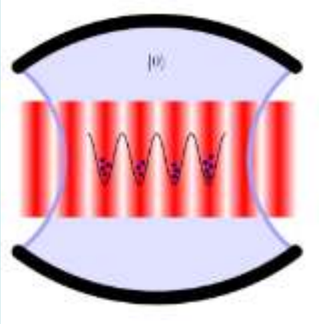
Innsbruck Physics Research Center

Foundations and Applications of Quantum Science

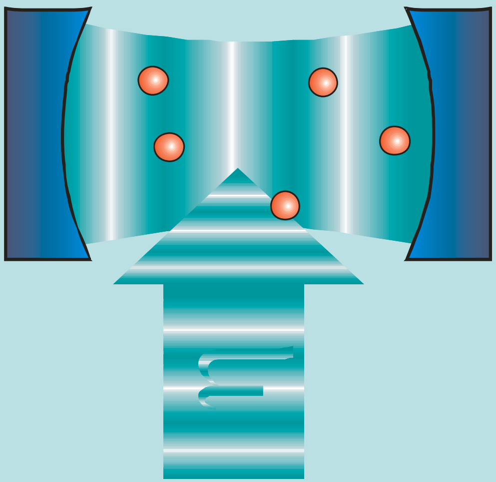
Selfordering of an (ultra-)cold gas in a resonator field

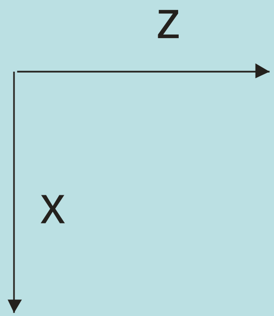


Der Wissenschaftsfonds.

Cooling and crystallisation of large ensembles through superradiant light scattering

Geometry: transverse direct excitation of atoms from side !





phase of excitation light depends on position x

$$\dot{\sigma}_i = (i\Delta_A - \gamma)\sigma_i - g(z_i)a + \eta_x + \xi_A$$

$$\dot{a} = (i\Delta_C - \kappa)a + \sum_{i=1}^{N} g^*(z_i)\sigma_i + \xi_i \, .$$

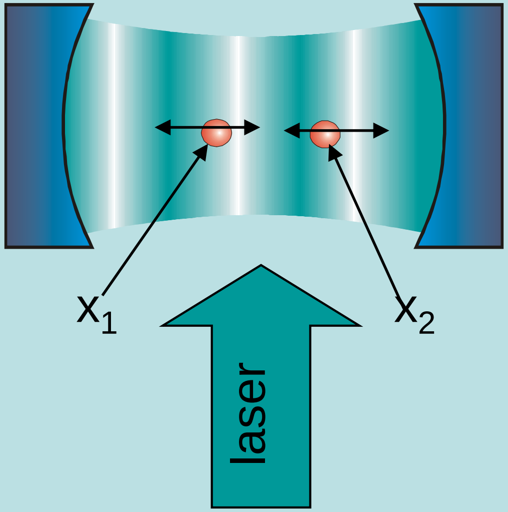
collective pump strength ***R***

Field in cavity generated only by atoms

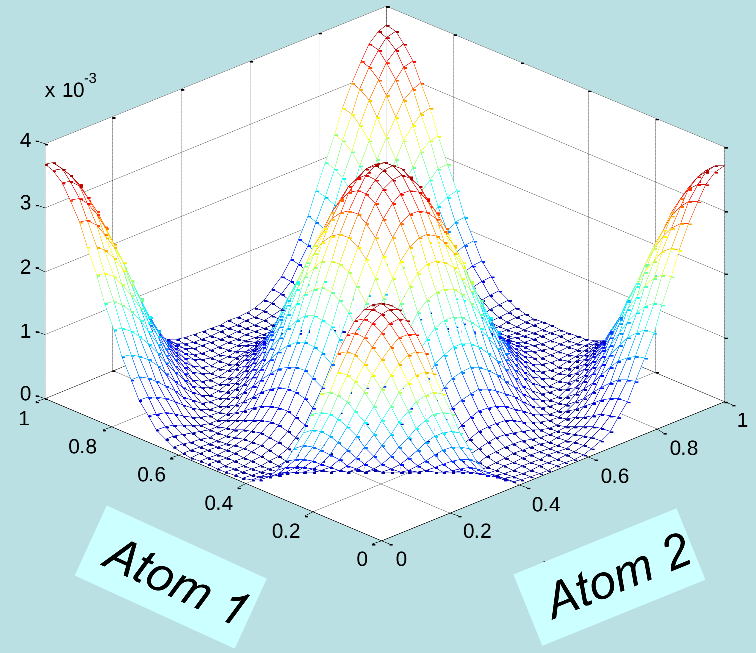
R *= 0 for random atomic distribution*

R ~ Ng *for regular lattice (Bragg)*

Two classical atoms at fixed positions



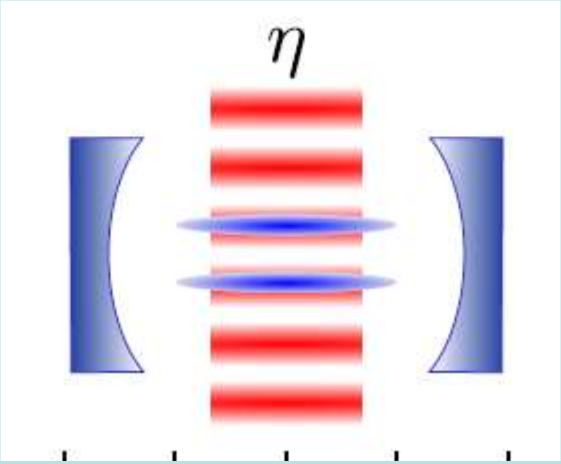
Cavity field as a function of positions for two atoms

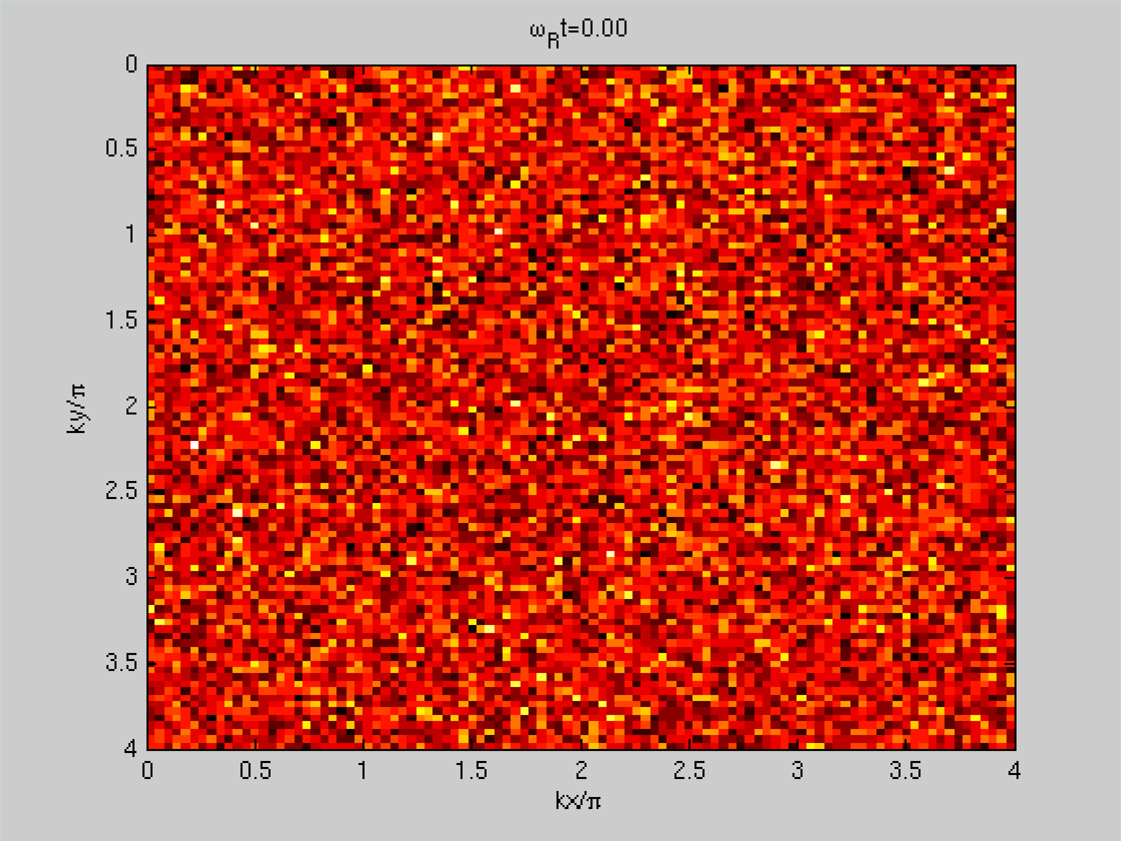


Maximum photon number for 0 and λ distance
Minimum photon number for λ/2 distance

=> for high field seekers λ ordering is energetically favorable

Numerical simulations of coupled dynamics including atomic motion (start with random distribution at Doppler temperature)





Atom-field dynamics for very large particle number :
=> Vlasov equation for particle distribution

Continuous density approximation for cold cloud: single particle distribution function

$$f_s(x,p,t) := \frac{1}{N_s}\left\langle \sum_{j_s=1}^{N_s} \delta(x - x_{j_s}(t))\delta(p - p_{j_s}(t)) \right\rangle$$

$$\Phi_s(x,\alpha) = \hbar U_{0,s}|\alpha|^2 \sin^2(kx) + \hbar\eta_s(\alpha + \alpha^*)\sin(kx)$$

Vlasov + field equation

$$\frac{\partial f_s}{\partial t} + \frac{p}{m_s}\frac{\partial f_s}{\partial x} - \frac{\partial \Phi_s(x,\langle\alpha\rangle)}{\partial x}\frac{\partial f_s}{\partial p} = 0$$

$$\dot{\alpha} = (i\Delta_c - \kappa)\,\alpha - i\sum_s \int \left(\alpha\, U_{0,s}\sin^2(kx) + \eta_s \sin(kx)\right) f_s\, dx\, dp$$

stability threshold of homogeneous distribution:

$$\frac{N\eta^2}{k_B T}\,\mathrm{vp}\int_{-\infty}^{\infty} \frac{g'(\xi)}{-2\xi}\,\mathrm{d}\xi < \frac{\delta^2 + \kappa^2}{\hbar|\delta|}$$

threshold at thermal equilibrium

$$U_0 N V_{opt} > \kappa^2$$

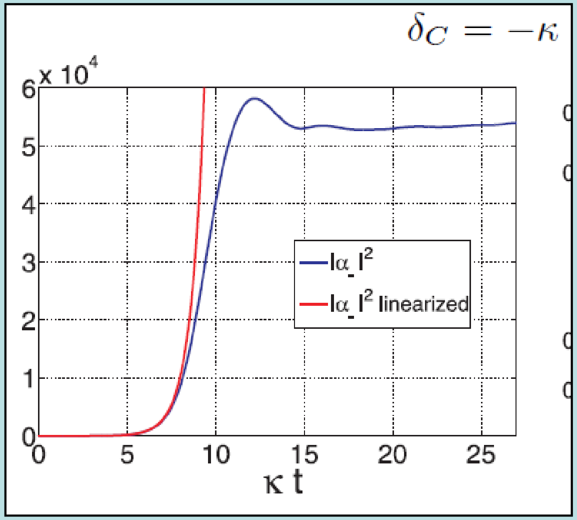
frequency shift of cavity → U_0

pump laser opt. potential → V_{opt}

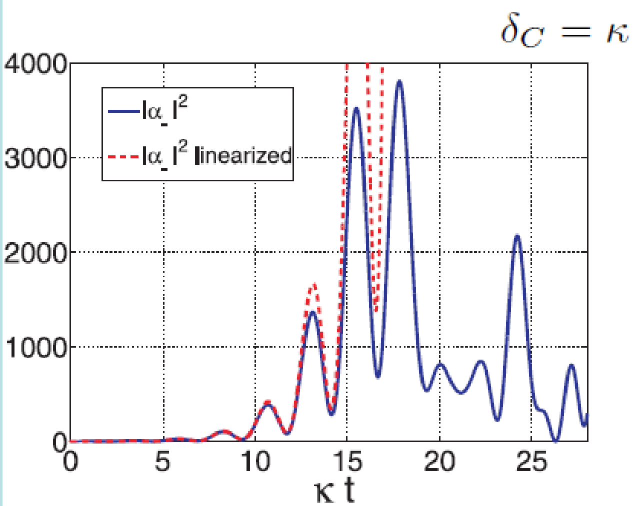
cavity damping → κ^2

time evolution of field intensity above threshold (~ δ_c^2)

negative detuning



positive detuning



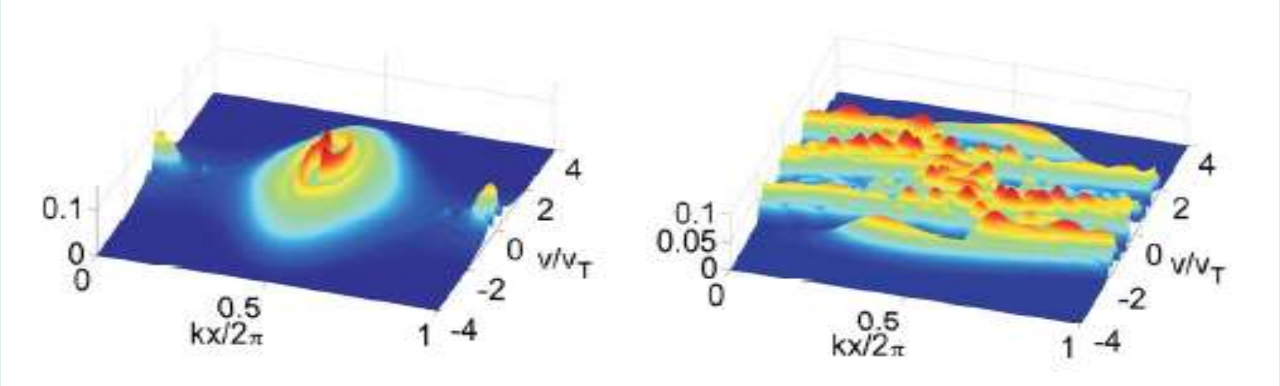
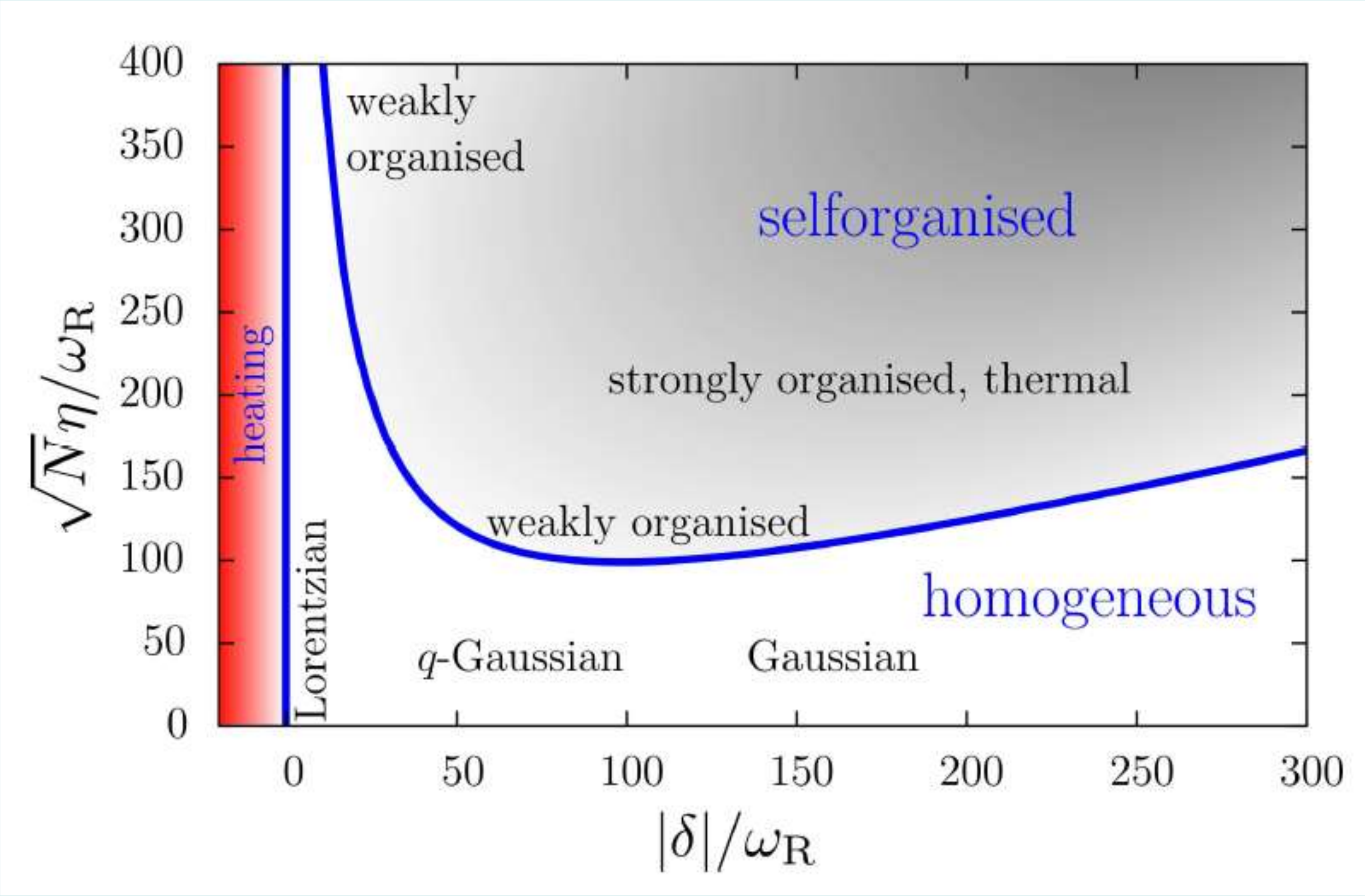


FIG. 24 Phase space densities of the particles for negative detuning $\delta_C = -\kappa$ (left) and positive detuning $\delta_C = \kappa$ (right)

- **instability leads to selfordering for negative detuning only**
- **dynamics driven by energy minimization via selftrapping**

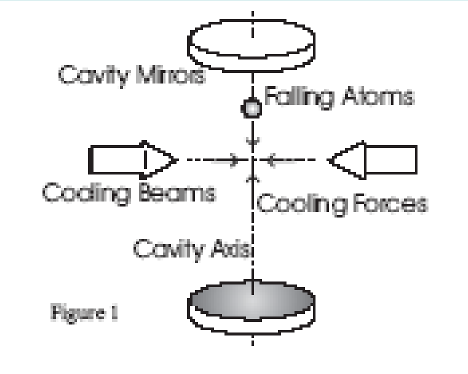
stability analysis including diffusion



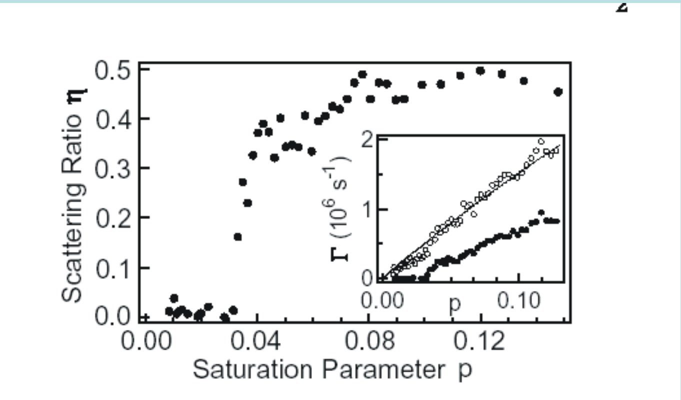
(Niedenzu, EPL 2011)

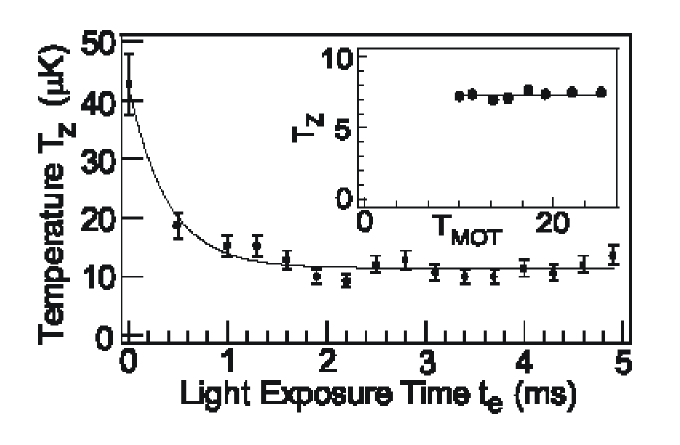
Experiment with atoms:

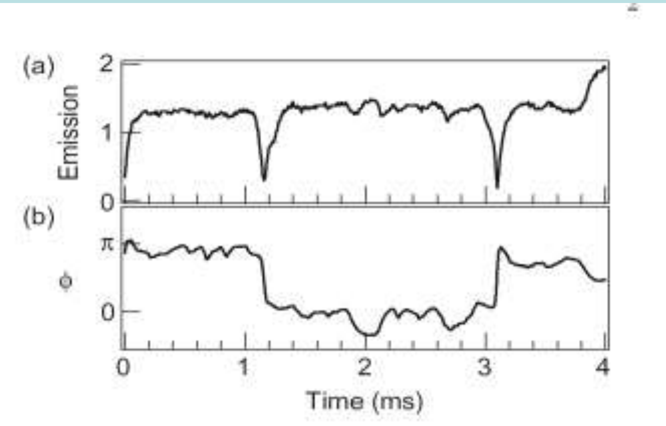
Vladan Vuletic: Stanford University (=>MIT)



10^6 Caesium atoms in resonator with transverse coherent pump field







Phase stability of coherent emission with Pi-jumps (bistable pattern)

- *>10^6 Atoms trapped and cooled to ~μK*
- *with simultaneous coherent light emission*

Phase memory of atomic system:
probability of Pi-jumps between sucessive jumps

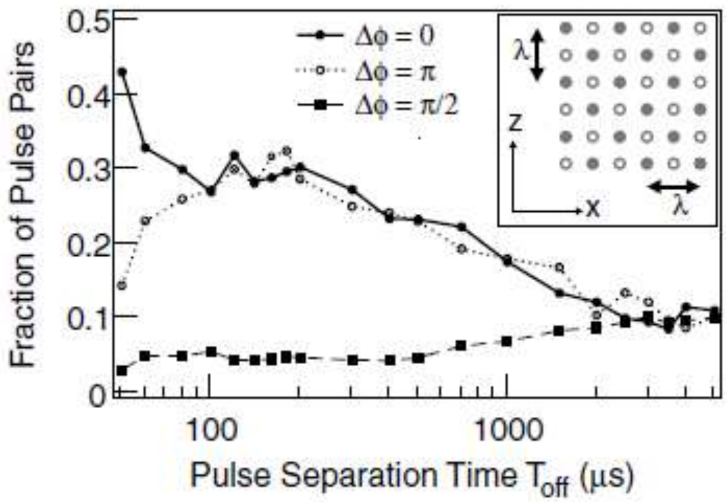
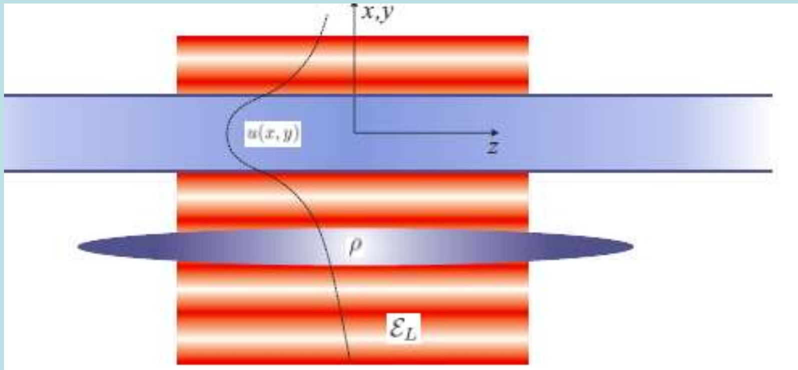


FIG. 3. The fraction of pulse pairs with relative phase shift $\Delta\phi$ is plotted versus pulse separation time. The solid circles, open circles, and solid squares correspond to $\Delta\phi = 0 \pm \pi/10$, $\pi \pm \pi/10$, and $\pi/2 \pm \pi/10$, respectively. The parameters are the same as for Fig. 2, except here $N = 1.3 \times 10^7$. The inset shows the two possible lattice configurations producing relative phase shift $\Delta\phi = \pi$ in the emitted light.

- ***Atomic cloud shows memory***
- ***Preparation of initial conditions needs great care***
- ***Many possible patterns lead to the same field:***
  only Δ_N = N_g – N_e counts
- ***Better memory could be expected if extra lattice is added***
- ***more recent experiments in Signapore and London (UCL)***

Crystalization in infinite (mirrorless) systems:
continous frequency band of modes



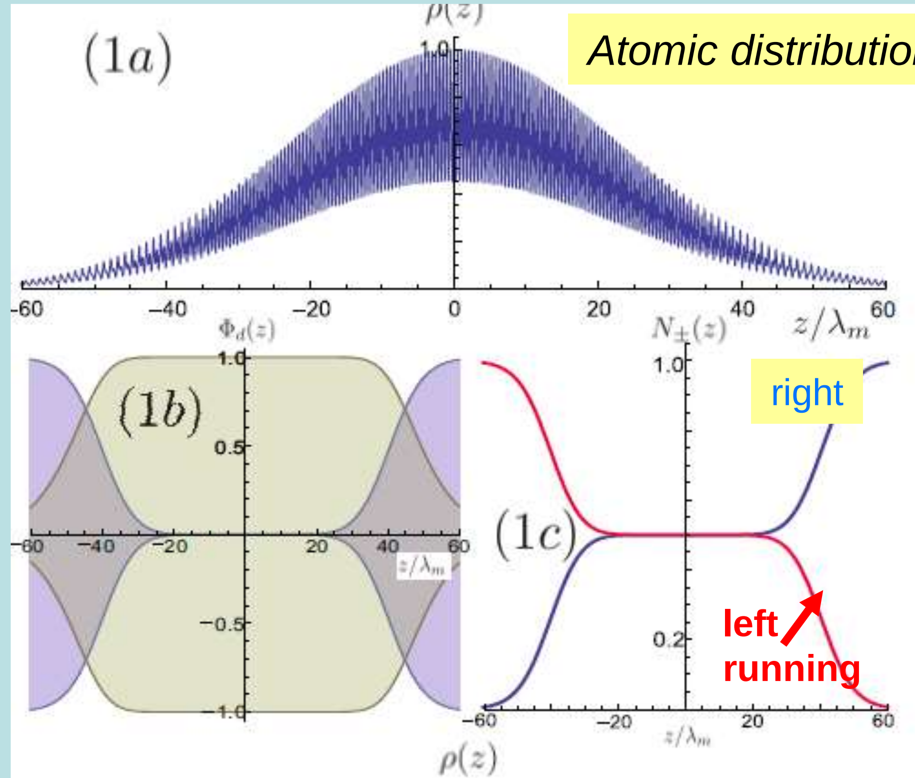
: Cigar-shaped atomic gas alongside optical nanofiber.

$$\frac{\partial^2 E}{\partial z^2} + \left(\beta_m^2 + k_L^2 \tilde{\chi}\right) E = -k_L^2 \tilde{\chi} E_L, \qquad ($$

$$\frac{\partial f}{\partial t} + \frac{p_z}{m}\frac{\partial f}{\partial z} - \frac{\partial}{\partial z}\left(U - \alpha\left[|E|^2 + 2E_L E_r\right]\right)\frac{\partial f}{\partial p_z} = 0$$

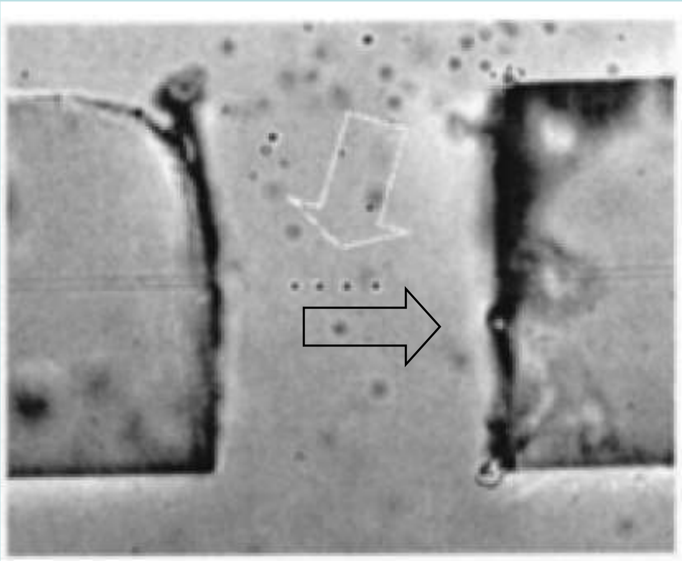
coupled Maxwell + mean field equations

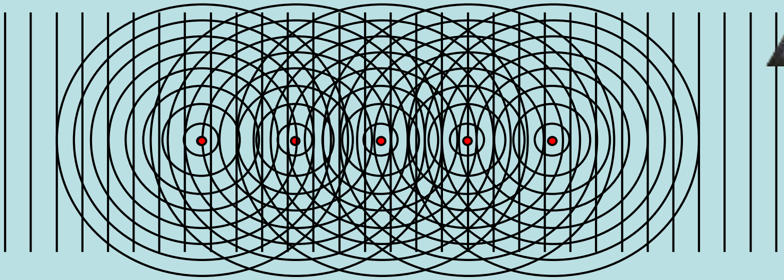
Atomic distribution

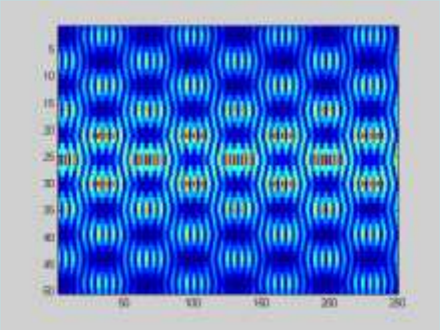


Field distribution

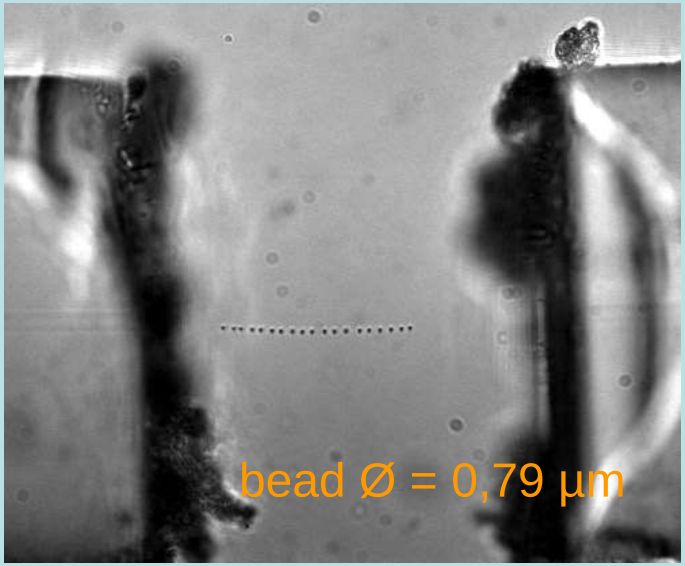
Beads in fiber optical trap (optical stretcher) by Singer et. al :

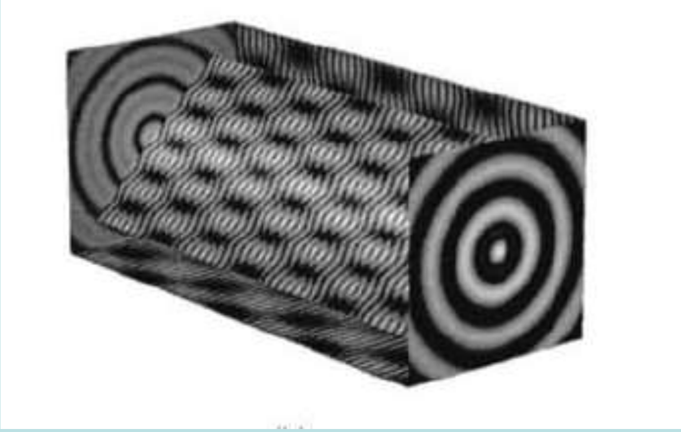






Interference pattern matches bead size to minimize energy





Is energy minimization a sufficient general principle here ?

REVIEWS OF MODERN PHYSICS, VOLUME 82, APRIL–JUNE 2010

Colloquium: Gripped by light: Optical binding

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(Published 3 June 2010)

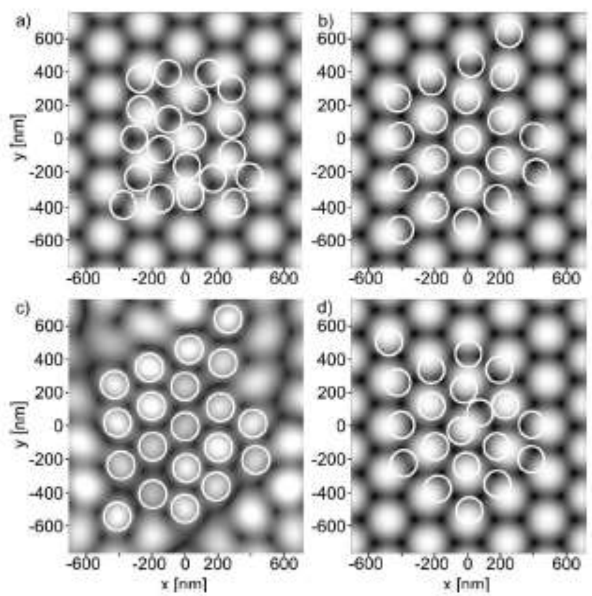


FIG. 9. Optical binding of infinite cylinders in the interference field of plane waves. Positions of 20 cylinders (white circles) and field distributions (background gray-scale images) for (a) random initial positions in a three-plane-wave interference pattern, background shows unperturbed incident field distribution; (b) organized final positions due to the trapping and binding forces, background shows unperturbed incident field distribution; (c) the same as (b) but final field distribution is shown at the background; and (d) organized final positions corresponding to another set of cylinders in initial positions. The parameters used are the following: $a=0.15\lambda$, $\lambda=546$ nm, $\varepsilon_p=2.56$, and $\varepsilon_m=1.69$. Adapted from Grzegorczyk *et al.*, 2006c.

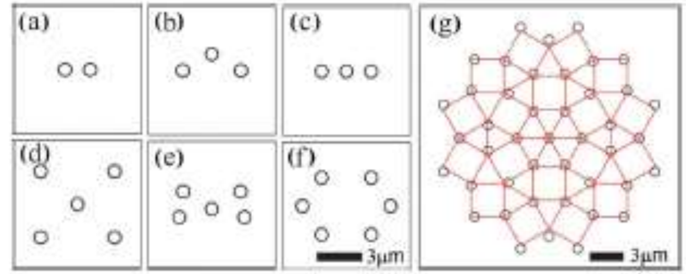


FIG. 11. (Color online) Stability of optical binding of multiple identical spheres. Examples of equilibrium configurations calculated for varying numbers of polystyrene ($\varepsilon_p=2.53$) spheres of radius $a=0.414$ μm placed in vacuum (air) and illuminated with horizontal polarization of the incident light with a wavelength $\lambda=0.52$ μm. Configurations (a)–(e) have all eigenmodes stable, (b) and (e) are in drifting equilibrium, and (f) and (g) have either stable or quasistable modes (see the text). From Ng, Lin, *et al.*, 2005.

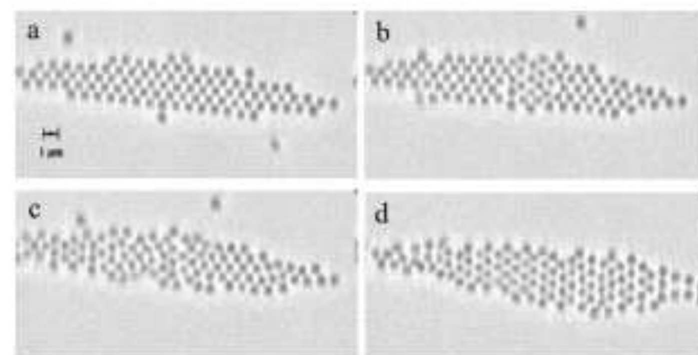
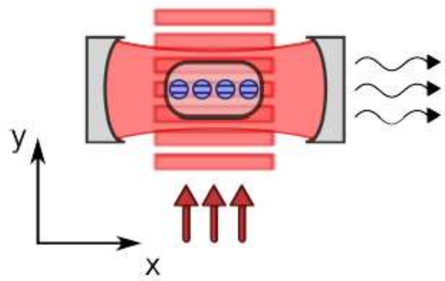


FIG. 13. Particle self-organization due to optical binding in the vicinity of a planar interface. Sequence of video frames of an array of 520 nm particles as a quarter-wave plate is rotated to change the polarization of one of the counterpropagating beams from s ($\phi=0°$) to p ($\phi=45°$). Hexagonal packing nucleating at the center of the array is seen at (b) and then the hexagonal crystalline structure grows outward toward the left- and right-hand sides of the array. From Mellor *et al.*, 2006.

- Particles try to adapt der positions until a local energy minimum is reached
- field gives long range interactions
  single mode cavity: infinite range
  multimode cavity : tailorable range
  free space: effective dipole-dipole (~ 1/r^n)

Multifrequency selfordering in a standing wave cavities

(S. Krämer)



- Modes $\{\omega_c^n; \hat{a}_n; \kappa_n\}$
- Many-Particle System $\{m; \hat{x}_i; \hat{\sigma}_+^i\}$
- Particles in trap $V(x)$
- Pump Laser $\{\omega_p^k; \eta_k\}$

Idea:
use frequency comb to pump at a large number of cavity frequencies

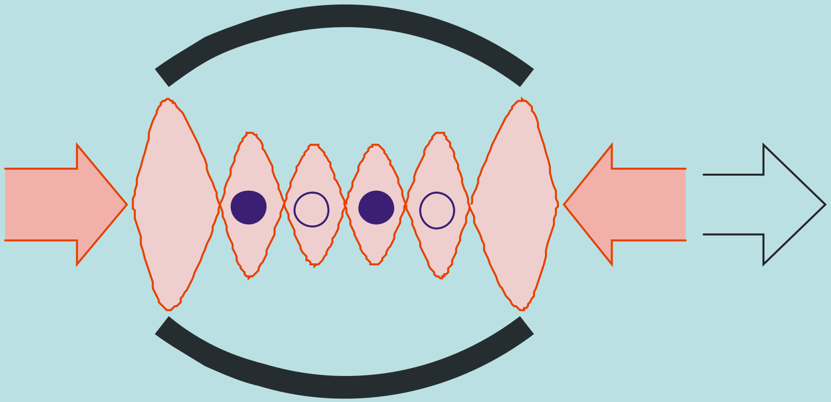
$$T_0 = \frac{\hbar}{2L\epsilon_0}\frac{\Delta_a}{\Delta_a^2+\gamma^2}|d_{eg}|^2$$

$$\eta_n = \frac{1}{2}\sqrt{\frac{\hbar\omega_c^n}{2L\epsilon_0}}E_0^n\frac{\Delta_a}{\Delta_a^2+\gamma^2}|d_{eg}|^2$$

$$H = -\sum_n \delta_n \hat{a}_n^\dagger \hat{a}_n + \sum_i\left(\frac{p_i^2}{2\mu} + V(\hat{x}_i)\right) + \sum_{ni} T_0\omega_n \sin^2(k_n(\hat{x}_i - L))\hat{a}_n^\dagger\hat{a}_n + \sum_{ni}\eta_n \sin(k_n(\hat{x}_i - L))(\hat{a}_n^\dagger + \hat{a}_n)$$

- Choice of frequencies and detunings allows to fix couplings
- Bias patterns via cavity pump – extra control inputs
- Output pattern reflects particle distribution

Quantum description of selforganization of atoms in a lattice

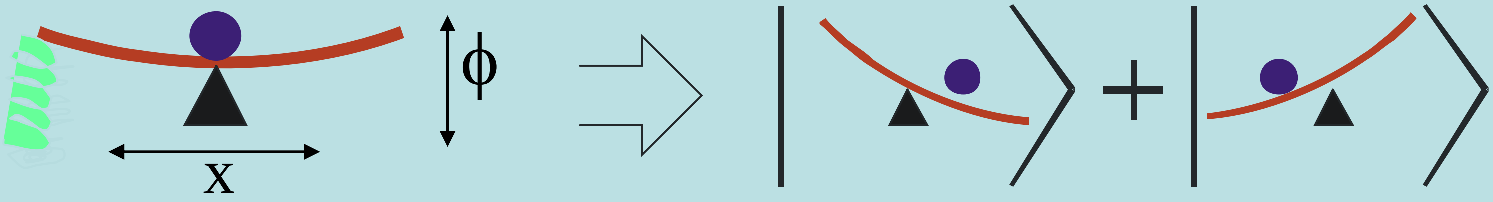


Atoms close to T=0 in
standing wave
(e.g. perpendicular to cavity)

How will selforganization happen here ?
(dynamics of a quantum phase transition)

Interlude

"decay of a quantum seesaw "



classical

quantum

Two degrees of freedom: tilt angle ϕ and particle position x

=> simple model Hamiltonian:

$$V(x,\varphi) = \omega_x^2 x^2 + \omega_\varphi^2 \varphi^2 - 2J \sin(\varphi) x.$$

$$H = \frac{1}{2}(P_x^2 + P_\varphi^2 + V(x,\varphi))$$

linear approx. in φ : X-x coupled oscillators

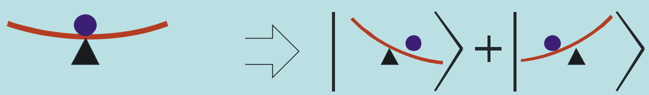
$$\hbar\omega_x a_x^\dagger a_x + \hbar\omega_\varphi a_\varphi^\dagger a_\varphi - \frac{J}{4}(a_\varphi^\dagger + a_\varphi)(a_x^\dagger + a_x)$$

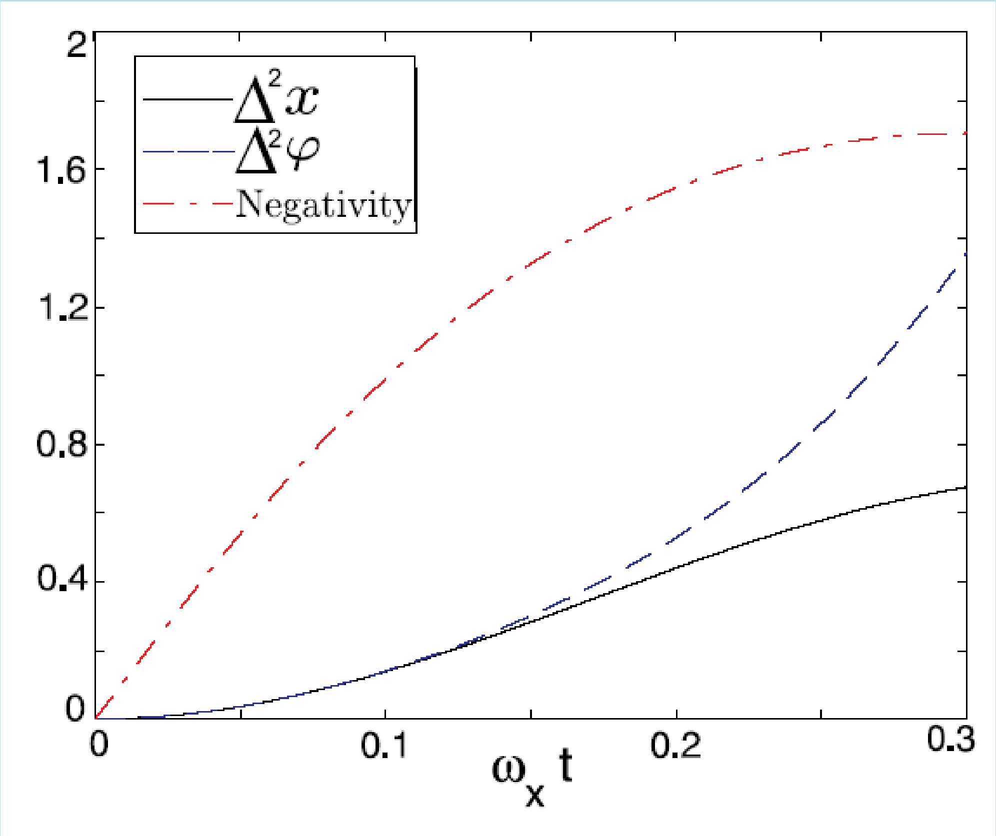
Note: classical equilibrium point at x=ϕ=0 has „long lifetime"

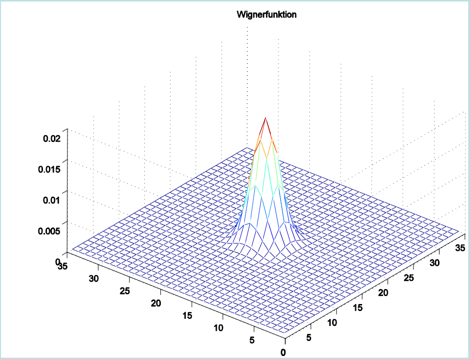
but

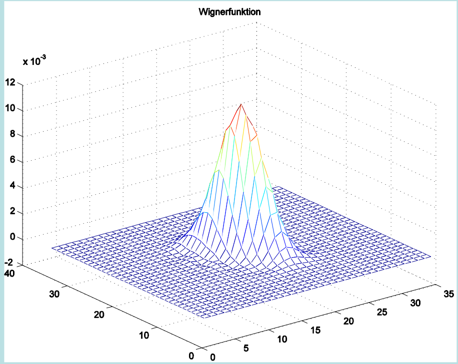
Quantum mechanical product state of oscillator ground states is not stationary

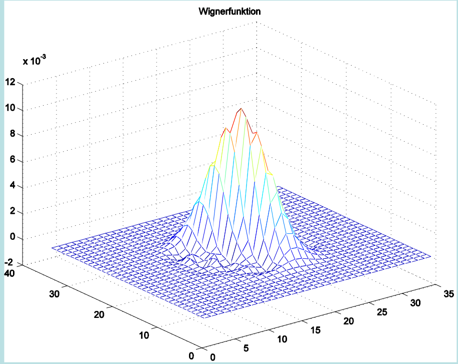
Quantum dynamics yields fast decay





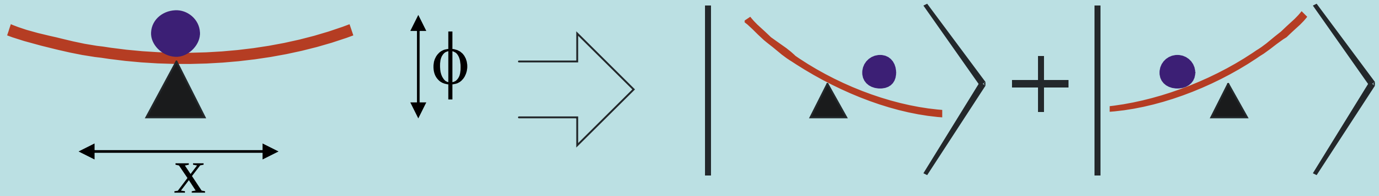






- ***instant growth of position spread and entanglement***
- ***possibilty of superpositions in a quantum seesaw allows tilting both ways simultaneously***

toy model implemented by aton + cavity field

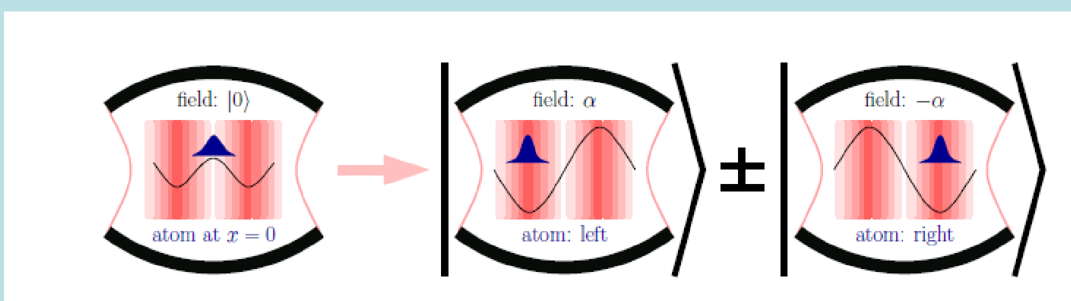


classical

quantum

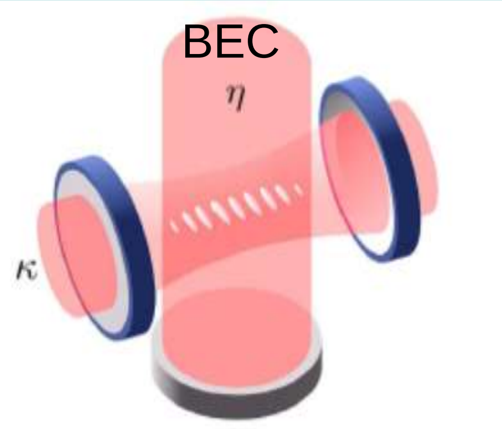
Two degrees of freedom: tilt angle ϕ and particle position x

Note: classical equilibrium point at x=ϕ=0
but
product state of oscillator ground states is no stationary state



field phase replaces tilt angle <> occupation difference replaces position

„mean field“ - dynamics of selforganization for transverse pump



$$H = -\Delta_C\, a^\dagger a + \int_0^L \Psi^\dagger(x)\Big[-\frac{\hbar}{2\,m}\frac{d^2}{dx^2} + U_0\, a^\dagger a \cos^2(kx) + i\eta_t \cos kx(a^\dagger - a)\Big]\Psi(x)dx,$$

Two-mode approximation (weak pump)
=> Tavis-Cummings model
=> superradiant phase transition

$$H = -\delta_C\, a^\dagger a + \omega_R \hat{S}_z + iy(a^\dagger - a)\hat{S}_x/\sqrt{N} + u a^\dagger a\left(\tfrac{1}{2} + \hat{S}_z/N\right)$$

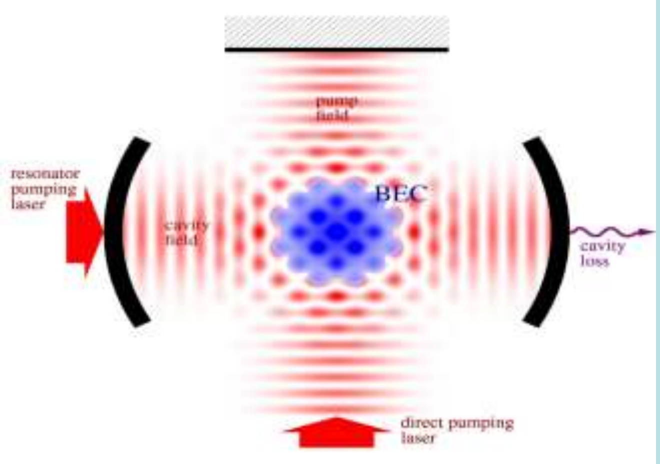
$$\Psi(x) = \frac{1}{\sqrt{L}} c_0 + \sqrt{\frac{2}{L}}\, c_1 \cos kx$$

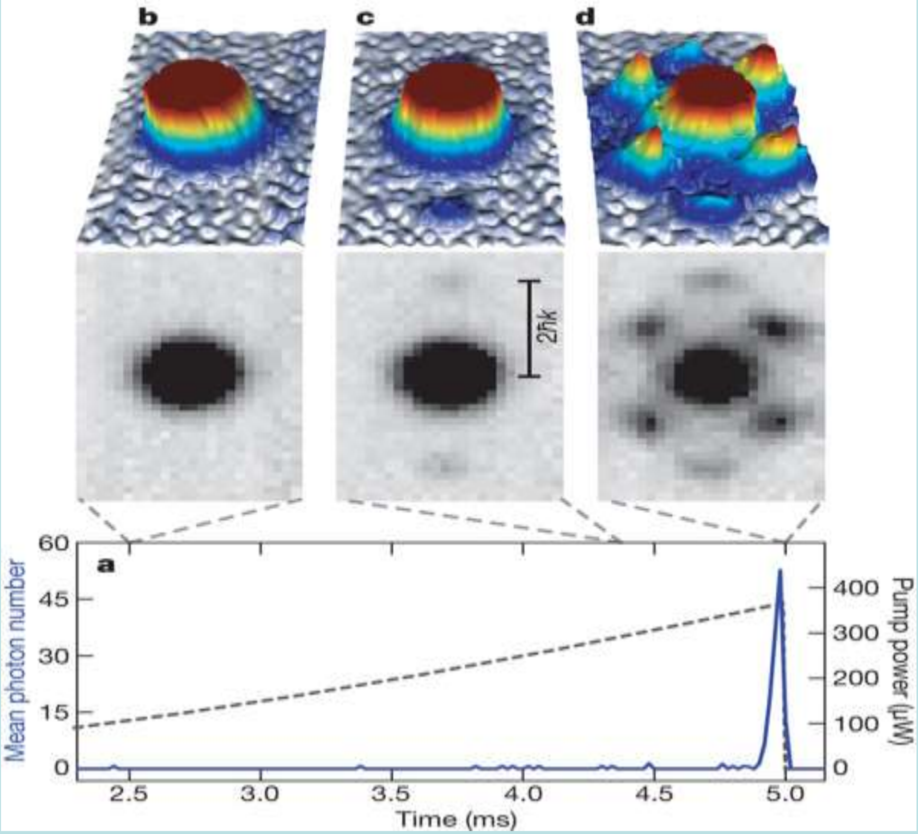
Nagy-Domokos,
PRL 104, 130401 (2010), NJP 2011

Full spatial dynamics:
generalized BH model

Fernandez-Vidal, Morigi
Phys. Rev. A 81, 043407 (2010)

Eperiment ETH:
Observation of the phase transition to new phase with coherence + ordering present ("supersolid phase")



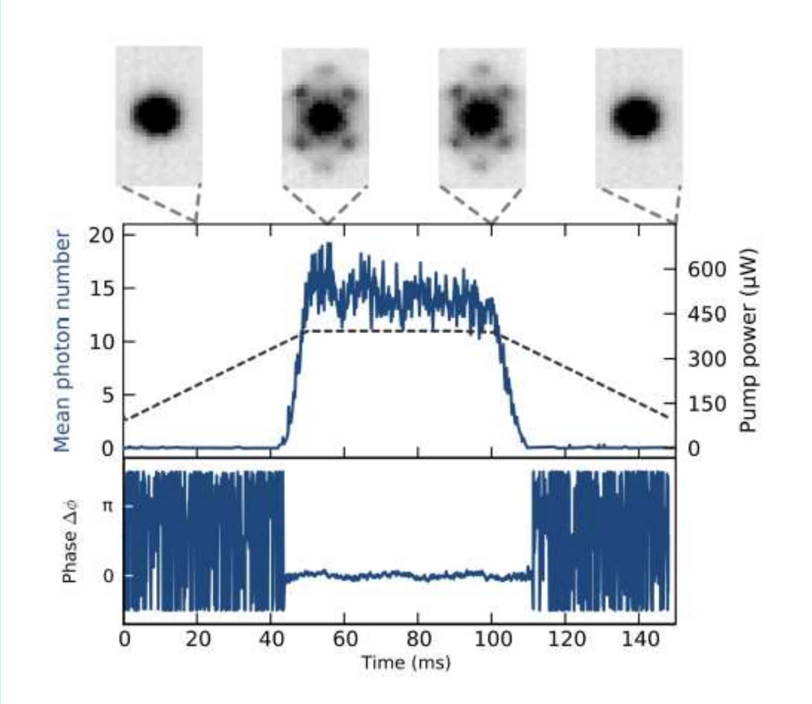


$$\Psi(x) = \frac{1}{\sqrt{L}} c_0 + \sqrt{\frac{2}{L}} c_1 \cos kx$$

Implementation of „Dicke Superradiant Phase" transition

K Baumann *et al.* *Nature* 464, 1301-1306 (2010) doi:10.1038/nature09009

Measurement of phase diagram



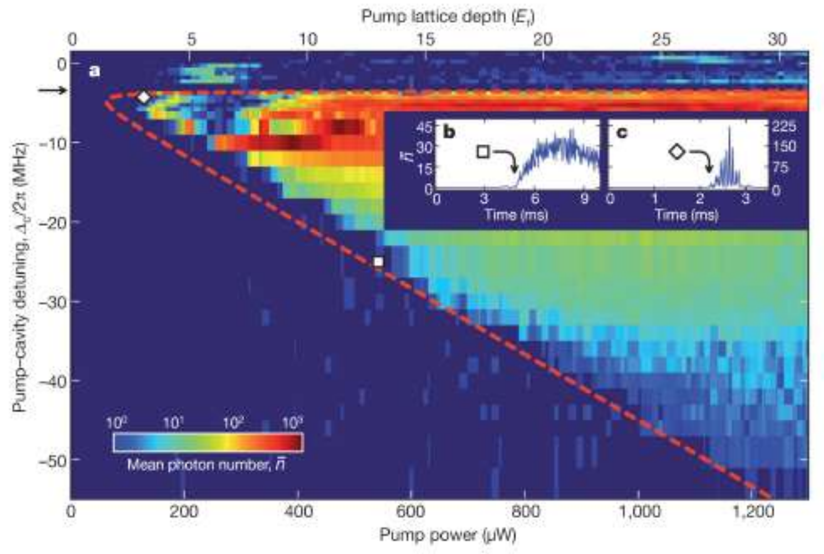


FIG. 40 *Phase diagram of the Dicke model*, from (Baumann *et al.*, 2010).

in ordered region:
coherence + ordering present: "supersolid phase"

probability of Pi - jumps between successive jumps is strongly reduced

Phase memory of quantum system ?

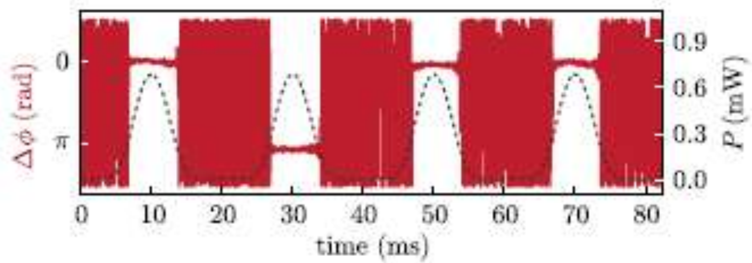


FIG. 34 (color online). Observation of symmetry breaking at the self-organization transition with a BEC. The relative pump-cavity phase $\Delta\phi$ monitored on a heterodyne detector while repeatedly entering the self-organized phase by tuning the transverse pump power P (dashed) is shown. The system organizes into one out of two possible checkerboard patterns corresponding to the two observed phase values differing by π. From Baumann *et al.*, 2010.

- No systematic study published
- Symmetry even harder to control

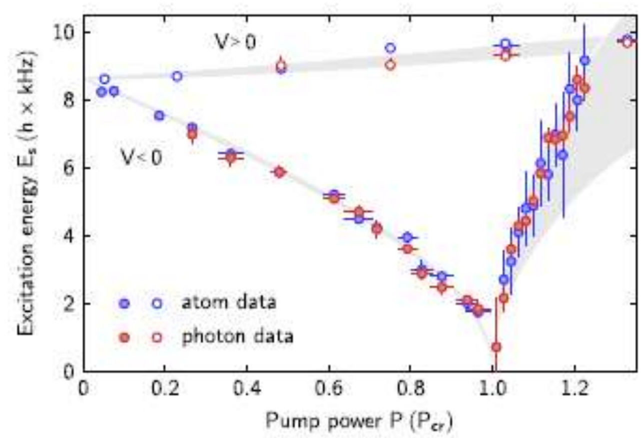
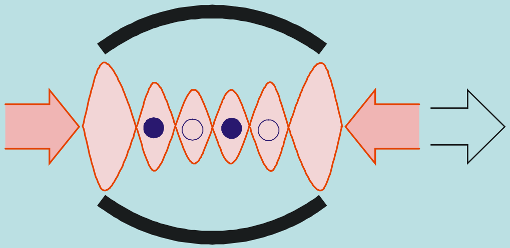


FIG. 35 (color online). Observation of mode softening induced by cavity-mediated atom-atom interactions in a Bose-Einstein condensate. The motional atomic excitation energy at momenta $(\pm\hbar k, \pm\hbar k)$ along the cavity and pump direction as a function of the transverse laser power P, which sets the modulus $|V|$ of the cavity-mediated atom-atom interaction, is shown. The sign of V is determined by the sign of δ_C. For negative interaction strength V, the system organizes at the critical pump power P_{cr}, while for positive interaction an increased excitation energy is observed in accordance with the absence of a phase transition. From Mottl *et al.*, 2012.

- Noise studies reveal characteristic fluctuations below threshold

Beyond mean field

multiparticle quantum description of selforganization in a lattice



- pump creates optical lattice with
- atoms in lowest band
- cavity field from scattered lattice light

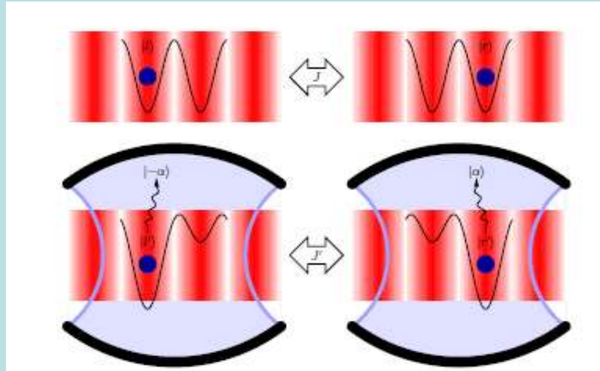
Effective Hamiltonian:

$$H = \sum_{k,l} E_{k,l} b_k^\dagger b_l + \hbar U_0 \eta' g \sum_{k,l} J_{k,l} b_k^\dagger b_l + \hbar \eta' \left(a + a^\dagger\right) \sum_{k,l} \tilde{J}_{k,l} b_k^\dagger b_l - \hbar \left(\Delta_c - U_0\right) a^\dagger a$$

pump amplitude determined by atomic distribution operator

How and when will selforganization happen here ?

Two degenerate states for single atom at two sites ...



Lowest energy eigenstates of double well

$$a = -i\frac{\eta'}{\kappa - i(\Delta_c - U_0)}\tilde{J}_0\left(b_1^\dagger b_1 - b_2^\dagger b_2\right)$$

$$a^\dagger a \sim \left(b_1^\dagger b_1 - b_2^\dagger b_2\right)^2$$

$$\frac{1}{\sqrt{2}}\left(|\text{left}\rangle\,|\alpha\rangle \pm |\text{right}\rangle\,|-\alpha\rangle\right)$$

... show atom field entanglement

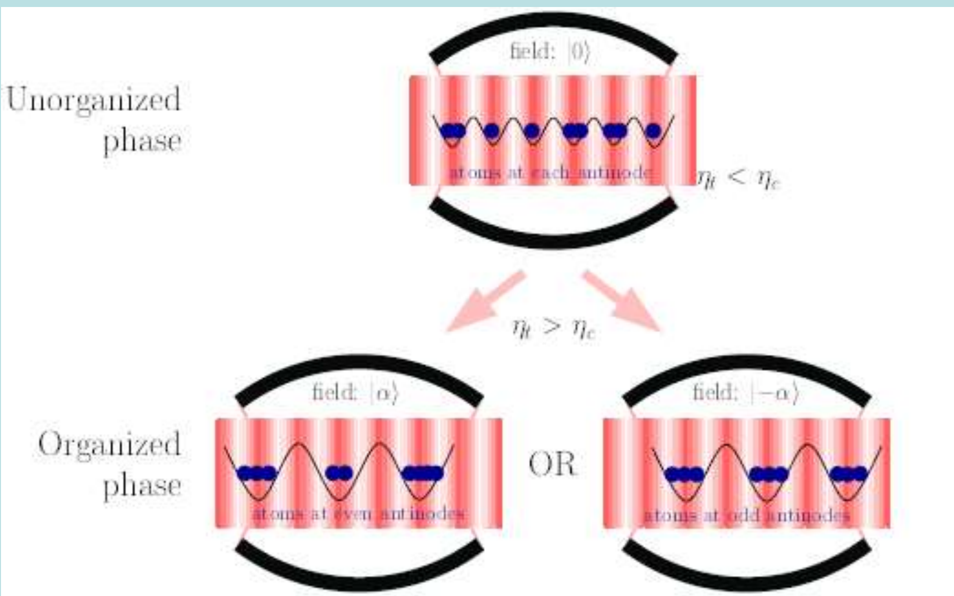
- strongly entangled ground state
- atom tunneling needs phase flip (stabilization)
- Symmetry leads to zero field amplitude but nonzero intensity (photons)

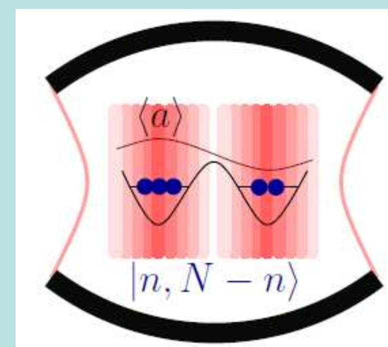
many atoms in lattice => two-effective sites needed

$$a = -i\frac{\eta'}{\kappa - i(\Delta_c - U_0)}\tilde{J}_0\left(b_1^\dagger b_1 - b_2^\dagger b_2\right)$$

$$a^\dagger a \sim \left(b_1^\dagger b_1 - b_2^\dagger b_2\right)^2$$

$$\frac{1}{\sqrt{2}}\left(|\text{left}\rangle\,|\alpha\rangle \pm |\text{right}\rangle\,|-\alpha\rangle\right)$$

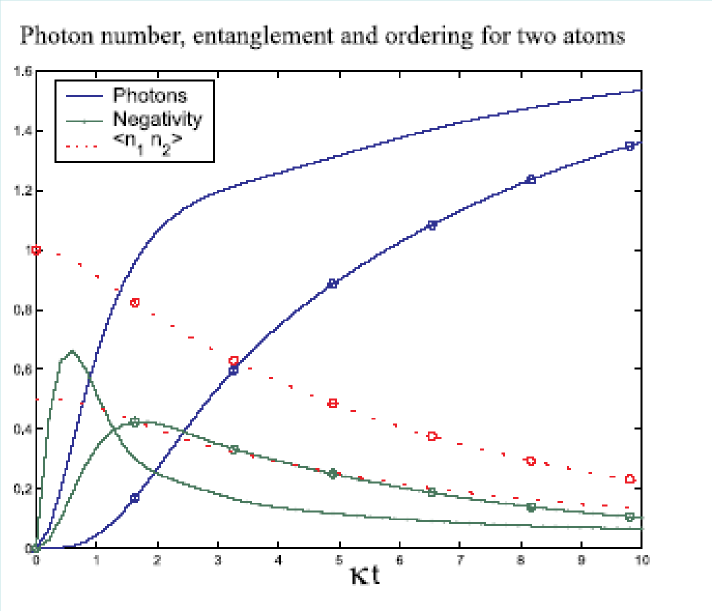


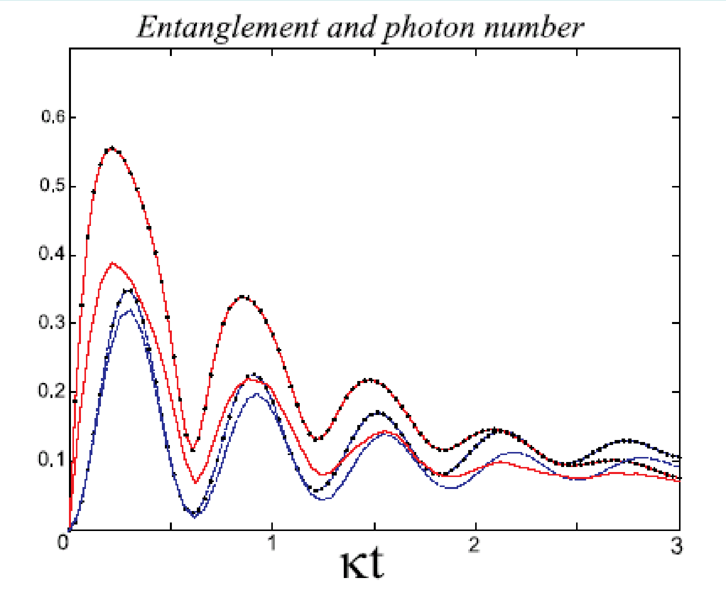


each distribution is
'local' stationary state for particles
without tunneling

Selforganization for **quantum field** is fast and involves entanglement and atomic qunatum statistics

Numerical solution for two atoms at two sites starting at equal population at right and left site





atom + field evolve in short time towards entangled cat state !

Note: superfluid selforganizes much faster than Mott insulator !!

Two „Hopfield“ neuron „ordering“

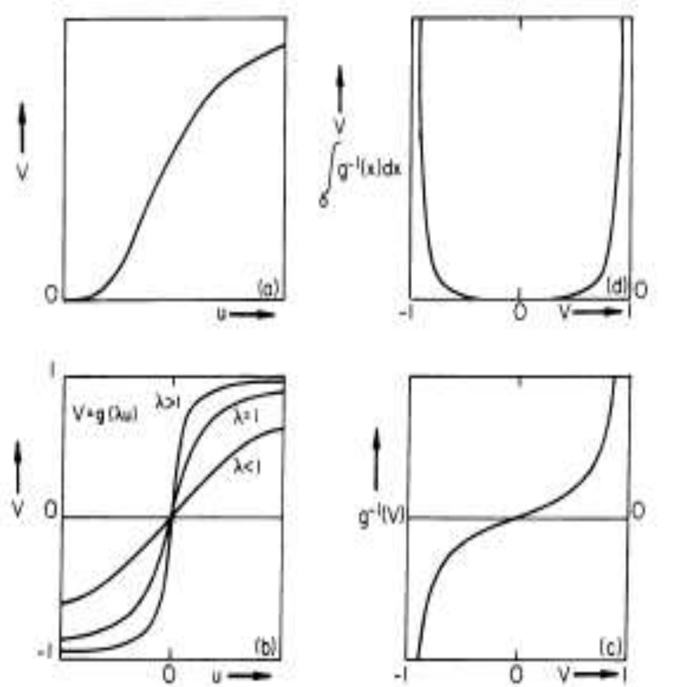


FIG. 1. (*a*) The sigmoid input–output relation for a typical neuron. All the $g(u)$ of this paper have such a form, with possible horizontal and vertical translations. (*b*) The input–output relation $g(\lambda u)$ for the "neurons" of the continuous model for three values of the gain scaling parameter λ. (*c*) The output–input relation $u = g^{-1}(V)$ for the g shown in *b*. (*d*) The contribution of g to the energy of Eq. **5** as a function of V.

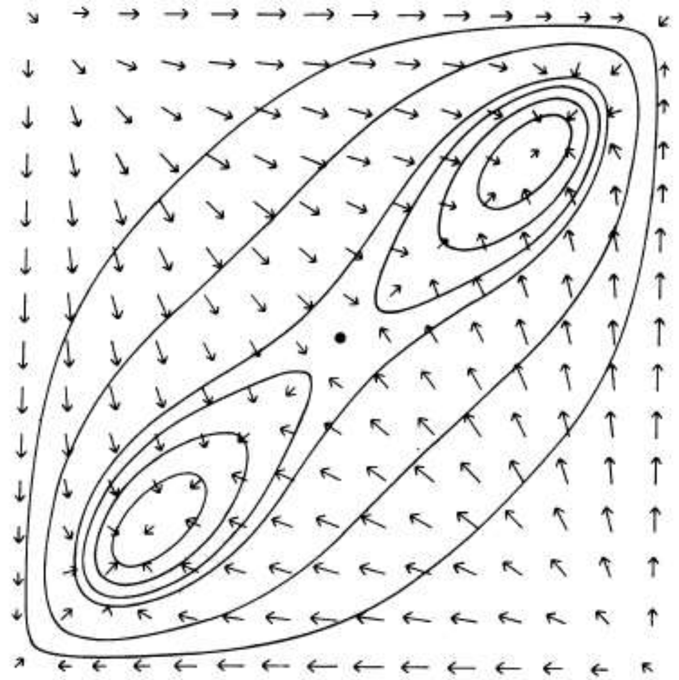
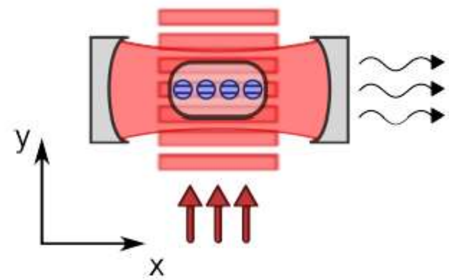


FIG. 3. An energy contour map for a two-neuron, two-stable-state system. The ordinate and abscissa are the outputs of the two neurons. Stable states are located near the lower left and upper right corners, and unstable extrema at the other two corners. The arrows show the motion of the state from Eq. **5**. This motion is not in general perpendicular to the energy contours. The system parameters are $T_{12} = T_{21} = 1$, $\lambda = 1.4$, and $g(u) = (2/\pi)\tan^{-1}(\pi\lambda u/2)$. Energy contours are 0.449, 0.156, 0.017, −0.003, −0.023, and −0.041.

- stationary states by „energy“ minimization
- dynamics does not follow energy surface

Multifrequency selfordering in a standing wave cavities
(S. Krämer)



- Modes $\{\omega_c^n; \hat{a}_n; \kappa_n\}$
- Many-Particle System $\{m; \hat{x}_i; \hat{\sigma}_+^i\}$
- Particles in trap $V(x)$
- Pump Laser $\{\omega_p^k; \eta_k\}$

Idea:
use frequency comb to pump at a large number of cavity frequencies

$$T_0 = \frac{\hbar}{2L\epsilon_0}\frac{\Delta_a}{\Delta_a^2+\gamma^2}|d_{eg}|^2$$

$$\eta_n = \frac{1}{2}\sqrt{\frac{\hbar\omega_c^n}{2L\epsilon_0}}E_0^n\frac{\Delta_a}{\Delta_a^2+\gamma^2}|d_{eg}|^2$$

$$H = -\sum_n \delta_n \hat{a}_n^\dagger \hat{a}_n + \sum_i \left(\frac{p_i^2}{2\mu} + V(\hat{x}_i)\right) + \sum_{ni} T_0\omega_n \sin^2(k_n(\hat{x}_i - L))\hat{a}_n^\dagger\hat{a}_n + \sum_{ni}\eta_n \sin(k_n(\hat{x}_i - L))(\hat{a}_n^\dagger + \hat{a}_n)$$

„Mean field“ model : multimode Tavis Cummings

$$H = -\sum_n \Delta_p^n \hat{a}_n^\dagger \hat{a}_n + \int dx \hat{\Psi}^\dagger(x)(\frac{-\Delta}{2m} + V(x))\hat{\Psi}(x)$$
$$+ \int dx \hat{\Psi}^\dagger(x) \sum_n T_0 \omega_n \sin^2(k_n(x+L)) \hat{a}_n^\dagger \hat{a}_n \hat{\Psi}(x)$$
$$+ \int dx \hat{\Psi}^\dagger(x) \sum_n \eta_n \sin(k_n(x+L))(\hat{a}_n^\dagger + \hat{a}_n)\hat{\Psi}(x)$$

Expand particle operators in trap eigenmodes

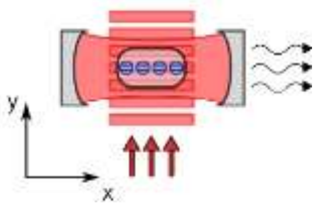
- $H_{particles} \Psi_k(x) = E_k \Psi_k(x)$
- Field operators: $\hat{\Psi}(x) = \sum_k \Psi_k(x) \hat{c}_k$

$$H = -\sum_n \Delta_p^n \hat{a}_n^\dagger \hat{a}_n + \sum_k E_k \hat{c}_k^\dagger \hat{c}_k + \sum_{nij} T_0 \omega_n A_{nij} \hat{c}_i^\dagger \hat{c}_j \hat{a}_n^\dagger \hat{a}_n + \sum_{nij} \eta_n B_{nij} \hat{c}_i^\dagger \hat{c}_j (\hat{a}_n^\dagger + \hat{a}_n)$$

$$A_{nij} = \int_{-\infty}^{\infty} \Psi_i^*(x)\Psi_j(x) \sin^2(k_n(x+L))dx$$
$$B_{nij} = \int_{-\infty}^{\infty} \Psi_i^*(x)\Psi_j(x) \sin(k_n(x+L))dx$$

Nonlinear coupled oscillator model with tailorable coupling:
pump amplitudes + detunings as control

2 Example: Box Potential



$$V(x) = \begin{cases} 0 & x \in [-a, a] \\ \infty & else \end{cases}$$

$$\Psi_i(x) = \begin{cases} \frac{1}{\sqrt{a}} \sin(K_i(x+a)) & x \in [-a, a] \\ 0 & else \end{cases}$$

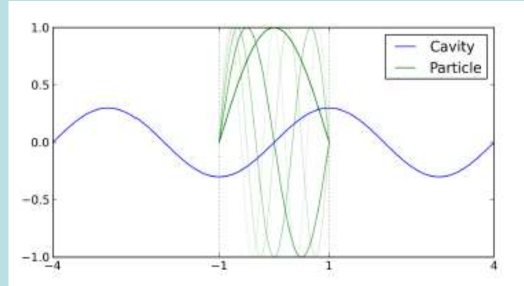
$$K_i = \frac{\pi}{2a} i \equiv K i$$

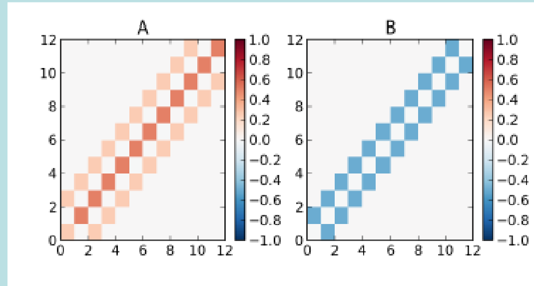
$$E_i = \frac{\hbar^2}{2m} \left(\frac{\pi i}{2a} \right)^2 = \frac{\hbar^2}{2m} K_i^2$$

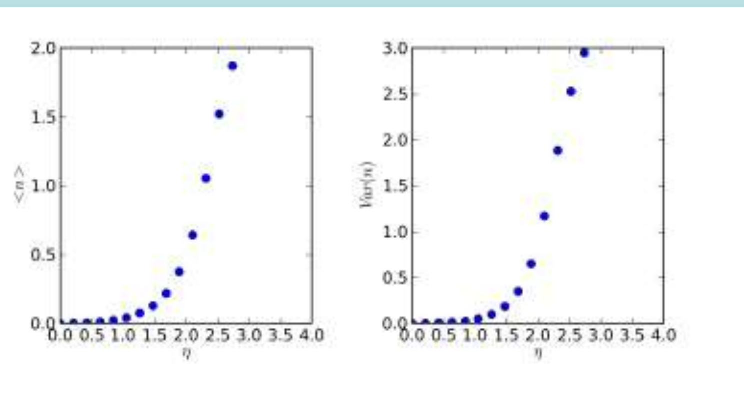
Overlap-Integrals 18, 19:

$$A_{nij} = \frac{1}{a} \int_{-a}^{a} \sin(Ki(x+a)) \sin(Kj(x+a)) \sin^2(kn(x+L)) dx$$

$$B_{nij} = \frac{1}{a} \int_{-a}^{a} \sin(Ki(x+a)) \sin(Kj(x+a)) \sin(kn(x+L)) dx$$







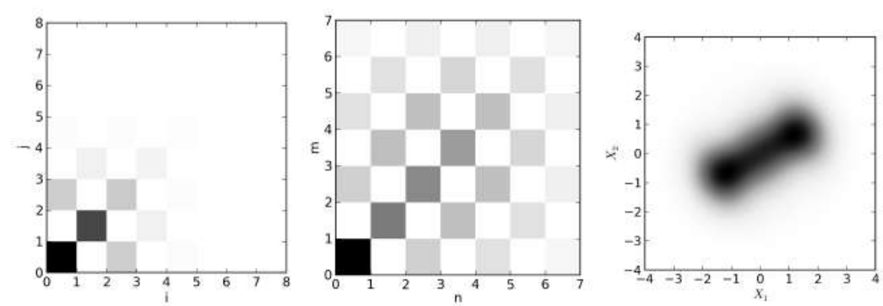


Figure 9: index=3

Degenerate mode selfordering of classical atoms

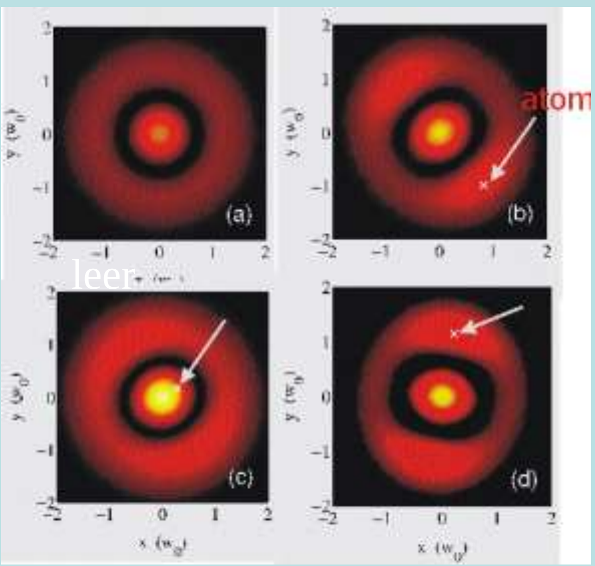
Single atom coupled to several degenerate modes

$$E(\vec{x},t) \cong \sum_{m=1}^{M} \alpha_m(t) u_m(\vec{x})$$

Atom couples modes and changes field distribution

Gauß-Laguerre modes in spherical mirror cavity

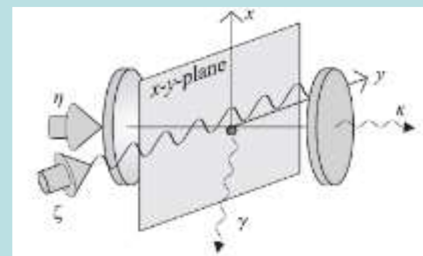
3 degenerate modes of:
(2,0), (0,1),(0,-1) family



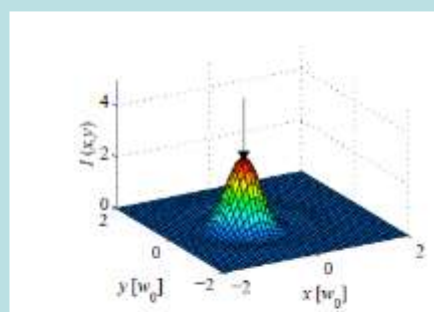
Horak, Rempe, H.R., PRL 2000

Gauß-Laguerre modes in confocal cavity

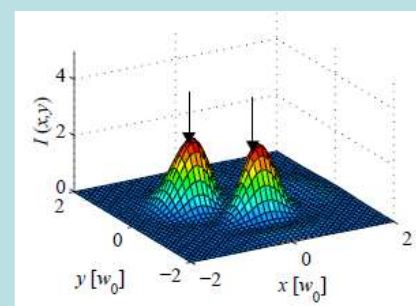
many degenerate modes



One atom



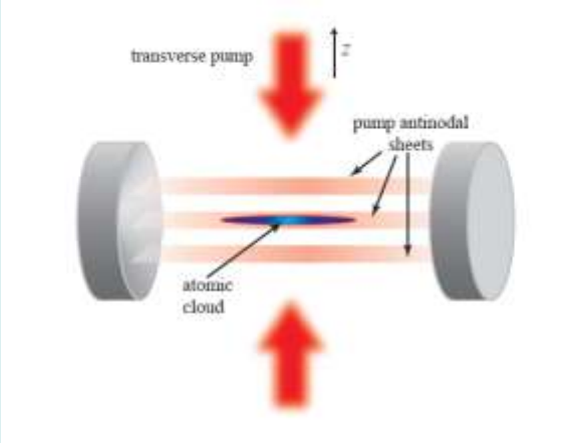
Two atoms



Large number of quasistationary states as local energy minima

Multiparticle selfordering in confocal resonator with many degenerate modes

S.Gopalakrishnan, B. L.Lev, P. M.Goldbart
Nat.Phys. 5, 845 (2009).



WICKENBROCK, HEMMERLING, ROBB, EMARY, AND RENZONI

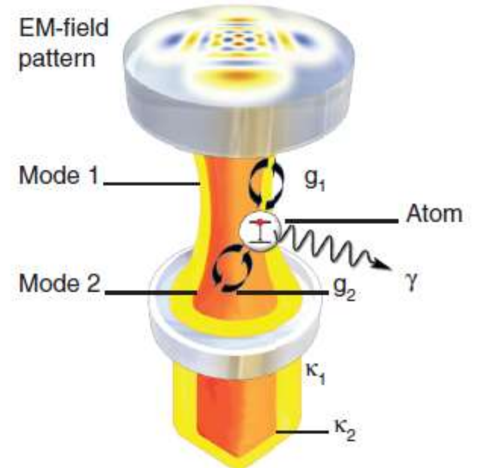
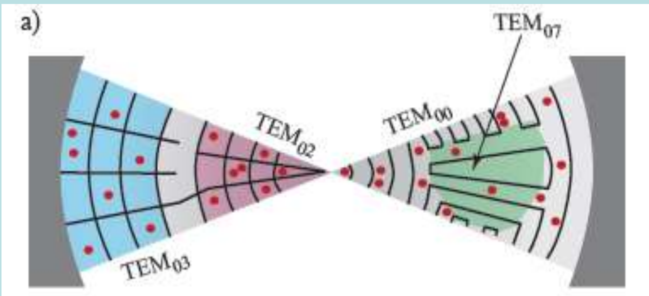


FIG. 1. (Color online) A two-level atom in an optical cavity interacting with different cavity modes depending on the spatially dependent couplings. Shown here are two higher-order transversal modes (modes 1 and 2) and one two-level atom exchange excitation with rates g_1 and g_2, respectively. The coherent process is damped by radiative coupling to the environment either via cavity decay of each mode through the mirrors (with rates κ_1 and κ_2) or atomic polarization decay (with rate γ).

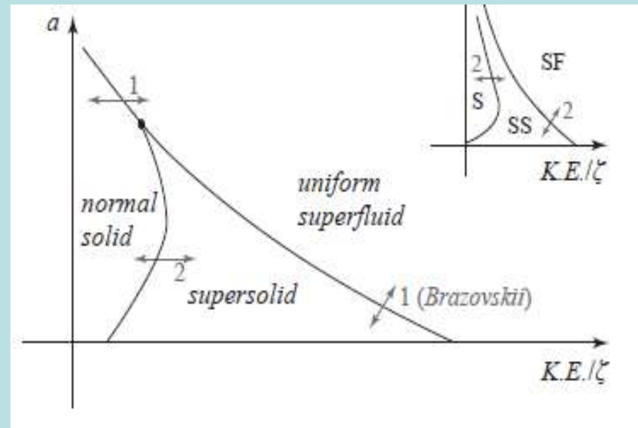
Generalization to multimode confocal cavity :

S.Gopalakrishnan, B. L.Lev, P. M.Goldbart
Nat.Phys. 5, 845 (2009).

„Quantum Brazovskii transition“



$$\frac{\Omega_{\rm th} - \Omega_{\rm th}^{\rm mf}}{\Omega_{\rm th}^{\rm mf}} \simeq 2.5 \left[\frac{\alpha U \sqrt{\hbar^2 K_0^2/2M}}{(\hbar \zeta N \chi)^{3/2}} \right]$$



P. Strack and S. Sachdev, P. Strack and W. Zwerger

- Dicke quantum spin glass of atoms and photons
- Exploring models of associative memory via cavity quantum electrodynamics

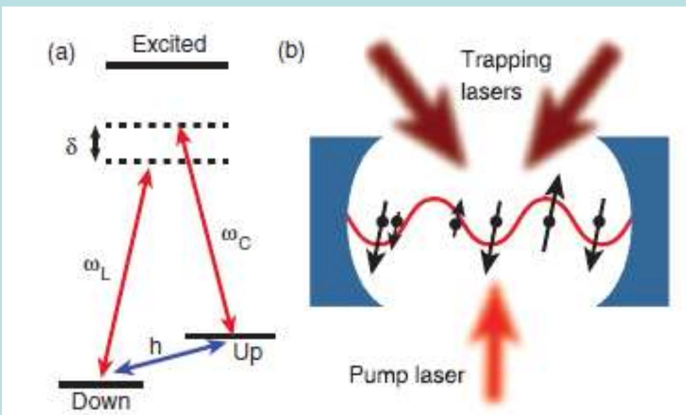
Philosophical Magazine
2011, 1–9, iFirst

Taylor & Francis
Taylor & Francis Group

Exploring models of associative memory via cavity quantum electrodynamics

Sarang Gopalakrishnan[a], Benjamin L. Lev[b] and Paul M. Goldbart[c*]

two hyperfine states per atom
with Raman coupling



cavity mediated coupling

$$H_{\mathrm{mm}} = -\zeta \sum_{\alpha, i \neq j, \mu} \Xi_\alpha(\mathbf{x}_i)\, \Xi_\alpha(\mathbf{x}_j)\, A^\mu S_i^\mu S_j^\mu + \cdots,$$

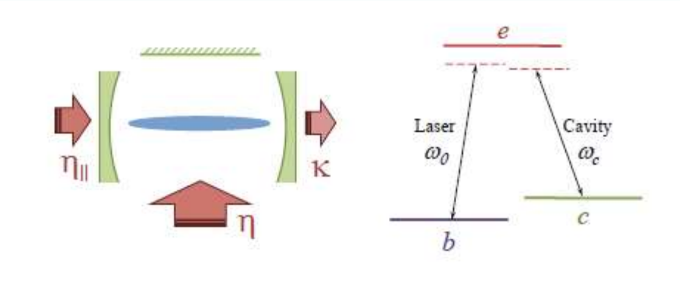
eliminate field

$$H_2 \sim \frac{g^4}{\Delta^2 \delta} \sum_{\alpha \neq \beta, i \neq j} \Xi_\alpha(\mathbf{x}_i)\Xi_\beta(\mathbf{x}_j)\Xi_\alpha(\mathbf{x}_j)\Xi_\beta(\mathbf{x}_i) S_i^c S_j^c + (\alpha \leftrightarrow \beta).$$

- direct implementation of Hopfield model with coupling determined by choice of modes !
- classical and quantum implementation depending on temperature
- NV-centers, Circuit QED versions

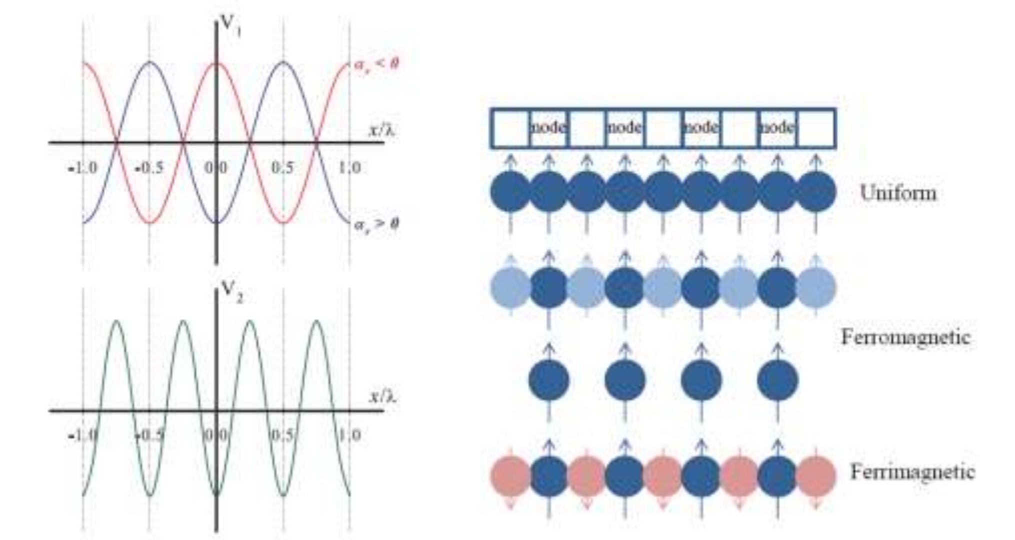
Raman superradiance and spin lattice of ultracold atoms in optical cavities

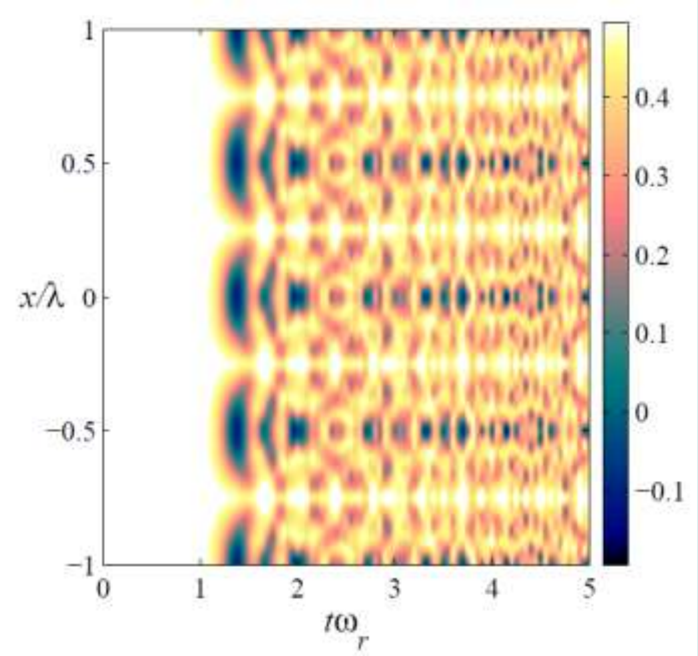
S Safaei[1], Ö E Müstecaplıoğlu[2] and B Tanatar[1]



$$\dot{\psi}_b = -\frac{i}{\hbar}\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V_b(x) + \frac{2\hbar h_0^2}{\Delta_0} + u_{bb}|\psi_b|^2 + u_{bc}|\psi_c|^2\right)\psi_b - \frac{i}{\hbar}V_1\psi_c$$

$$\dot{\psi}_c = -\frac{i}{\hbar}\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V_c(x) + \hbar\omega_{bc} + V_2 + u_{cc}|\psi_c|^2 + u_{bc}|\psi_b|^2\right)\psi_c - \frac{i}{\hbar}V_1\psi_b$$



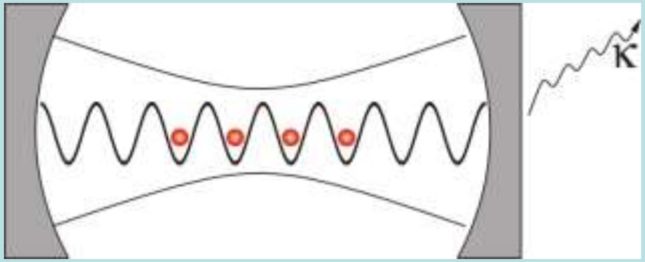


Formation of anti-ferromagnet


The end ?

measurement induced dynamics : transmission spectrum of single mode with quantum index

Only one mode: a_0 Standing wave cavity around partially filled lattice



$$a_0^\dagger a_0 = \frac{|\eta_0|^2}{(\Delta_p - \delta_0 \hat{D}_{00})^2 + \kappa^2}$$

detuning D_{00} is operator in particle space

$\Delta_p = \omega_{lp} - \omega_0$ ... probe to empty cavity detuning

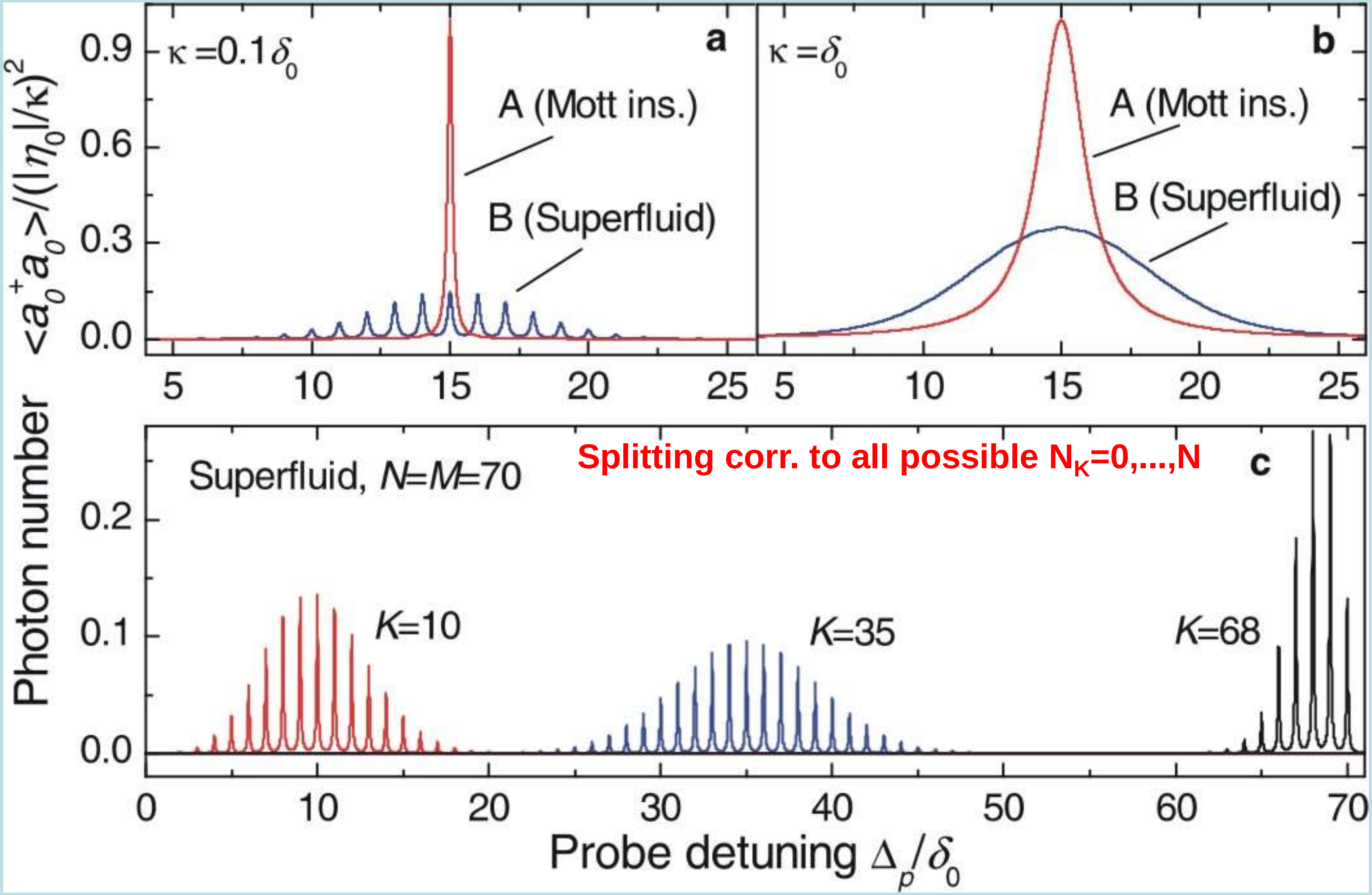
- Mott insulator: $\langle a_0^\dagger a_0 \rangle_{MI}$ single lorentzian with width κ and frequency shift $\delta_0 \langle \hat{D}_{00} \rangle_{MI} = \delta_0 N_K$

  ➡ classical result

- Superfluid: $\langle a_0^\dagger a_0 \rangle_{SF}$ different dispersion shifts corresponding to all possible atom number distributions

  ➡ comb-like structure

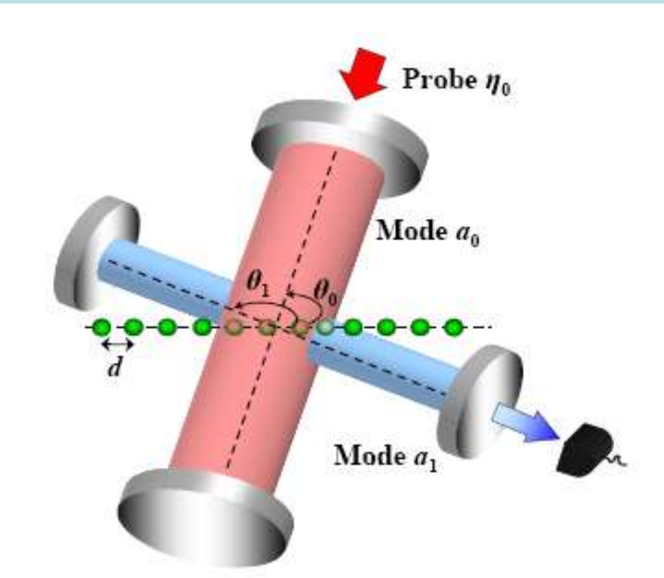
Numerical example: light transmission of a cavity lattice (K-sites)

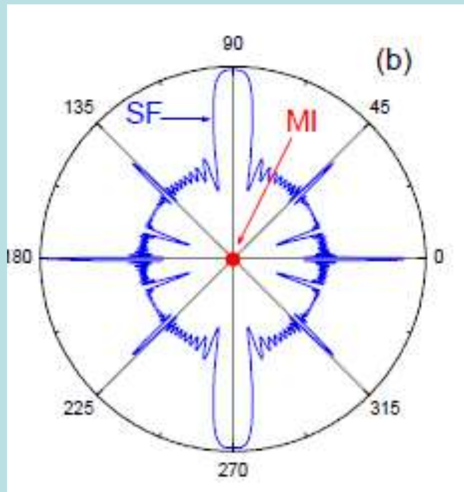


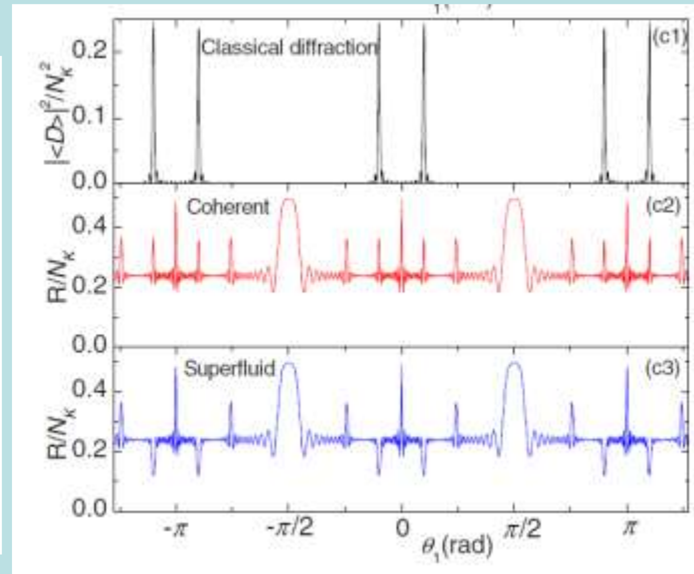
I.Mekhov, Nat. Physics 2007, PRA 2008
related work by P. Meystre + al.

Generalized setup for nondestructive measurements of atom distributions in different quantum phases of equal density

scattering spectra + distribution reflect quantum properties of atomic distribution







** Light scattering into Bragg minima exhibits nonclassical features (=> Morigi et.al 2011)*

** Experiments in 2D lattices show Bragg peaks: (Kuhr, Bloch and Ketterle group 2011)*

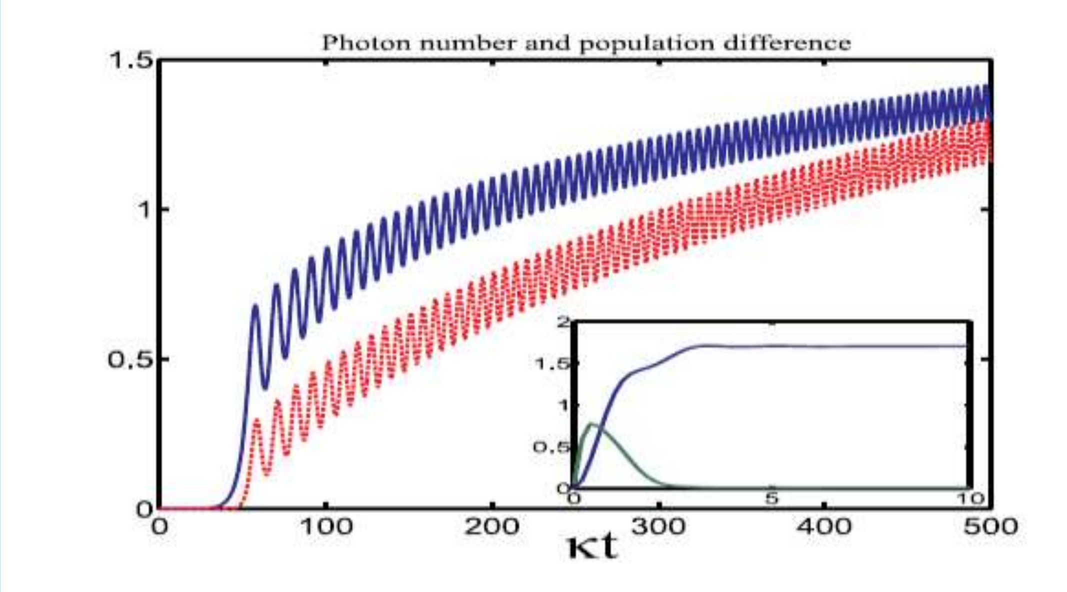
** Collective excitations incl. dipol-dipole interaction => super-/subradiance (Zoubi /Ritsch)*

‚Semiclassical‘ approximation of lattice field (n>>1) :
Quantum atoms on classical seesaw

$$\dot{\alpha}(t) = \left[i\left(\Delta_c - U_0 N\right) - \kappa\right]\alpha(t) - i\tilde{J}\left\langle b_l^\dagger b_l - b_r^\dagger b_r\right\rangle$$

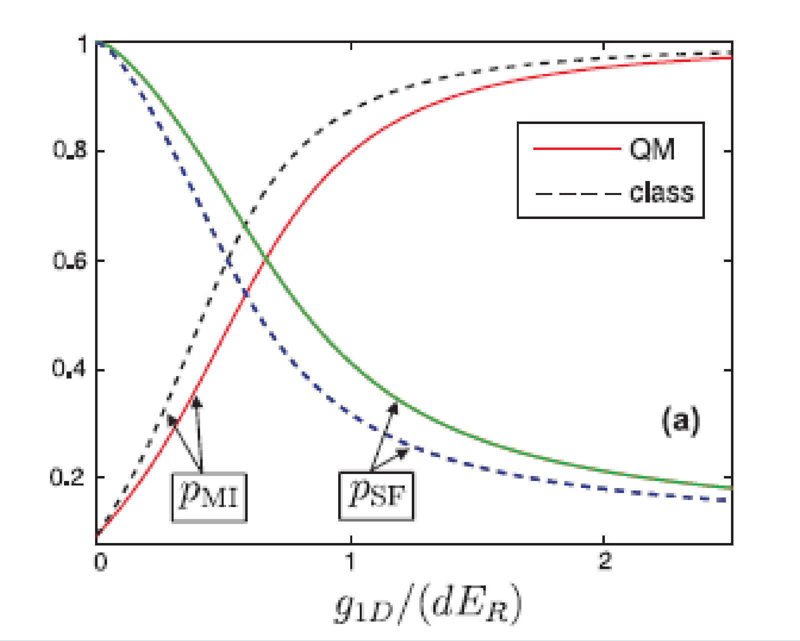
$$H = J\left(b_l^\dagger b_r + b_r^\dagger b_l\right) + \hbar\tilde{J}\left(b_l^\dagger b_l - b_r^\dagger b_r\right) 2\mathrm{Re}\left\{\alpha(t)\right\}$$

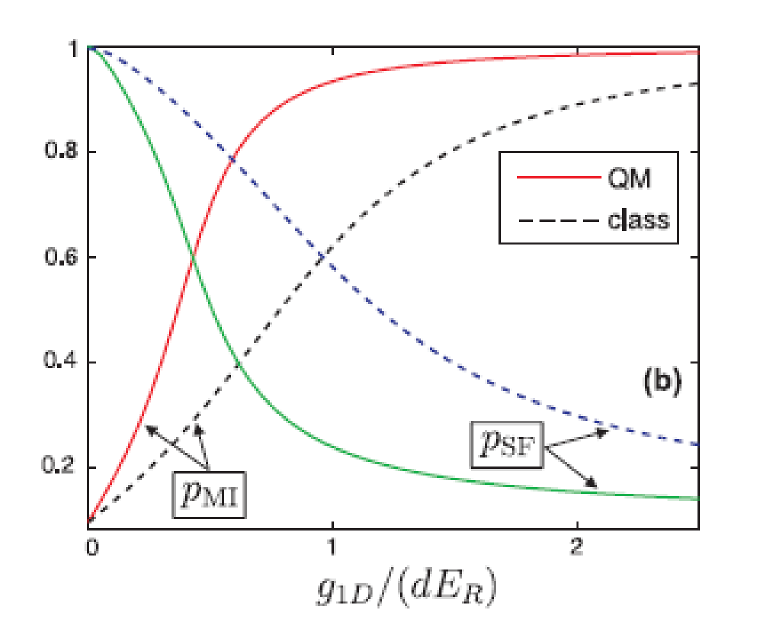
For symmetric initial condition (e.g. Superfluid, Mott-insulator)
no fields is created =>
symmetric initial state is stationary !



Population is stable for long time and only eventually organizes to ordered state

Contribution of "Mott-insulator" and "superfluid" for 4 atoms in 4 wells





$$\Delta_c - U_0 J_0 N = \kappa.$$

blue detuning

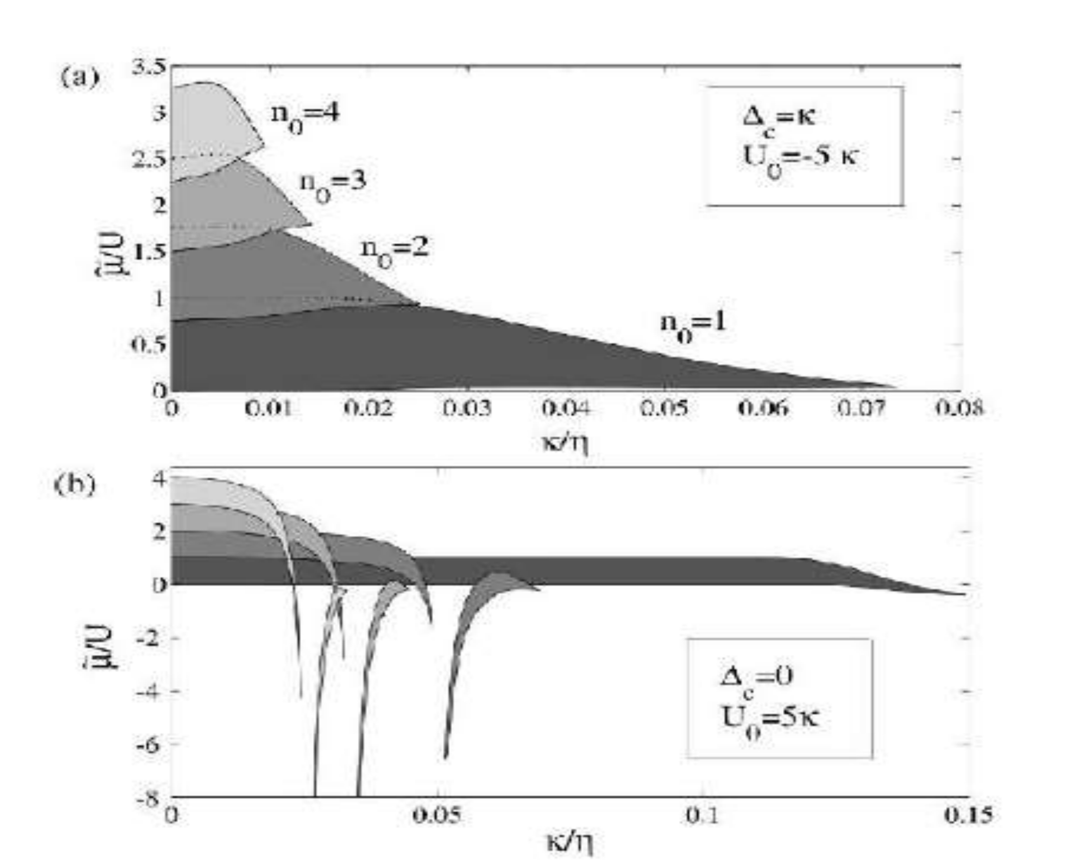
$$\Delta_c - U_0 J_0 N = -\kappa.$$

red detuning

Do superpositions of Mott and Superfluid phase survive for large N ?

Thermodynamic limit and phases of cavity generated lattices

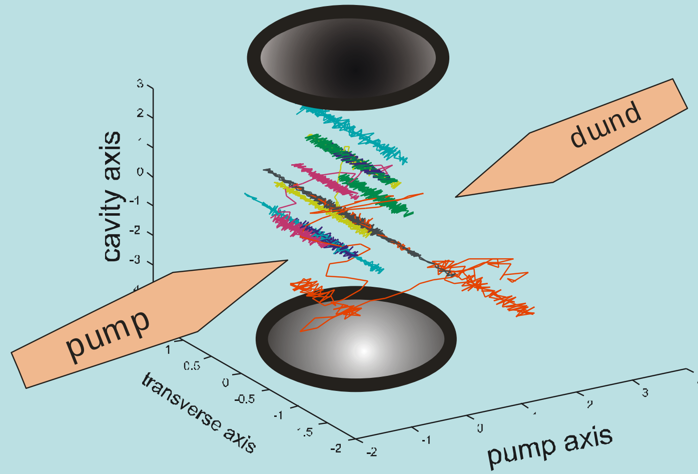
Cavity creates extra effective attraction or repulsion : bistable phases
=> phase superpositions of Mott + Superfluid in principle possible !?



M. Lewenstein, G. Morigi et. al. (PRL 2007, 2008)
Phase diagram in thermodynamic limit

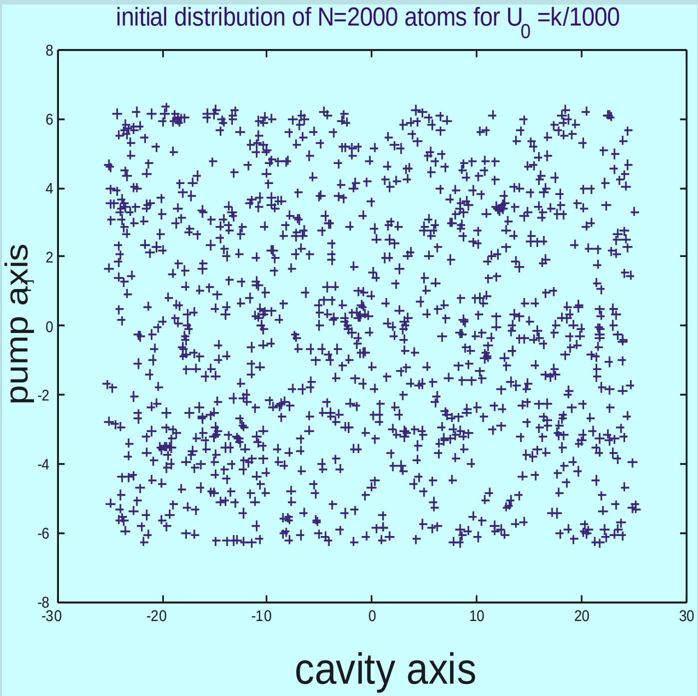
Generalization to fermions, Morigi PRA 2008

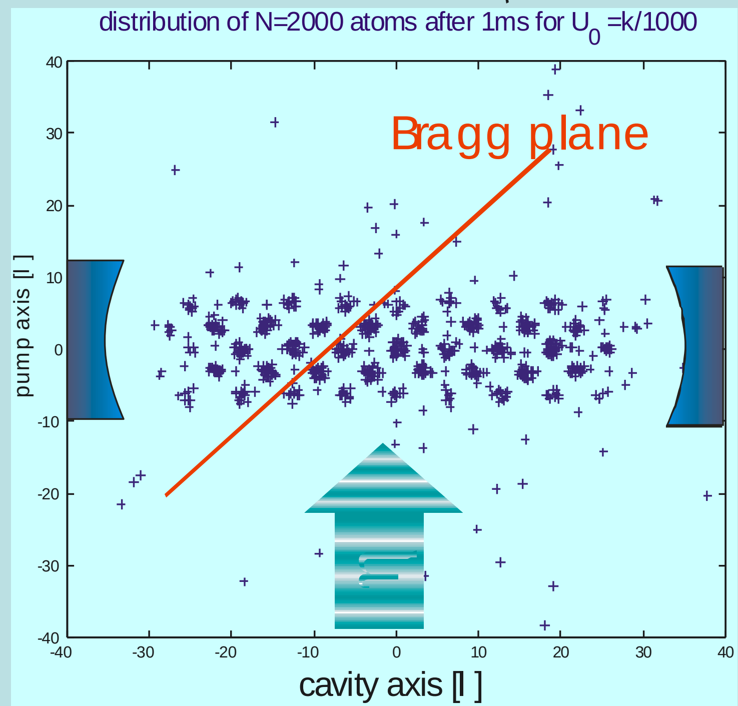
Selforganisation in 3D



Simulation:

coherent light emission
in connection with cooling

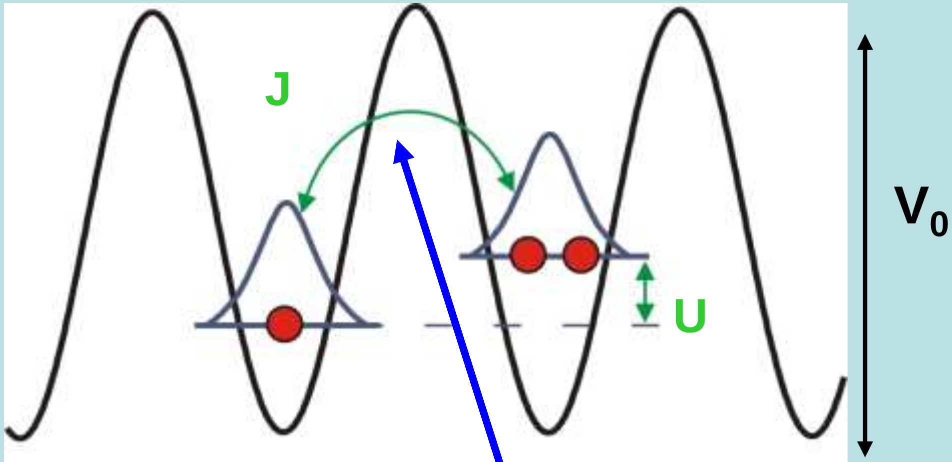




Atoms order in regular tube-lattice structure with Bragg planes optimizing scattering to the cavity

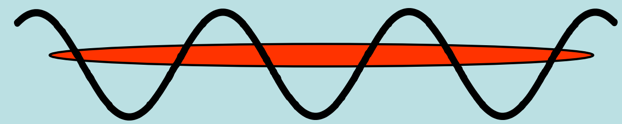
** analogy to self gravitating systems*

Ultracold Atoms in optical lattices



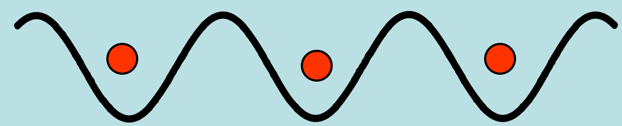
Tunneling J depends on laser power (V_0)

- **Superfluid Phase** J>>U



weakly interacting system; delocalized atoms

- **Mott-Insulator Phase:** J<<U



Theory: Fisher *et al*. (1989), Jaksch *et al.* (1998)
Experiment: Greiner *et al.* (2002)

Effective Hamiltonian

$$H = -J \sum_{\langle n,m \rangle} b_n^\dagger b_m + \frac{U}{2} \sum_n b_n^\dagger b_n \left(b_n^\dagger b_n - 1 \right) + \sum_i (\varepsilon_n - \mu) b_n^\dagger b_n$$

Bose Hubbard model
for a single standing wave mode resonator

effective single atom Hamiltonian

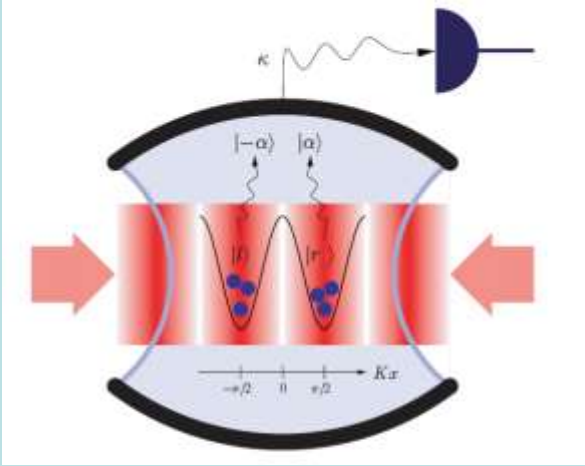
quantized light potential

extra classical potential

$$H_{\text{eff}} = \frac{p^2}{2m} + \cos^2(kx)\left(\hbar U_0 a^\dagger a + V_{cl}\right) - \hbar\Delta_c a^\dagger a$$
$$- \; i\hbar\eta\left(a - a^\dagger\right) + \hbar\eta_{\text{eff}} h(y)\cos(kx)\left(a + a^\dagger\right)$$

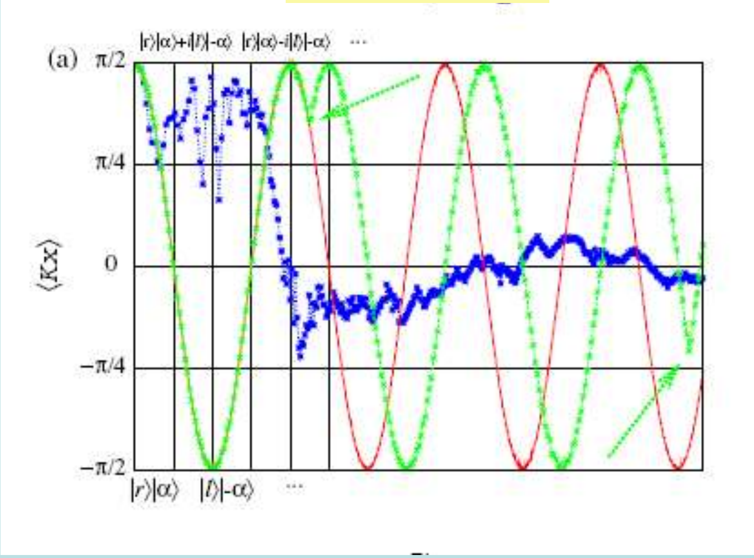
Quantum Model for field and atoms

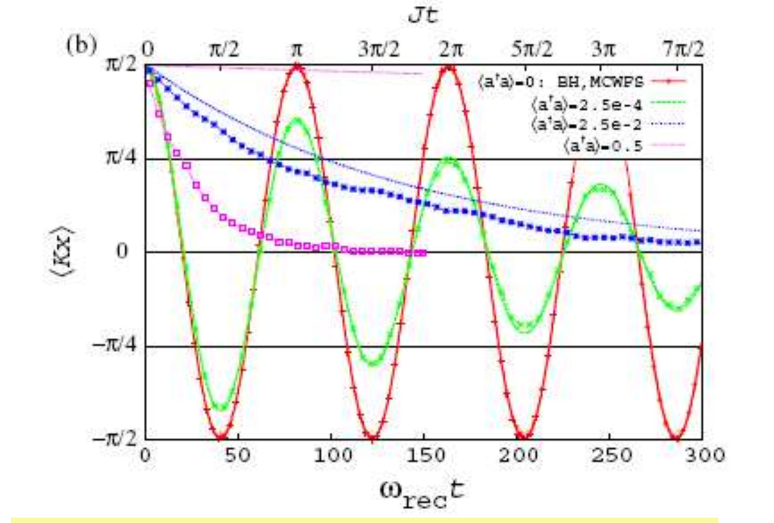
Microscopic dynamics of selforganization



How do the atoms evolve into an ordered state at T=0 ?

single atom





fast decay towards entangled state

Single atom - single photon kaleidoscope

Single atom coupled to several degenerate modes

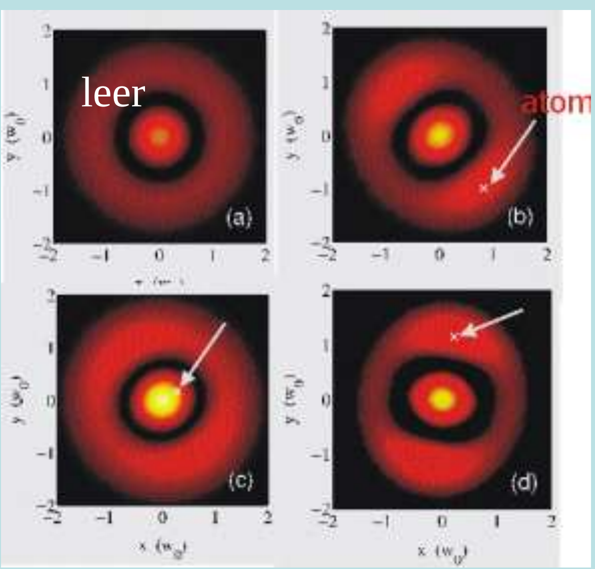
$$E(\vec{x},t) \cong \sum_{m=1}^{M} \alpha_m(t) u_m(\vec{x})$$

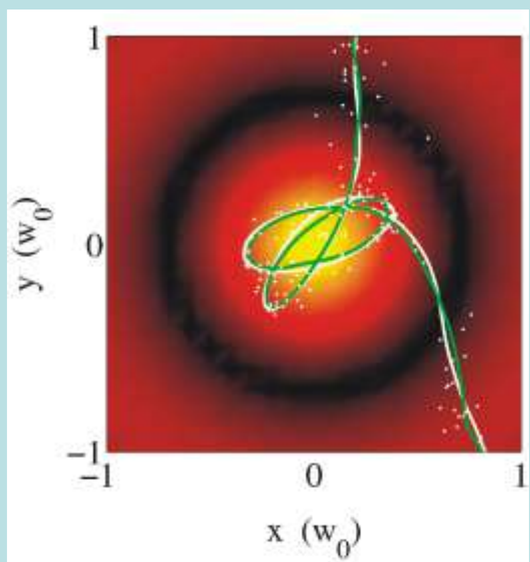
z.b.: Gauß-Laguerre Moden for fixed n=2 p+|m|.

$$u_{pm}(\rho,\theta,z) = C_{pm}^{\mathrm{LG}} \cos(kz) e^{-\frac{\rho^2}{\mathrm{w_0}^2}+im\theta}(-1)^p L_p^{|m|}\left(\frac{\sqrt{2}\rho^2}{\mathrm{w_0}^2}\right),$$

stationary state for :
(2,0), (0,1),(0,-1) family

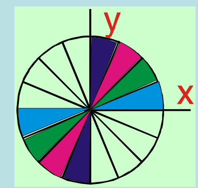
simul. atomic trajectory (green)



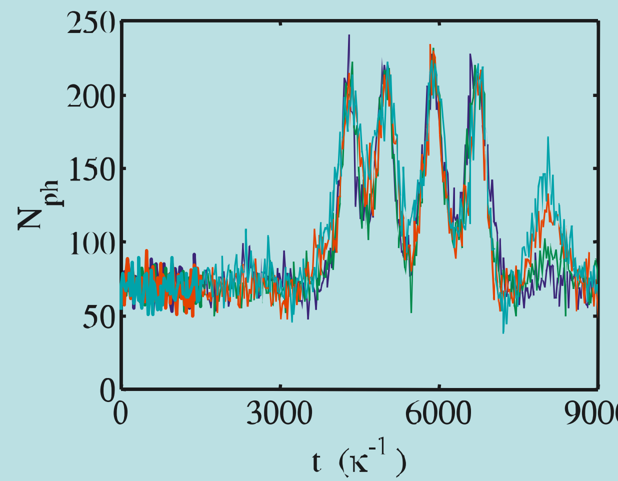


Atom couples modes and changes field distribution

Spatially resolved photon detection of emitted field via 4-segment detector (Simulation)



(Rempe MPQ-Munich)



reconstruction of path from
4 noisy currents => white line
with subwavelength accuracy