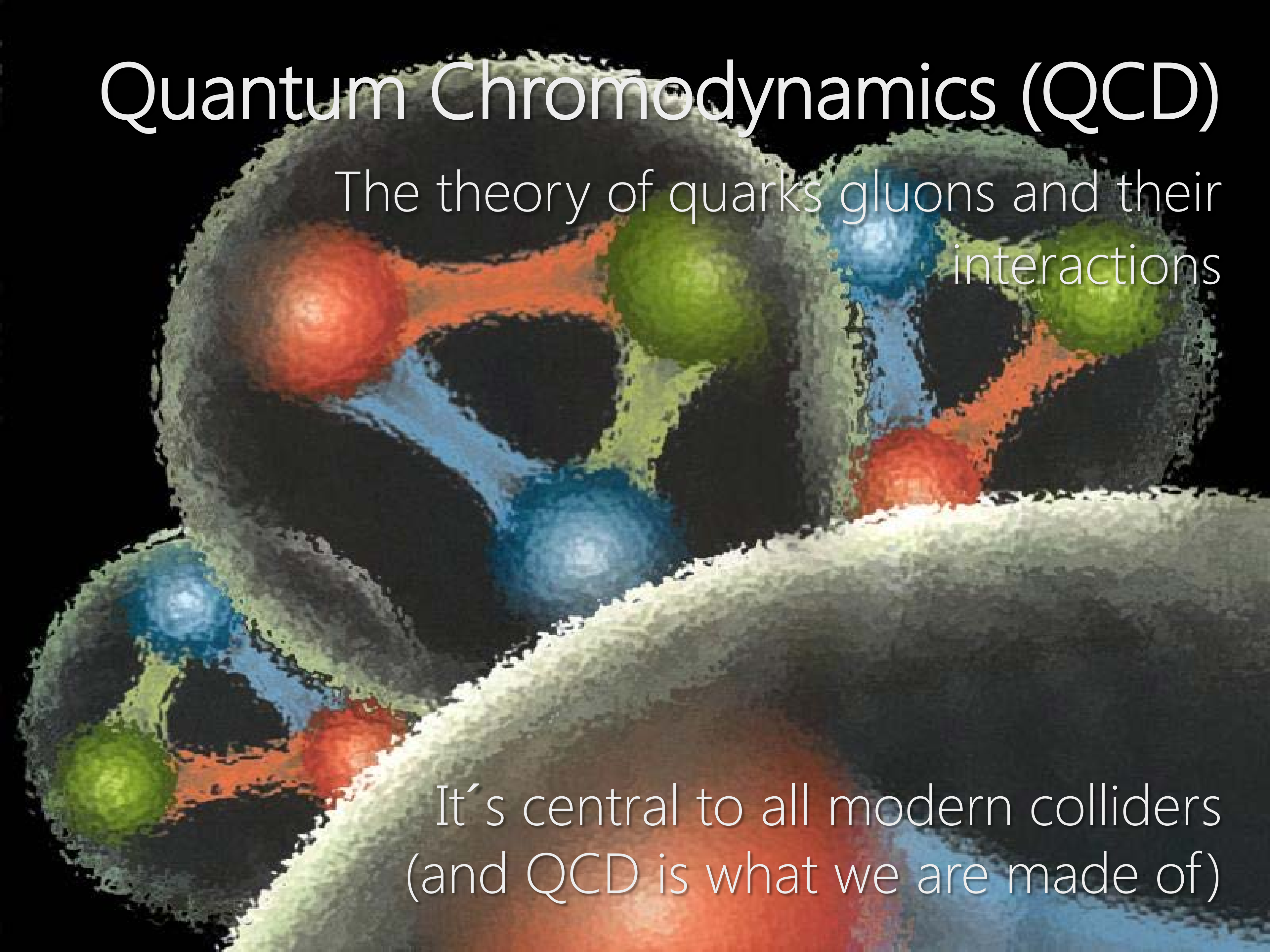
CENTRO DE CIENCIAS DE BENASQUE PEDRO PASCUAL

QCD, jets and Monte Carlo

Taller de Altas Energías TAE2014, September 2014



Quantum Chromodynamics (QCD)

The theory of quarks gluons and their interactions

It´s central to all modern colliders
(and QCD is what we are made of)

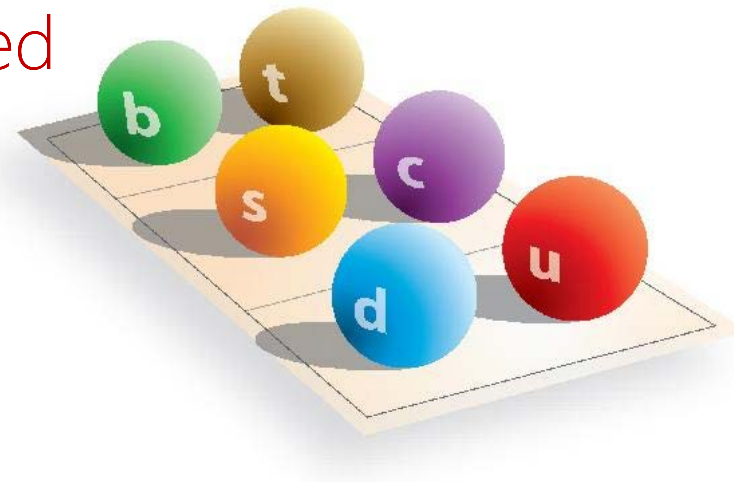
Outline

1. QCD Lagrangean, and IR divergences in e+e-.
2. pQCD at hadron colliders
3. New methods in pQCD: helicity, colour order and generalized unitarity
4. The collinear limit of QCD
5. Parton distribution functions
6. Jets and Monte Carlo

QCD and e^+e^- colliders

The ingredients of QCD

QCD is a gauge invariant QFT, based on a local SU(3) symmetry group



- **Quarks** (and anti-quarks): six flavours
  - they come in 3 colours
- **Gluons**: massless gauge bosons
  - a bit like photons in QED
  - but there are 8 of them, and they are colour charged
- And the **coupling** $\boldsymbol{\alpha_S(\mu)}$
  - that´s not so small and runs fast
  - at the LHC, in the range 0.08 @ 5 TeV to O(1) at 0.5 GeV

Quark Lagrangean + colour

The quark part of the Lagrangean

$$\mathcal{L}_q = \bar{\psi}_i \; \left(\delta_{ij}(i \not{\partial} - m) + g_S \, T^a_{ij} \, \not{A}^a \right) \psi_j$$

- where quarks carry three colours $\psi_i = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}$

- SU(3) local gauge symmetry: 8 (= 3^2 - 1) generators $T_{ij}{}^1 \ldots T_{ij}{}^8$ corresponding to 8 gluons $A_\mu{}^1 \ldots A_\mu{}^8$

- The fundamental representation: $\mathbf{T}^a = \frac{1}{2}\lambda^a$, Traceless and Hermitian

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda^8 = \begin{pmatrix} \frac{1}{\sqrt{3}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{3}} & 0 \\ 0 & 0 & \frac{-2}{\sqrt{3}} \end{pmatrix}$$

Gluon Lagrangean

The gluon part of the Lagrangean

$$\mathcal{L}_g = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a}$$

where the field tensor is

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + i\, g_S \left(-i\, f_{abc}\right) A^a_\mu\, A^c_\nu$$

$$[\mathbf{T}^a, \mathbf{T}^b] = i f_{abc} \mathbf{T}^{\mathbf{c}}$$

f_{abc} are the structure constant of SU(3): antisymmetric in all indices.
Needed for gauge invariance of the Lagrangean

Gluon propagator: $\frac{1}{k^2 + i0}\, d^{\mu\nu}(k)$

Feynman gauge $d^{\mu\nu}(k) = -g^{\mu\nu}$ simpler but requires ghosts

Axial gauge $d^{\mu\nu}(k,n) = -g^{\mu\nu} + \frac{k^\mu n^\nu + n^\mu k^\nu}{n \cdot k}\,, \qquad n^2 = 0$

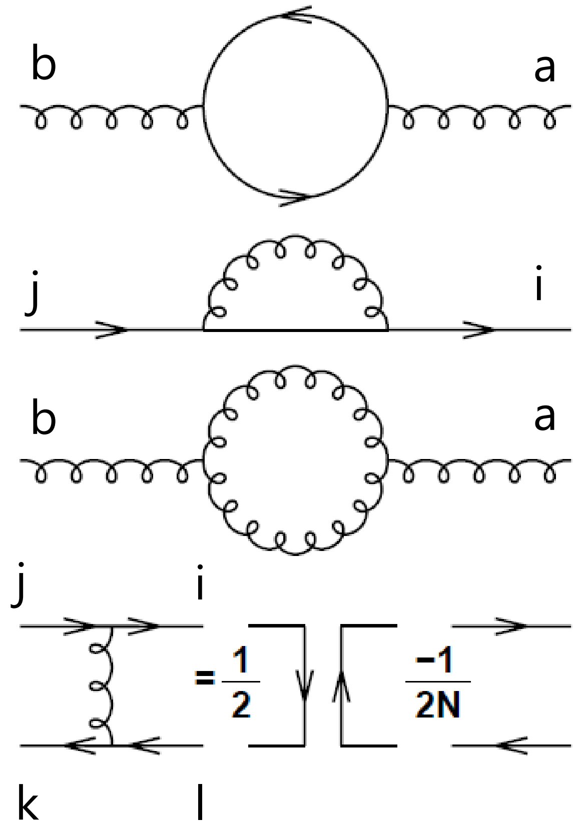
Colour algebra

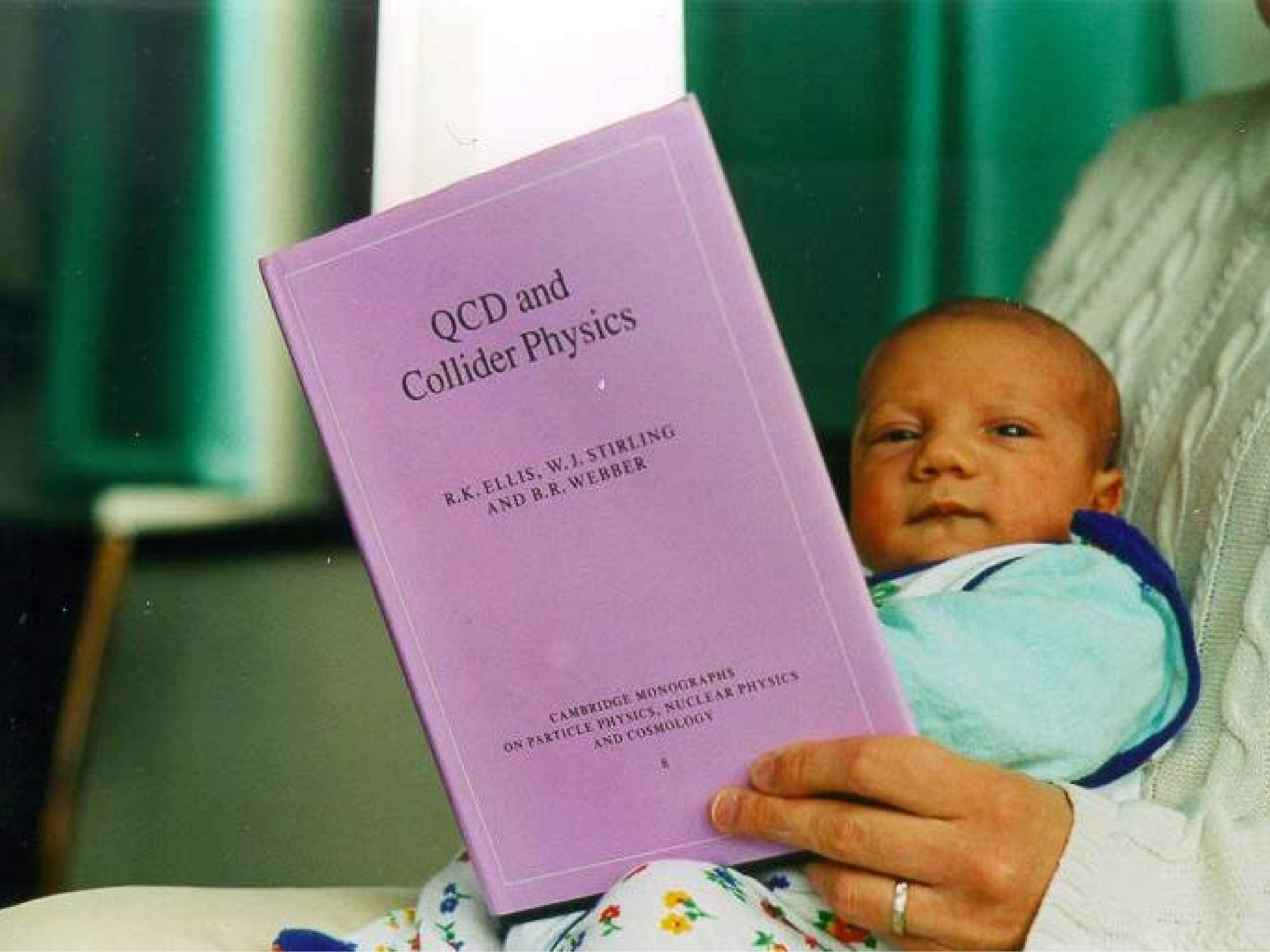
$$\mathrm{Tr}(\mathbf{T}^a\mathbf{T}^b) = T_R\,\delta^{ab}\ , \qquad T_R = \frac{1}{2}$$

$$\sum_a T^a_{ik}T^a_{kj} = C_F\,\delta_{ij}\ , \qquad C_F = \frac{N_C^2-1}{2N_C} = \frac{4}{3}$$

$$\sum_{c,d} f^{acd}f^{bcd} = C_A\,\delta^{ab}\ , \qquad C_A = N_C = 3$$

$$T^a_{ij}\,T^a_{kl} = \frac{1}{2}\left(\delta_{jk}\delta_{il} - \frac{1}{N_C}\delta_{ij}\delta_{kl}\right)\ , \qquad \text{Fierz}$$




QCD and
Collider Physics
R.K. ELLIS, W.J. STIRLING
AND B.R. WEBBER
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AND COSMOLOGY
8

Peturbation Theory

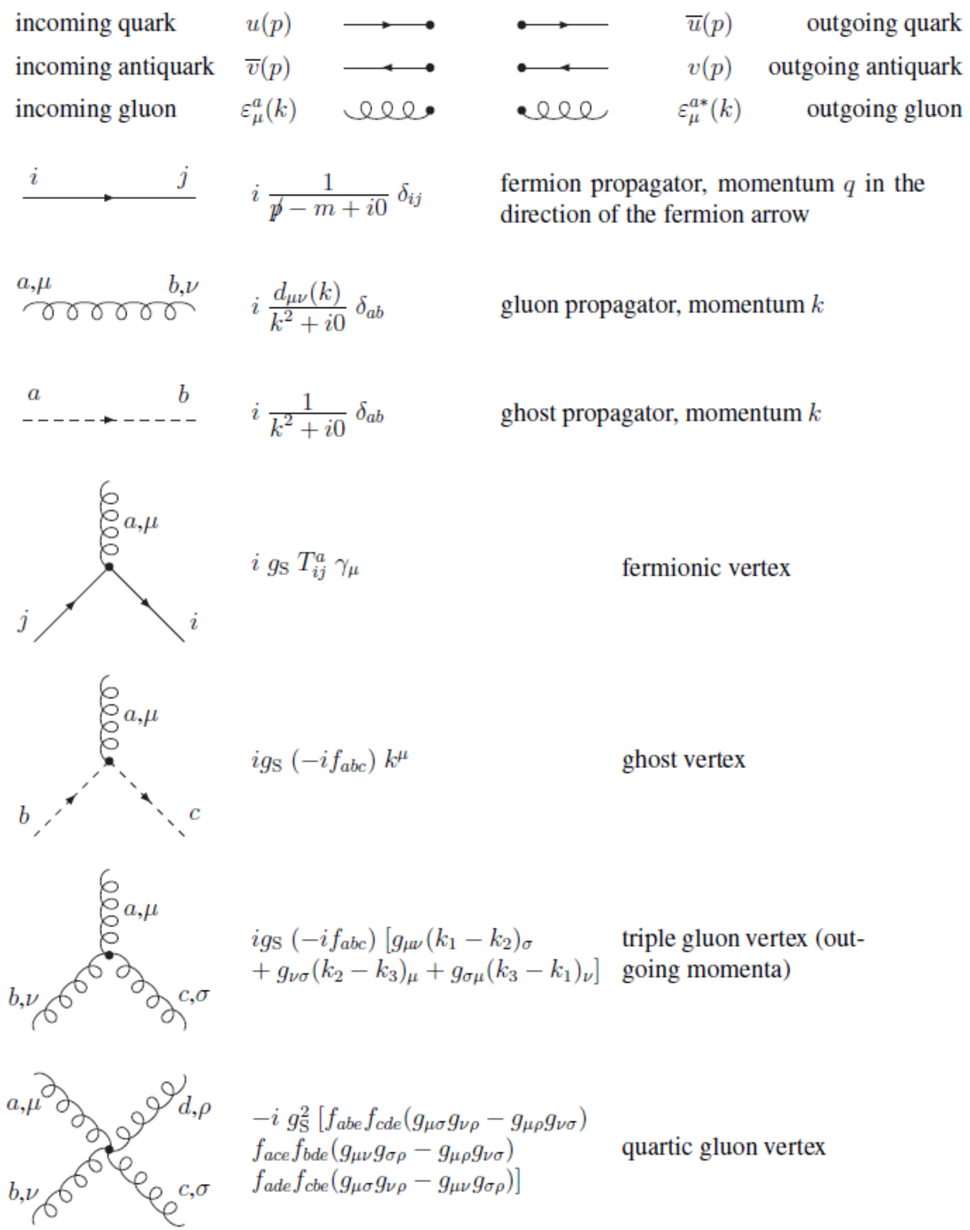
- Relies on the idea of order-by-order expansion in the small coupling $\alpha_S \ll 1$

$$\alpha_S + \alpha_S^2 + \alpha_S^3 + \cdots$$

small

smaller

negligible ?



How big is the coupling ?

All the SM couplings (including $\overline{MS}$ mass/Yukawa) depend on the energy scale (obey Renormalization Group Equation RGE),
and the QCD coupling run fast

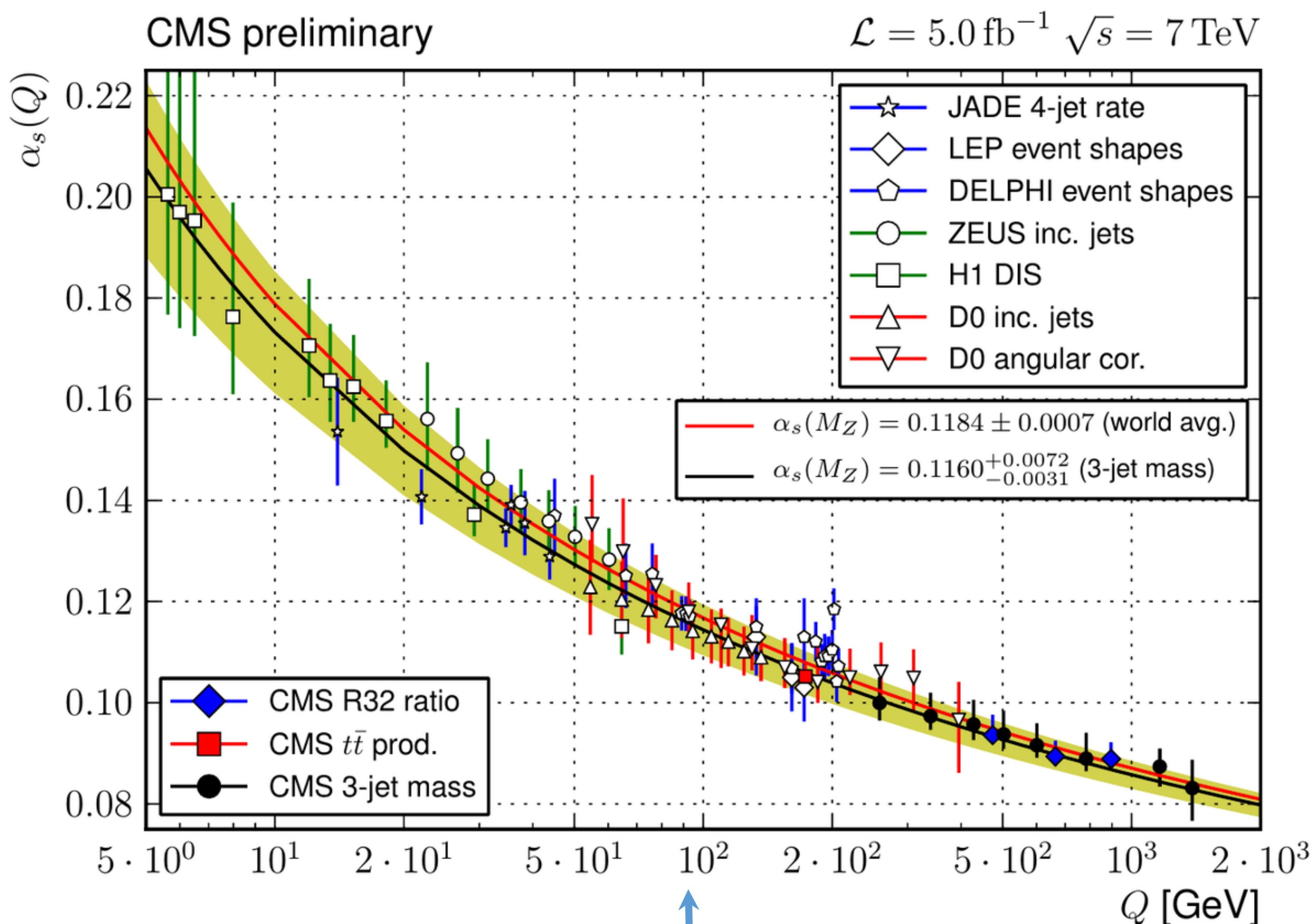
$$\frac{\partial a_S}{\partial \log \mu^2} = \beta(a_S) = -a_S^2 (b_0 + a_S\, b_1 + a_S^2\, b_2 + \ldots) \,, \qquad a_S = \frac{\alpha_S}{\pi}$$

$$\frac{\partial \log m_q}{\partial \log \mu^2} = \gamma_m(a_S) = -a_S(g_0 + a_S\, g_1 + a_S^2\, g_2 + \cdots) \,,$$

$$b_0 = \frac{1}{12}(11 C_A - 2 N_F) \,, \qquad b_1 = \frac{1}{24}\left(17 C_A^2 - (5 C_A + 3 C_F) N_F\right)$$

$$g_0 = 1 \qquad g_1 = \frac{1}{16}\left(\frac{202}{3} - \frac{20}{9} N_F\right)$$

- Sign $\beta(\alpha_S) < 0$: Asymptotic Freedom due to gluon self-interactions
  [Nobel Prize 2004, Gross, Politzer, Wilczek]
- **At high scales:** coupling becomes small, quarks and gluons are almost free, strong interactions are weak
- **At low scales:** coupling becomes large, quarks and gluons interact strongly, confined into hadrons, perturbation theory fails

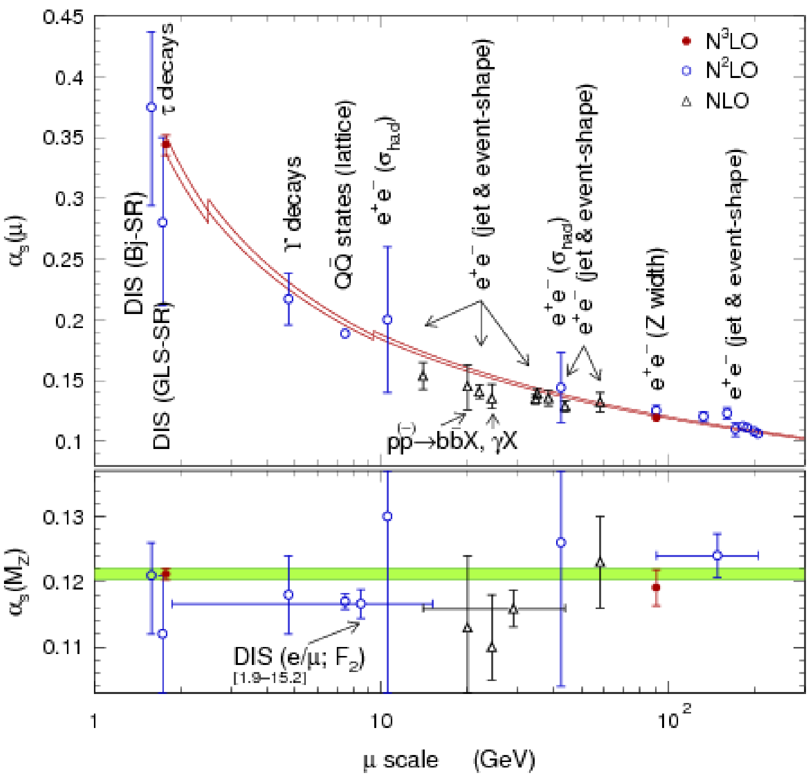

CMS preliminary
$\mathcal{L} = 5.0\,\mathrm{fb}^{-1}$ $\sqrt{s} = 7\,\mathrm{TeV}$
$\alpha_s(Q)$
0.22
0.20
0.18
0.16
0.14
0.12
0.10
0.08
JADE 4-jet rate
LEP event shapes
DELPHI event shapes
ZEUS inc. jets
H1 DIS
D0 inc. jets
D0 angular cor.
$\alpha_s(M_Z) = 0.1184 \pm 0.0007$ (world avg.)
$\alpha_s(M_Z) = 0.1160^{+0.0072}_{-0.0031}$ (3-jet mass)
CMS R32 ratio
CMS $t\bar{t}$ prod.
CMS 3-jet mass
$5 \cdot 10^0$
10^1
$2 \cdot 10^1$
$5 \cdot 10^1$
10^2
$2 \cdot 10^2$
$5 \cdot 10^2$
10^3
$2 \cdot 10^3$
Q [GeV]

m_Z

Flavour thresholds

$$a_S^{(N_F)}(\mu_{\rm th}) = a_S^{(N_F-1)}(\mu_{\rm th}) \left[1 + \sum C_k(x) \left(a_S^{(N_F-1)}(\mu_{\rm th})\right)^k\right]$$

$$m_q^{(N_F)}(\mu_{\rm th}) = m_q^{(N_F-1)}(\mu_{\rm th}) \left[1 + \sum H_k(x) \left(a_S^{(N_F-1)}(\mu_{\rm th})\right)^k\right] , \qquad x = \log(\mu_{\rm th}^2/m_q^2)$$



$$C_1 = \frac{x}{6} , \qquad C_2 = -\frac{11}{72} + \frac{19}{24}x + \frac{x^2}{36}$$

$$H_1 = 0 , \qquad H_2 = -\frac{89}{432} + \frac{5}{36}x - \frac{x^2}{12}$$

- The $\beta(\alpha_S)$ and $\gamma_m(\alpha_S)$ functions depend on N_F
- Interpret it in the context of Effective Theories with different number of active flavours, and match the couplings at threshold
- Matching is independent of $\mu_{\rm th}$(up to higher orders)

- α_S might become discontinuous, is that a problem ?
- Similar discussion for PDFs

Exercises:

1. Integrate analytically the one-loop and two-loop RGE for the strong coupling, and one-loop for a quark mass
2. Calculate $\alpha_S(10\ \mathrm{GeV})$ and $\alpha_S(1\ \mathrm{TeV})$ from $\alpha_S(m_Z) = 0.1184\ \pm 0.0007$
3. If $m_b(m_b) = 4.2 \pm 0.1\ \mathrm{GeV}$, what is $m_b(m_Z)$
4. Hint

$$a_S(\mu) = \frac{a_S(\mu_0)}{1 + b_0\, a_S(\mu_0) \log \frac{\mu^2}{\mu_0^2}} \qquad \alpha_S(\mu) = \frac{\pi}{b_0 \log \frac{\mu^2}{\Lambda^2_{\mathrm{QCD}}}}$$

Then calculate Λ_{QCD}, the "fundamental" scale of QCD, at which coupling blows up (NB: it is not unambiguously defined at higher order)

The infrared problem

- Soft divergences (=IR) because gluons are massless and can be emitted with zero energy (same phenomenon as in QED with soft photons)
- Collinear divergences (=mass singularities): when either gluons or massless quarks are produced with parallel momenta

  Formally could keep $m_q \neq 0$ but perturbative results will depend on large $\log(m_q)$, and are not trustworthy

Ultraviolet divergences are removed by renormalization
Soft and collinear divergences should cancel ➔ results dominated by large virtualities

Theorems about cancellation of divergences

- BN (Block-Nordsieck): QED (with finite fermion mass) IR divergences cancel is sum over soft (unobserved) photons in the **final state**
- KLN (Kinoshita, Lee, Nauenberg): IR and collinear divergences cancel if sum over degenerate **final** and **initial** states ($\gamma^* \rightarrow$hadrons need only sum in final state)

Definition of infrared and collinear safety

For an observable´s distribution to be calculable in [fixed order] perturbation theory, the observable should be infrared safe, i.e. insensitive to the emission of soft or collinear gluons. In particular if $\vec{p}_i$ is any momentum occurring in its definition, it must be invariant under the branching

$$\vec{p}_i \to \vec{p}_j + \vec{p}_k$$

whenever $\vec{p}_j$ *and* $\vec{p}_k$ are parallel (collinear) or one of them is small (soft)
[Ellis, Stirling, Webber, QCD and Collider Physics]

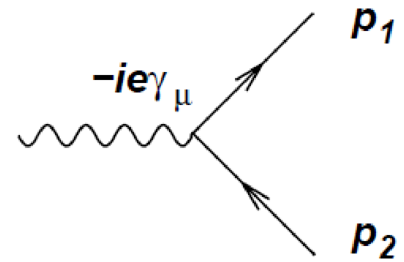
Examples

- Multiplicity of gluons — not IRC safe, modified by soft/collinear splitting
- Energy of hardest particle — not IRC safe, modified by collinear splitting
- Energy flow into a cone — is IRC safe, soft emissions don´t change energy flow and collinear emissions don´t change its direction

e^+e^-: soft-collinear gluon amplitude

- At leading-order (LO):

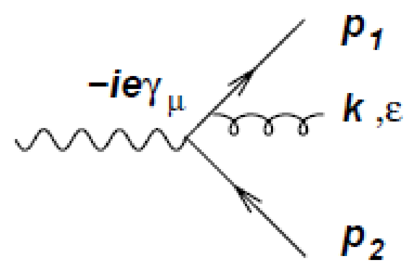
$$M^{(0)}_{q\bar{q}} = (-ie_q)\, \bar{u}(p_1)\, \gamma^\mu\, v(p_2)$$

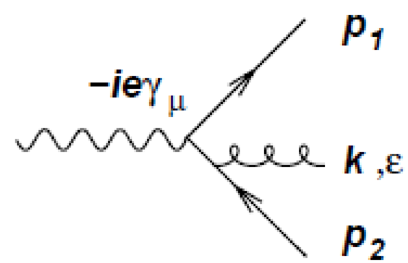


- Then emit a gluon

$$M^{(0)}_{q\bar{q}g} = (-ie_q)(i\, g_S)\, \mathbf{T}^a\, \bar{u}(p_1) \left(\not{\varepsilon}(k)\, \frac{i}{\not{p}_1 + \not{k}}\, \gamma^\mu - \gamma^\mu\, \frac{i}{\not{p}_2 + \not{k}}\, \not{\varepsilon}(k) \right) v(p_2)$$

Using equation of motion $\not{p}_2\, v(p_2) = 0$
and $\not{p}_2\, \not{\varepsilon} = 2\varepsilon \cdot p_2 - \not{\varepsilon}\, \not{p}_2$
in the soft ($\not{k} \to 0$) and
collinear ($\not{k}\, v(p_2) \to 0$) limits





$$(\not{p}_2 + \not{k})\, \not{\varepsilon}(k)\, v(p_2) \simeq 2\varepsilon \cdot p_2\, v(p_2)$$

Then

$$M^{(0)}_{q\bar{q}g} \simeq (-ie_q)\, (i\, g_S)\, \mathbf{T}^a\, \bar{u}(p_1)\, \gamma^\mu\, v(p_2) \left(\frac{p_1 \cdot \varepsilon}{p_1 \cdot k} - \frac{p_2 \cdot \varepsilon}{p_2 \cdot k} \right)$$

e^+e^-: square amplitude

$$|M^{(0)}_{q\bar{q}g}|^2 \simeq \sum_{a,pol} \left| i\, g_S \mathbf{T}^a M^{(0)}_{q\bar{q}} \left(\frac{p_1 \cdot \varepsilon}{p_1 \cdot k} - \frac{p_2 \cdot \varepsilon}{p_2 \cdot k} \right) \right|^2$$

$$= -|M^{(0)}_{q\bar{q}}|^2\, g_S^2\, C_F \left(\frac{p_1}{p_1 \cdot k} - \frac{p_2}{p_2 \cdot k} \right)^2 = |M^{(0)}_{q\bar{q}}|\, g_S^2\, C_F \frac{2p_1 \cdot p_2}{(p_1 \cdot k)(p_2 \cdot k)}$$

Include phase space

$$d\Phi_{q\bar{q}g} |M^{(0)}_{q\bar{q}g}|^2 \simeq \left(d\Phi_{q\bar{q}} |M^{(0)}_{q\bar{q}}|^2 \right) \frac{d^3k}{2E(2\pi)^3}\, g_S^2\, C_F \frac{2p_1 \cdot p_2}{(p_1 \cdot k)(p_2 \cdot k)}$$

Note factorization into hard and soft-collinear-gluon emission

e^+e^-: square amplitude

The squared matrix element in terms of energy and angle

$$\frac{2p_1 \cdot p_2}{(p_1 \cdot k)(p_2 \cdot k)} = \frac{4}{E^2(1-\cos^2\theta)}$$

- It diverges for $E \to 0$: infrared (or soft) emission
- It diverges for $\theta \to 0$ and $\theta \to \pi$: collinear singularities

Use dimensional regularization to integrate analytically over the soft and collinear region of the phase-space

$$\frac{d^3k}{2E(2\pi)^3} \to \frac{d^{d-1}k}{2E(2\pi)^{d-1}} \qquad d = 4 - 2\epsilon$$

Leads to poles in $1/\epsilon^2$, $1/\epsilon$, and a finite remainder

- **Slicing method:** split phase-space in two regions

$$\int_0^1 \frac{f(x)}{x} \quad \to \quad \int_0^1 x^{-1+\epsilon} f(x) \simeq f(0) \int_0^{\omega} x^{-1+\epsilon} + \int_w^1 \frac{f(x)}{x}$$

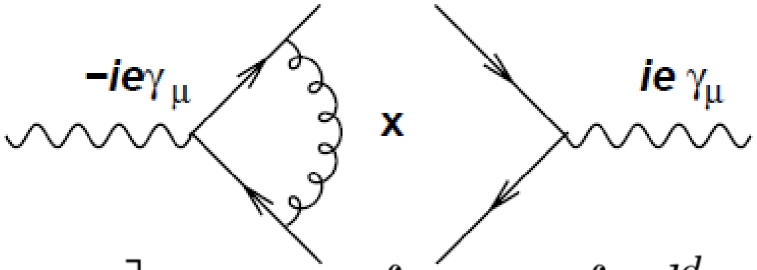
$$= \quad f(0) \left(\frac{1}{\epsilon} + \log w \right) + \int_w^1 \frac{f(x)}{x}$$

- **Subtraction method:** add and subtract back an approximation having the same singular behaviour

$$\int_0^1 x^{-1+\epsilon} f(x) = f(0) \int_0^1 x^{-1+\epsilon} + \int_0^1 \frac{f(x) - f(0)}{x}$$

e⁺e⁻: virtual amplitude

- The one-loop amplitude:



$$M^{(1)}_{q\bar{q}} = (-ie_q)\, g_S^2\, C_F\, \bar{u}(p_1)$$
$$\times \left[\int_q \frac{\gamma^\nu\, (\not{q} - \not{p}_1)\, \gamma^\mu\, (\not{q} + \not{p}_2)\, \gamma_\nu}{[(q-p_1)^2 + i0][(q+p_2)^2 + i0](q^2 + i0)} \right] v(p_2) \qquad \int_q = -i \int \frac{d^d q}{(2\pi)^d}$$

- Set the virtual gluon on-shell $\quad \frac{1}{q^2 + i0} \to -2\pi\, i\, \theta(q_0)\, \delta(q^2) = -\tilde{\delta}(q)$

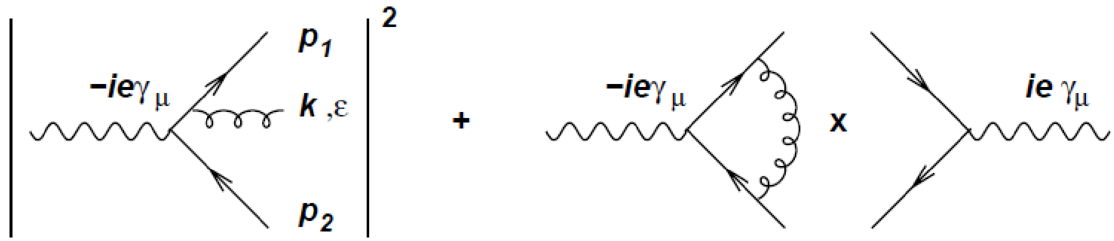
$$(\not{q} + \not{p}_2)\gamma^\nu v(p_2) = [2(q+p_2)^\nu - \gamma^\nu \not{q}] v(p_2)$$

$$M^{(1)}_{q\bar{q}} \simeq -g_S^2\, C_F\, M^{(0)}_{q\bar{q}} \int_q \frac{p_1 \cdot p_2}{(q\cdot p_1)(q \cdot p_2)}\, \tilde{\delta}(q)$$

Total cross-section must be finite: if real part has poles in $1/\epsilon$, integration of the virtual part should exhibit the same poles of opposite sign (Unitarity, conservation of probability)

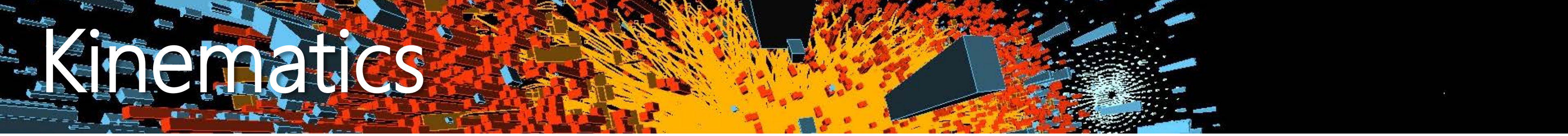
e^+e^-:total cross-section

The total cross-section is the sum of all real and virtual diagrams



- Corrections to σ_{tot} come from hard ($E{\sim}Q$) large-angle gluons, and large virtualities ($q{\sim}Q$): physics at short-distance
- Soft gluons are emitted on long timescale $\sim 1/(E\,\theta^2)$ relative to the collision scale ($1/Q$) and cannot influence the cross-section
- Transition to hadrons also occurs on long time scale ($1/\Lambda_{\mathrm{QCD}}$) and then is factorized
- Correct renormalization scale for α_S is $\mu{\sim}Q$

QCD at the LHC



Kinematics

Transverse plane

- Azimuthal angle ϕ
- Transverse momentum $p_T = \sqrt{p_x^2 + p_y^2}$
- Transverse mass $m_T = \sqrt{p_T^2 + m^2}$

Longitudinal variables

- Rapidity: $$y = \frac{1}{2} \log \left(\frac{E + p_z}{E - p_z} \right)$$
- Pseudo-rapidity: $$\eta = - \log \left(\tan(\theta/2) \right)$$

$$p^\mu = (m_T \cosh(y), p_T \cos(\phi), p_T \sin(\phi), m_T \sinh(y))$$

Exercises:
1. Show that $\eta = y$ for massless particles
2. Show that $\Delta y = y_i - y_j$ is invariant under boost