Quantum-Enhanced Sensing

Morgan W. Mitchell[1,2]

[1] ICFO – Institute of Photonic Sciences
[2] ICREA – Institucio Catalana de Recerca I Estudis Avancats

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Languages

Quantum-Enhanced Sensing

From Wikipedia, the free encyclopedia

In physics, quantum-enhanced sensing is a component of quantum metrology, the study and use of quantum physics in metrology. Quantum-enhanced sensing employs squeezing, entanglement, and other quantum effects to improve the sensitivity of measurements.

Quantum metrology

$$I = \langle [\partial_B \ln P_i(B)]^2 \rangle$$

Fisher Information

History · Timeline

Branches	[show]
Fundamentals	[show]
Formulations	[show]
Core topics	[show]
Rotation	[show]
Scientists	[show]

v · T · E

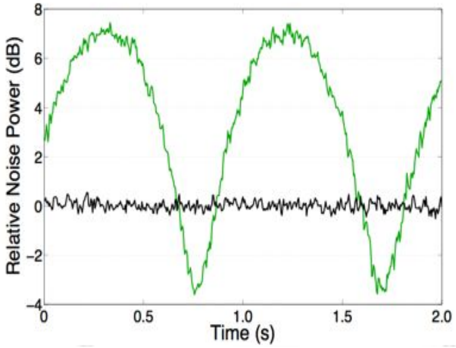


Diagram of a squeezing trace (green), showing noise below the standard quantum limit (black)

Contents [hide]

History [edit]

Some Greek philosophers of antiquity, among them Aristotle,

QUANTUM METROLOGY

What is quantum metrology ?

Quantum Measurement Division

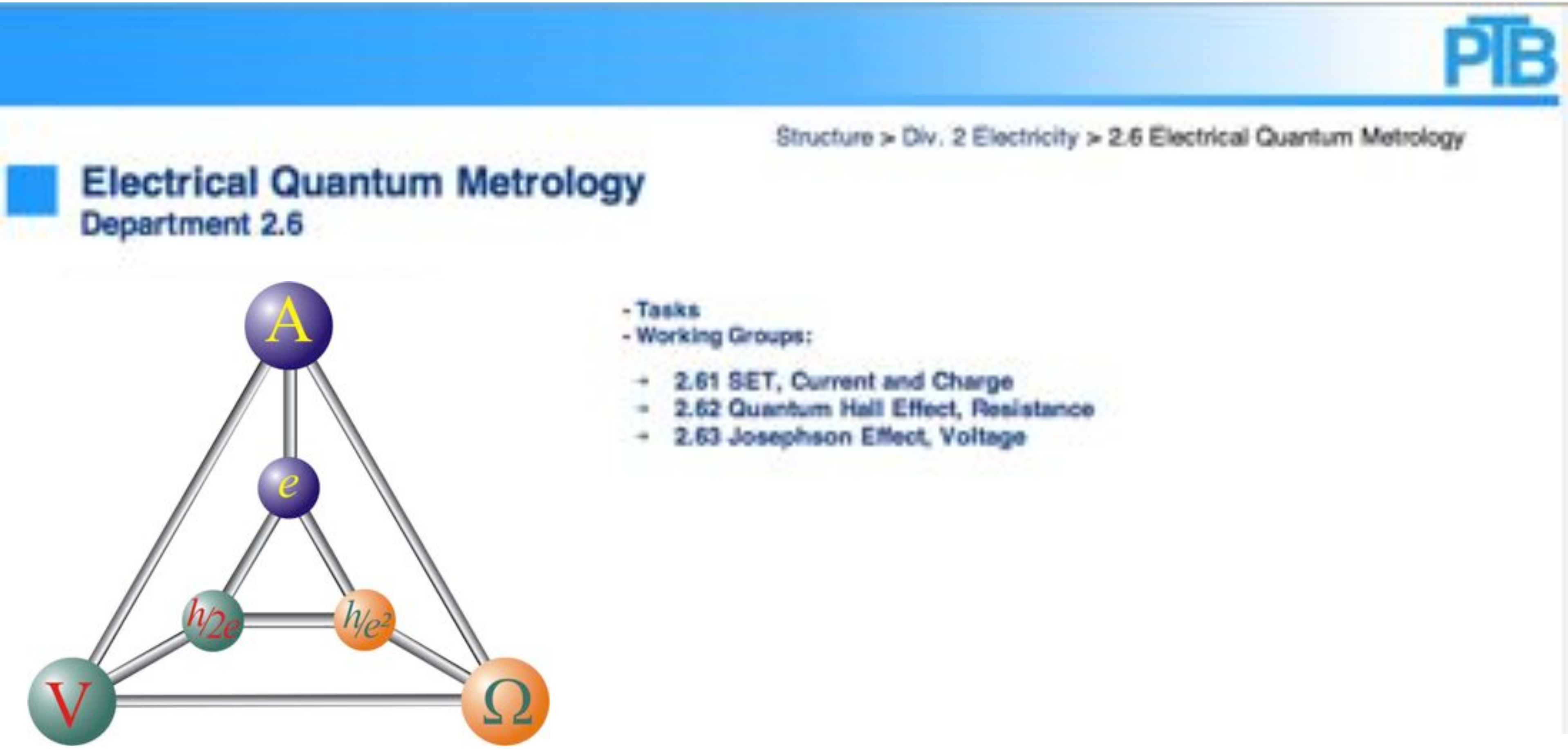
Topic Areas

- Laser Cooling and Cold Atomic Matter
- Nanoscale and Quantum Metrology
- Quantum Nature of Light and Matter
- Critically Evaluated Atomic Data
- Electrical and Mass Metrology

Welcome

The Quantum Measurement Division (QMD) provides measurement and data support for a broad range of national needs, conducts metrology research enabling more accurate determination of SI units and fundamental constants, performs basic electrical, mass, force and torque calibrations, distributes its findings widely and effectively, and maintains an active schedule of services, partnerships and collaborations with industry, academe and government.

What is quantum metrology ?



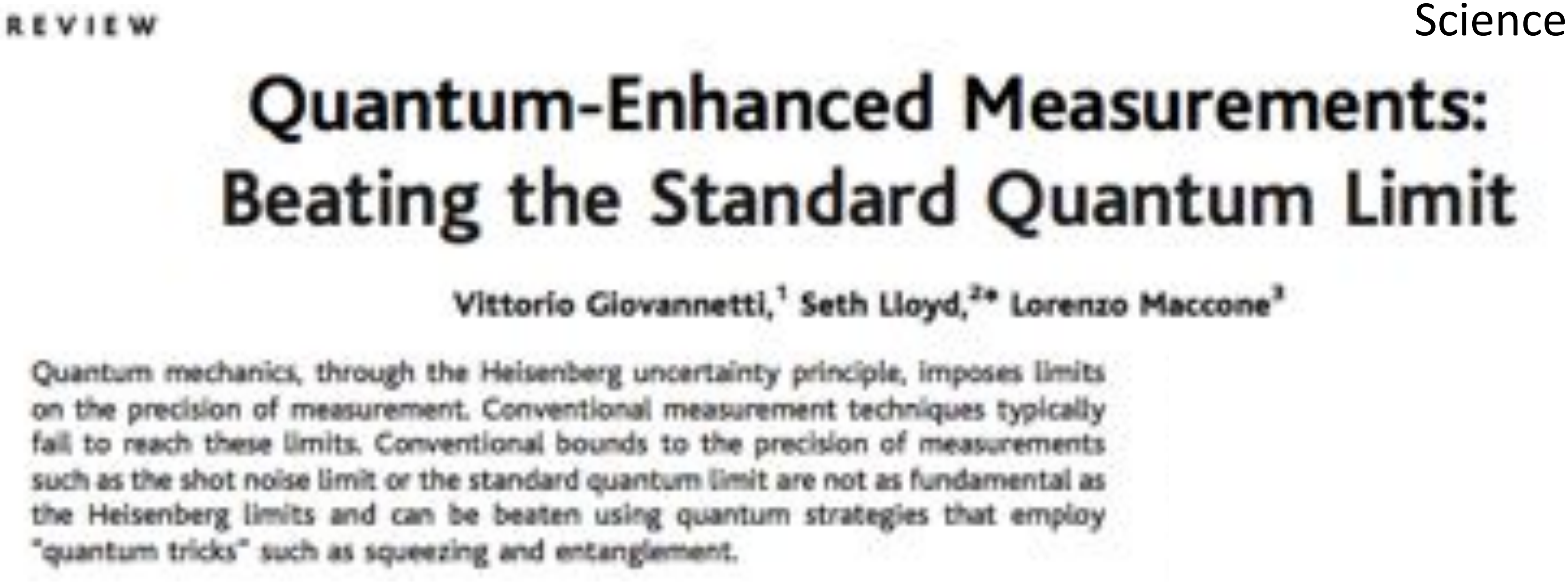
Structure > Div. 2 Electricity > 2.6 Electrical Quantum Metrology

Electrical Quantum Metrology
Department 2.6

- Tasks
- Working Groups:
  - 2.61 SET, Current and Charge
  - 2.62 Quantum Hall Effect, Resistance
  - 2.63 Josephson Effect, Voltage

Quantum-enhanced sensing

Science, 2004



REVIEW

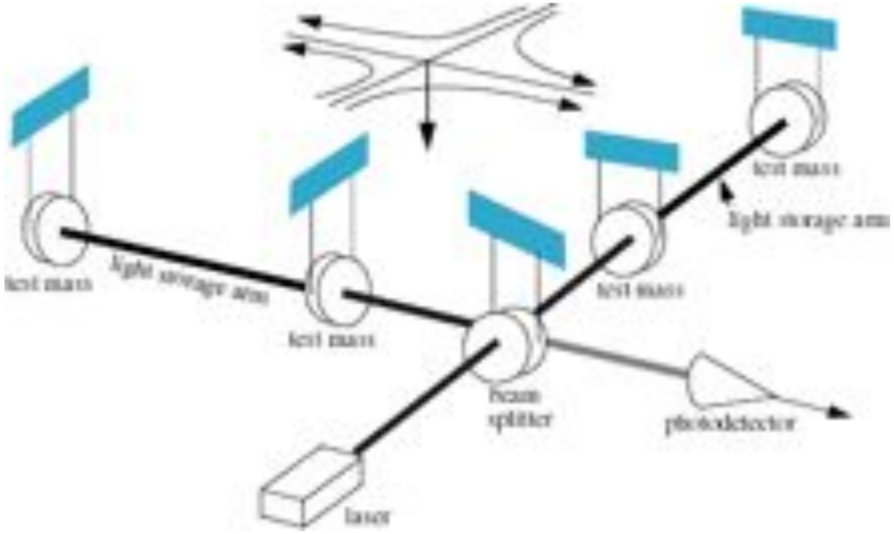
Quantum-Enhanced Measurements: Beating the Standard Quantum Limit

Vittorio Giovannetti,[1] Seth Lloyd,[2*] Lorenzo Maccone[3]

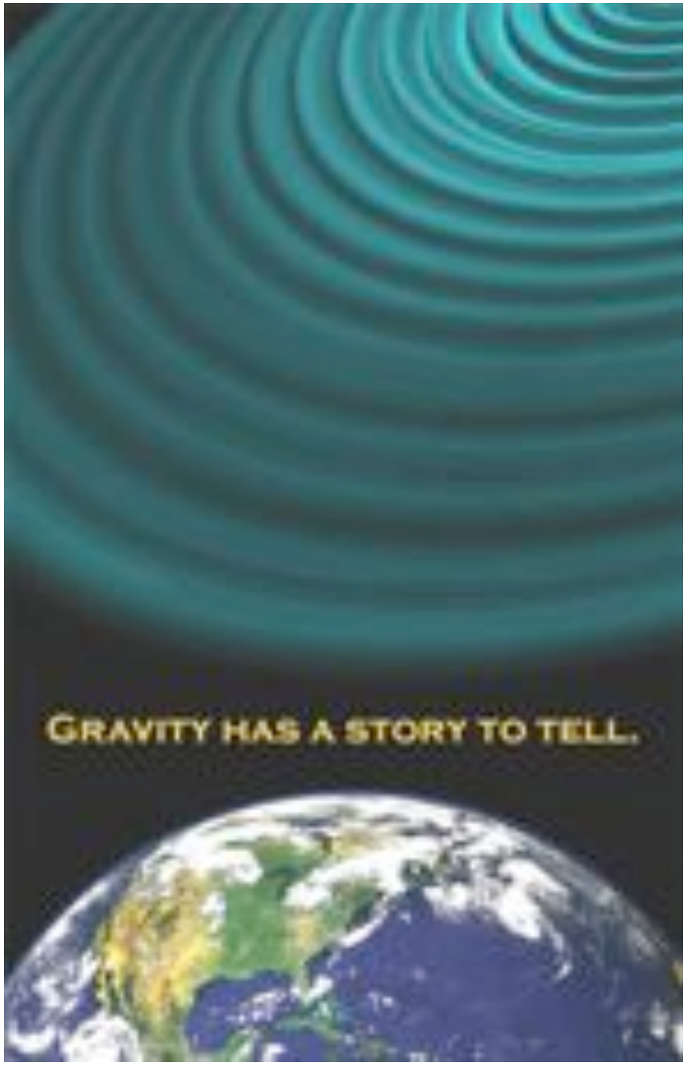
Quantum mechanics, through the Heisenberg uncertainty principle, imposes limits on the precision of measurement. Conventional measurement techniques typically fail to reach these limits. Conventional bounds to the precision of measurements such as the shot noise limit or the standard quantum limit are not as fundamental as the Heisenberg limits and can be beaten using quantum strategies that employ "quantum tricks" such as squeezing and entanglement.

HISTORY

Sensing gravitational waves

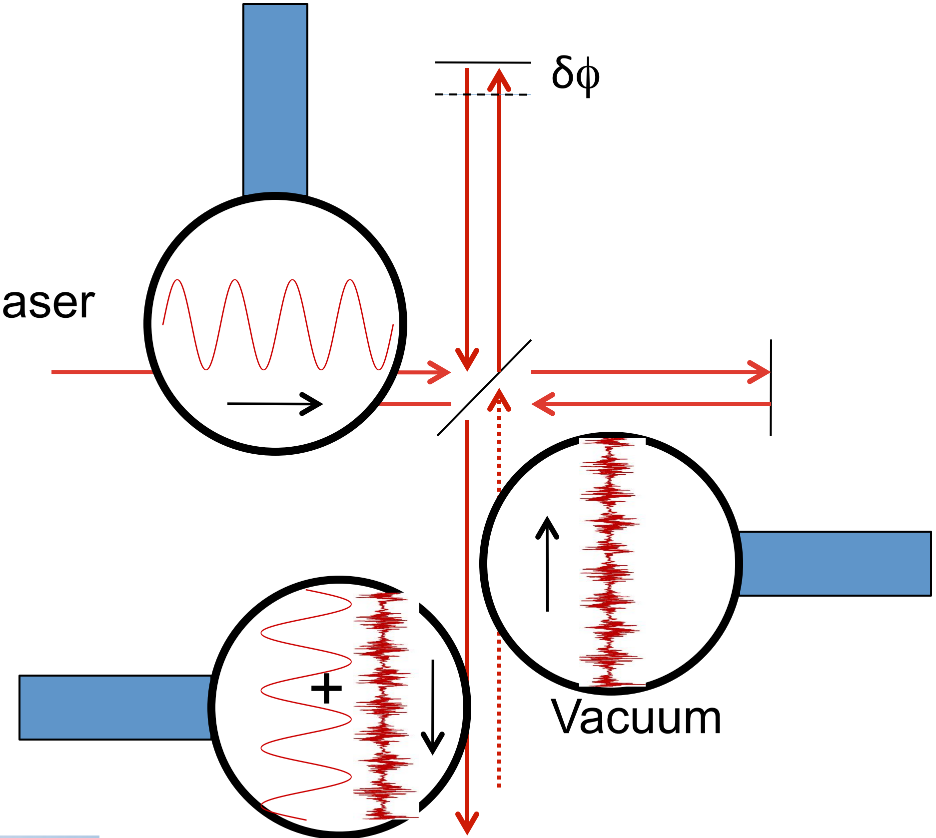


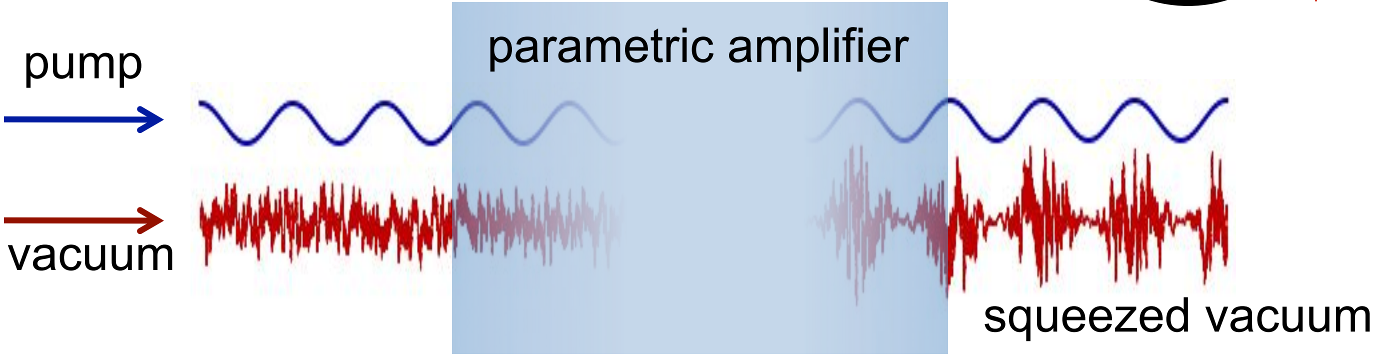
Very large Michelson interferometer,
e.g. LIGO, VIRGO, GEO, TAMA, LISA
sensitivity $\delta L = 10^{-18}$ m or $\delta L/L = 10^{-23}$



Beating shot noise in interferometry

Carlton M. Caves





Caves,
PRD 1981

What is quantum metrology ?

Quantum Optics

We're from ~~the government~~ and we're here to help.

Quantum Science Implementations, Benasque, 17 July 2014

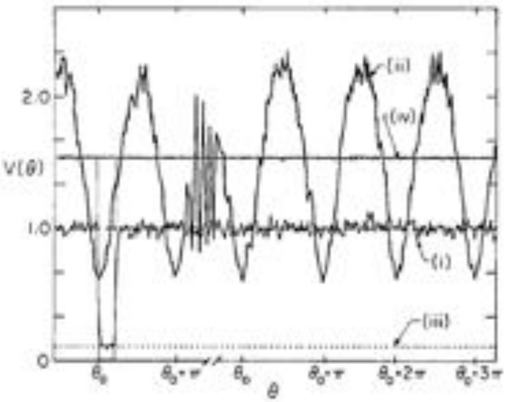
Morgan W. Mitchell, ICFO

Path of technology development in Q. Metrology

1981

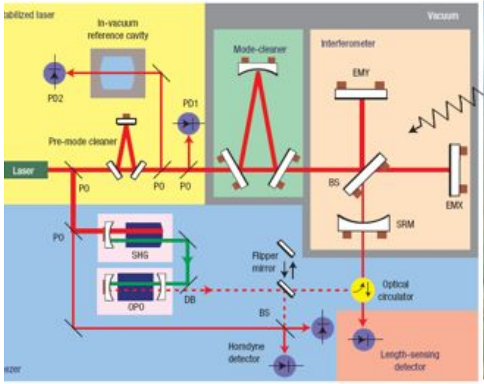
Caves proposes squeezing for GW interferometry

1985



Slusher, Kimble first squeezed light

2000s

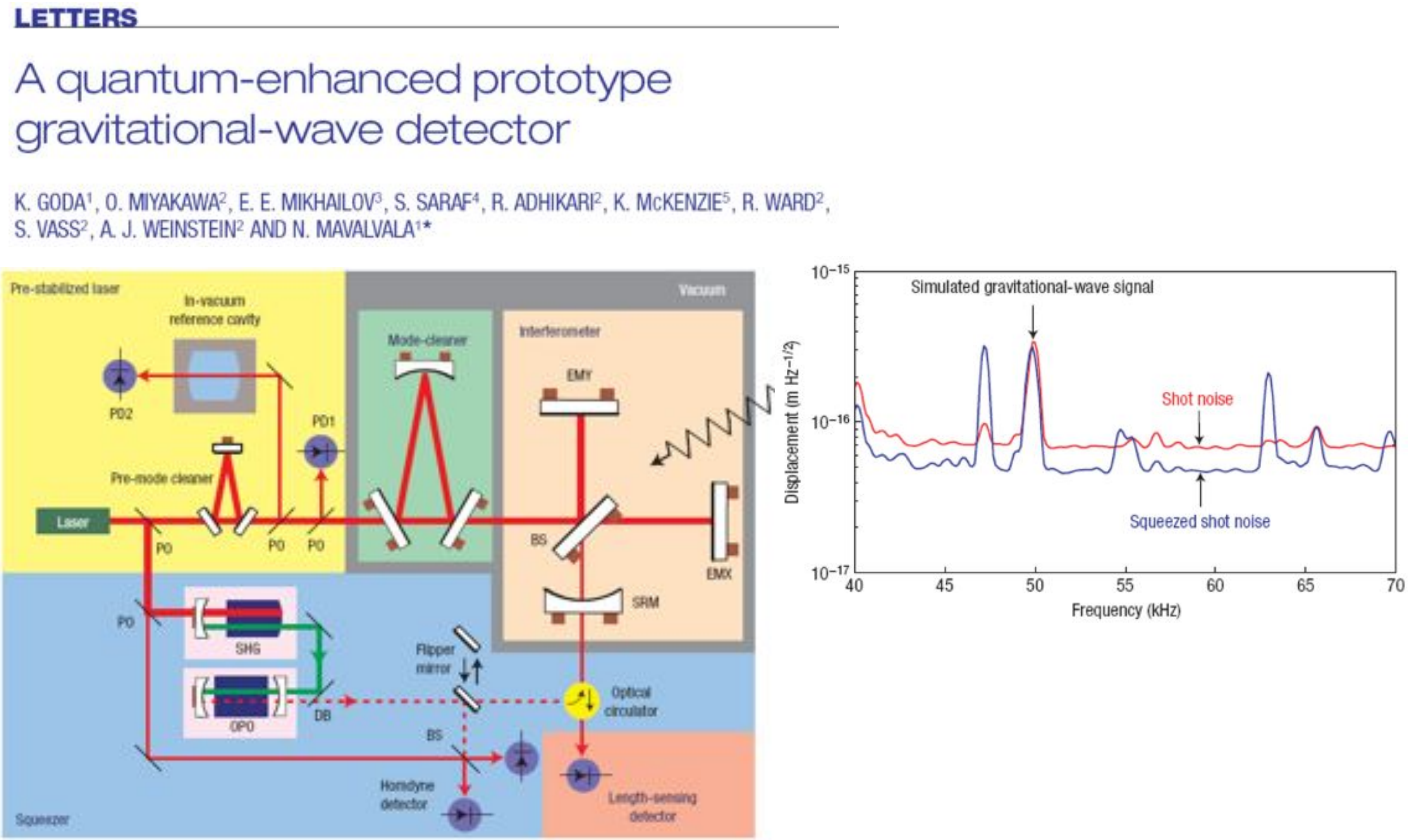


Prototype squeezed-light GW detectors

2011-2015

GEO600
Advanced LIGO
Advanced VIRGO

Squeezed-light GW detector



LETTERS

A quantum-enhanced prototype gravitational-wave detector

K. GODA[1], O. MIYAKAWA[2], E. E. MIKHAILOV[3], S. SARAF[4], R. ADHIKARI[2], K. McKENZIE[5], R. WARD[2], S. VASS[2], A. J. WEINSTEIN[2] AND N. MAVALVALA[1]*

Nature Physics **4**, 472 (2008)

related work from ANU, Hanover

GEO 600 sensitivity boost

A gravitational wave observatory operating beyond the quantum shot-noise limit

N.Phys 2011

The LIGO Scientific Collaboration †*

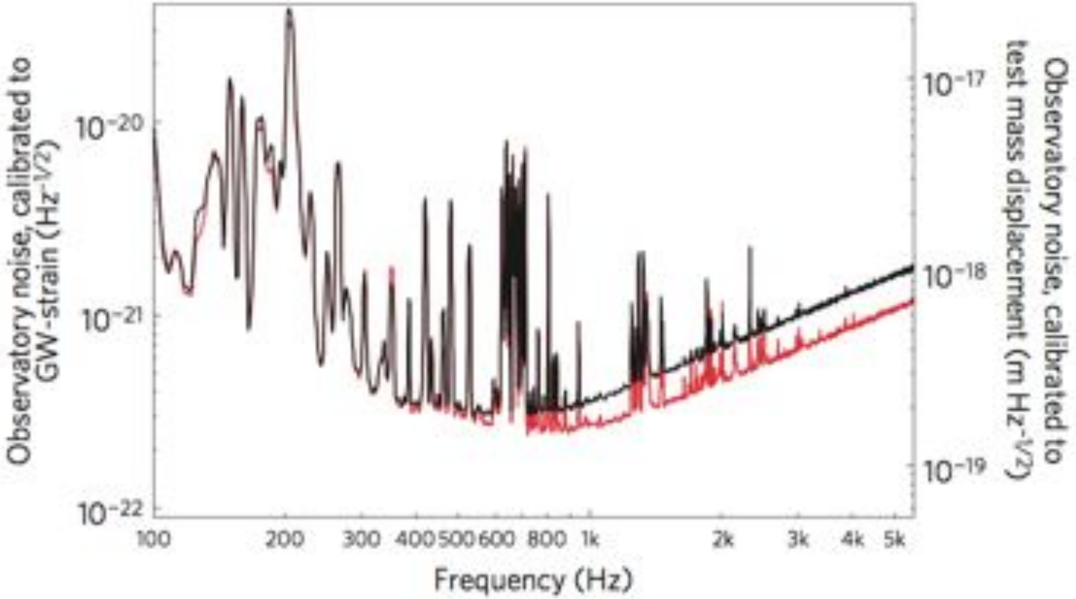


Figure 3 | Nonclassical reduction of the GEO 600 instrumental noise using squeezed vacuum states of light.

GEO 600 sensitivity boost

Enhanced sensitivity of the LIGO gravitational wave detector by using squeezed states of light

The LIGO Scientific Collaboration*

N.Phot 2013

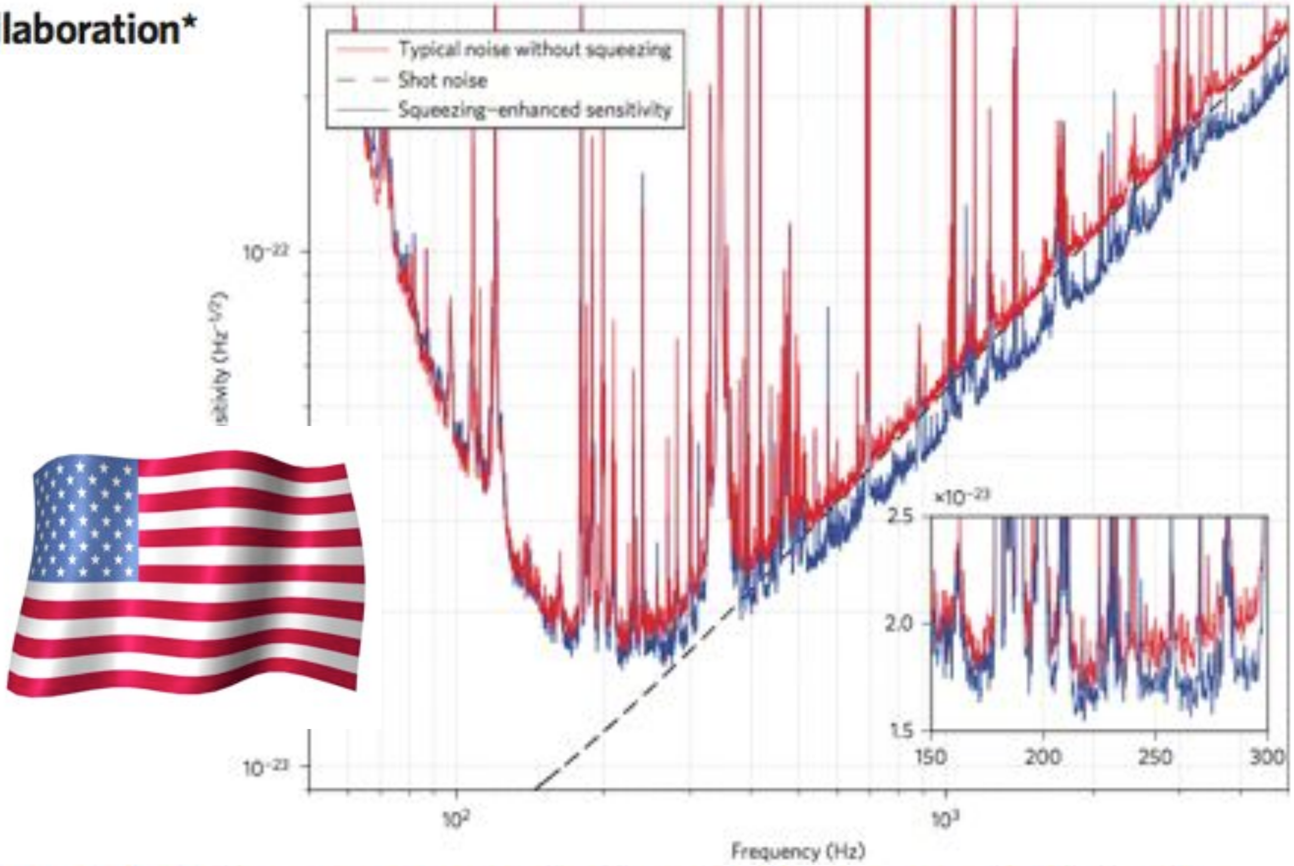


Figure 2 | Strain sensitivity of the H1 detector measured with and without squeezing injection.

GEO 600 sensitivity boost

A gravitational wave observatory operating beyond the quantum shot-noise limit

N.Phys 2011

The LIGO Scientific Collaboration †*

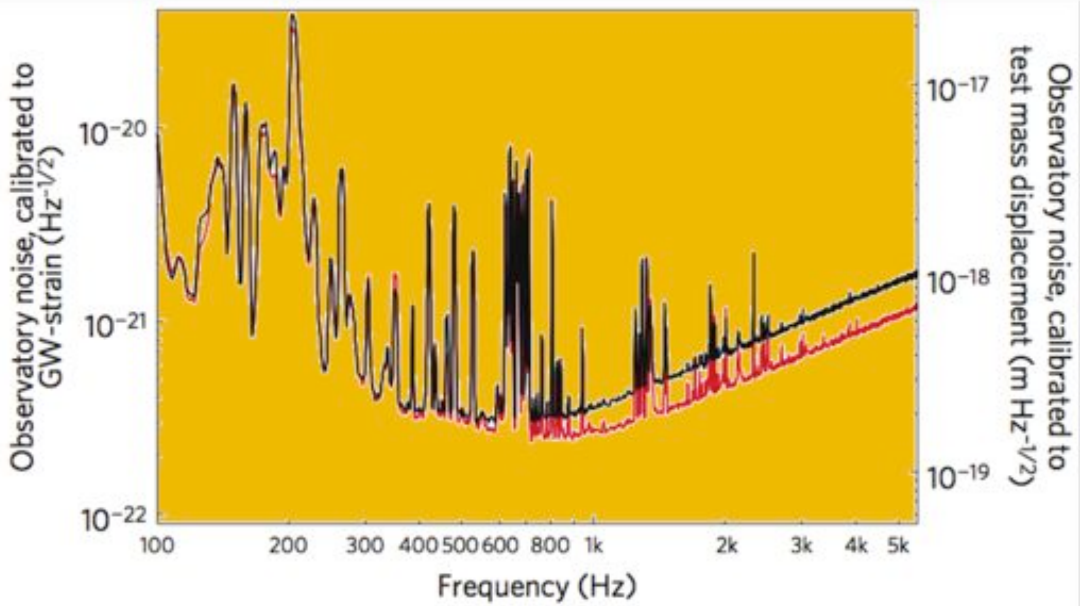


Figure 3 | Nonclassical reduction of the GEO 600 instrumental noise using squeezed vacuum states of light.

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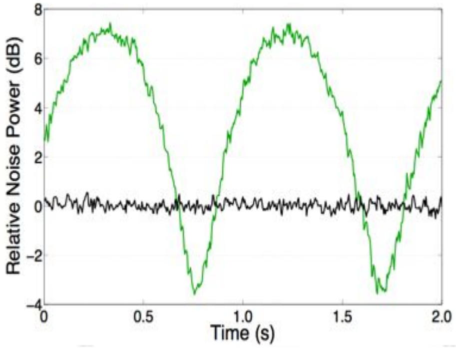

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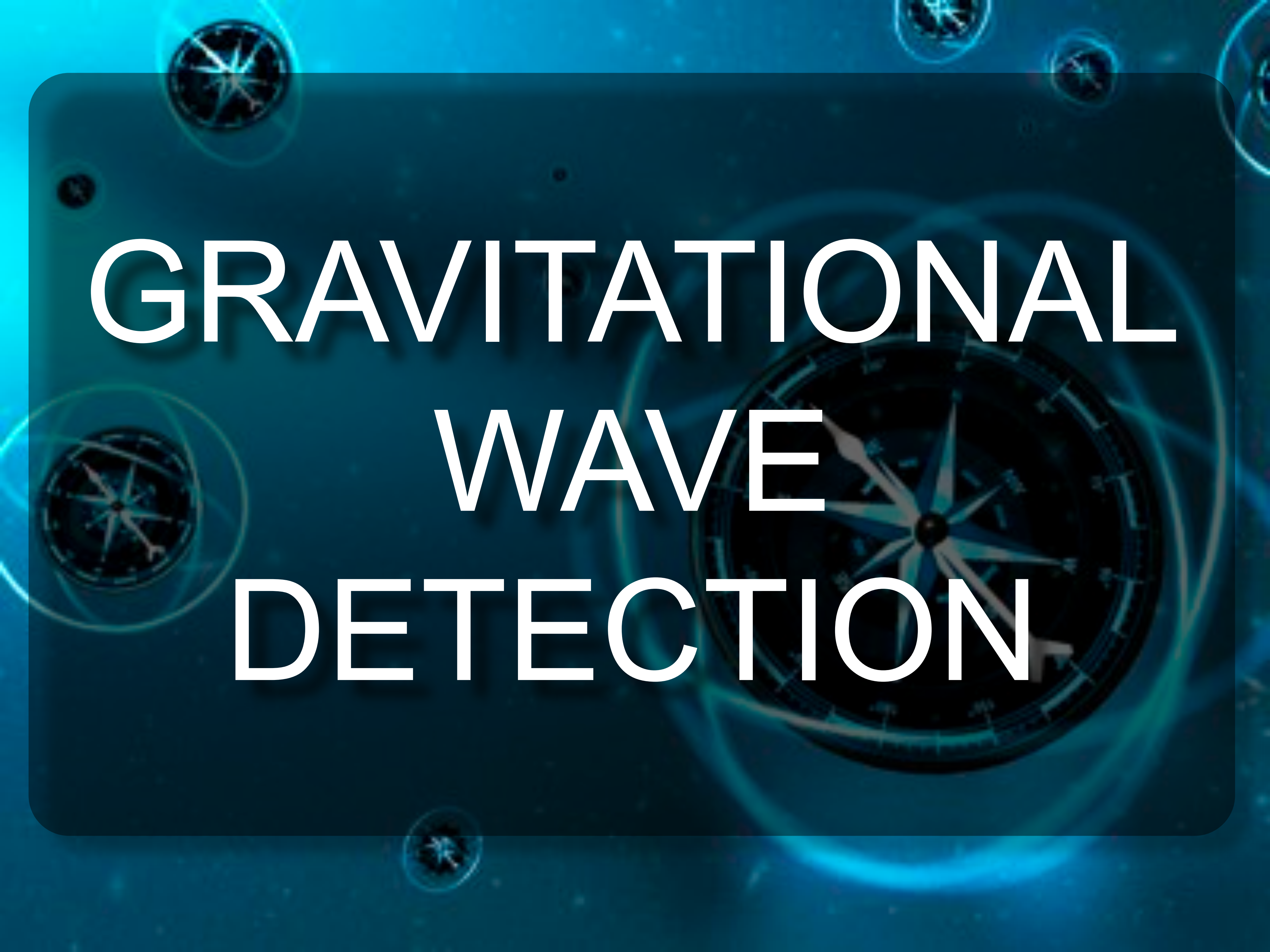


GRAVITATIONAL WAVE DETECTION

Gravitational wave detectors

LIGO (USA)

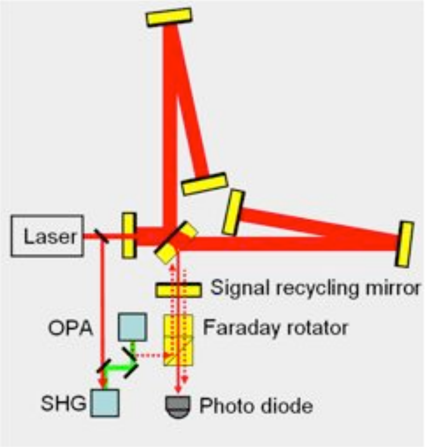
4km

Hanford, Washington

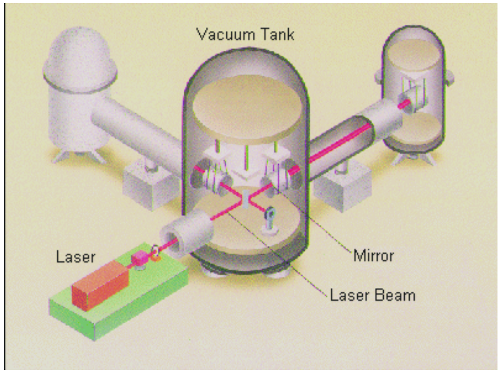
Livingston, Louisiana

VIRGO (Italy)

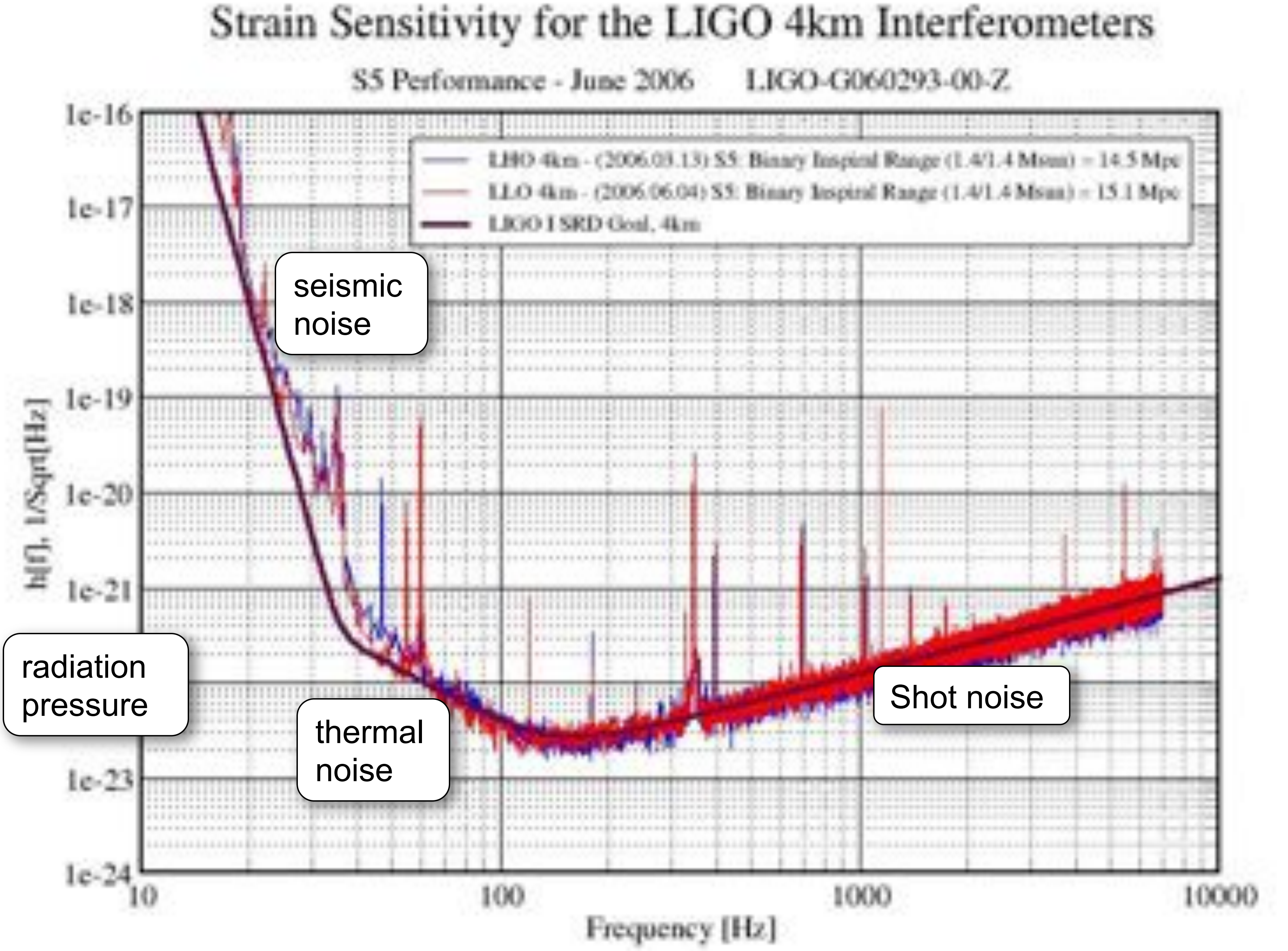
3km



GEO (Germany)



TAMA (Japan)



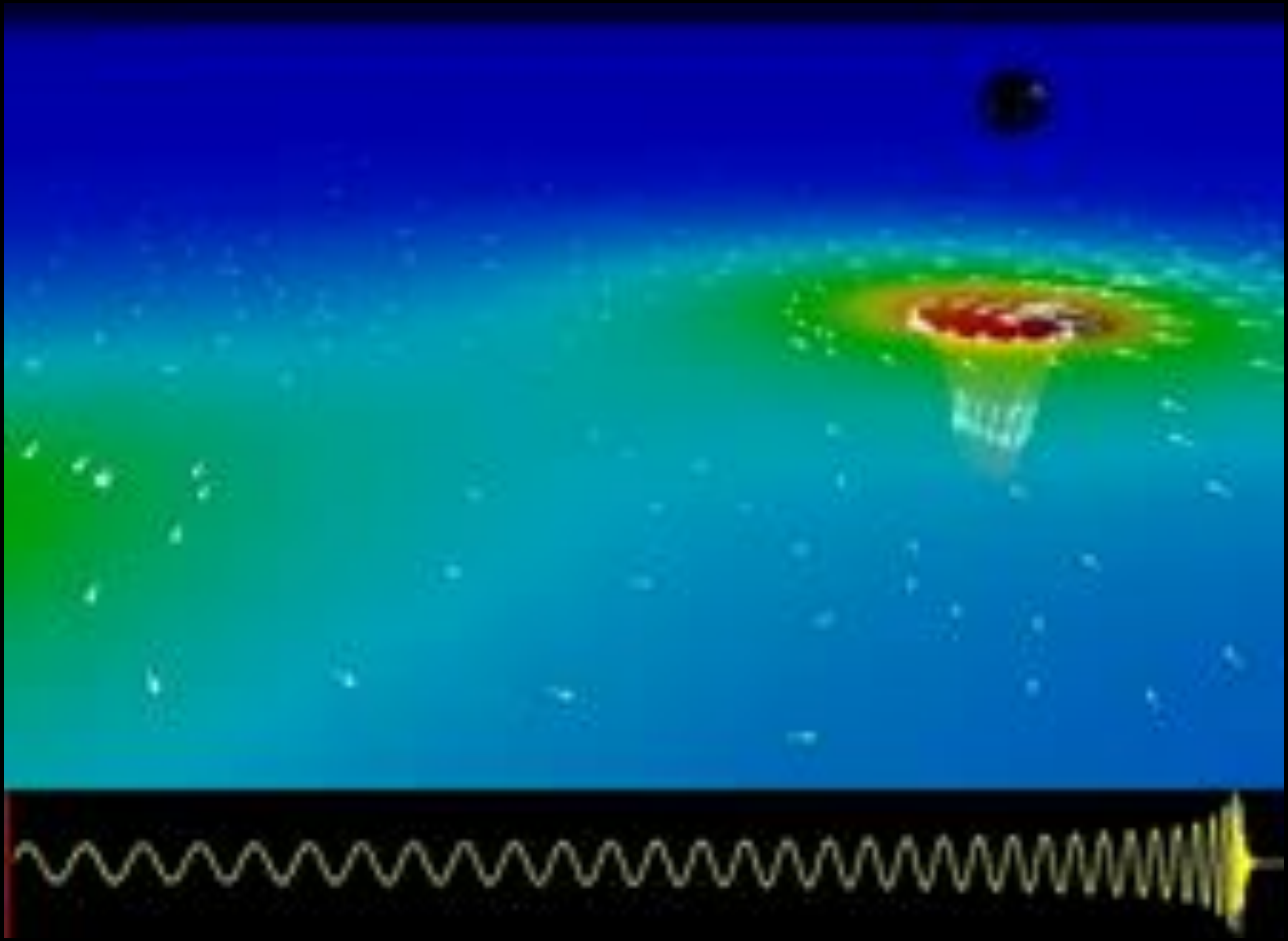
seismic noise

radiation pressure

thermal noise

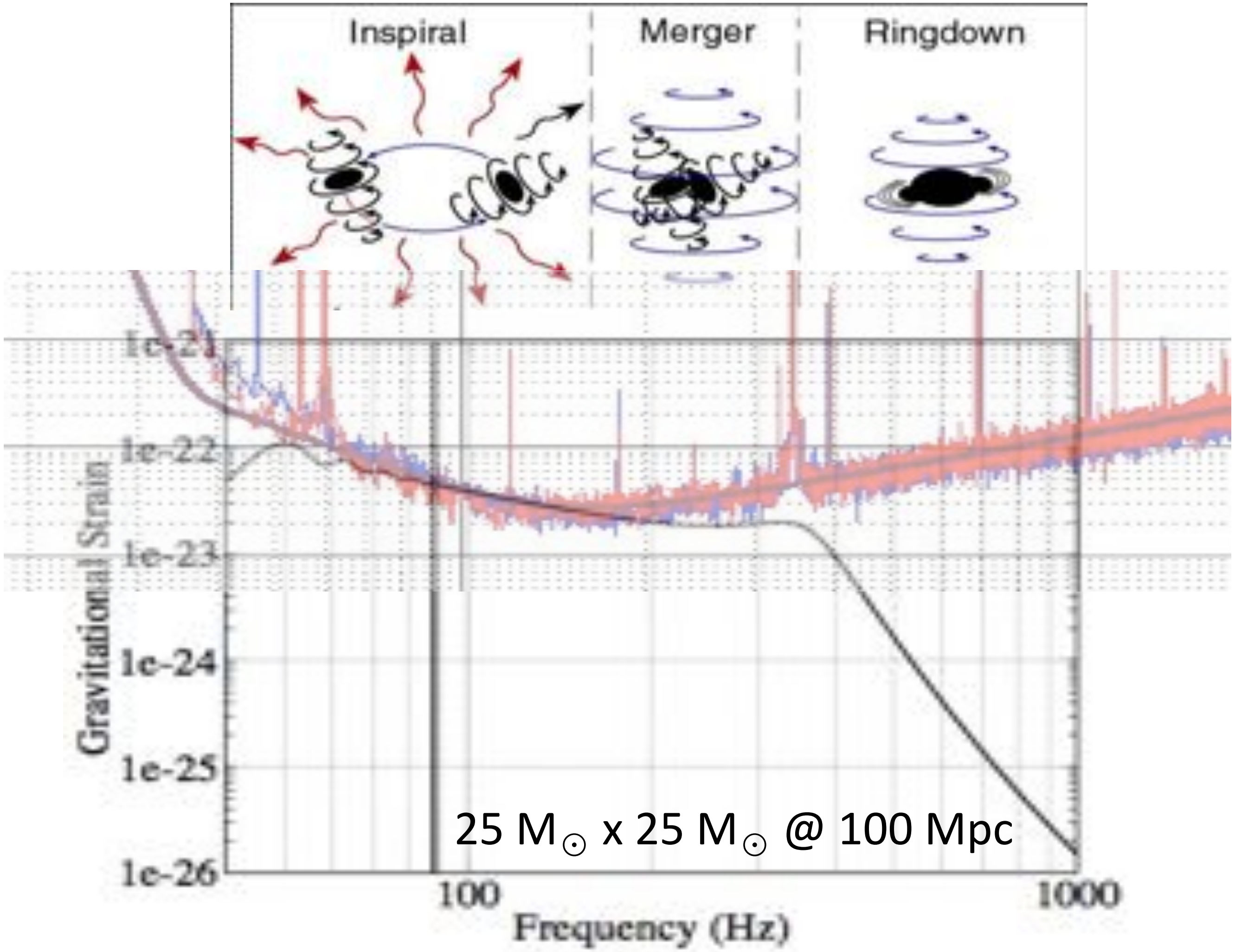
Shot noise

Gravitational waves from black holes



Gravitational wave detection

Search for gravitational waves from binary black hole inspiral, merger and ringdown. Phys. Rev. D 83, 122005 (2011)



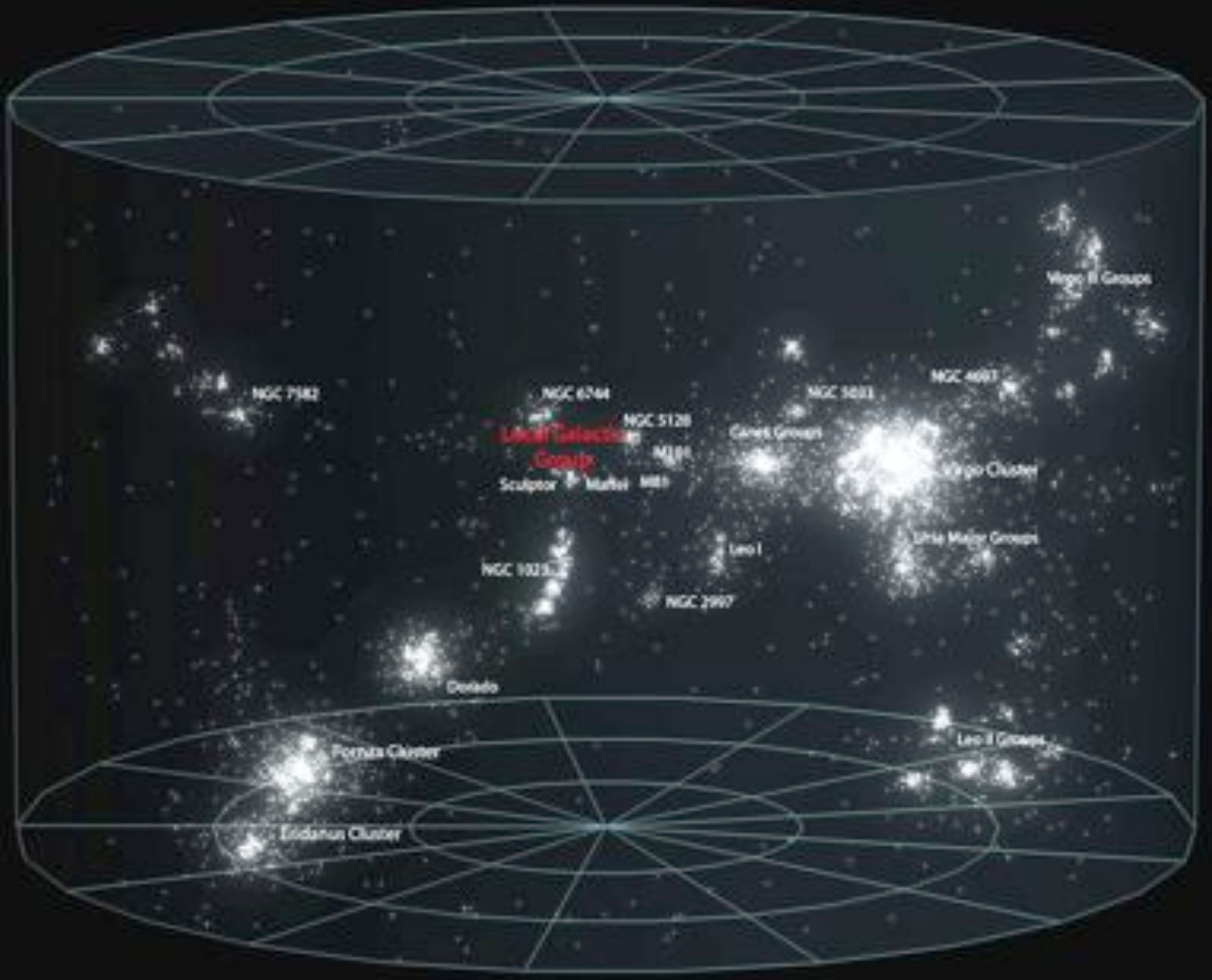
$25\ M_\odot$ x $25\ M_\odot$ @ 100 Mpc

Search for gravitational waves from binary black hole inspiral, merger and ringdown

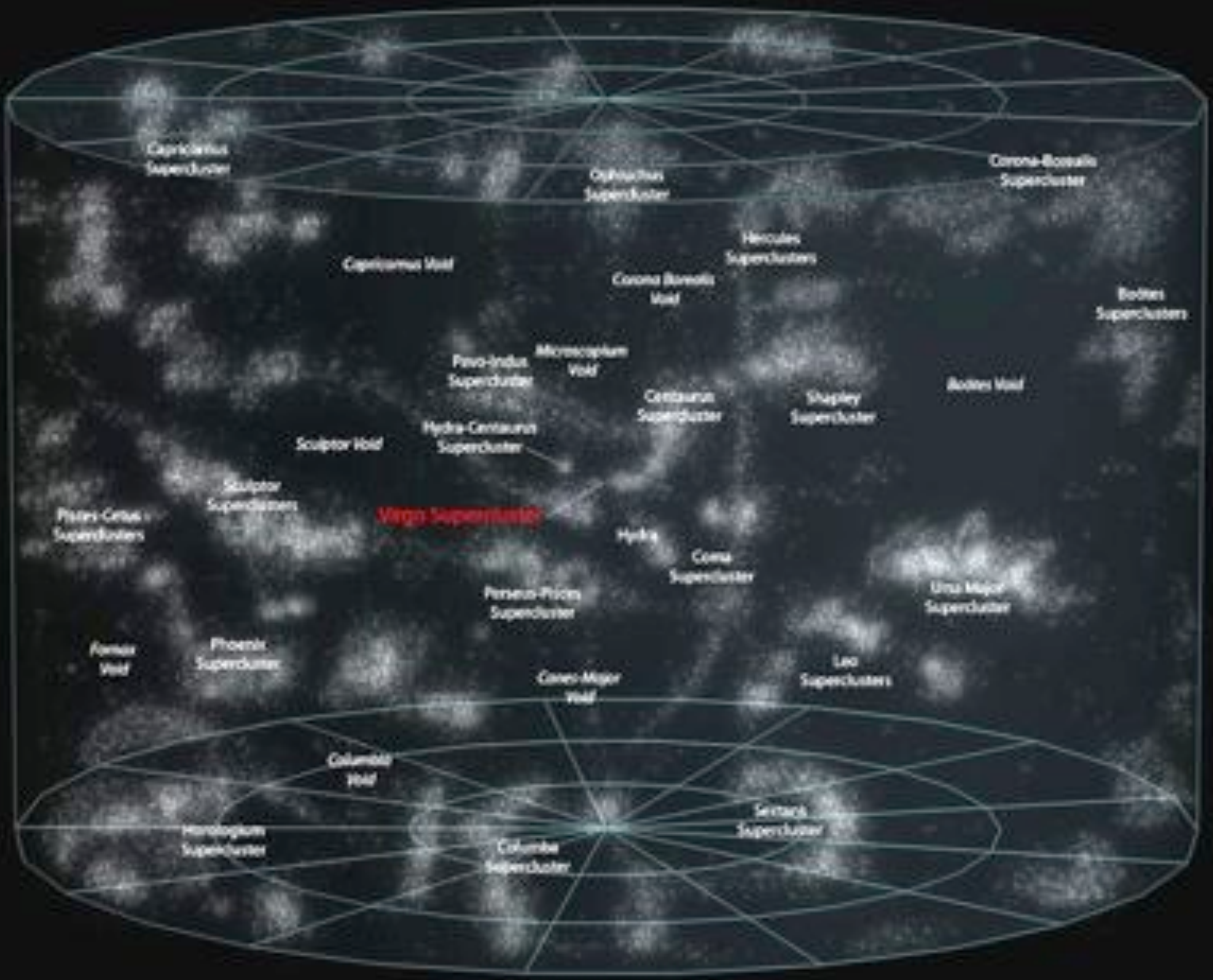
arXiv:1102.3781v1 [gr-qc] 18 Feb 2011

LIGO + VIRGO

Virgo Supercluster



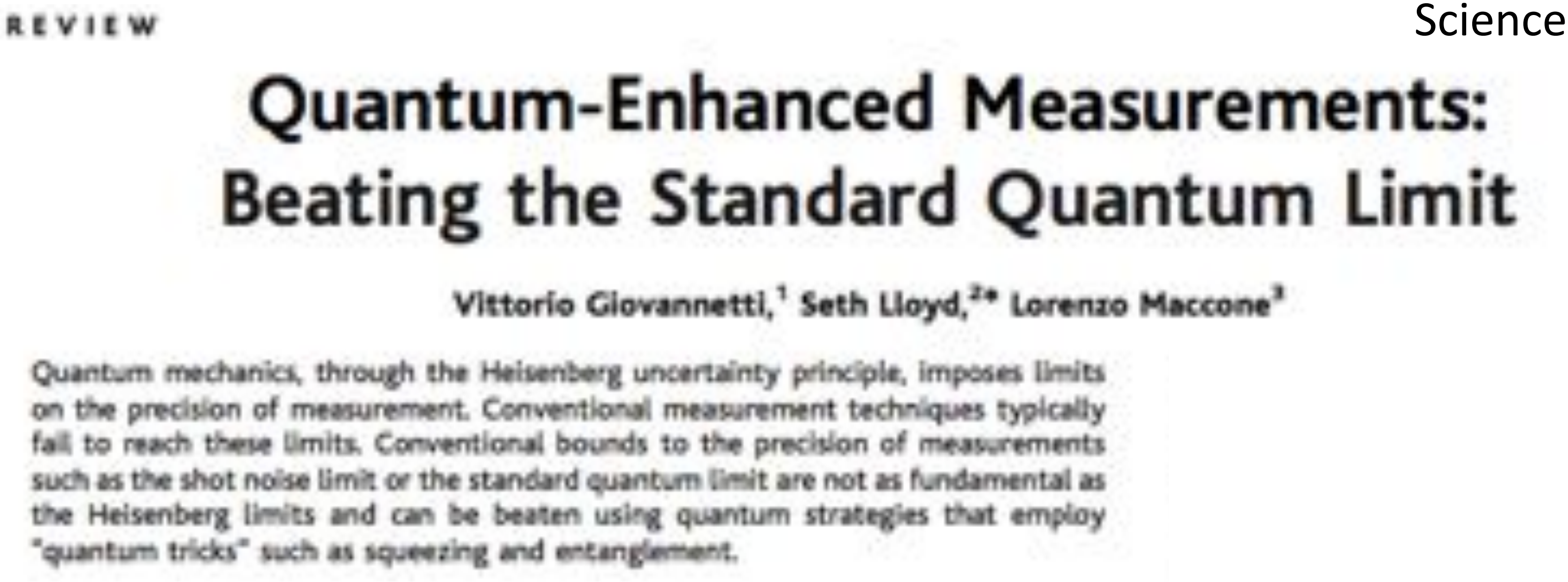
Local Superclusters



STANDARD THEORETICAL FORMULATION

Formalization of quantum metrology

Science, 2004



REVIEW

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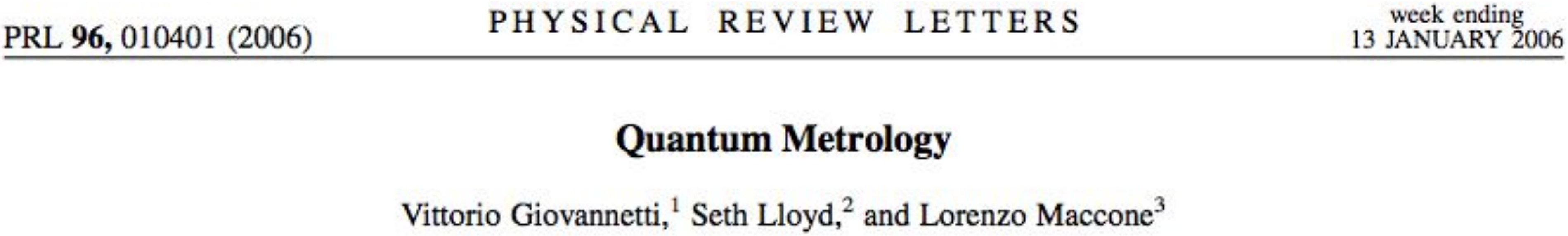
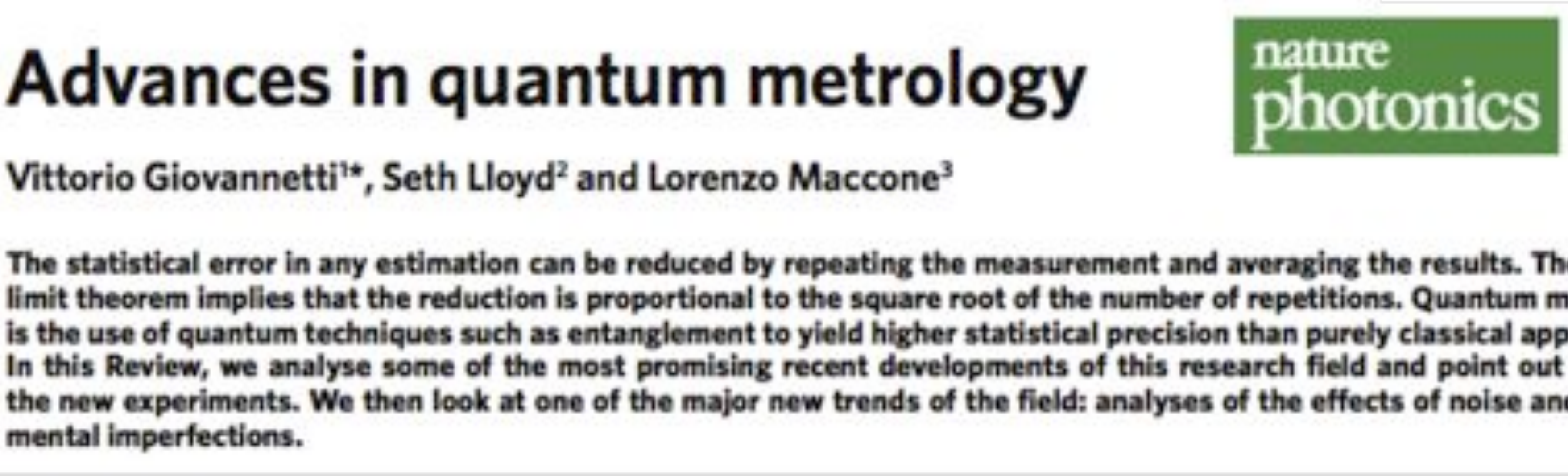
Formalization of quantum metrology



PRL **96,** 010401 (2006) PHYSICAL REVIEW LETTERS week ending 13 JANUARY 2006

Quantum Metrology

Vittorio Giovannetti,[1] Seth Lloyd,[2] and Lorenzo Maccone[3]



nature photonics

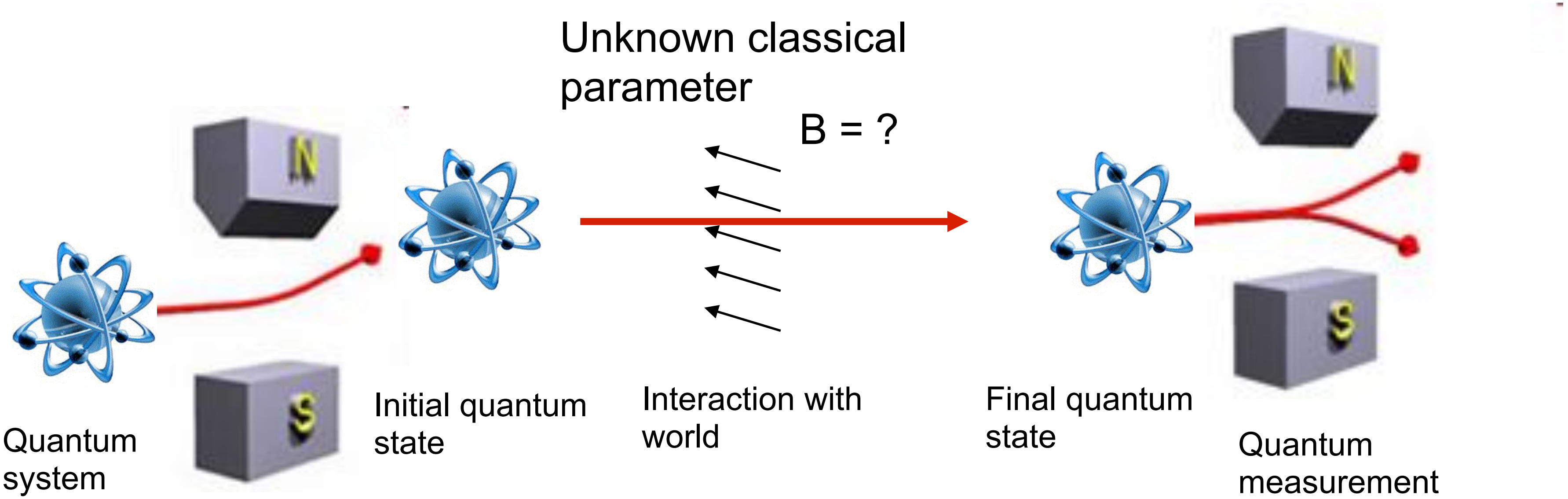
Advances in quantum metrology

Vittorio Giovannetti[1]*, Seth Lloyd[2] and Lorenzo Maccone[3]

The statistical error in any estimation can be reduced by repeating the measurement and averaging the results. The central limit theorem implies that the reduction is proportional to the square root of the number of repetitions. Quantum metrology is the use of quantum techniques such as entanglement to yield higher statistical precision than purely classical approaches. In this Review, we analyse some of the most promising recent developments of this research field and point out some of the new experiments. We then look at one of the major new trends of the field: analyses of the effects of noise and experimental imperfections.

2011

Formalization of quantum metrology



Unknown classical parameter

B = ?

Quantum system

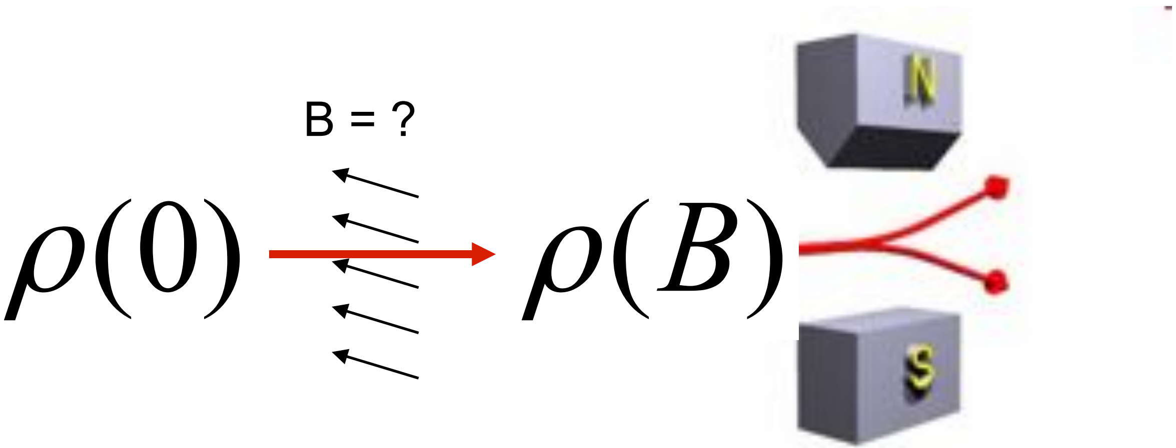
Initial quantum state

Interaction with world

Final quantum state

Quantum measurement

Contemporary Quantum Metrology


B = ?

$$\rho(0) \rightarrow \rho(B)$$

INITIAL Quantum
STATE

FINAL Quantum
STATE

Quantum
measurement

Outcomes $i = 1 \ldots N$

POVM elements A_i

Outcome probabilities

$$P_i(B) = Tr[\rho(B) A_i]$$

Estimator $\breve{B}$

e.g. Maximum
likelihood

Contemporary Quantum Metrology

Outcomes ⟶ Estimator $\breve{B}$ e.g. Maximum likelihood

Classical Fisher Information: $I(B) \equiv \left\langle [\partial_B \ln P_i(B)]^2 \right\rangle$

- Cramer-Rao Bound: $\mathrm{var}(\breve{B}) \geq 1/I(B)$
- Additive $I_{X,Y}(B) = I_X(B) + I_Y(B)$

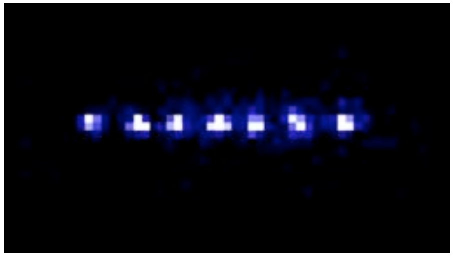
Quantum Fisher Information:
(info available in the state) $I_Q(B) \equiv \left\langle \left[R_\rho^{-1}(\partial_B \rho) \right]^2 \right\rangle$

$$R_\rho(A) \equiv (A\rho + \rho A)/2$$

- Quantum Cramer-Rao bound: $\mathrm{var}(\breve{B}) \geq 1/I_Q(B) \geq 1/I(B)$

Braunstein and Caves PRL 1994 “Statistical Distance and the Geometry of Quantum States”

Provable results in phase estimation



Given an N-qubit state

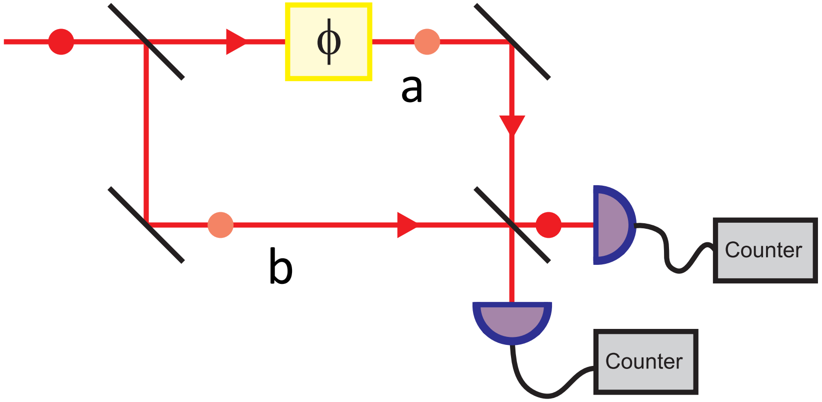
subject to a unitary evolution

$$U_\phi = \bigotimes_{i=1}^{N} u_\phi$$

$$u_\phi \equiv |0\rangle\langle 0| + e^{i\phi}|1\rangle\langle 1|$$

	separable	any
Fisher information is upper bounded by	N	N^2
asymptotic uncertainty $\delta\phi \geq$	$N^{-1/2}$	N^{-1}

Provable results in phase estimation



Given an N-photon two-mode state

subject to a unitary evolution

$$U_\phi = e^{i\phi a^\dagger a}$$

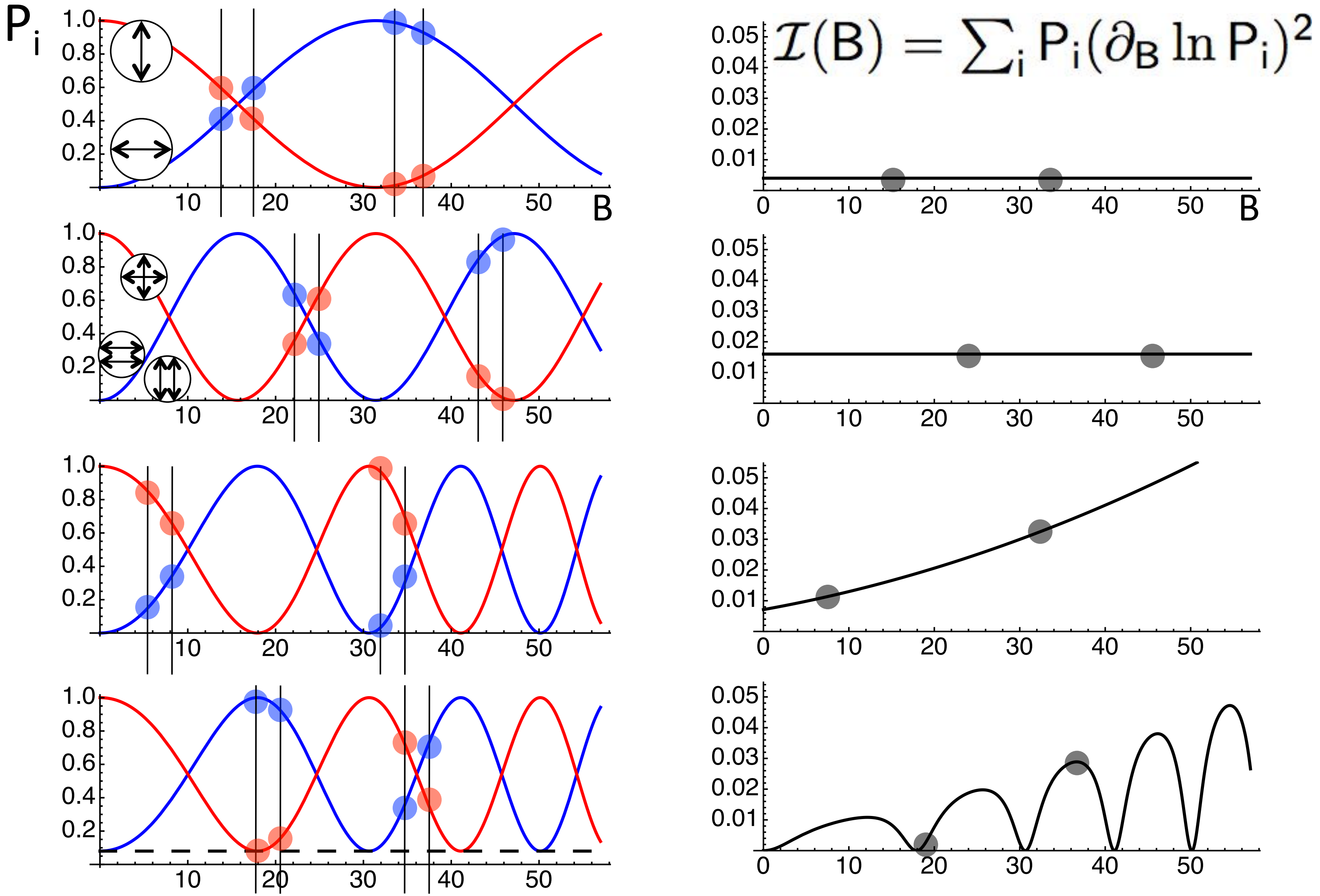
		separable	any
Fisher information is upper bounded by		N	N^2
asymptotic uncertainty	$\delta\phi \geq$	$N^{-1/2}$	N^{-1}

Say something about scaling

		shot-noise limit	"Heisenberg limit"
asymptotic uncertainty	$\delta\phi \geq$	$N^{-1/2}$	N^{-1}

FISHER INFORMATION

Fisher information



$$\mathcal{I}(B) = \sum_i P_i (\partial_B \ln P_i)^2$$

HISTORICAL ANALOGY

Metaphor

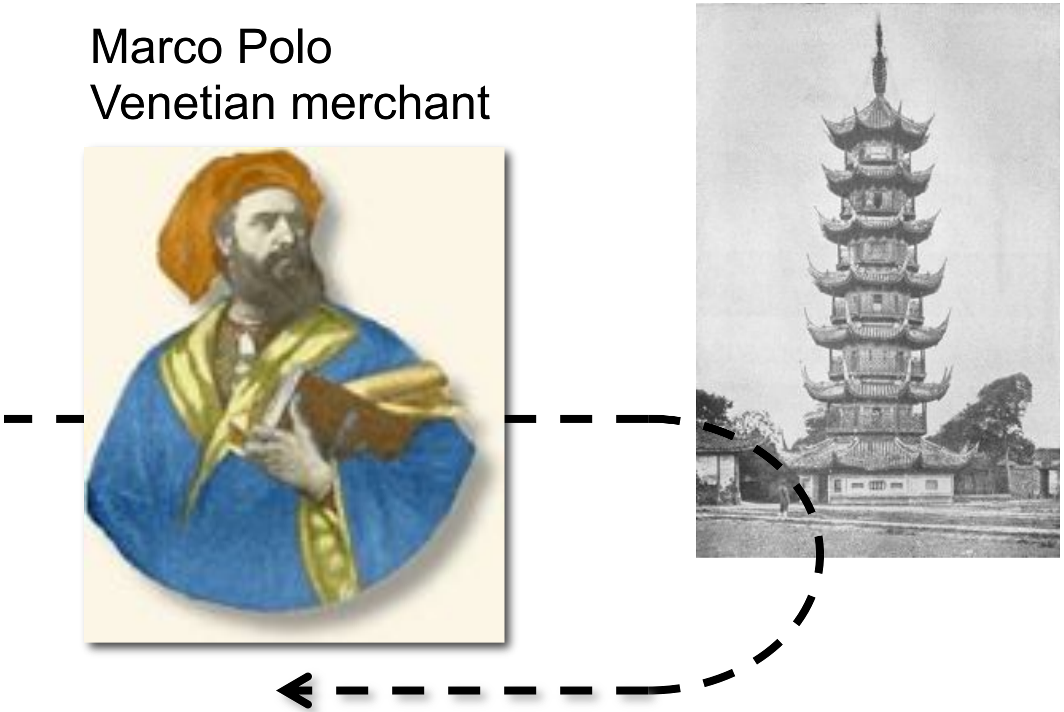
Kublai Khan 1215-1294
Mongol Emperor 1260-1294
grandson of Genghis Kahn

Mongol Empire: 33 million km^2

Metaphor

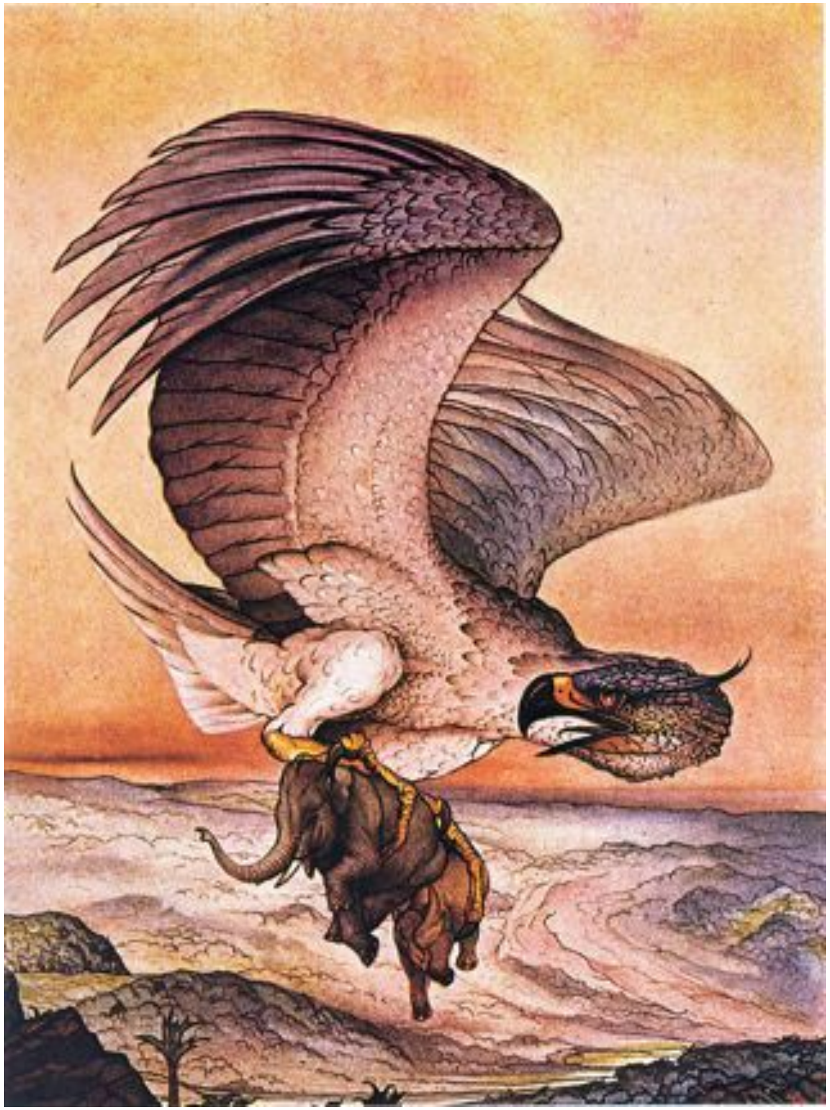
Kublai Khan 1215-1294
Mongol Emperor 1260-1294
grandson of Genghis Kahn

Marco Polo
Venetian merchant



Ambassador in Burma, India, many parts of China

Metaphor

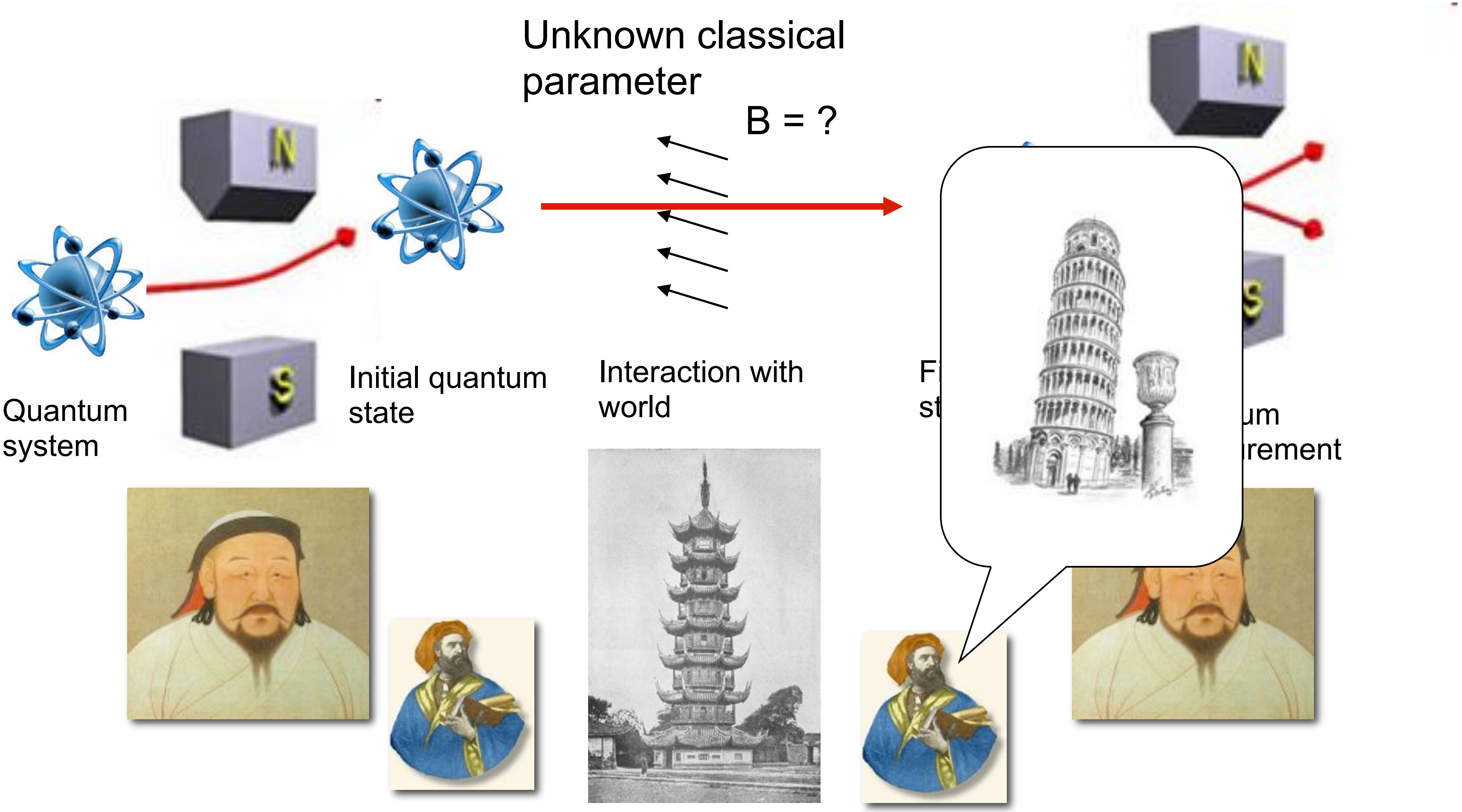


"It is so strong that it will seize an elephant in its talons and carry him high into the air, and drop him so that he is smashed to pieces; having so killed him the bird gryphon swoops down on him and eats him at leisure."

It is not that Kublai Khan believed all the things that Marco Polo told him… but the Emperor did listen to the young Venetian with more curiosity and attention than to his other ambassadors.

- Italo Calvino "Invisible Cities"

Quantum Metrology



Unknown classical parameter

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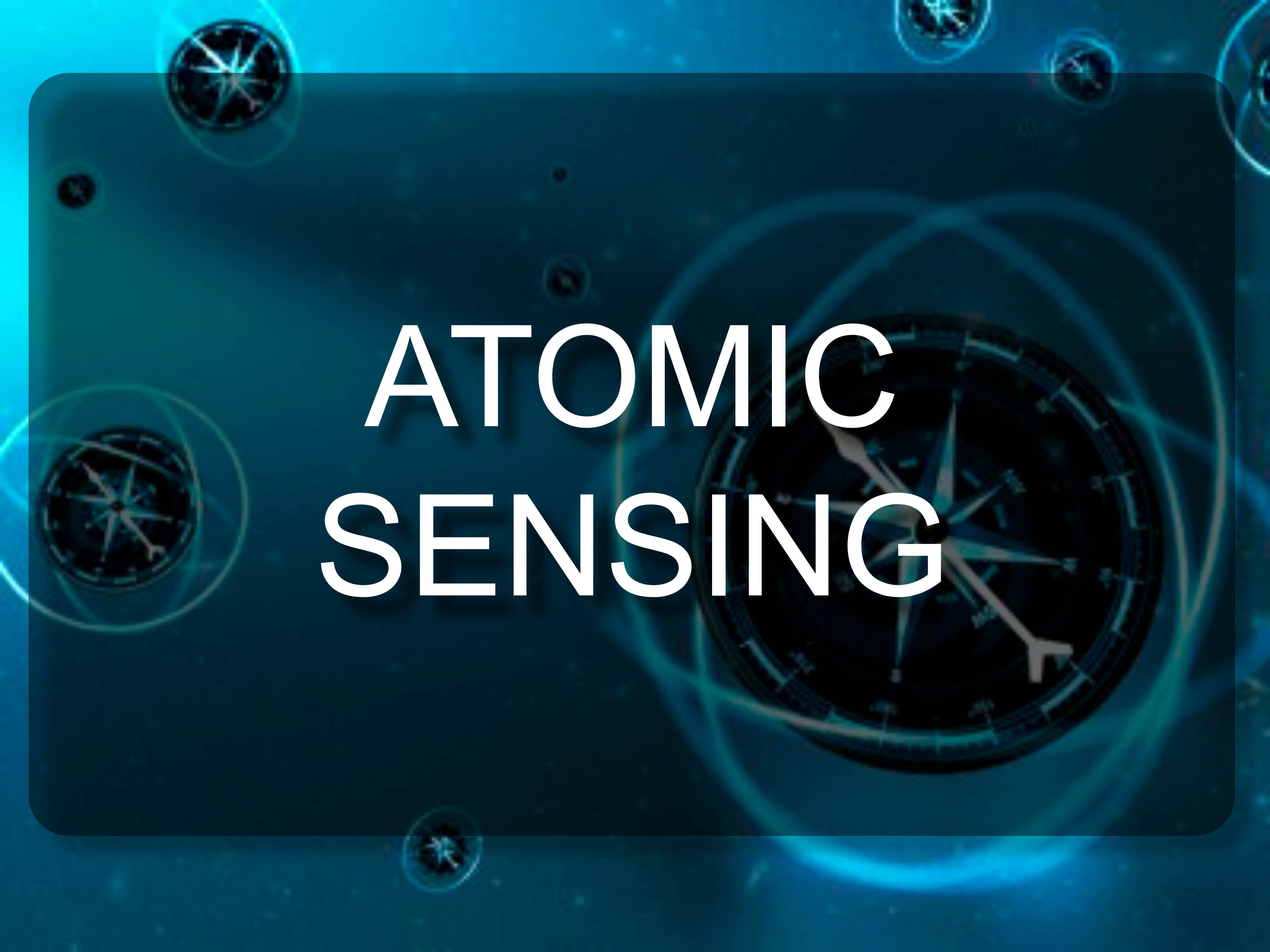
Quantum system

Initial quantum state

Interaction with world

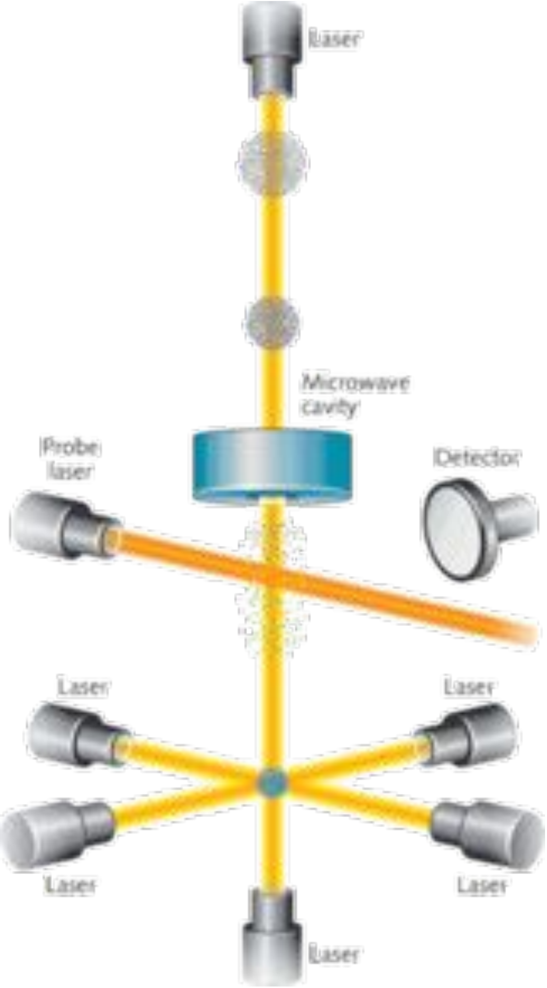
Fi st

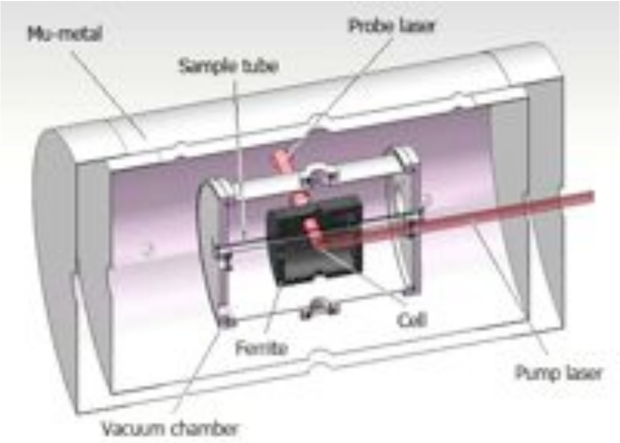
um rement

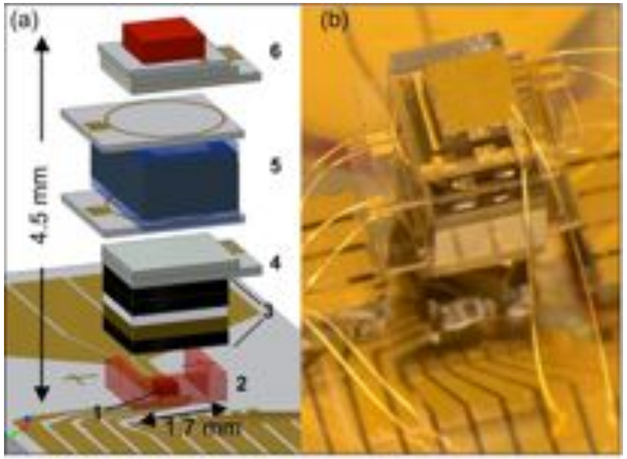


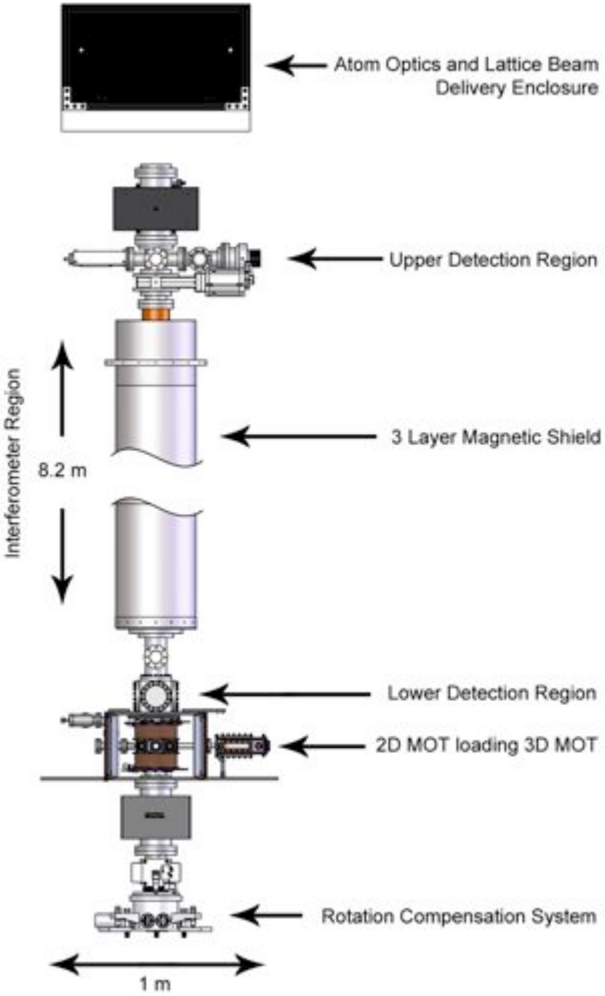
ATOMIC SENSING

Atom interferometers





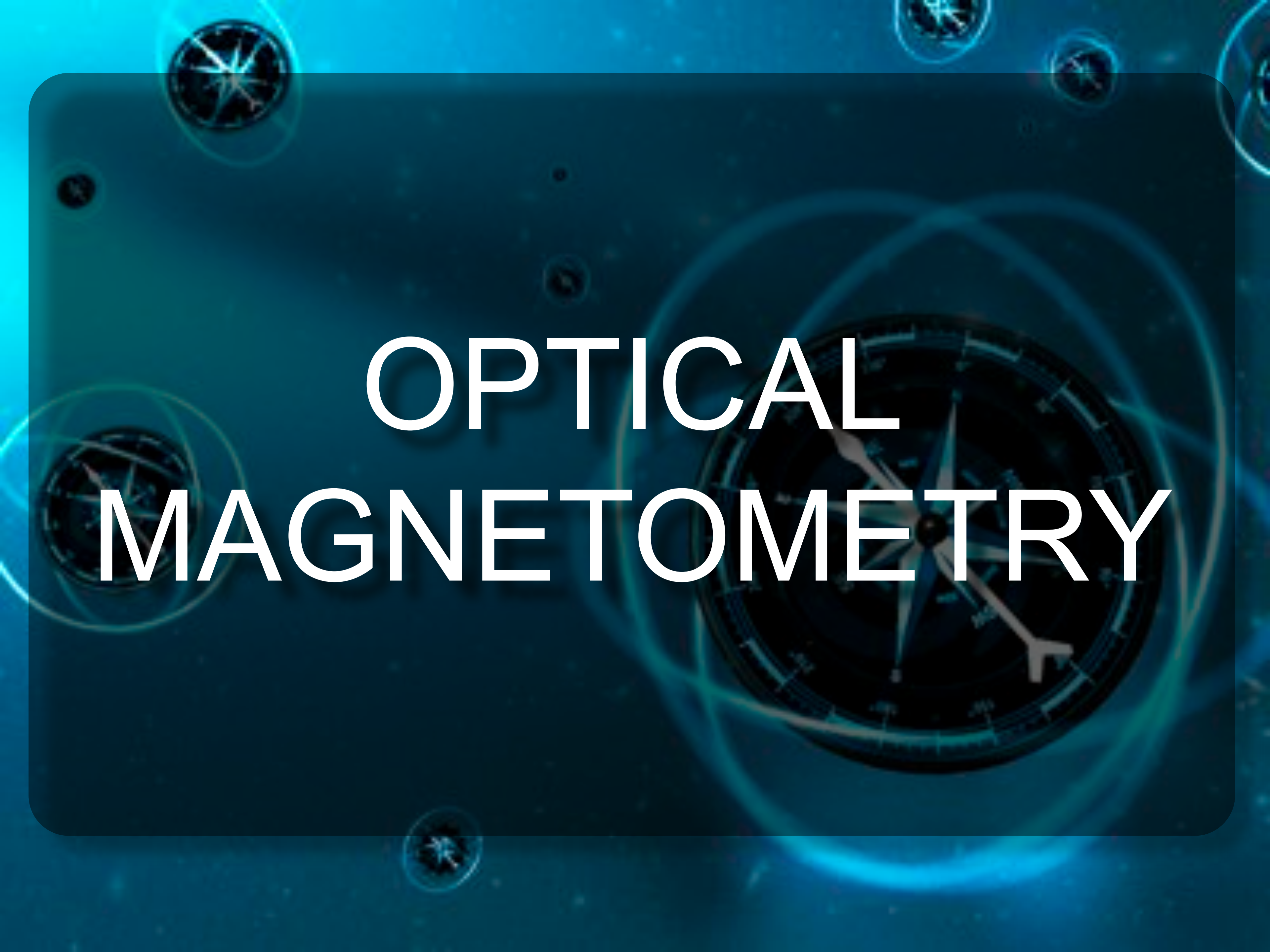




atomic clock

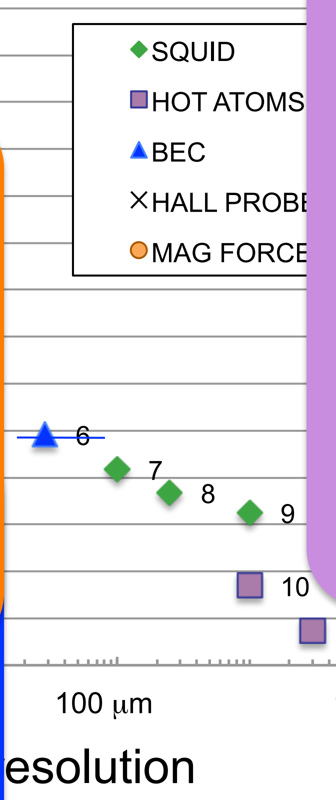
optical magnetometers

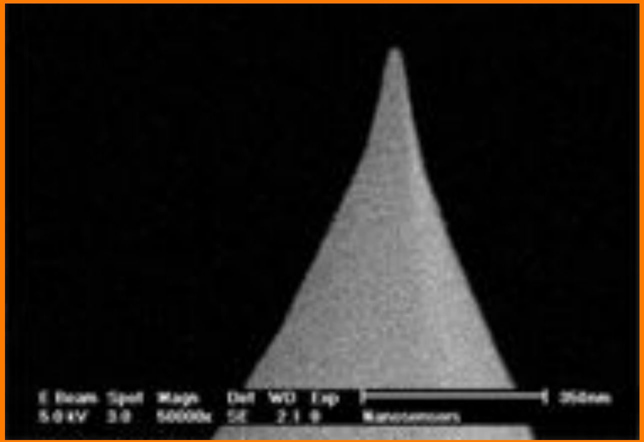
atomic gravimeter



OPTICAL MAGNETOMETRY

Atomic magnetometers are best-in-class sensors

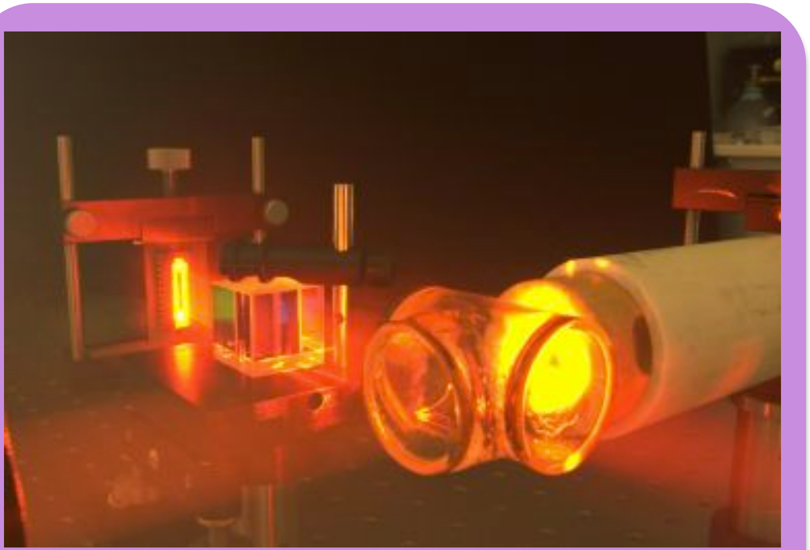




magnetic force microscope tip

cold atoms

Stamper-Kurn
Schmiedmayer
Gawlik, Mitchell



hot atoms

Romalis, Kitching

)

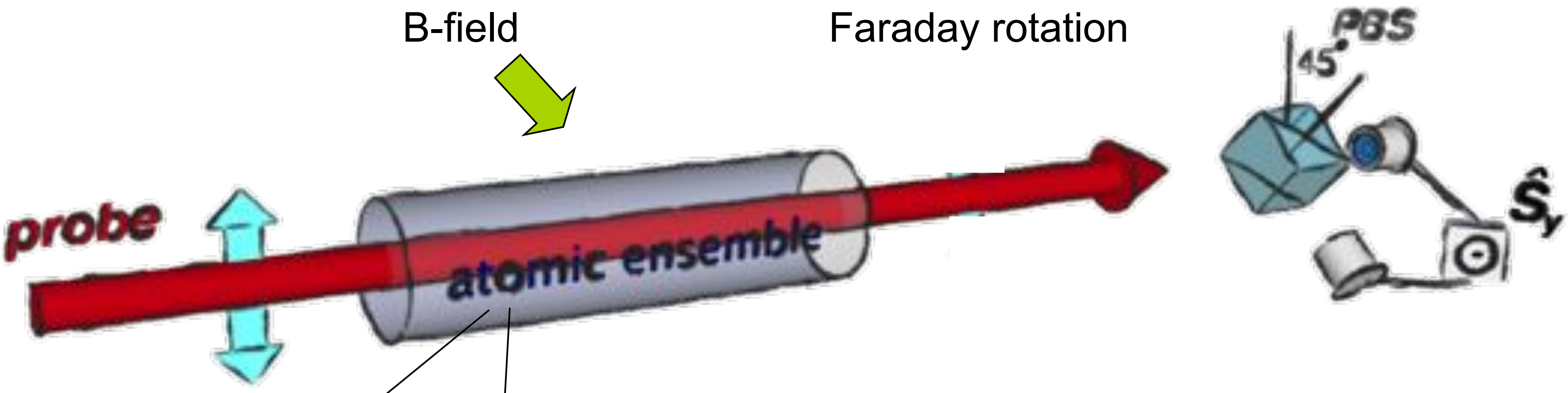
2005)

10. Griffith et al Opt. Express **18** 27167 (2010)
11. Kominis, et al Nature, **422** 596 (2003)
12. Dang et al. Appl. Phys. Lett. **97** 151110 (2010)
13. Koschorreck et al. Appl. Phys. Lett. **98** 074101 (2011)
14. Sewell et al. Phys. Rev. Lett. **109**, 253605 (2012)
15. Behbood et al. Appl. Phys. Lett. **102** 173504 (2013)

Optical magnetometer

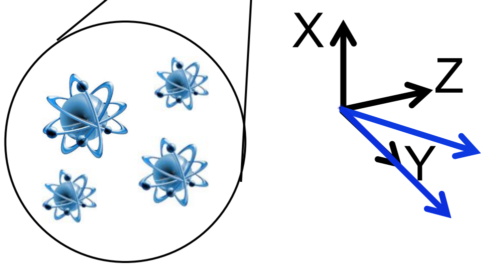
B-field

Faraday rotation



Individual spins

$\mathbf{f}^{(i)}$



collective spin

$\mathbf{F} = \sum_i \mathbf{f}^{(i)}$

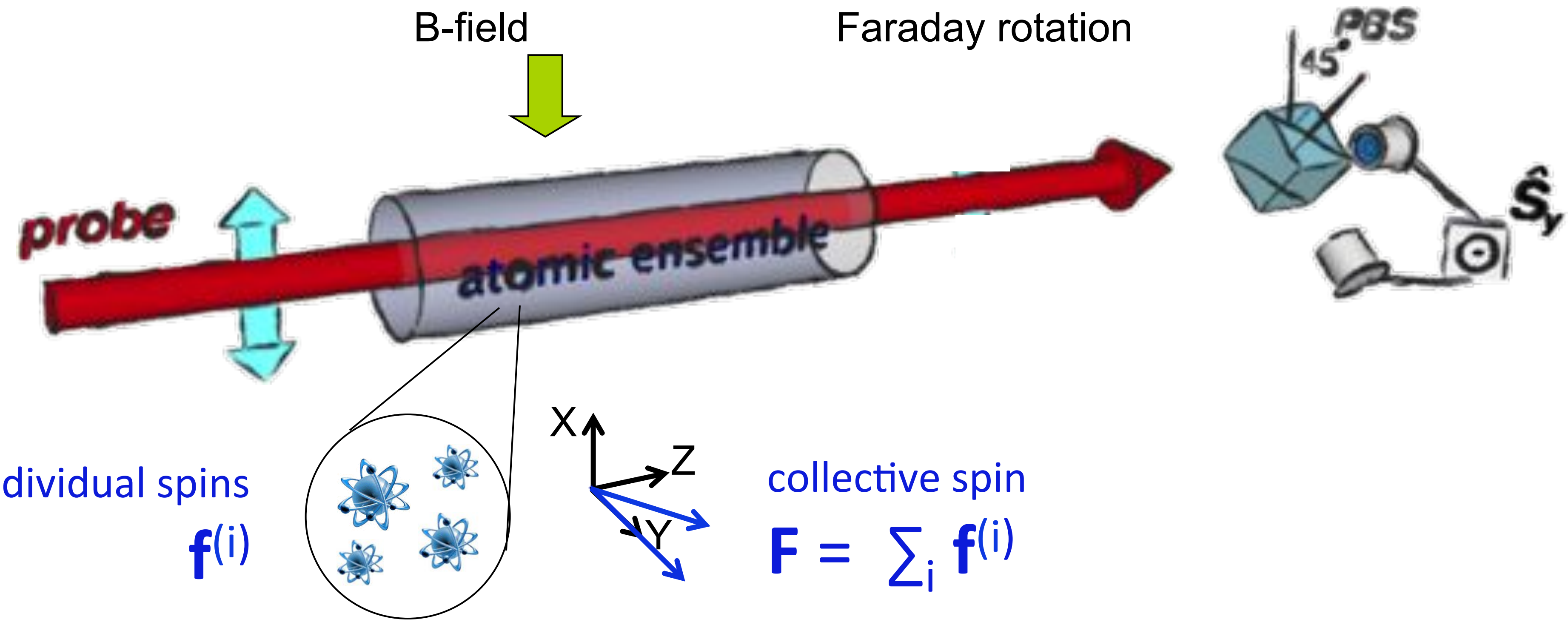
Stokes operators

$$S_X = (n_H - n_V)/2$$
$$S_Y = (n_{\nearrow} - n_{\nwarrow})/2$$
$$S_Z = (n_L - n_R)/2$$

$$[S_x, S_y] = iS_z$$

$$H_{\text{eff}} = \alpha^{(1)} S_z F_z$$

Faraday rotation optical magnetometer



B-field

Faraday rotation

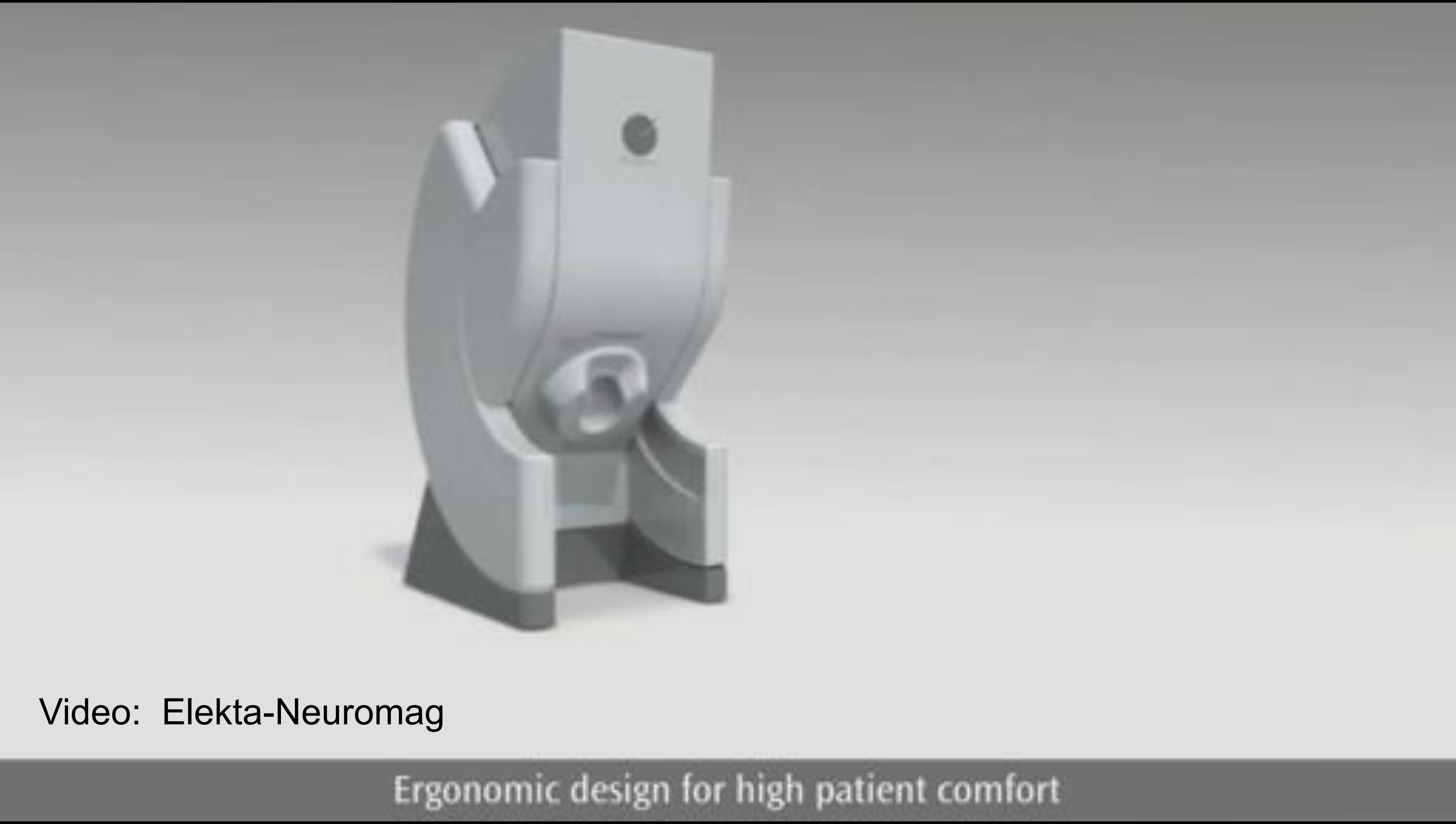
Individual spins

$\mathbf{f}^{(i)}$

collective spin

$\mathbf{F} = \sum_i \mathbf{f}^{(i)}$

Cranial magnetism

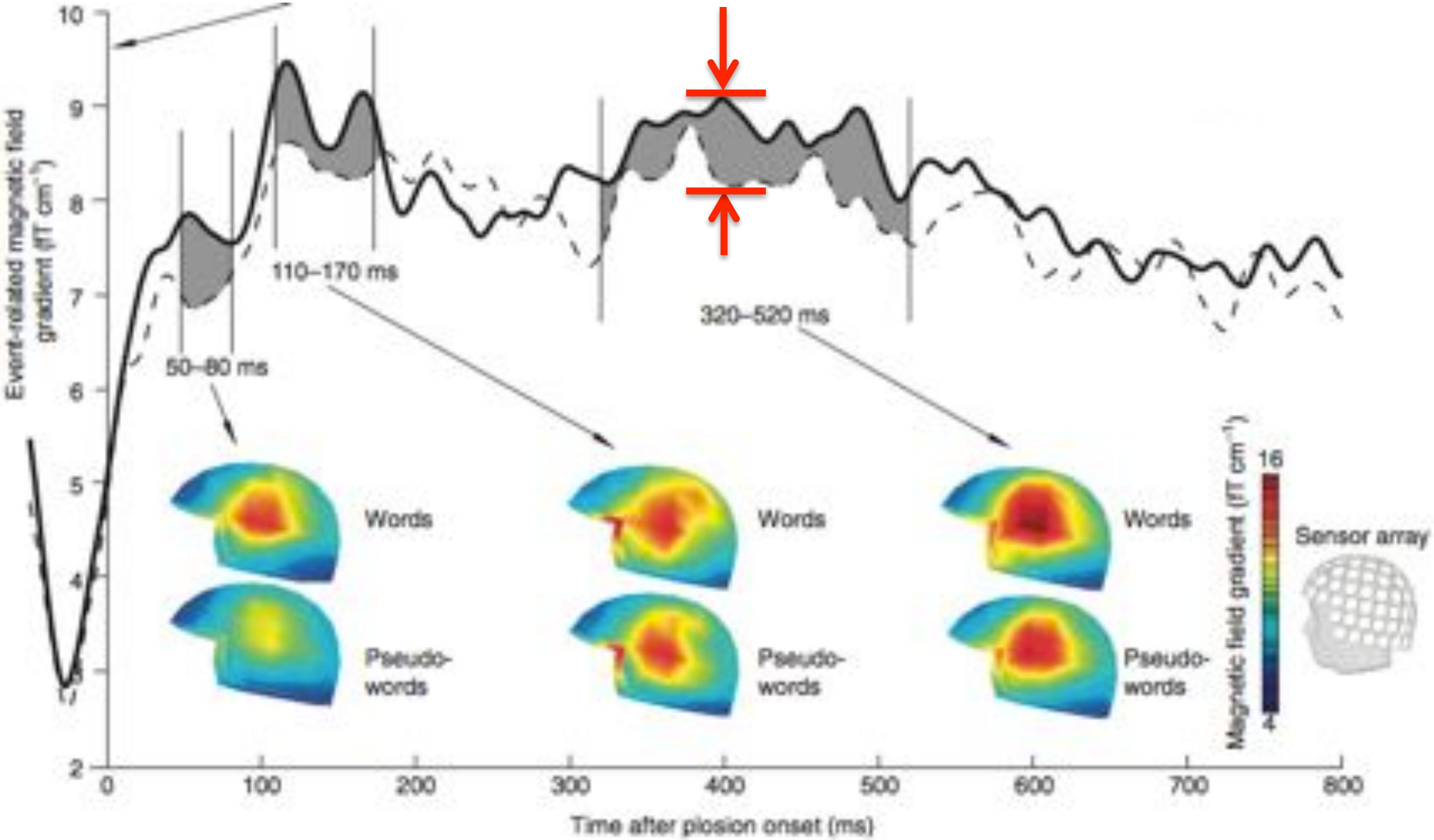


Video: Elekta-Neuromag

magnetometers detect mental events

words:
"boat"
"joke"

pseudo-words:
"boak"
"jote"



Lucy macGregor, et al. "Ultra-rapid access to words in the brain," *Nature Comm*. 2012

magnetometers detect mental events

arXiv.org > physics > arXiv:1307.2357

Physics > Instrumentation and Detectors

A Compact, High Performance Atomic Magnetometer for Biomedical Applications

Vishal K. Shah, Ronald T. Wakai

(Submitted on 9 Jul 2013)

QuSpin Inc.

+ U. Wisconsin

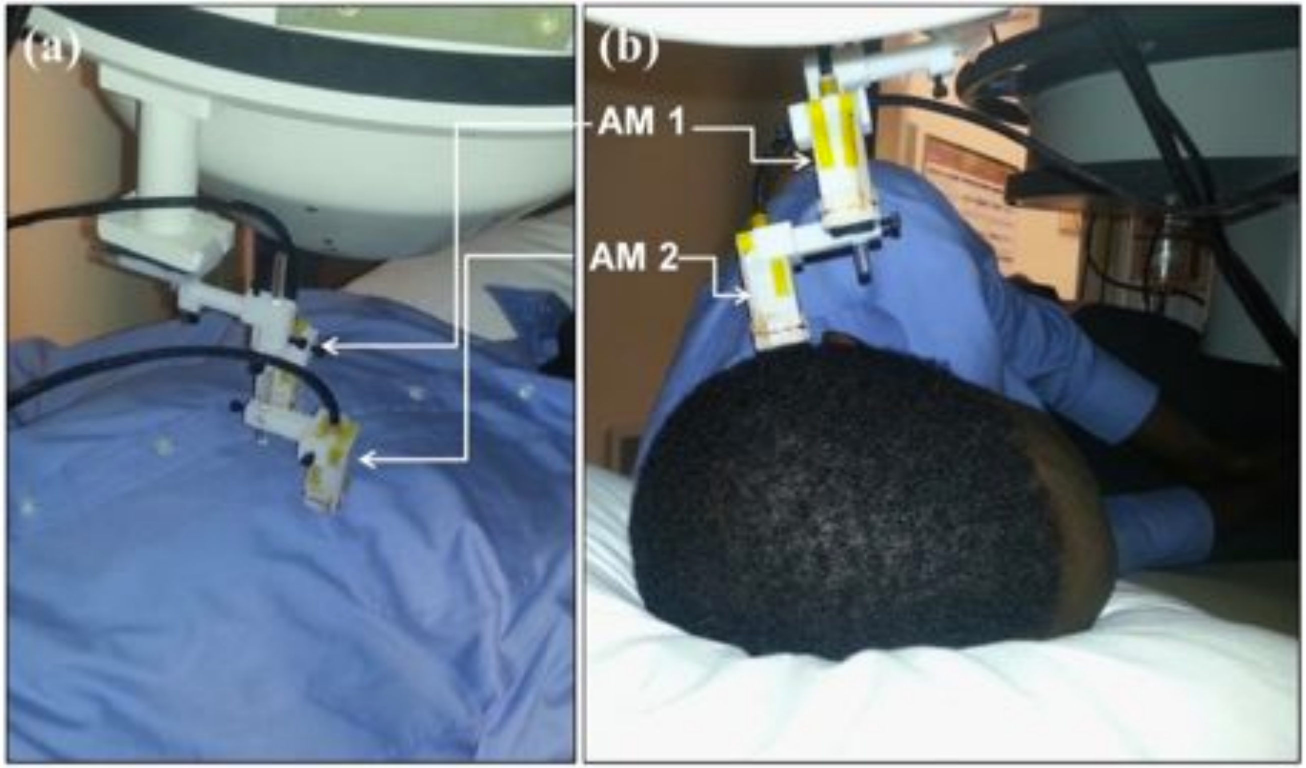
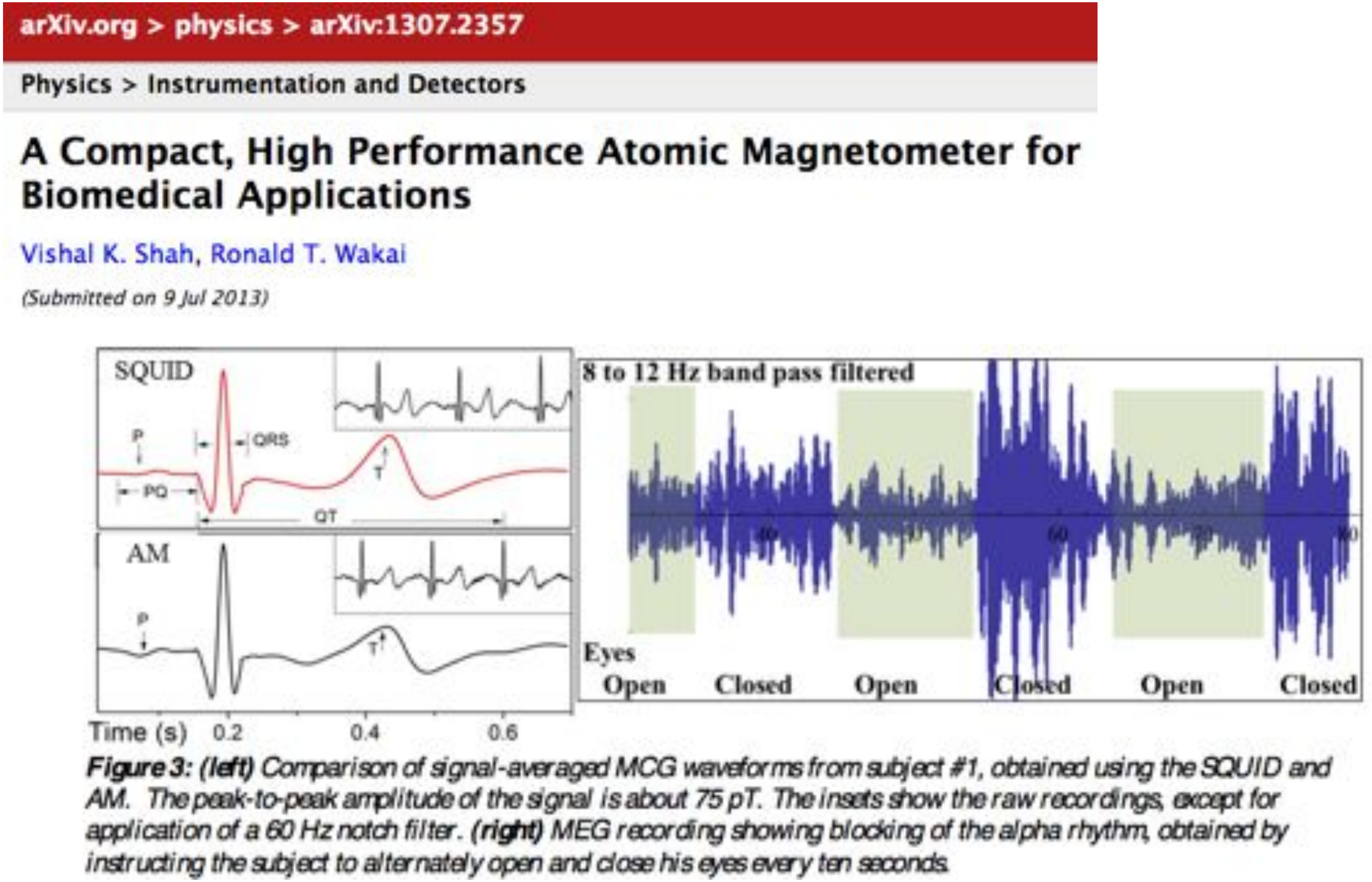


Figure 2: **(a)** A picture of two AMs positioned roughly over the chest of a subject for recording MCG. **(b)** A close-up picture of two AMs positioned over the parietal cortex in use for MEG-AER recordings. AM 2 which is closer to head was used for the actual measurements while AM 1 was used as reference sensor for background noise cancellation.

magnetometers detect mental events



arXiv.org > physics > arXiv:1307.2357

Physics > Instrumentation and Detectors

A Compact, High Performance Atomic Magnetometer for Biomedical Applications

Vishal K. Shah, Ronald T. Wakai

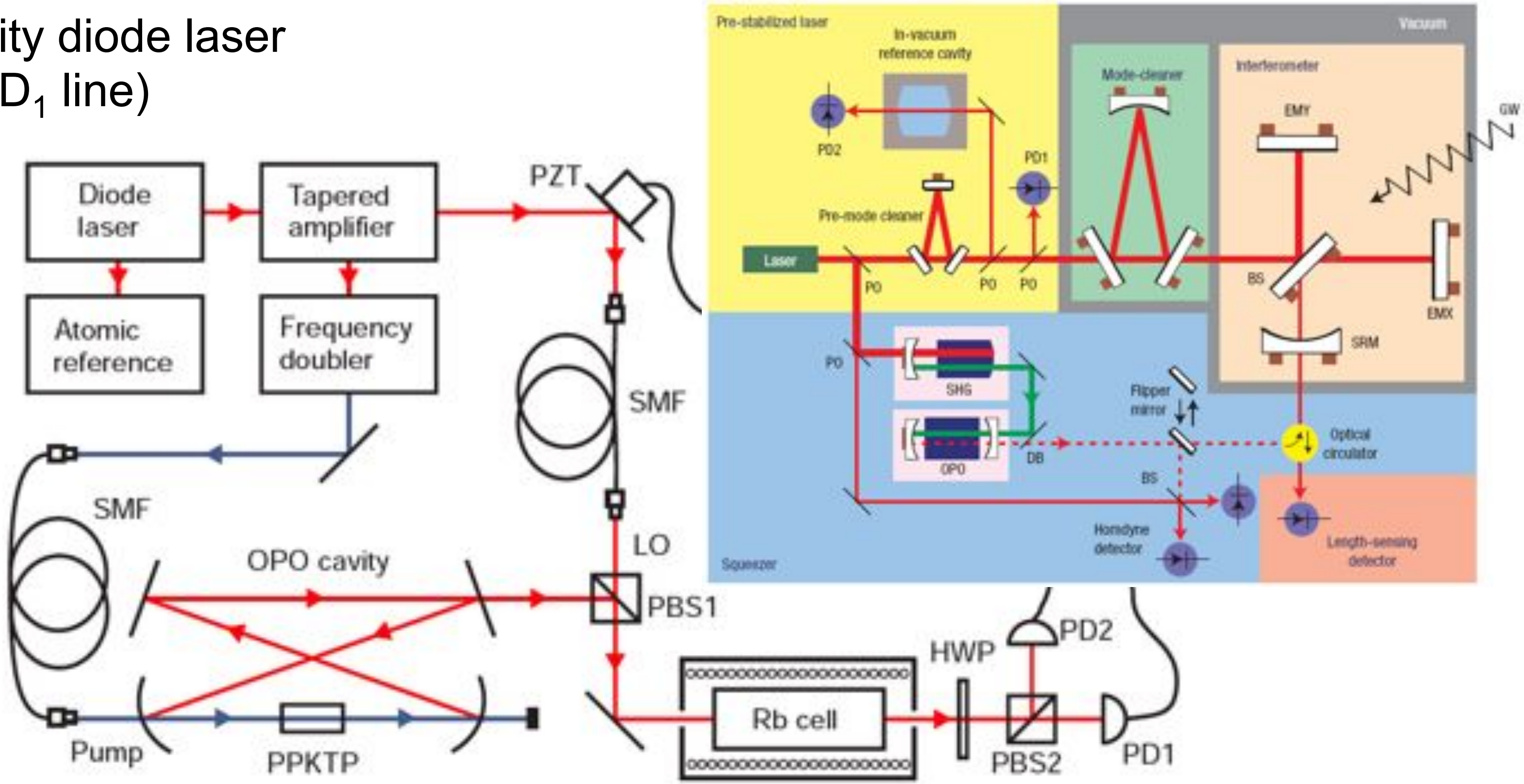
(Submitted on 9 Jul 2013)

Figure 3: ***(left)*** *Comparison of signal-averaged MCG waveforms from subject #1, obtained using the SQUID and AM. The peak-to-peak amplitude of the signal is about 75 pT. The insets show the raw recordings, except for application of a 60 Hz notch filter.* ***(right)*** *MEG recording showing blocking of the alpha rhythm, obtained by instructing the subject to alternately open and close his eyes every ten seconds.*

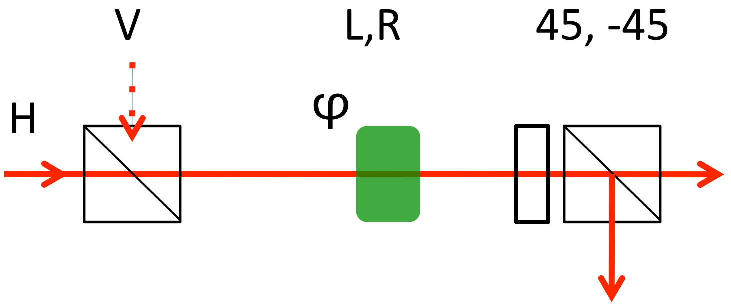
SQUEEZED LIGHT MAGNETOMETRY

Squeezed-light magnetometer

External cavity diode laser
795 nm (Rb D_1 line)

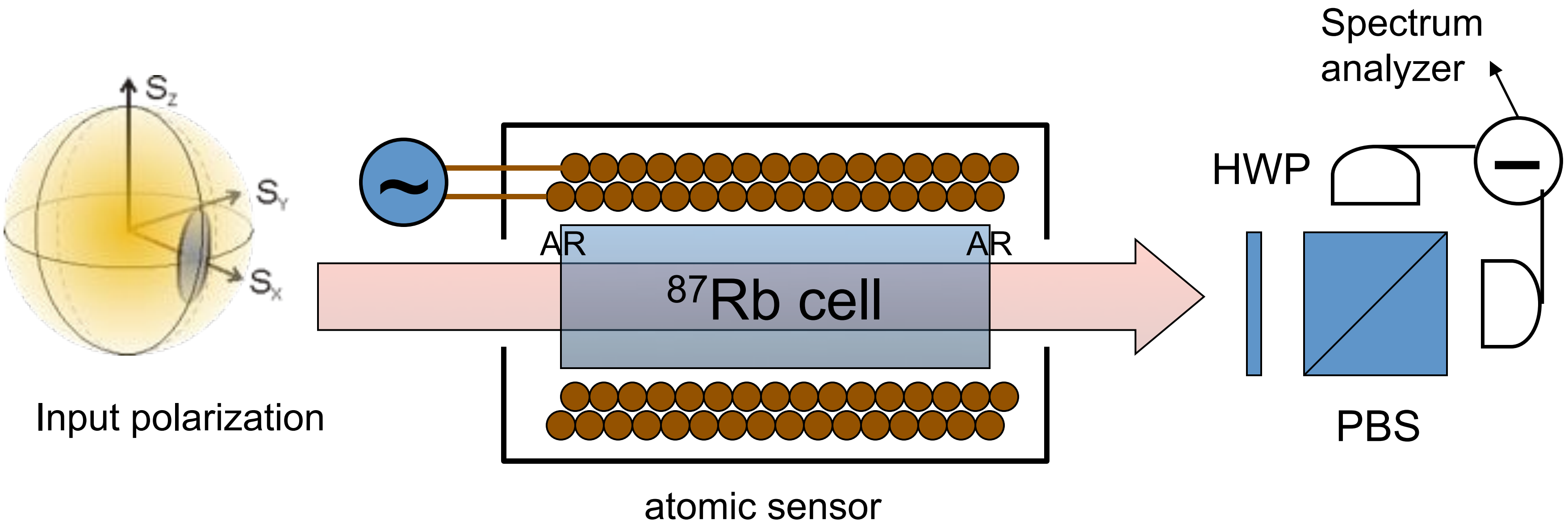


PPKTP OPO
cavity bandwidth 8 MHz
Parametric gain 4.6

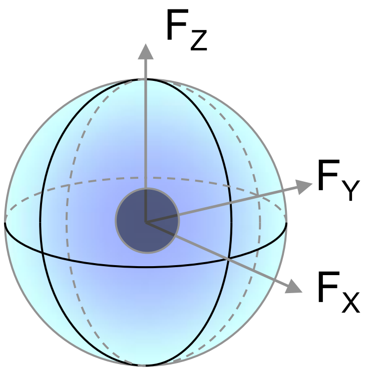


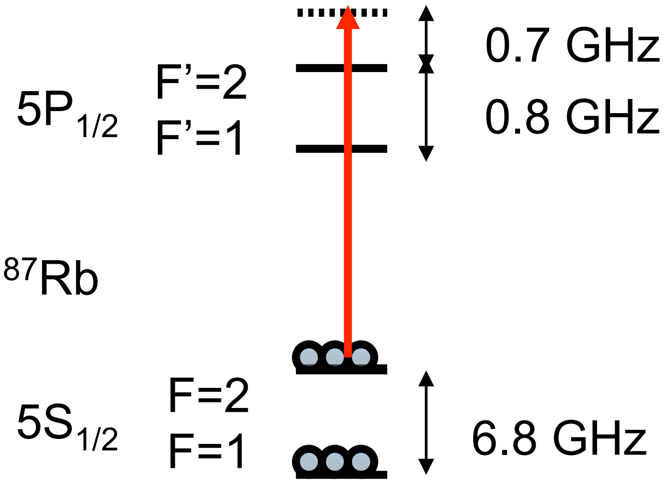
Shot-noise limited
balanced polarimeter

Prototype optical magnetometer

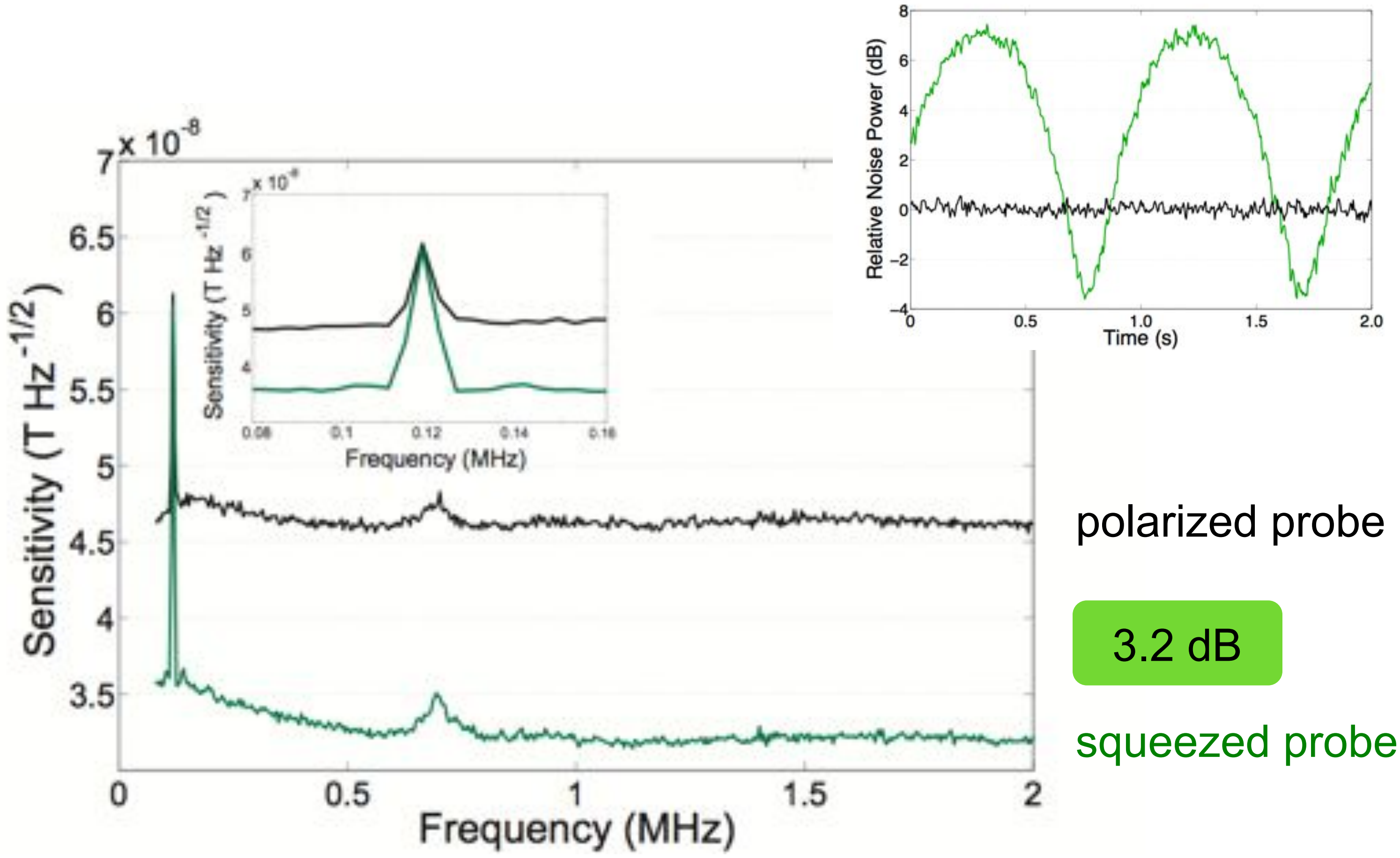


^{87}Rb purity	> 99%
Temperature:	21°
Atomic state:	thermal
Buffer gas:	none
Cell coating:	none
Optical losses:	4%
Probe power:	620 μW
Probe waist:	950 μm





Improved SNR with squeezing



squeezer ON

squeezer OFF

3.6 dB

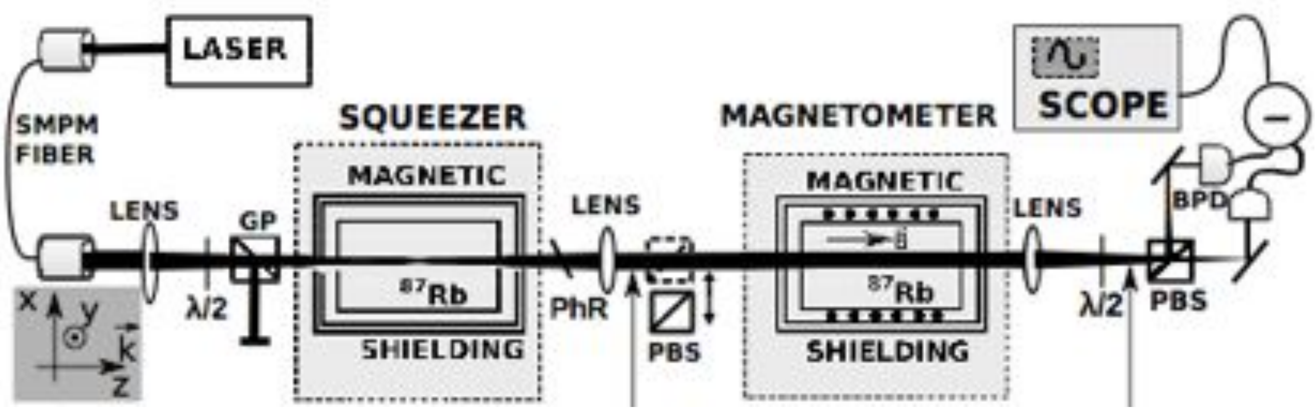
polarized probe

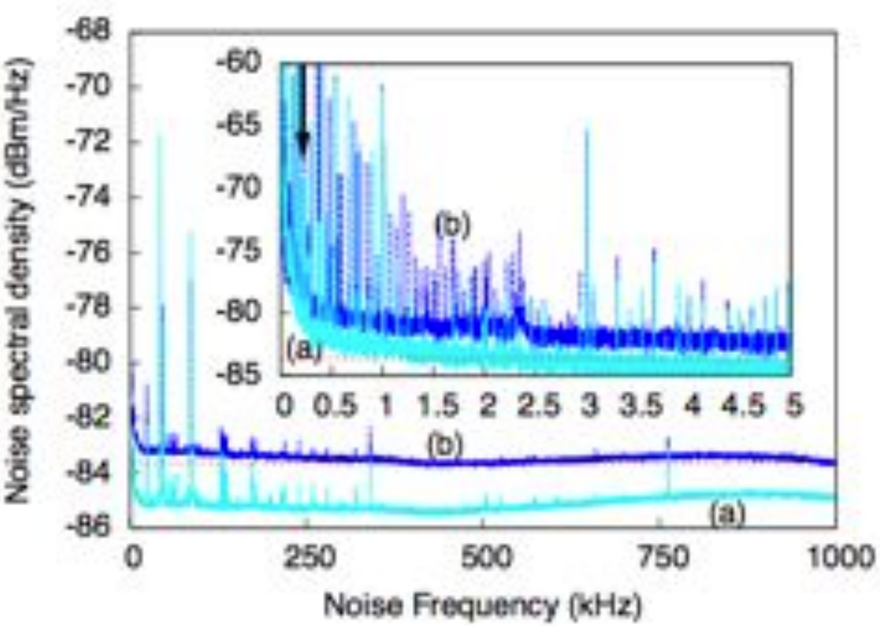
3.2 dB

squeezed probe

Wolfgramm, Cerè, Beduini, Predojević, Koschorreck, MWM Phys. Rev. Lett. 105, 053601 (2010)

Another squeezed-light magnetometer





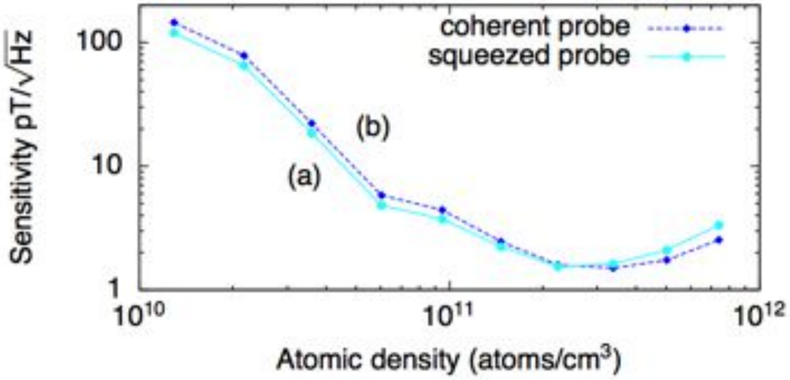
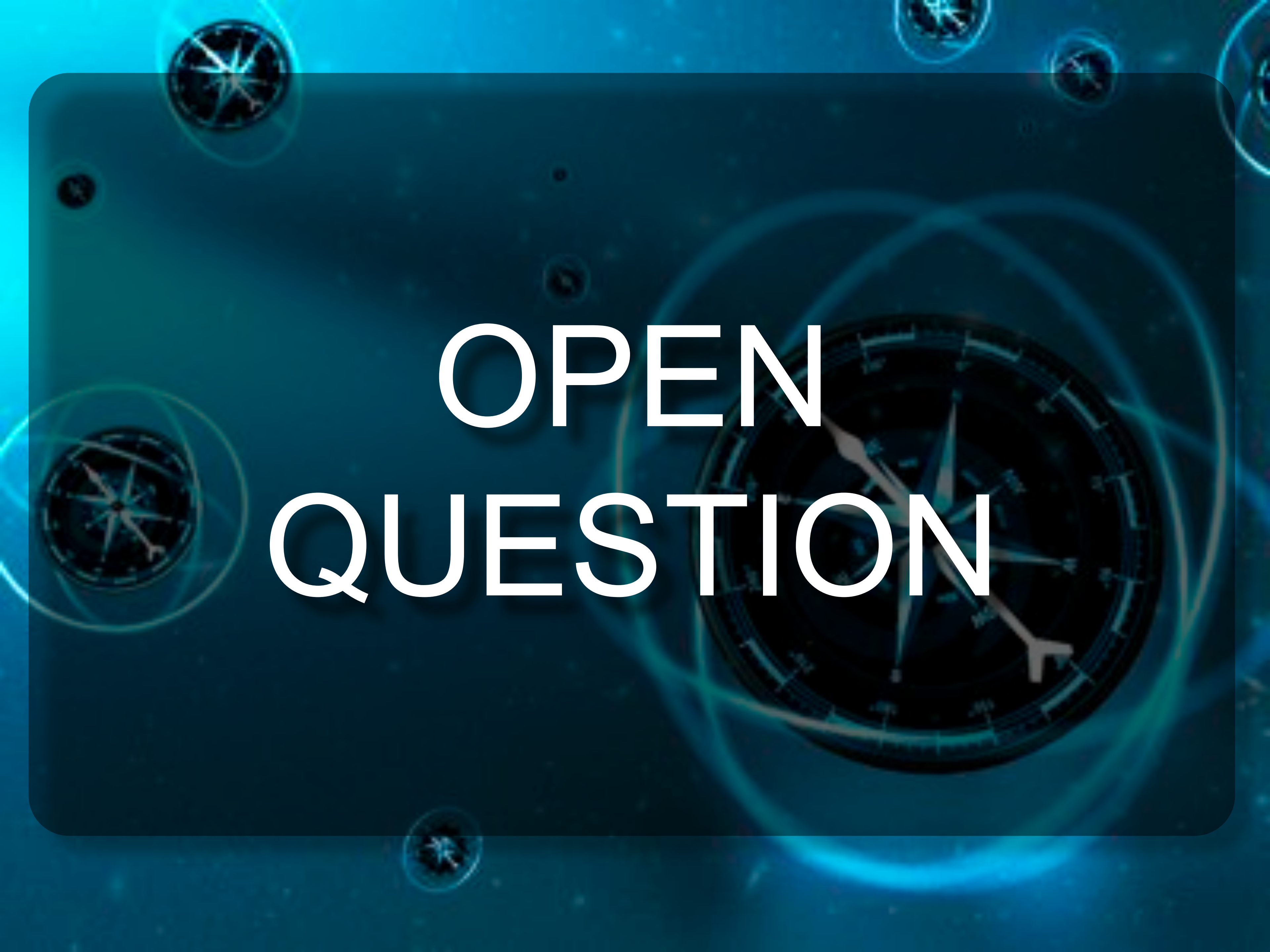


FIG. 8. (Color online) NMOR magnetometer sensitivity as a function of the atomic density with polarization-squeezed (a) and coherent (b) (shot-noise-limited) optical probes. Errorbars are smaller than the size of the markers. Laser probe power is 6 mW. Detection frequency is 500 kHz.

Horrom, Singh, Dowling and Mikhailov PRA (2012)



OPEN QUESTION

Another squeezed-light magnetometer

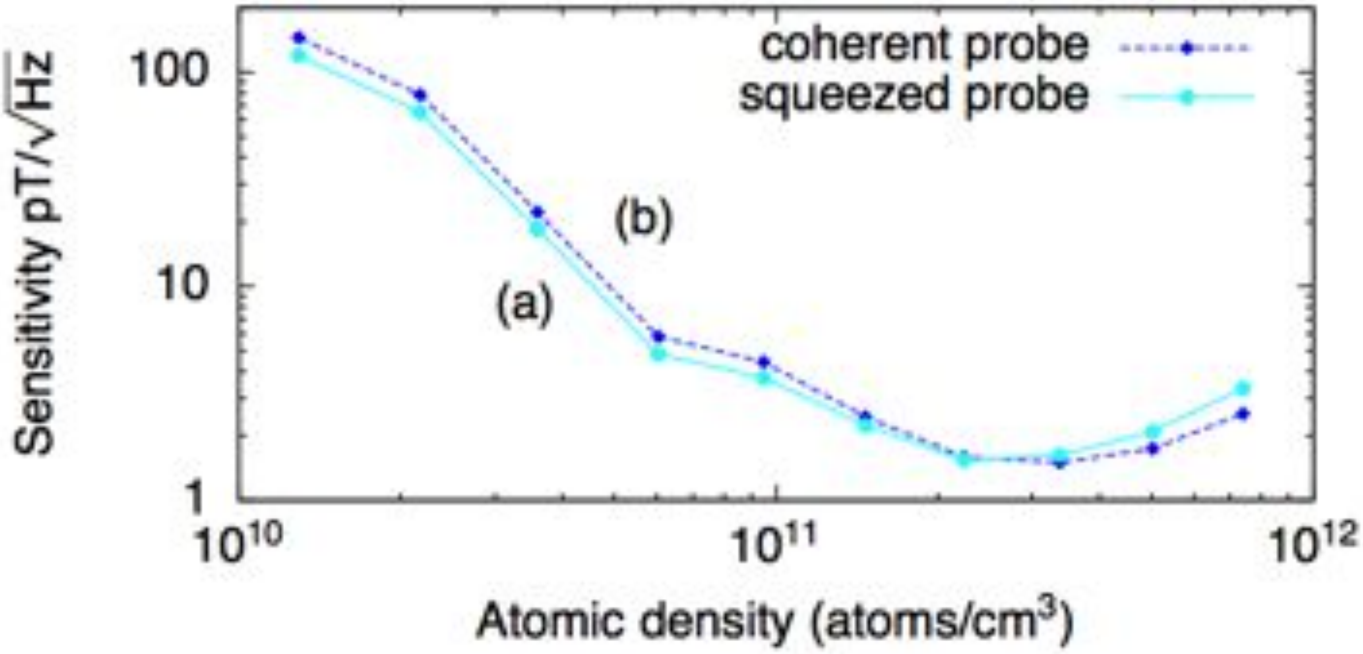


FIG. 8. (Color online) NMOR magnetometer sensitivity as a function of the atomic density with polarization-squeezed (a) and coherent (b) (shot-noise-limited) optical probes. Errorbars are smaller than the size of the markers. Laser probe power is 6 mW. Detection frequency is 500 kHz.

Horrom, Singh, Dowling and Mikhailov PRA (2012)

SYSTEM: HOT ATOMS

Another squeezed-light magnetometer

Is this a “quantum science implementation” ?

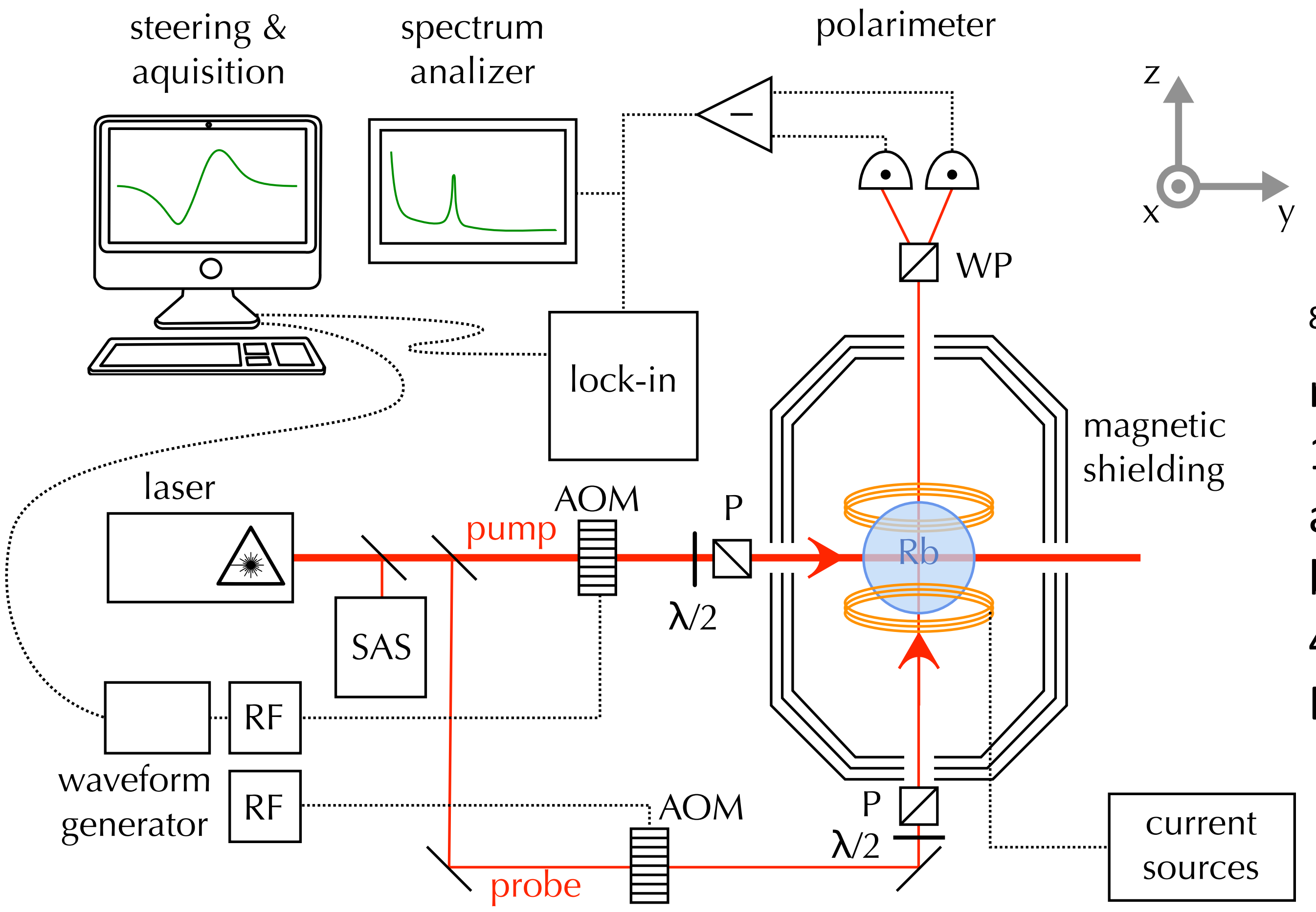
Vapor cell
- High density → signal ☺, collisions ☺/☹
- High temperature →Doppler shifts ☹, thermal state ☹

Optical quantum noise
- Beam shape

Atomic quantum noise
- Multi-level system
- Diffuse in/out of beam

Magnetometer functioning
- Modulation strategies
- Sensitivity / dynamic range / bandwidth

Toward a record sensitivity with squeezing

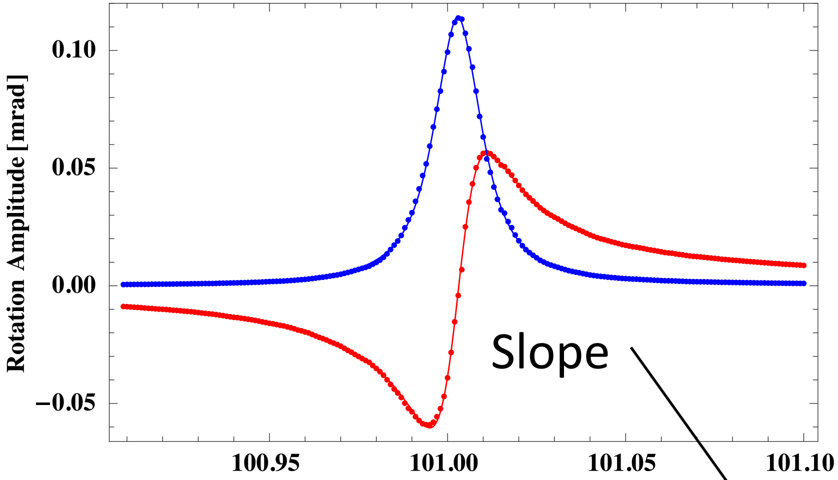


^{85}Rb
room temp
10 cm diameter
anti-relaxation coating
box solenoid
4-layer shielding
D_1 line

Gawlik group, Krakow

Toward a record sensitivity with squeezing

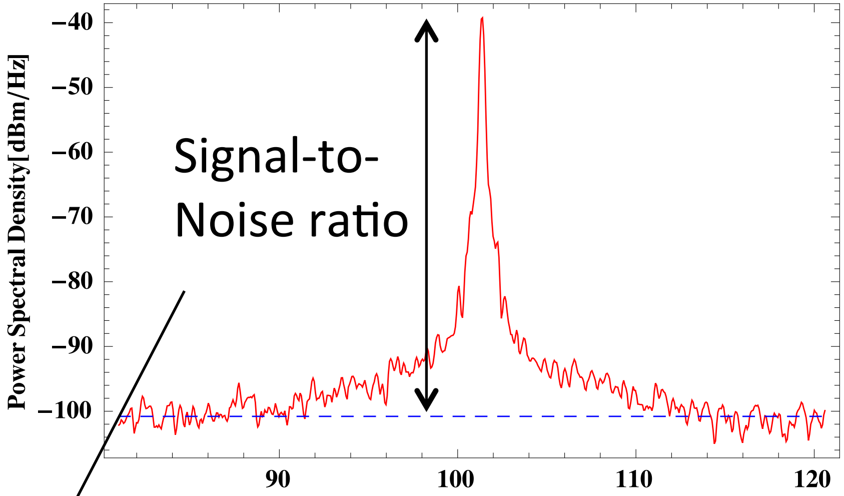
Rotation (sine & cosine)



Slope

mod. frequency (kHz)

Detected power

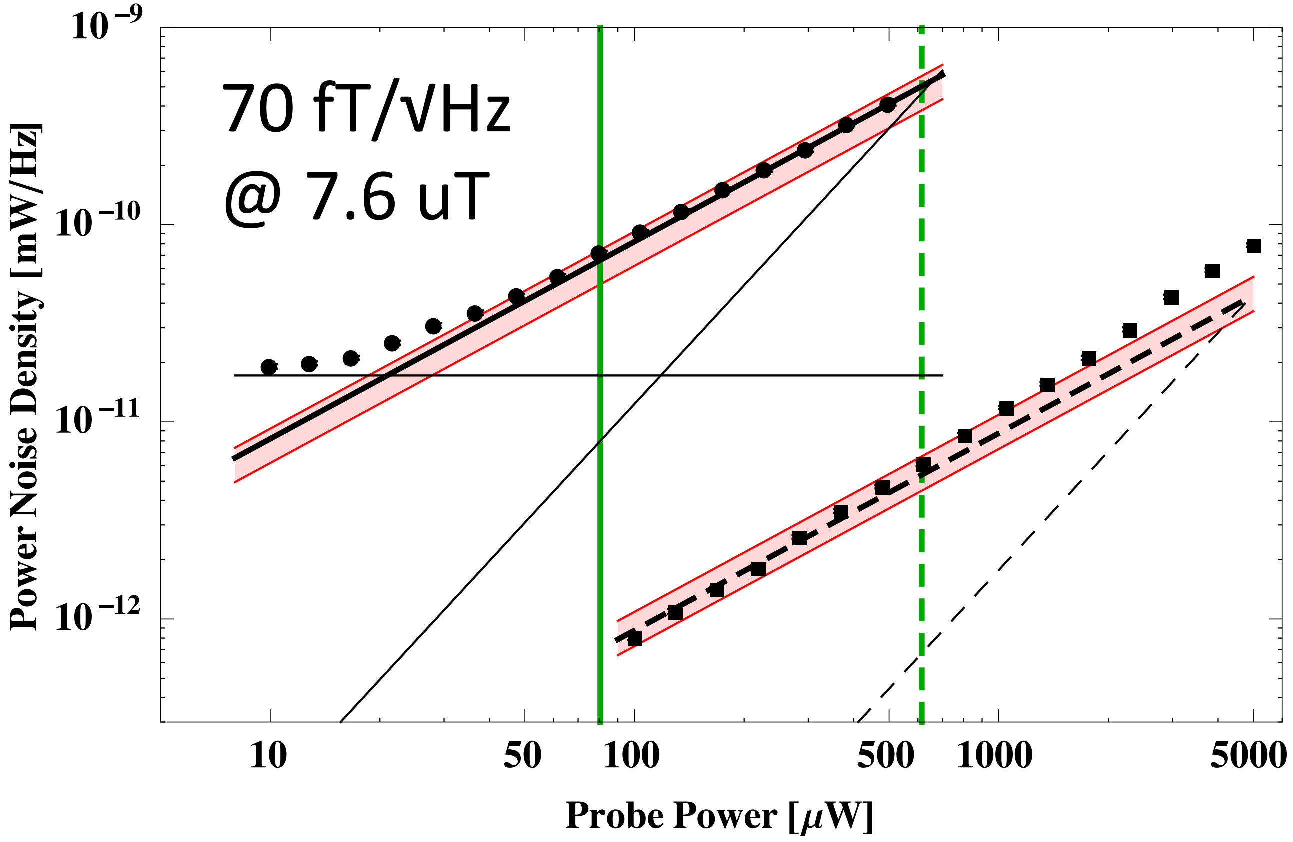


Signal-to-Noise ratio

Detection Frequency (kHz)

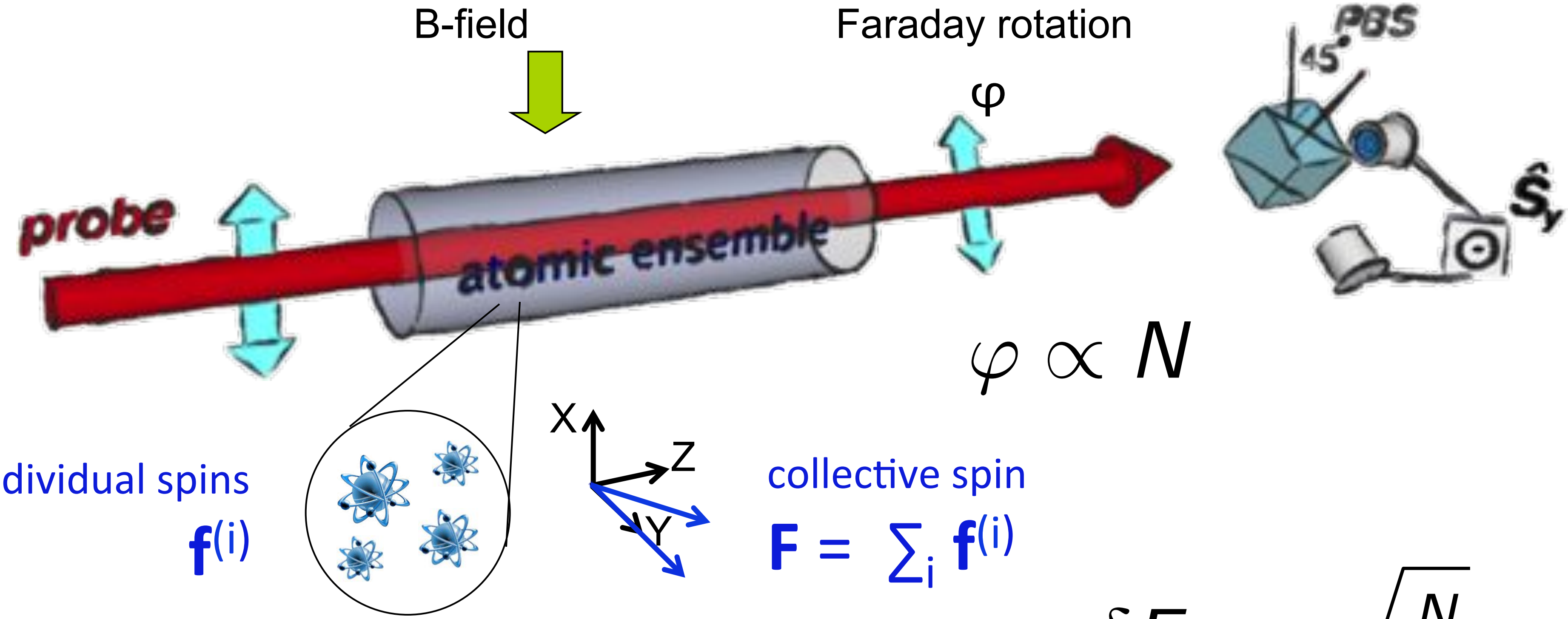
Sensitivity (T/√Hz)

Toward a record sensitivity with squeezing



SYSTEM: COLD ATOMS

Faraday rotation optical magnetometer



B-field

Faraday rotation

φ

$$\varphi \propto N$$

Individual spins

$\mathbf{f}^{(i)}$

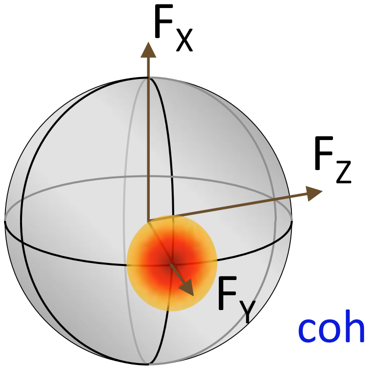
collective spin

$\mathbf{F} = \sum_i \mathbf{f}^{(i)}$

$$[F_x, F_y] = i\, F_z$$

and cycl. permutations

$$\delta F_x\, \delta F_z \geq \tfrac{1}{2}\, |\langle F_y \rangle|$$



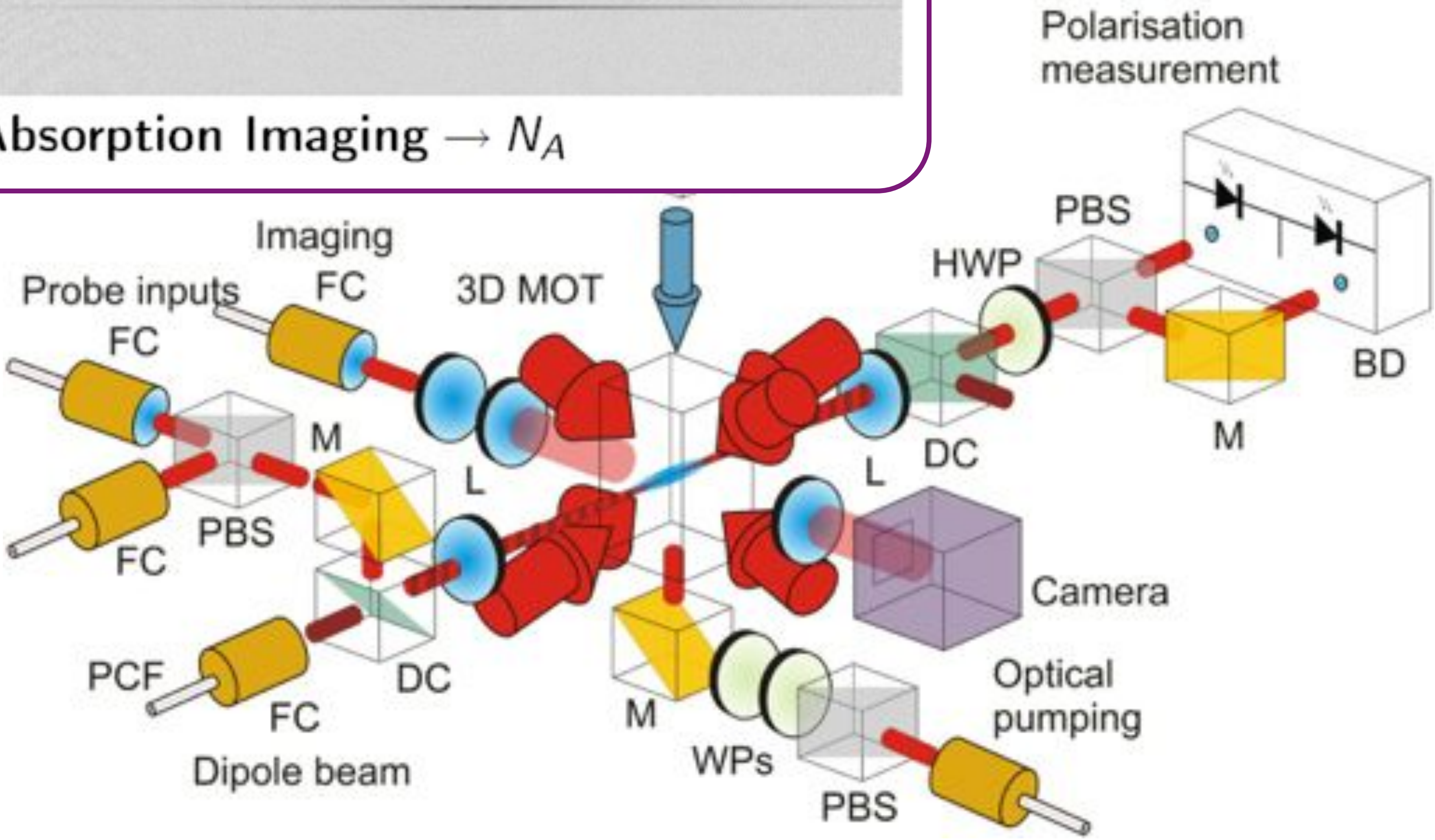
coherent
spin state

$$\delta F_z = \sqrt{\frac{N}{2}}$$

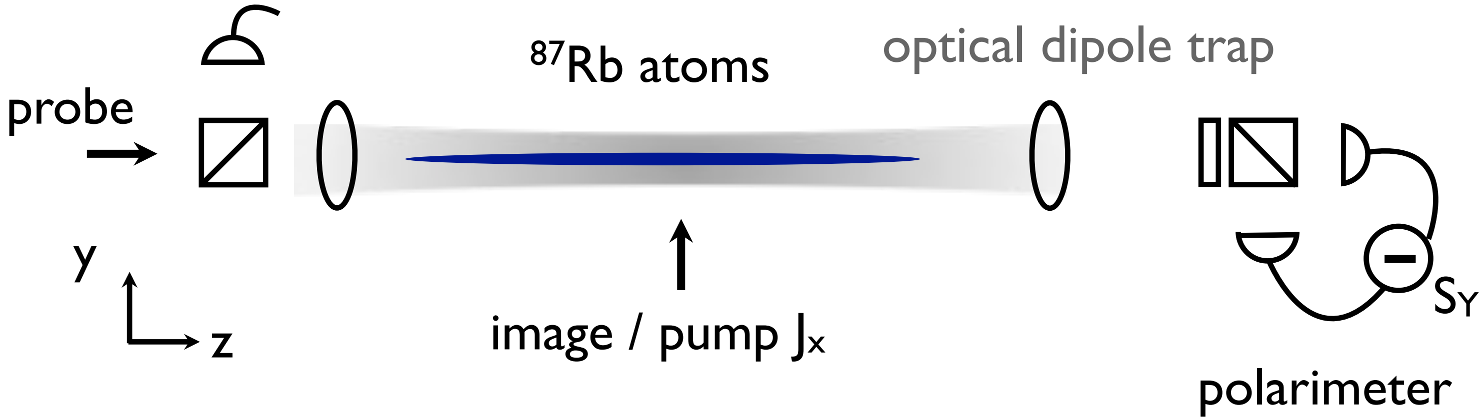
standard quantum limit
(F=1)

Cold atom magnetometer

Absorption Imaging $\rightarrow N_A$



Quantum interface with cold ^{87}Rb ensemble



1 μs long pulses
linearly polarized
"mode matched" to atoms
0.7 GHz from D_2 line

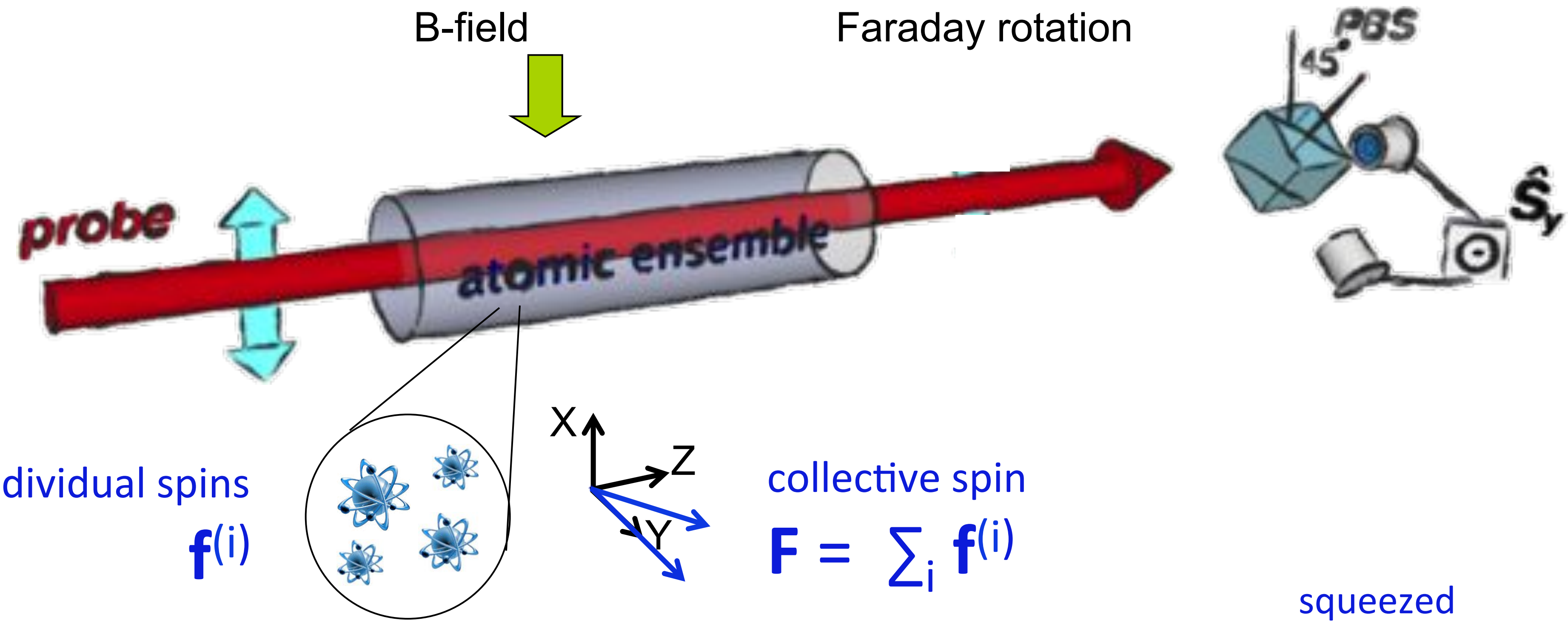
[1] effective OD > 50
[2] Sensitivity 512 spins, < SQL
[3] QND measurement
[4] spin squeezing

~10^6 ^{87}Rb atoms at 25μK
f=1 ground-state

1 Kubasik, et al. PRA 79, 043815 (2009)
2 Koschorreck, et al. PRL (2010)
3 Koschorreck, et al. PRL (2010),
Sewell, et al. N. Phot. (2013)
4 Sewell, et al. arXiv (2011)

SQUEEZING BY QUANTUM NON-DEMOLITION

Faraday rotation optical magnetometer



Individual spins

$\mathbf{f}^{(i)}$

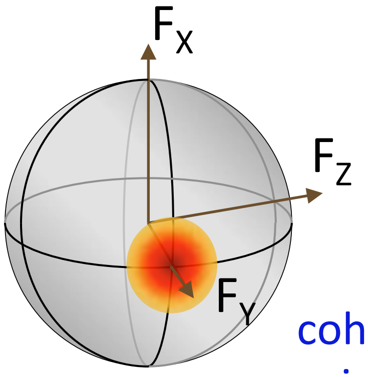
collective spin

$\mathbf{F} = \sum_i \mathbf{f}^{(i)}$

$[F_x,F_y] = i\, F_z$

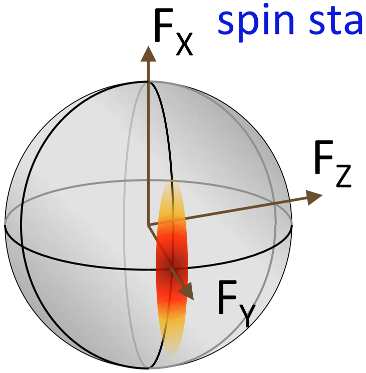
and cycl. permutations

$\delta F_x\, \delta F_z \geq ½\ |\langle F_y\rangle|$

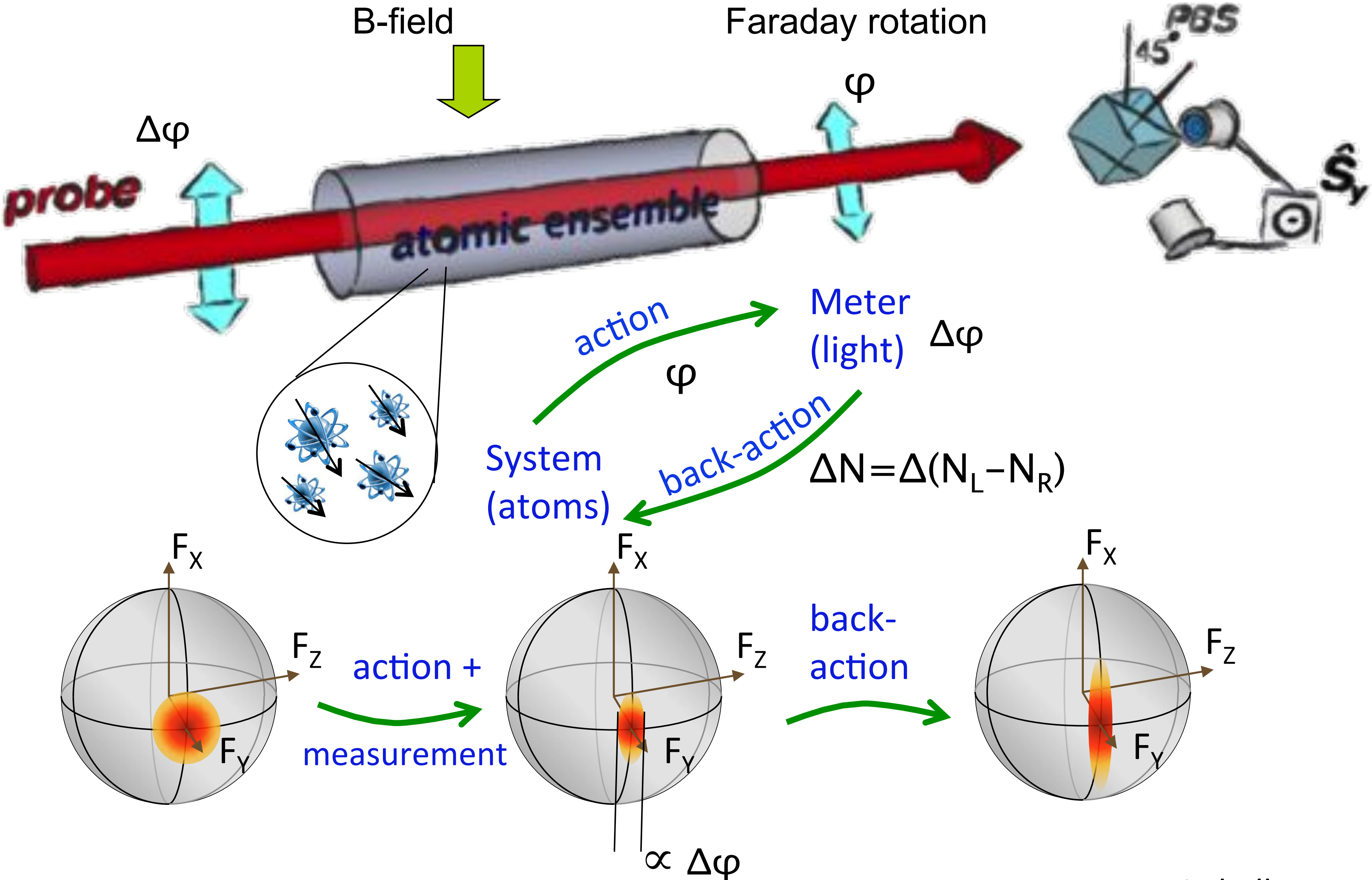


coherent spin state

squeezed spin state



QND in optical magnetometer



B-field

Faraday rotation

φ

$\Delta\varphi$

action

φ

Meter (light)

$\Delta\varphi$

System (atoms)

back-action

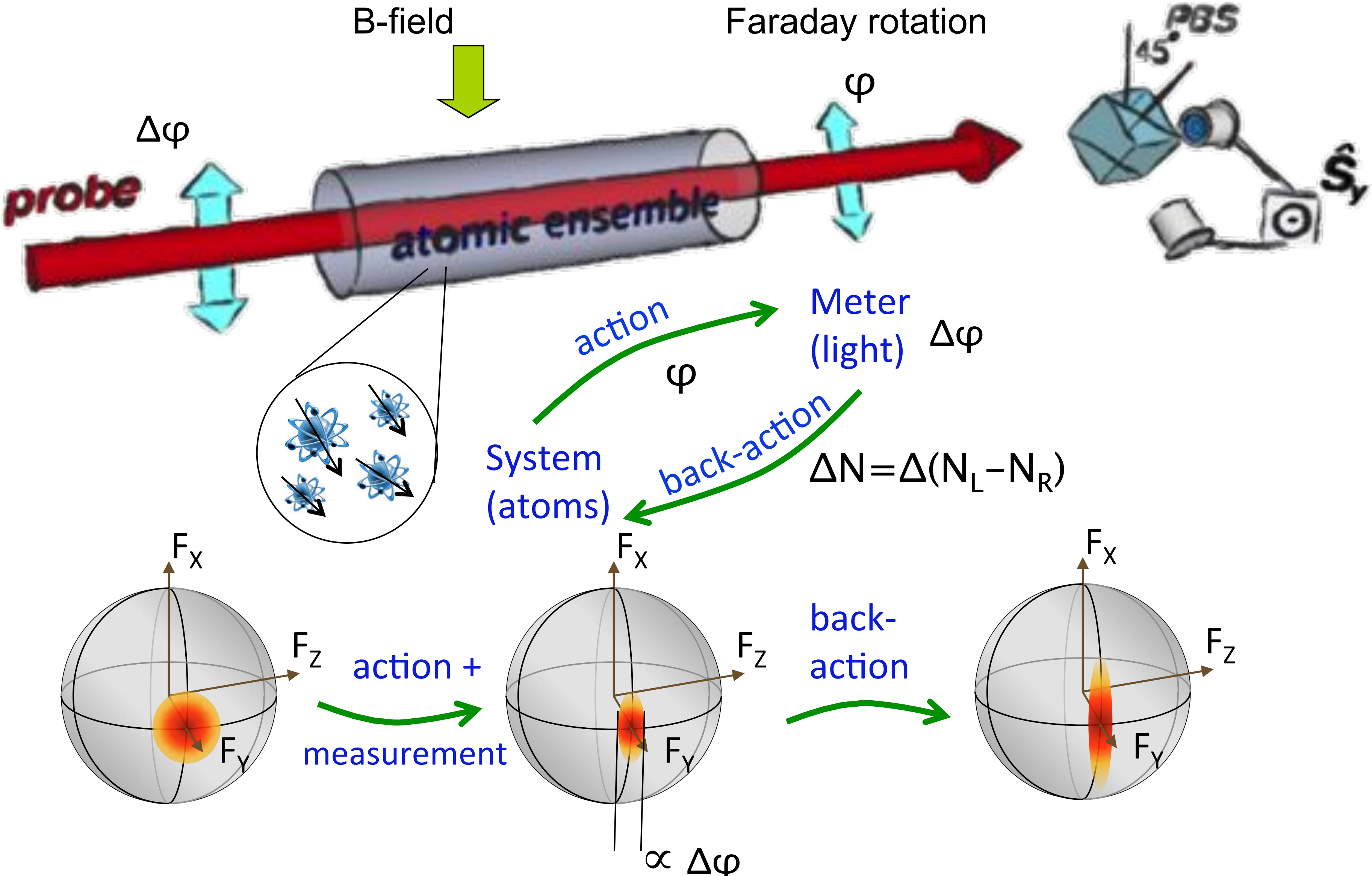
$\Delta N = \Delta(N_L - N_R)$

action + measurement

back-action

$\propto \Delta\varphi$

QND in optical magnetometer



B-field

Faraday rotation

φ

$\Delta\varphi$

action

φ

Meter (light)

$\Delta\varphi$

System (atoms)

back-action

$\Delta N = \Delta(N_L - N_R)$

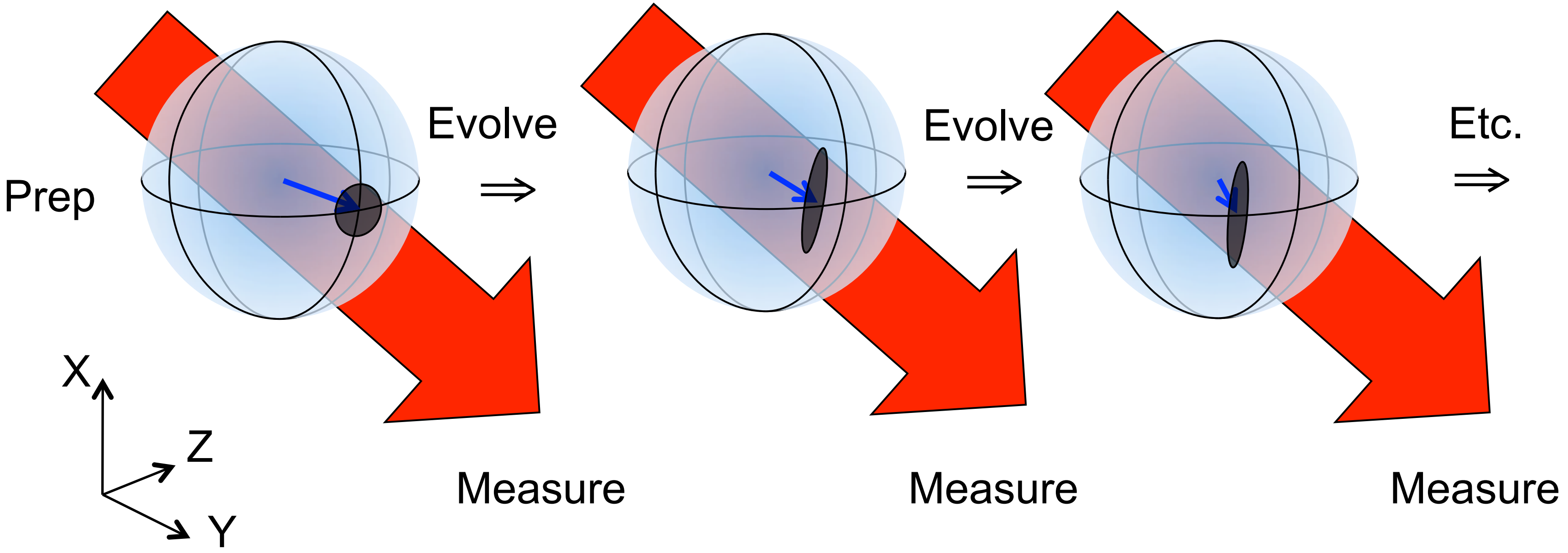
F_X, F_Z, F_Y

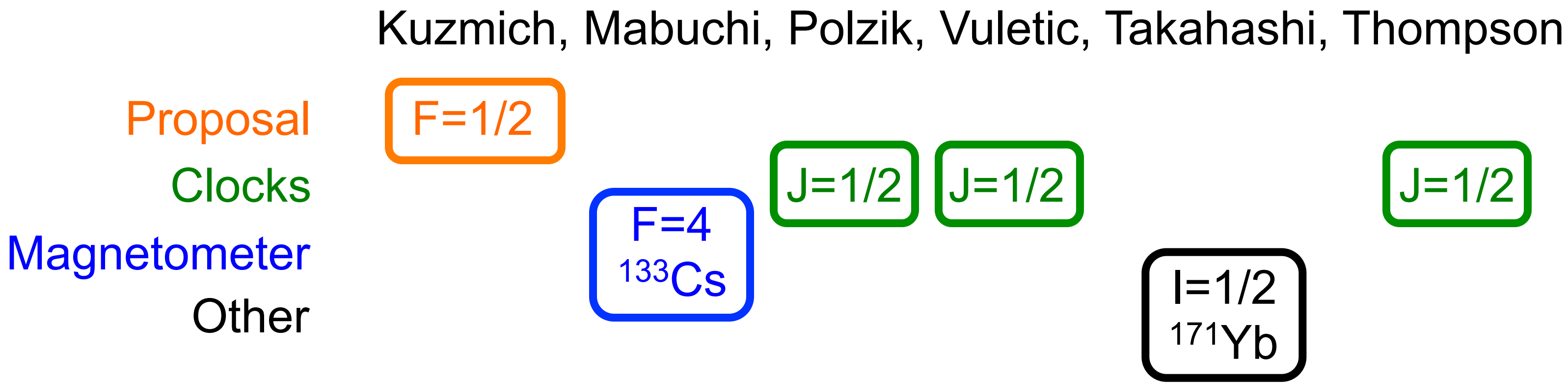
action + measurement

back-action

$\propto \Delta\varphi$

Measurement-induced squeezing





Kuzmich, Mabuchi, Polzik, Vuletic, Takahashi, Thompson

Proposal
Clocks
Magnetometer
Other

F=1/2

F=4
^{133}Cs

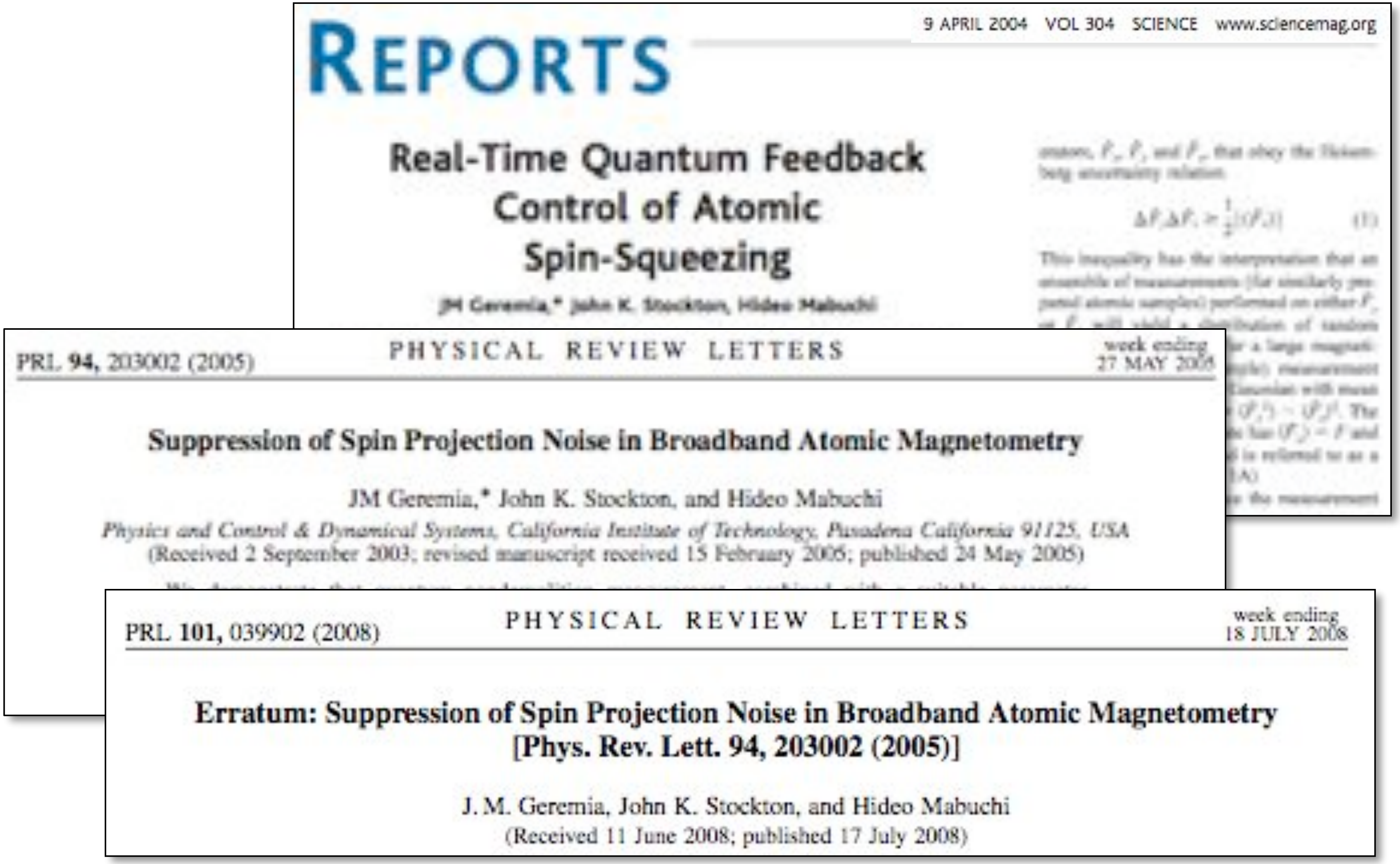
J=1/2

J=1/2

I=1/2
^{171}Yb

J=1/2

To boldly go where others have gone before



9 APRIL 2004 VOL 304 SCIENCE www.sciencemag.org

REPORTS

Real-Time Quantum Feedback Control of Atomic Spin-Squeezing

JM Geremia,* John K. Stockton, Hideo Mabuchi

PRL 94, 203002 (2005) PHYSICAL REVIEW LETTERS week ending 27 MAY 2005

Suppression of Spin Projection Noise in Broadband Atomic Magnetometry

JM Geremia,* John K. Stockton, and Hideo Mabuchi

Physics and Control & Dynamical Systems, California Institute of Technology, Pasadena California 91125, USA

(Received 2 September 2003; revised manuscript received 15 February 2005; published 24 May 2005)

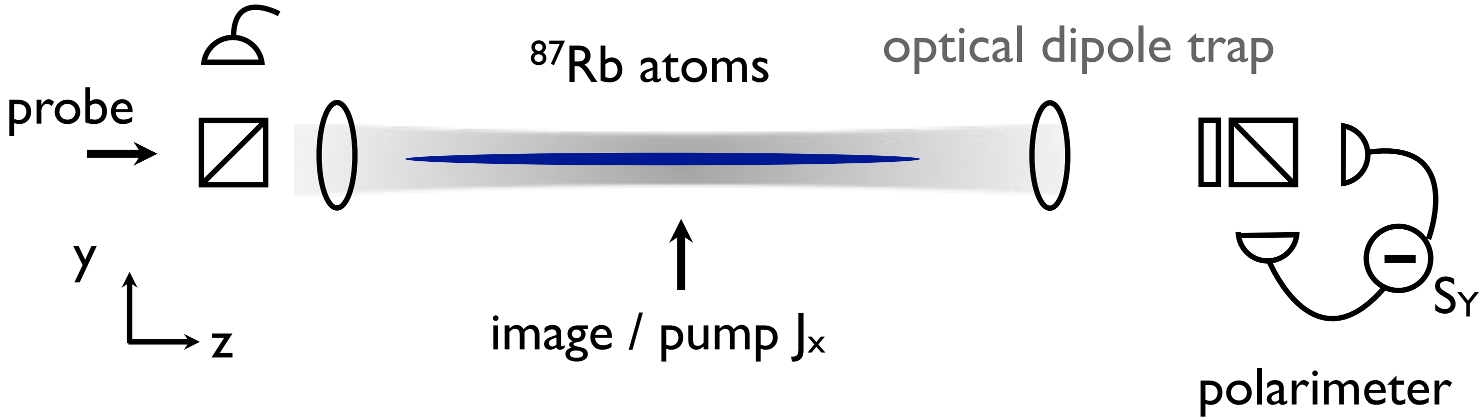
PRL 101, 039902 (2008) PHYSICAL REVIEW LETTERS week ending 18 JULY 2008

Erratum: Suppression of Spin Projection Noise in Broadband Atomic Magnetometry [Phys. Rev. Lett. 94, 203002 (2005)]

J. M. Geremia, John K. Stockton, and Hideo Mabuchi

(Received 11 June 2008; published 17 July 2008)

Quantum interface with cold ^{87}Rb ensemble



1μs long pulses
linearly polarized
"mode matched" to atoms
0.7 GHz from D_2 line

^{1} effective OD > 50
^{2} Sensitivity 512 spins, < SQL
^{3} QND measurement
^{4} spin squeezing

~10^6 ^{87}Rb atoms at 25μK
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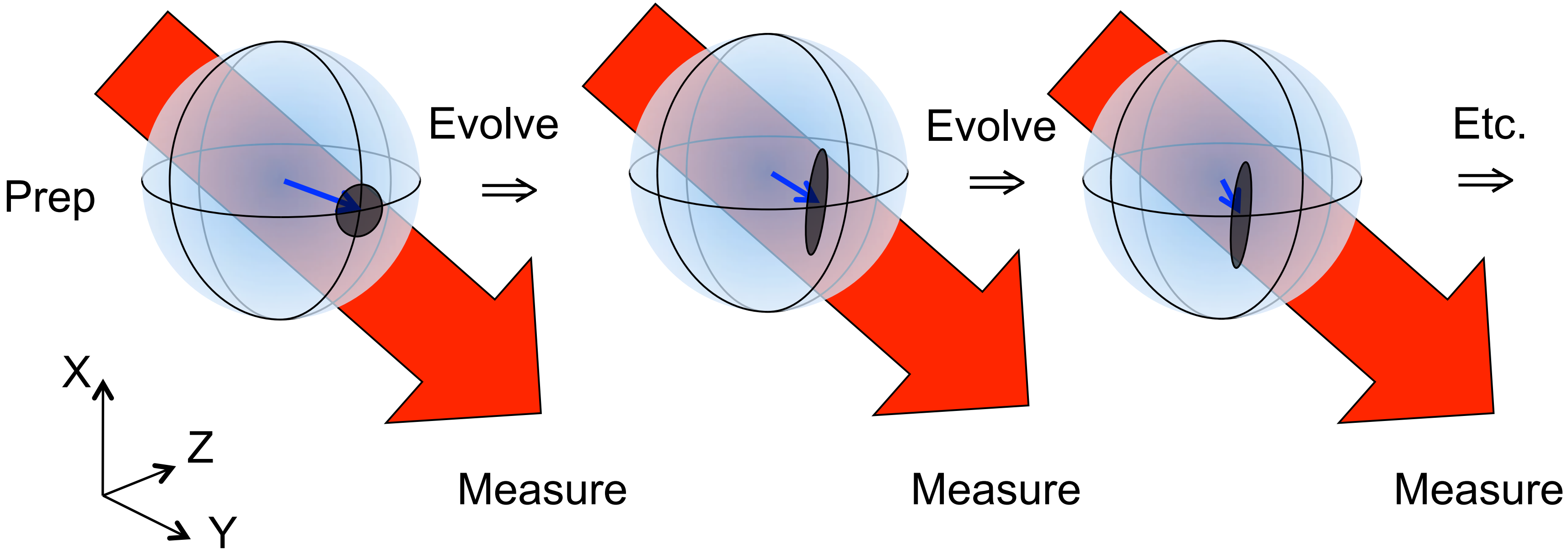
3 Koschorreck, et al. PRL (2010),
 + Sewell, et al. N. Phot. (2013)
4 Sewell, et al. PRL (2012)

Measurement-induced squeezing



Prep

Evolve ⇒

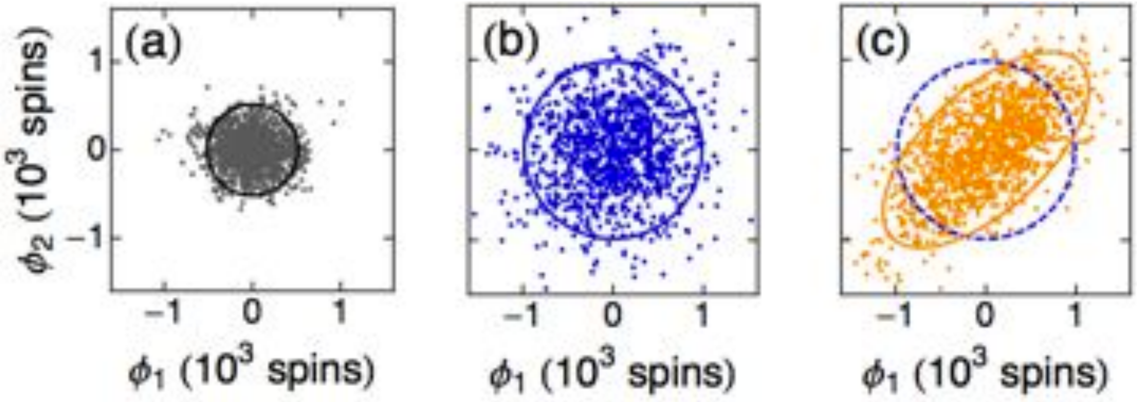
Evolve ⇒

Etc. ⇒

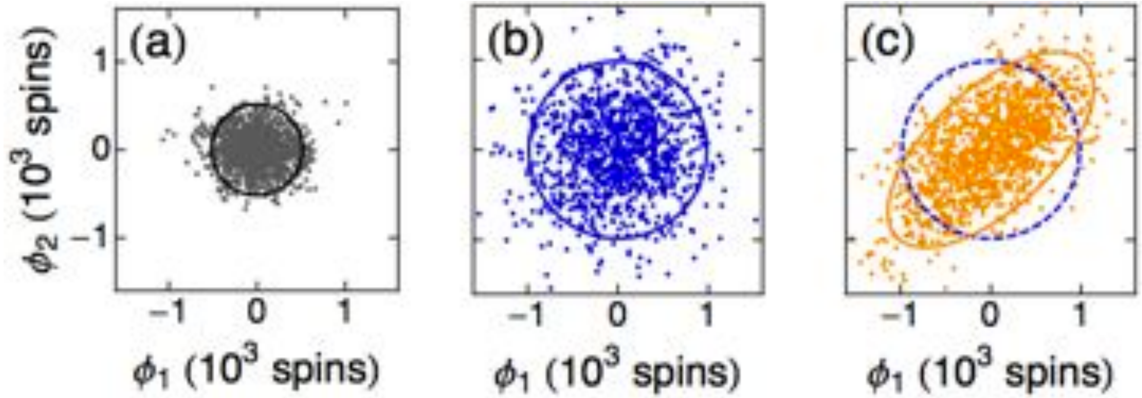
Measure

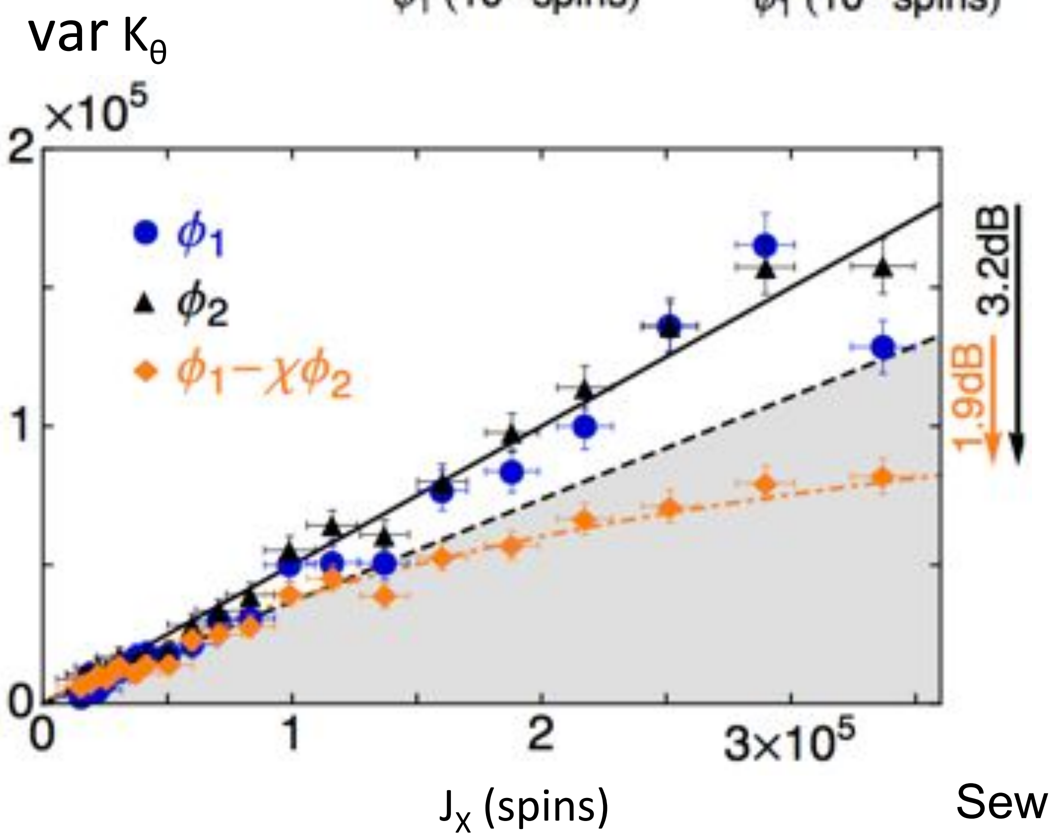
Measure

Measure



Squeezing of spin alignment-orientation

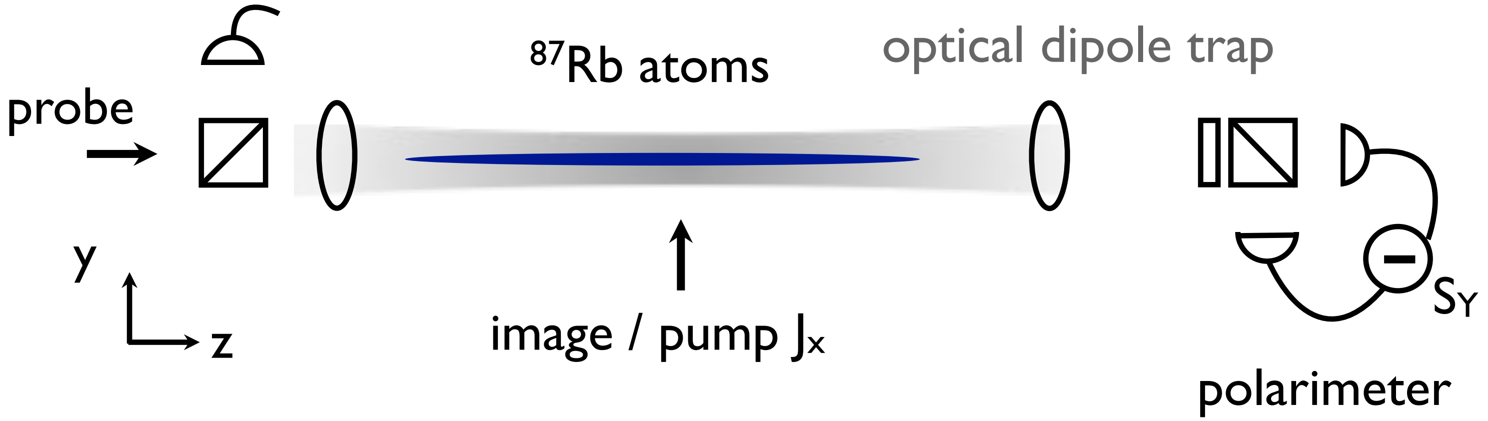


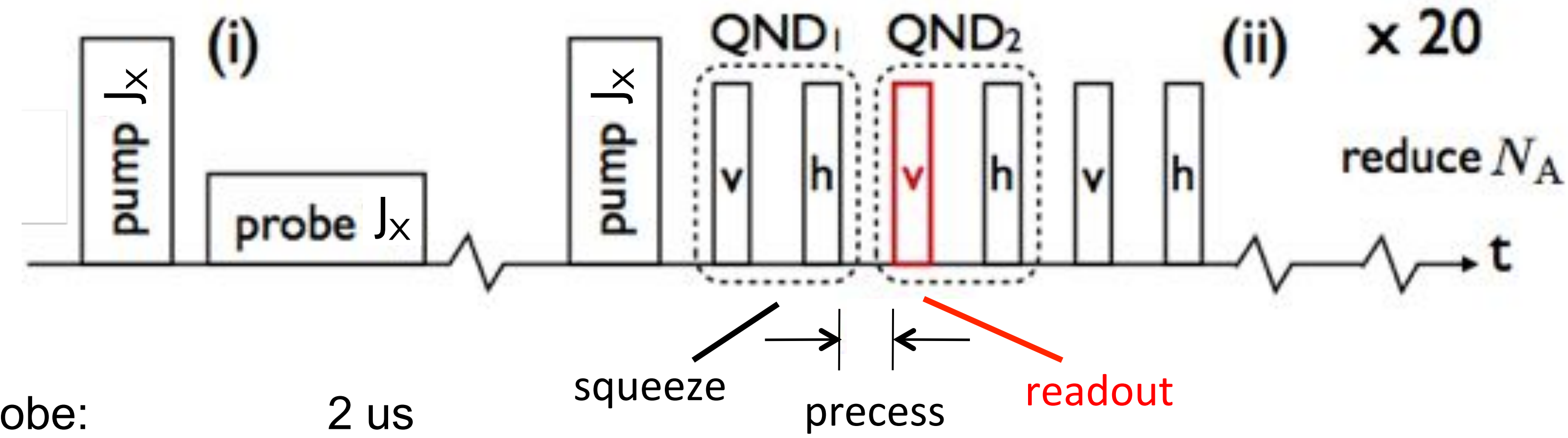


squeezing of
alignment-orientation
variable K_θ

Sewell et al. PRL **109**, 253605 (2012)

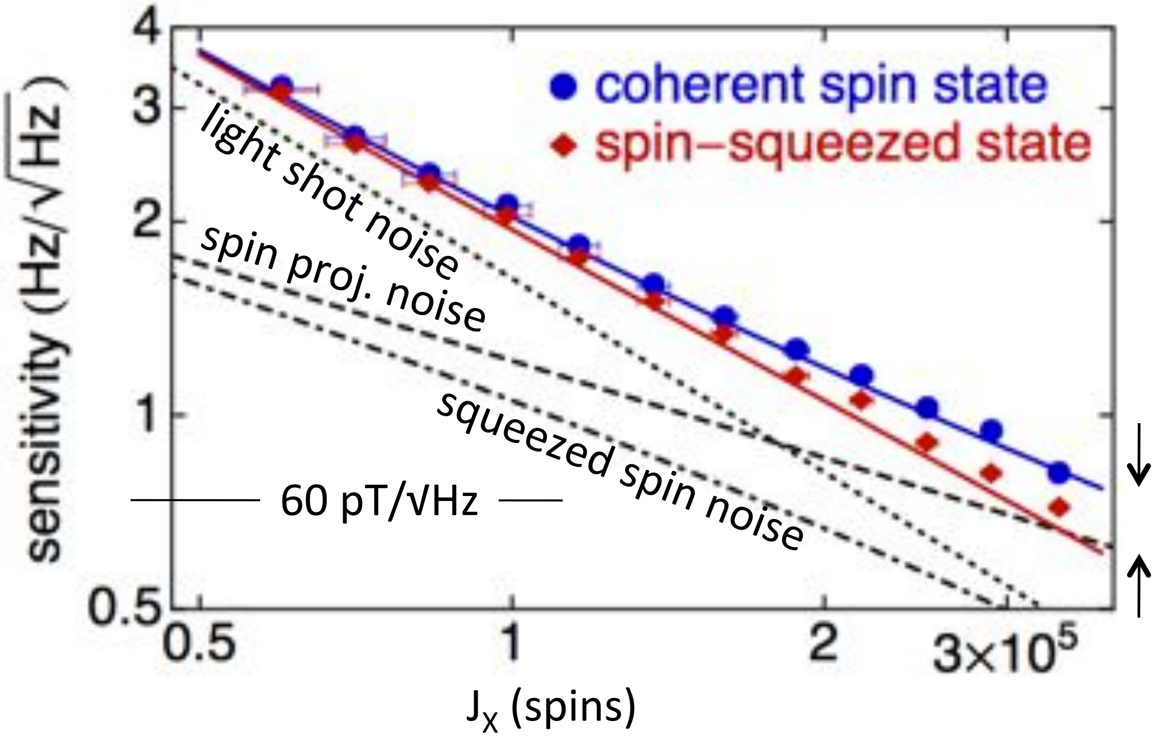
Measurement sequence

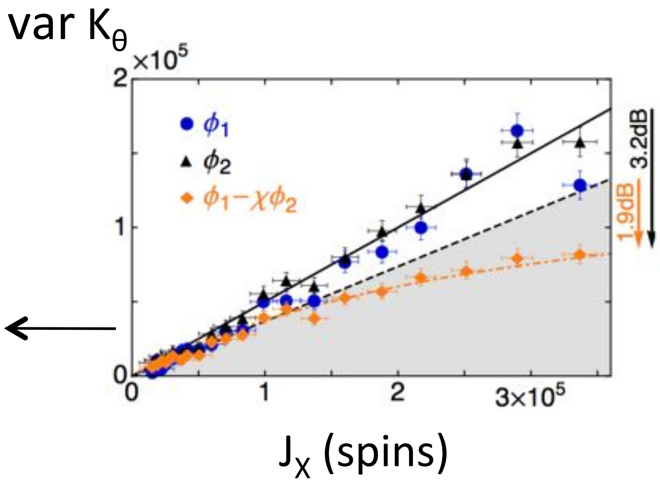




probe:	2 us
precession:	5 us
detuning:	600 MHz

Squeezed-atom magnetometry





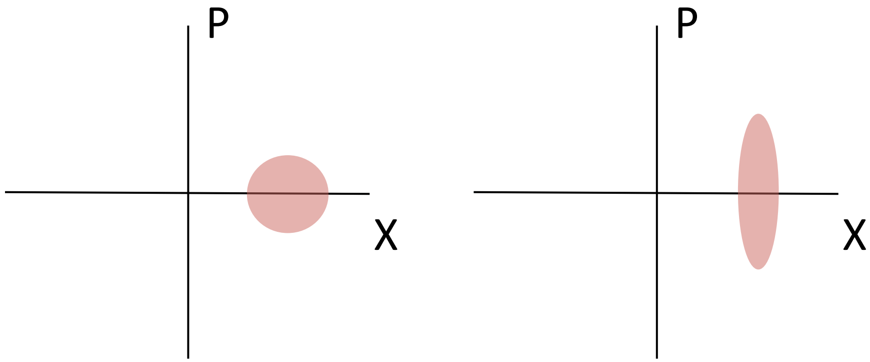
Sewell et al. PRL **109**, 253605 (2012)

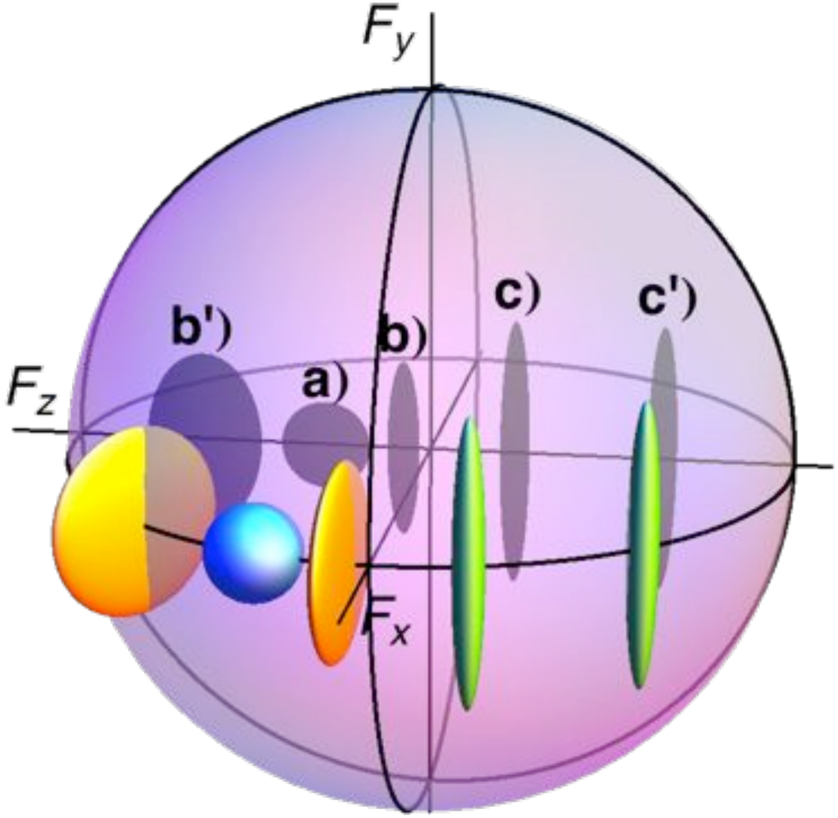
EXOTIC SQUEEZED STATES

Spin squeezing is different than light squeezing

$$[X, P] = i$$

$$\delta X \delta P \geq \frac{1}{2}$$



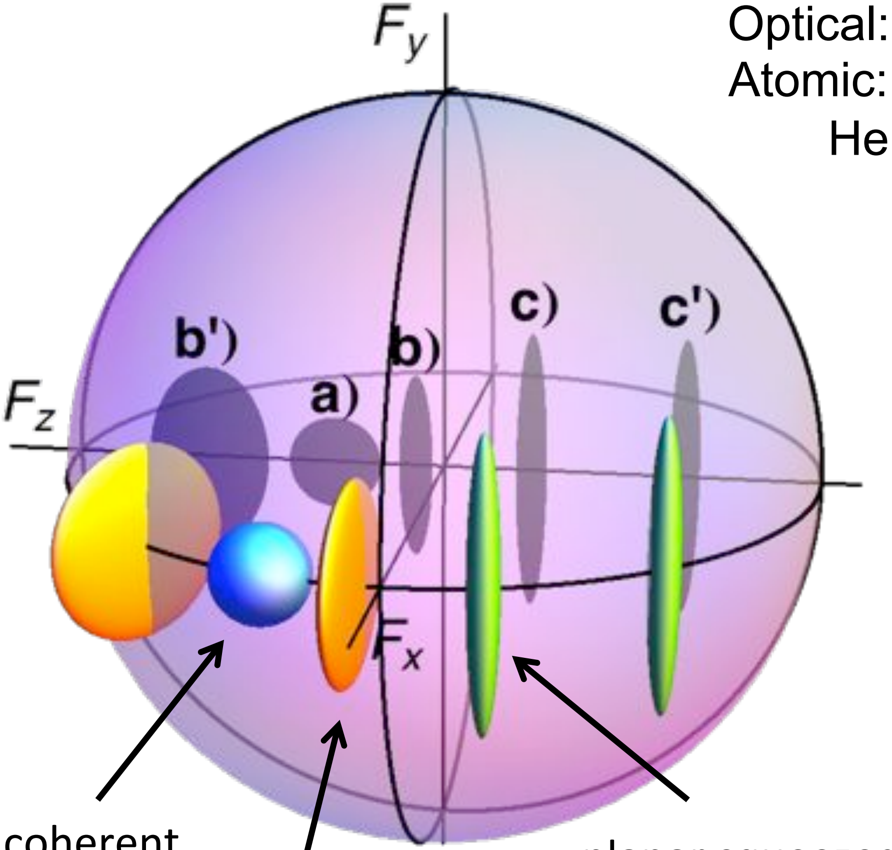


$$[F_z, F_x] = iF_y$$

$$\delta F_z \delta F_x \geq \frac{1}{2} |\langle F_y \rangle|$$

Planar squeezed states

Optical: Korolkova, Leuch, Schnabel, Bachor, Lam
Atomic: He, Peng, Drummond and Reid PRA 2011
He, Vaughan, Drummond and Reid NJP 2012



$$[F_z, F_x] = iF_y$$

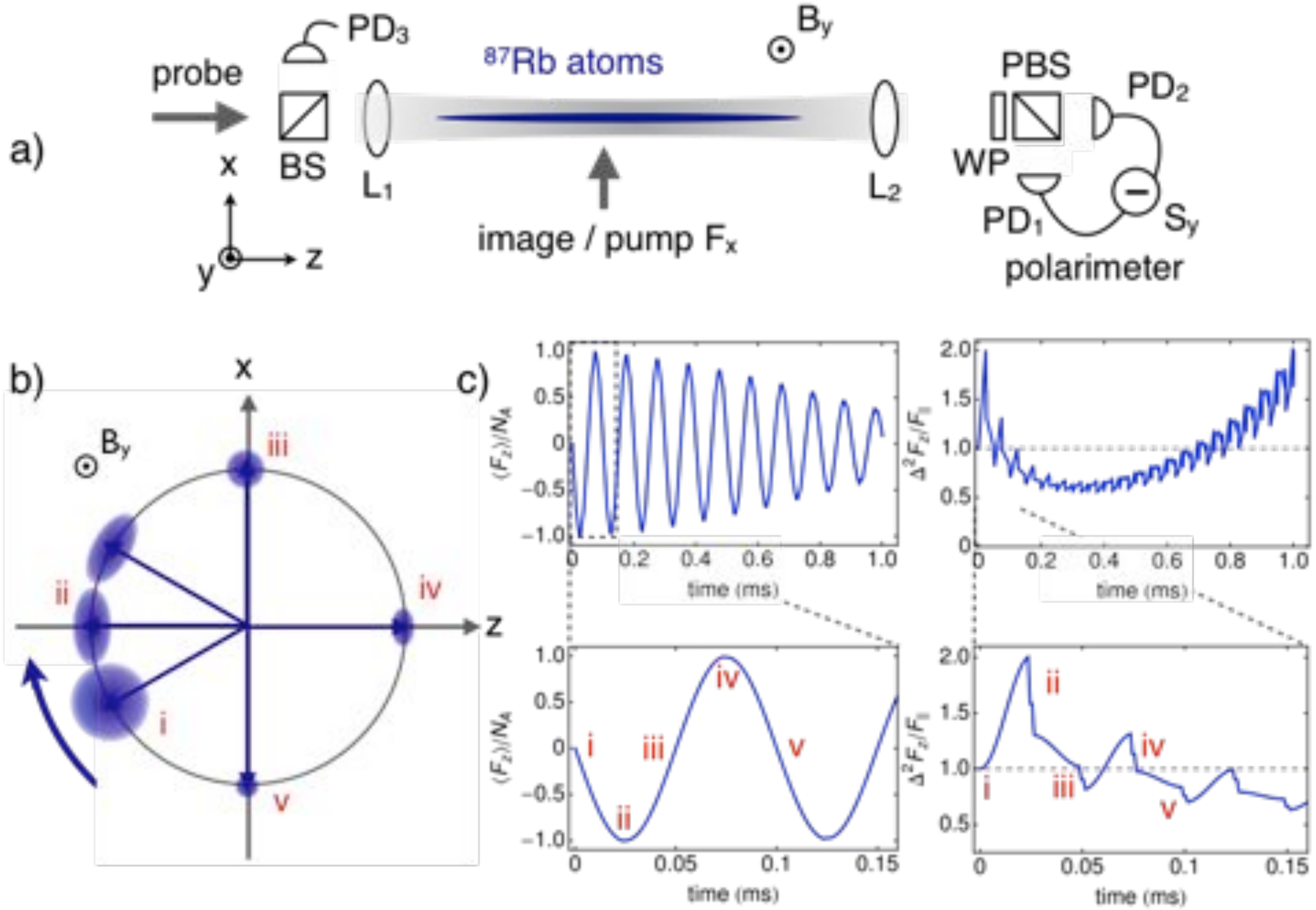
$$\delta F_z \delta F_x \geq \frac{1}{2}|\langle F_y \rangle|$$

coherent spin state

squeezed
R. Sewell PRL 2012

planar squeezed state
G. Puentes et al. NJP 2013
G. Colangelo et al. NJP 2013

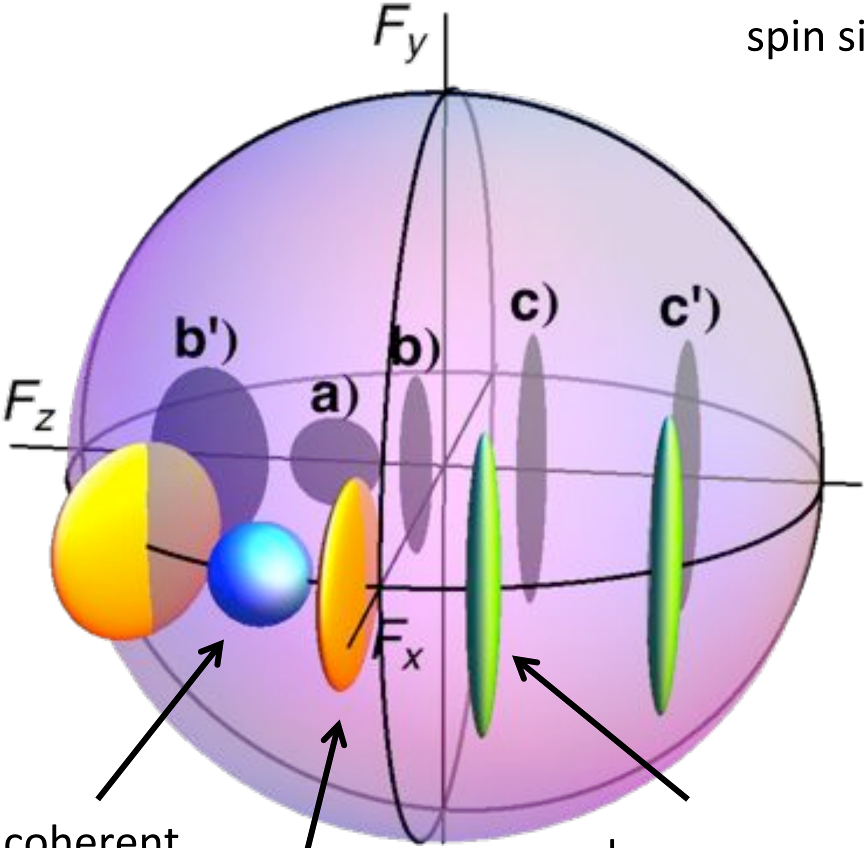
Planar squeezed states



G. Puentes et al. NJP 2013

G. Colangelo et al. NJP 2013

Beyond planar squeezing

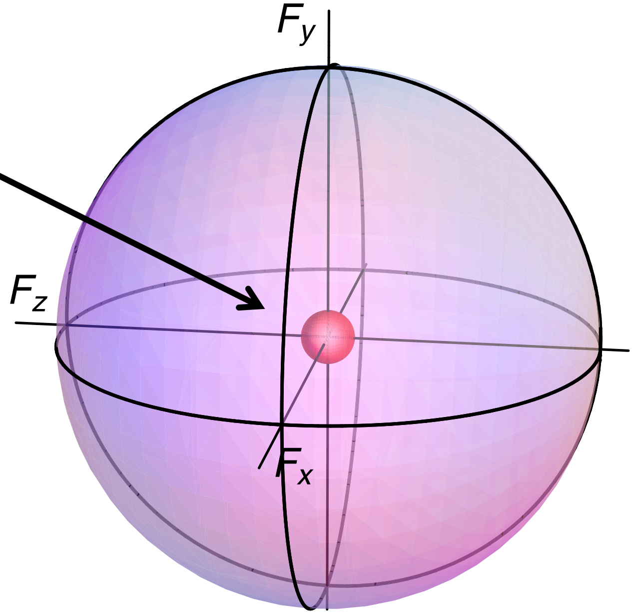


coherent spin state

squeezed
R. Sewell PRL 2012

planar squeezed state
G. Puentes et al. NJP 2013
G. Colangelo et al. NJP 2013

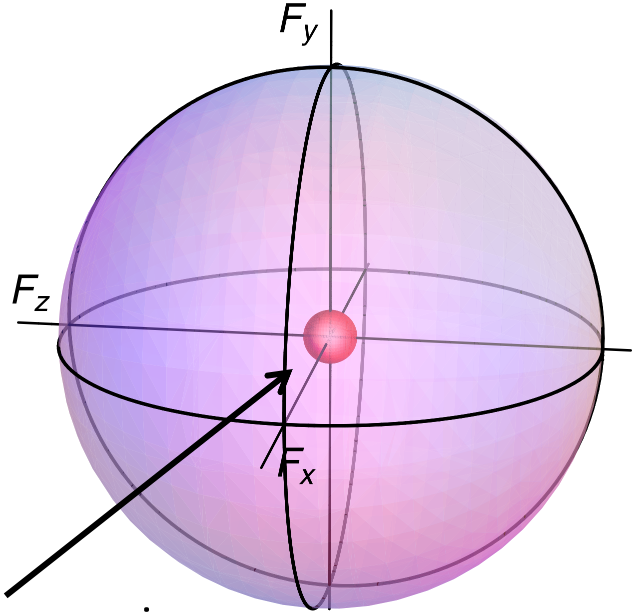
macroscopic spin singlet



$$\delta F_x \delta F_y \geq 0$$
$$\delta F_y \delta F_z \geq 0$$
$$\delta F_z \delta F_x \geq 0$$

G. Toth, MWM, NJP **12** 053007 (2010)
Phys. Rev. A 87, 021601(R) (2013)

Measurement-based spin entanglement



macroscopic
spin singlet

$$\delta F_x \delta F_y \geq \frac{1}{2}|\langle F_z \rangle|$$

$$\delta F_x \delta F_y \geq 0$$
$$\delta F_y \delta F_z \geq 0$$
$$\delta F_z \delta F_x \geq 0$$

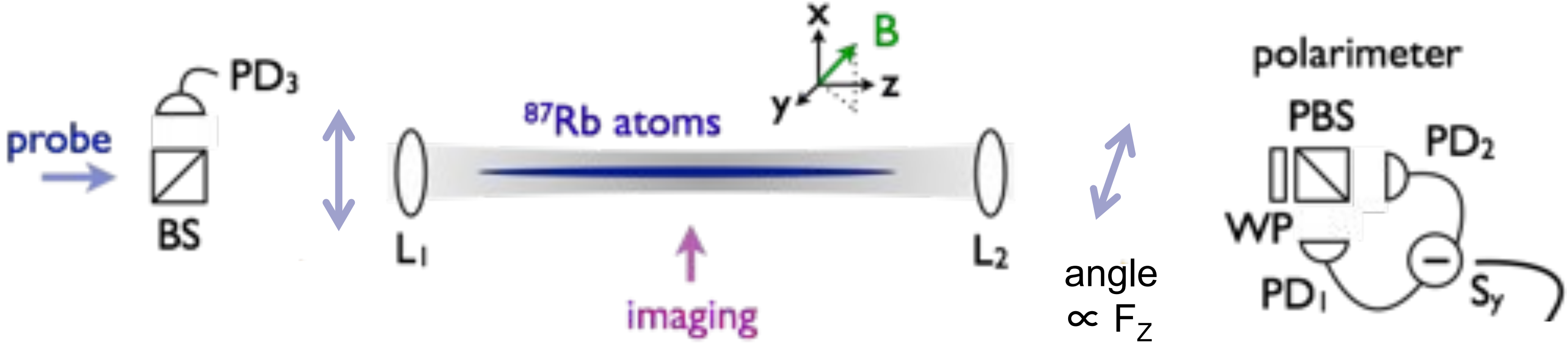
spin squeezing parameter $\xi^2 \equiv \dfrac{|\Delta \vec{F}|^2}{N_A f}$

condition for squeezing $\xi^2 < 1$

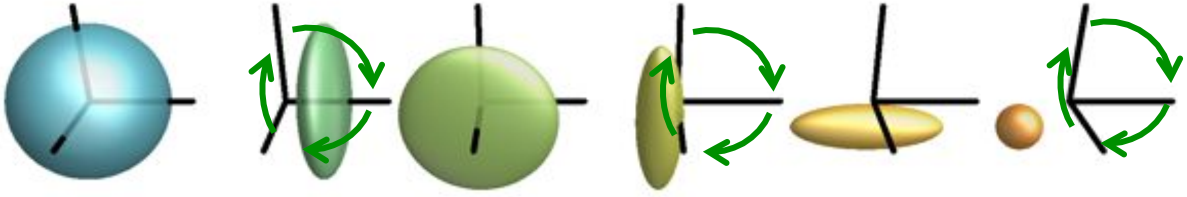
number of atoms in singlets $N_A(1 - \xi^2)$

NJP **12**, 053007 (2010)
PRA **87**, 021601(R) (2013)

Measurement-induced squeezing in 3D



angle $\propto F_z$

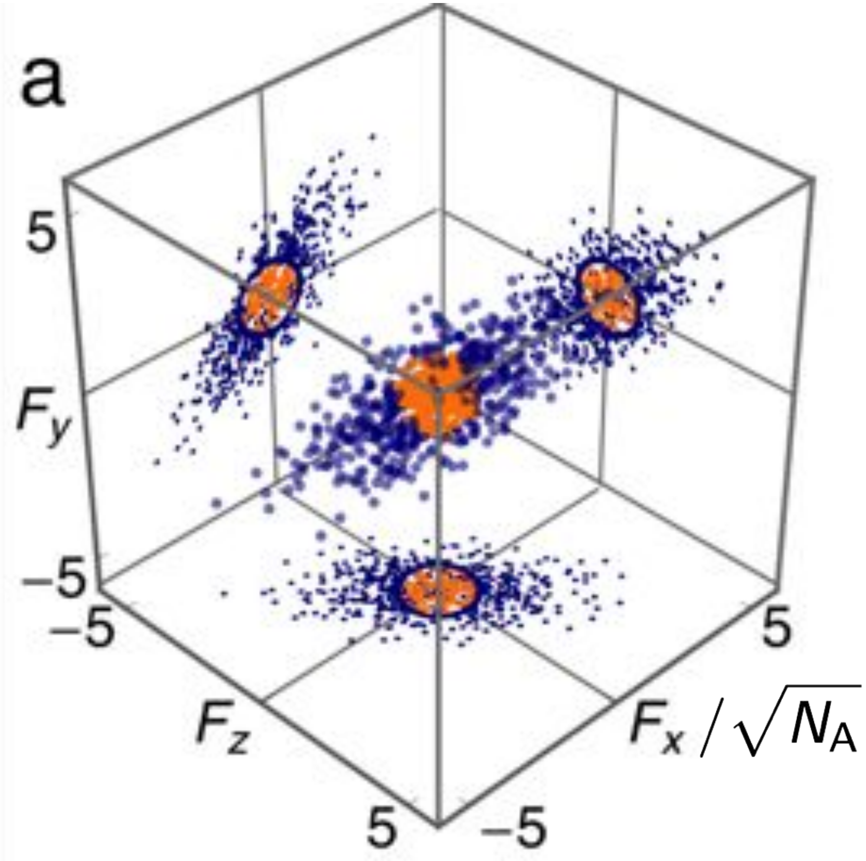


measure measure measure

repeat

arXiv:1403.1964 (2014)
to appear in PRL

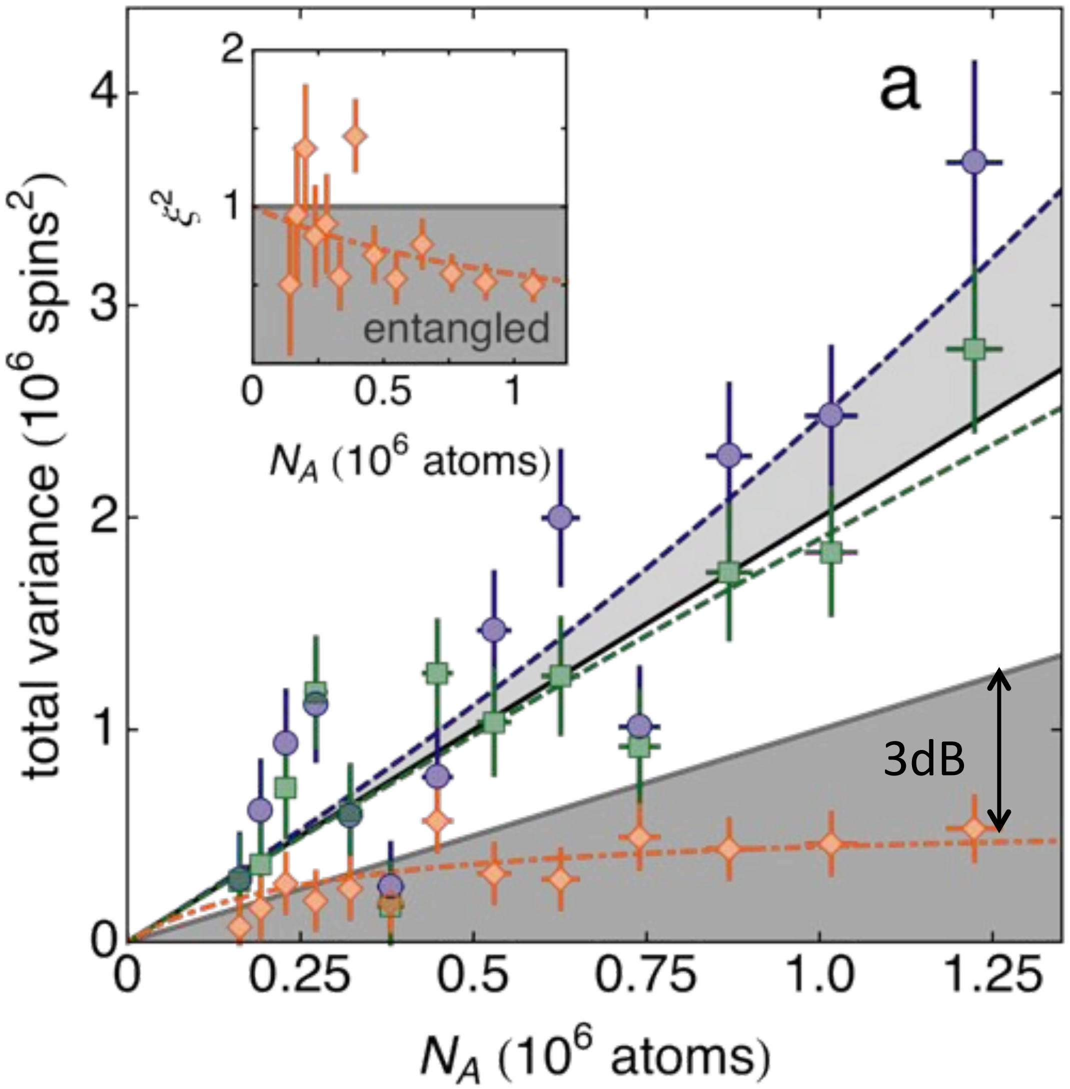
Vector non-demolition measurements



first vector measurement

arXiv:1403.1964 (2014)
to appear in PRL

Quantifying squeezing by conditional variance



$|\Delta \mathbf{F}|^2$ (1st measurement)

$|\Delta \mathbf{F}|^2$ (2nd measurement)

standard quantum limit

conditional variance

arXiv:1403.1964 (2014)
to appear in PRL

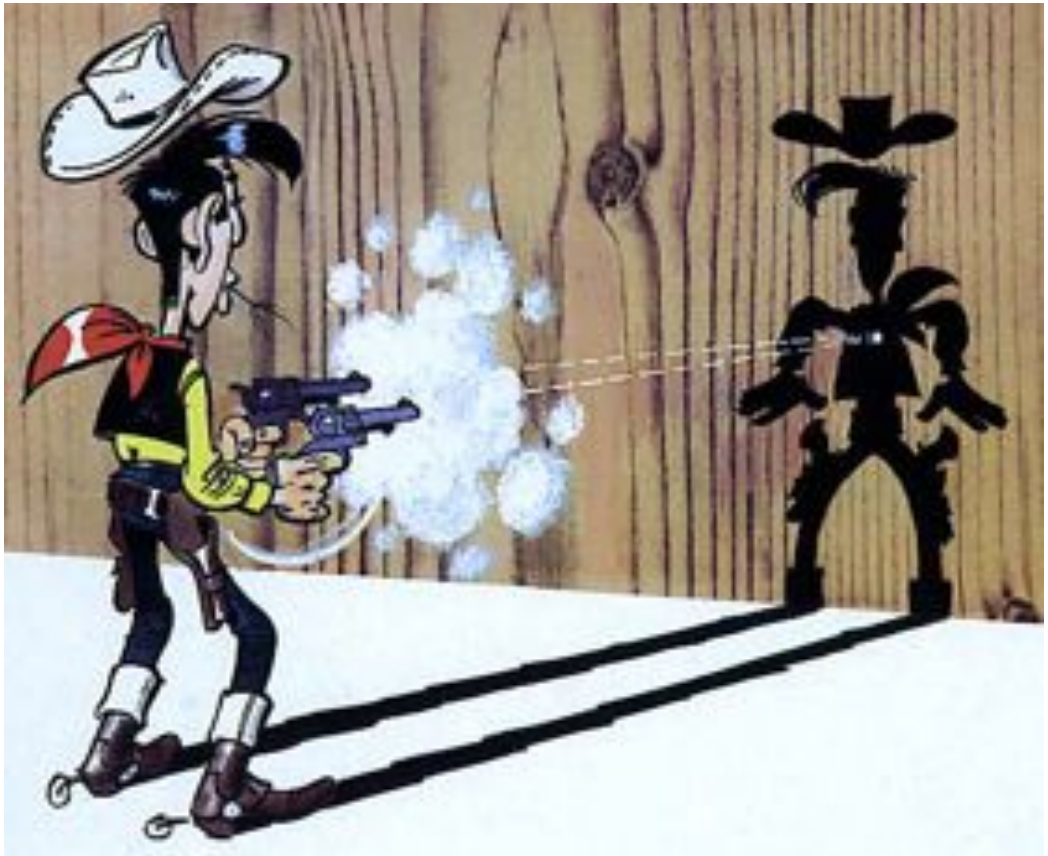
NOON STATES

NOON States

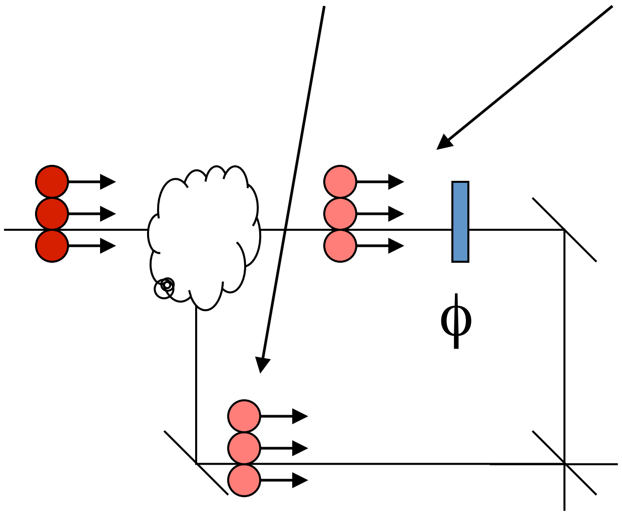
High-NOON States by Mixing Quantum and Classical Light

Itai Afek, Oron Ambar and Yaron Silberberg*

Science 2010

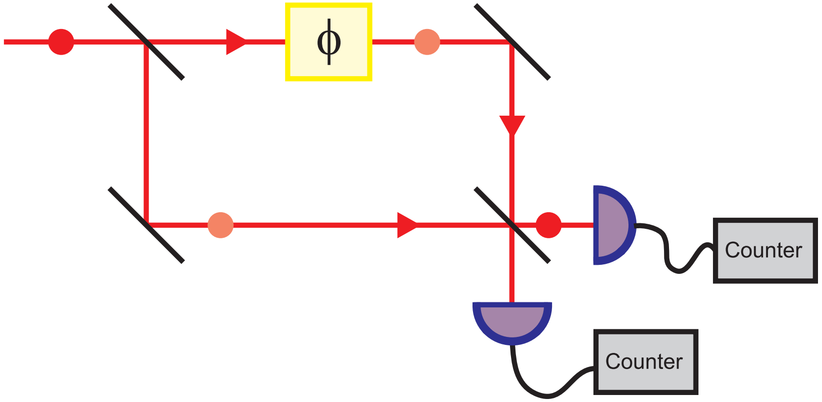


$$|\mathrm{NOON}\rangle = |N,0\rangle + |0,N\rangle$$

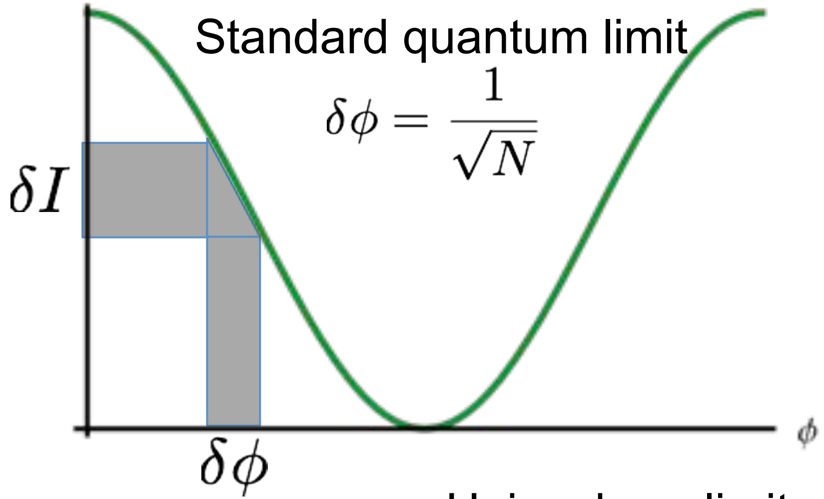


NooN interferometry

Classical light / Independent single photons

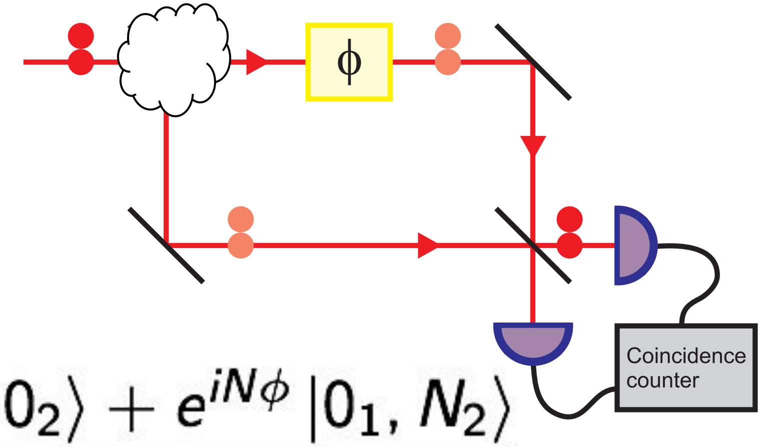


Standard quantum limit



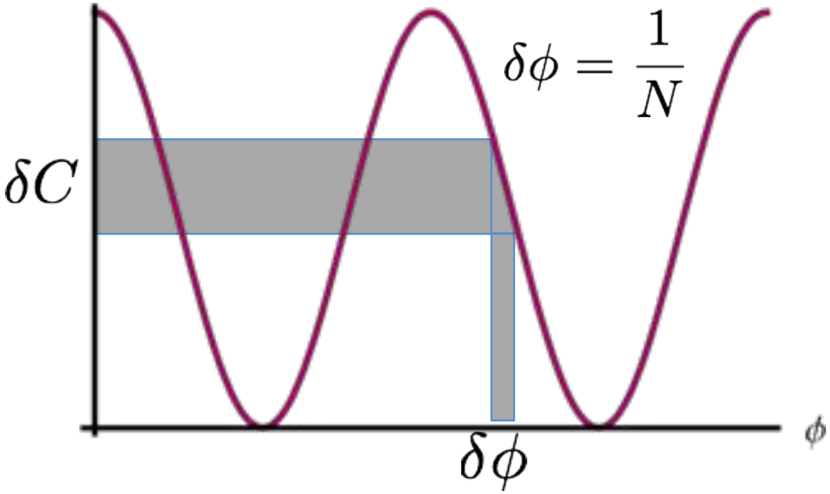
$$\delta\phi = \frac{1}{\sqrt{N}}$$

NOON states



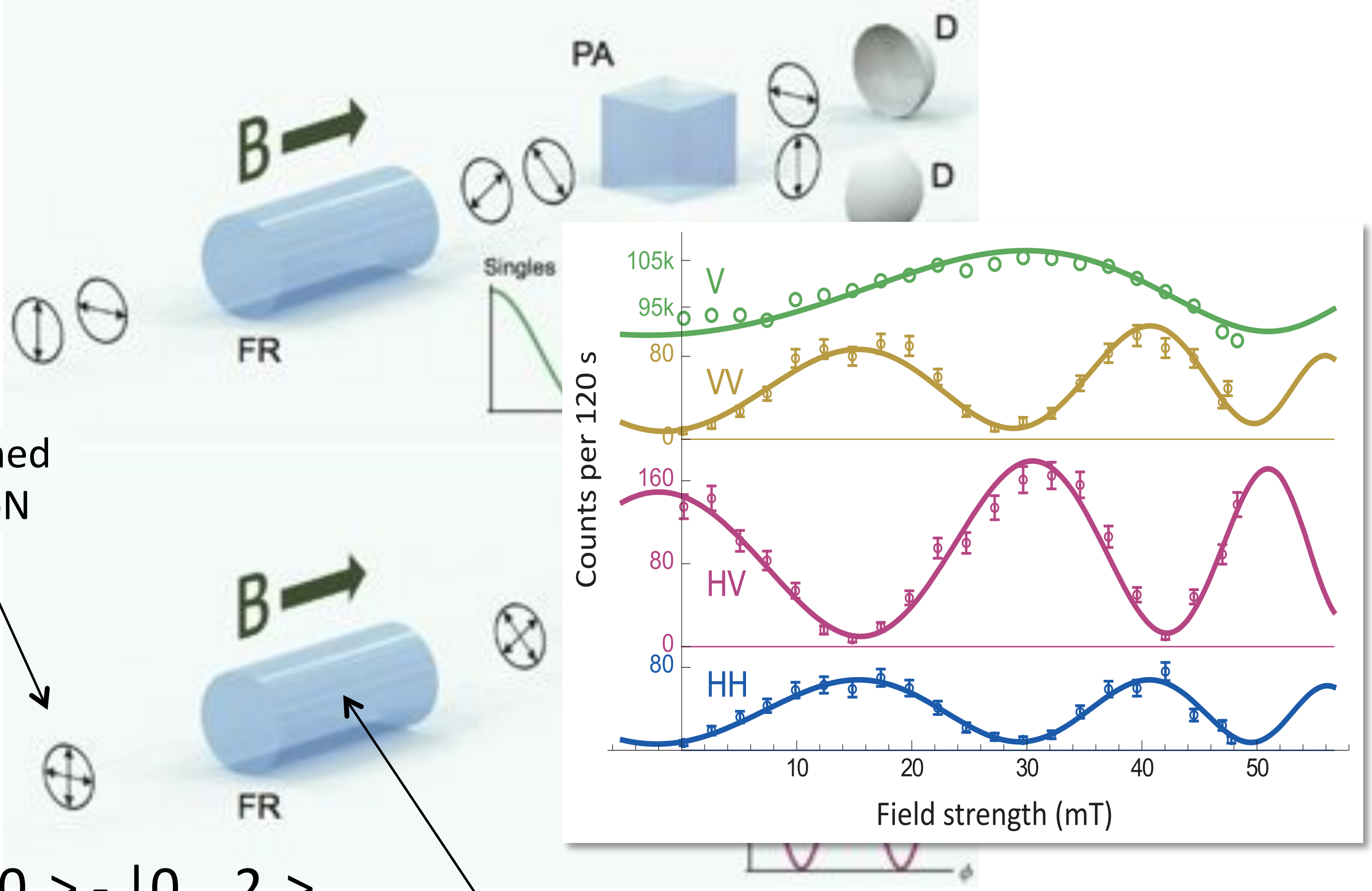
$$\frac{|N_1,0_2\rangle + e^{iN\phi}\,|0_1,N_2\rangle}{\sqrt{2}}$$

Heisenberg limit



$$\delta\phi = \frac{1}{N}$$

NooN states probing of an atomic Faraday rotator



^{87}Rb -tuned
N=2 NooN
state

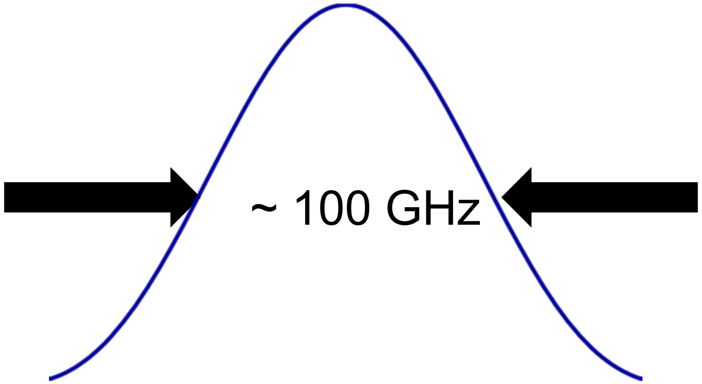
$= |2_L, 0_R\rangle - |0_L, 2_R\rangle$

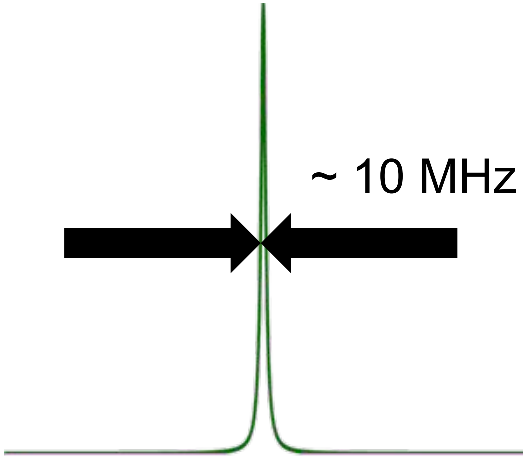
^{85}Rb ensemble

GENERATING NOON STATES

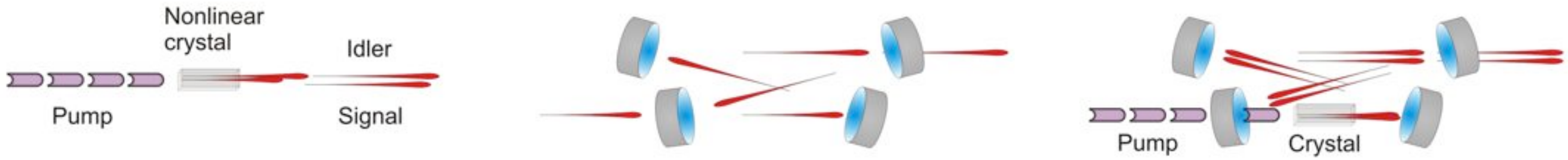
Spectral mismatch

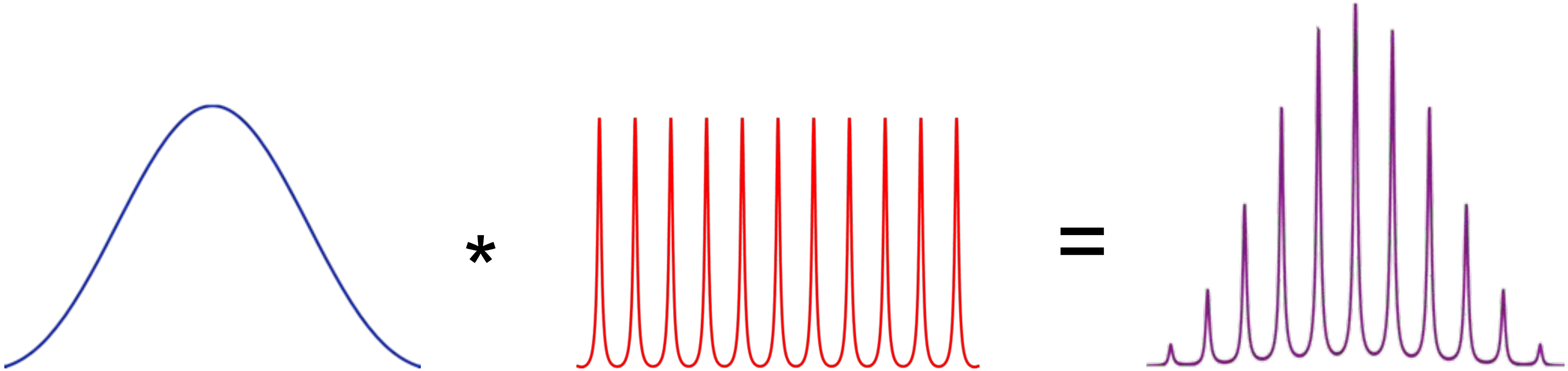
Challenge: SPDC spectrum doesn't match atomic spectrum.





Cavity-enhanced SPDC

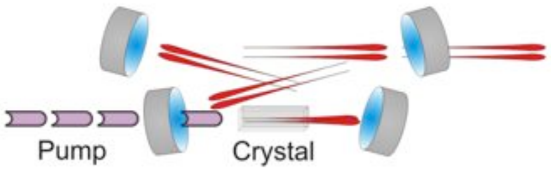


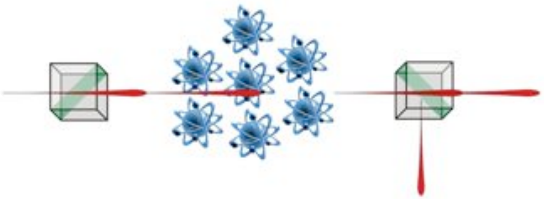


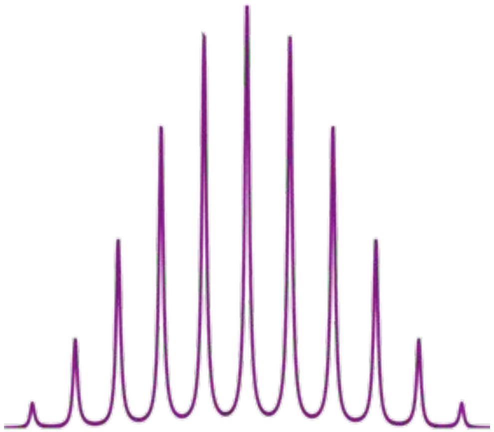
Bright filter-free source of indistinguishable photon pairs
F. Wolfgramm, et al. Opt. Express (2008)

NOON states from cavity-enhanced down-conversion: high quality and super-resolution
F. Wolfgramm, et al. JOSA B (2010)

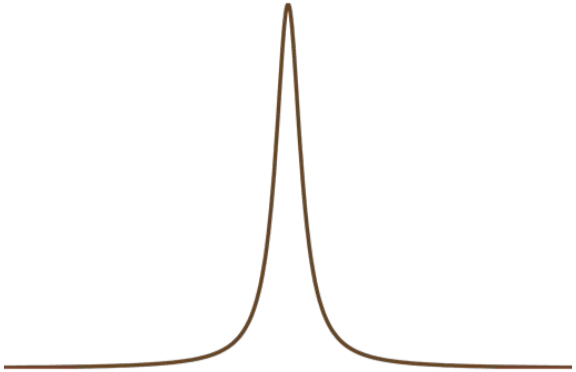
Selecting single mode with filter







*



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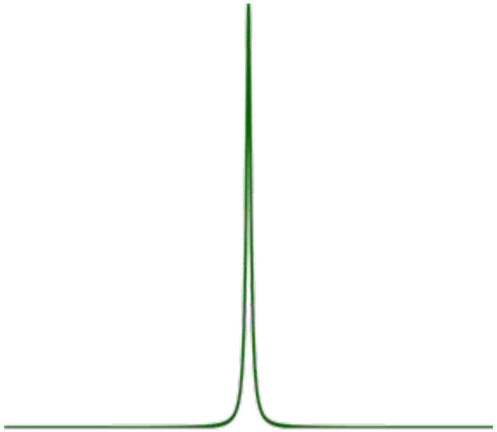
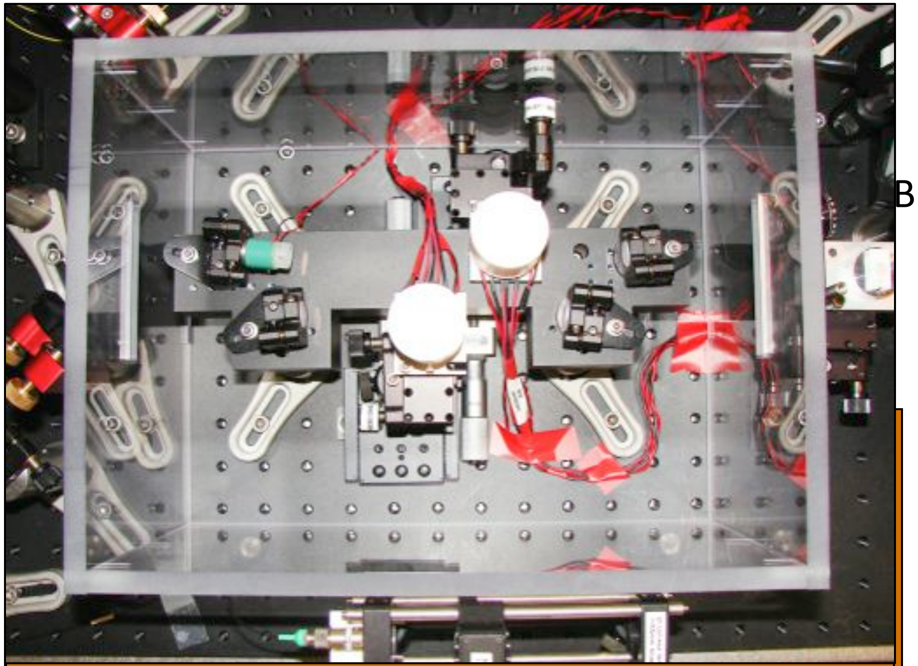


Narrowband tunable filter based on velocity-selective optical pumping in an atomic vapor
A. Cerè, et al. Opt. Lett. (2009)

Atom-resonant heralded single photons by interaction-free measurement
F. Wolfgramm, et al. PRL (2011)

Ultra-narrow Faraday rotation filter at the Rb D_1 line
Joanna A. Zielińska et al. Opt. Lett. (2012)

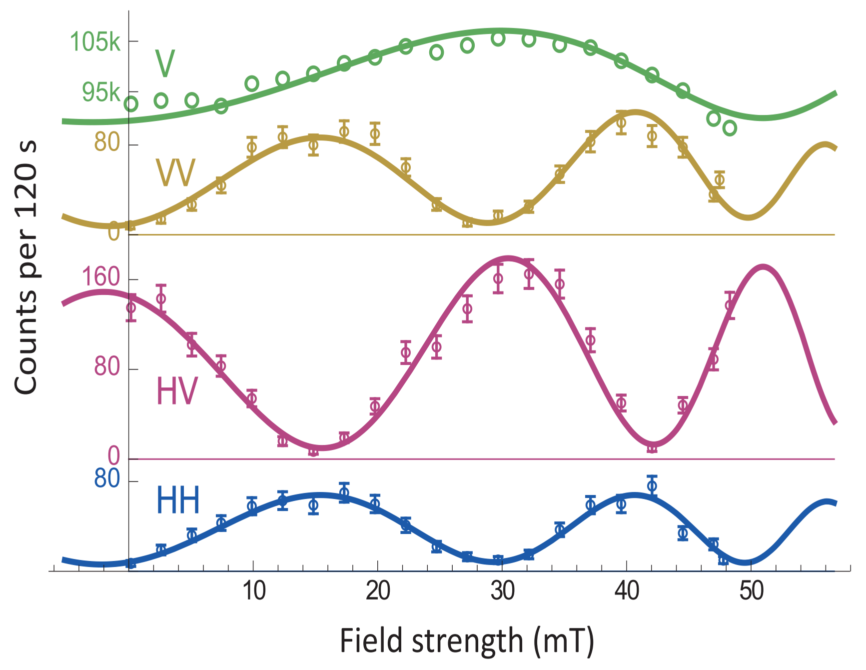
Narrowband entangled-photon technology



Laser wavelength:	794.7 nm
Cavity bandwidth:	7 MHz
FSR	490 MHz
Finesse:	70
Phase-matching:	Type II
Nonlinear crystal:	PPKTP
Compensation crystal:	KTP
Pump power:	200 uW
Brightness	~1000 pairs/s
NOON Fidelity	99%

Cavity-enhanced Type-II SPDC source

Bright filter-free source of indistinguishable photon pairs
F. Wolfgramm, et al. Opt. Express (2008)

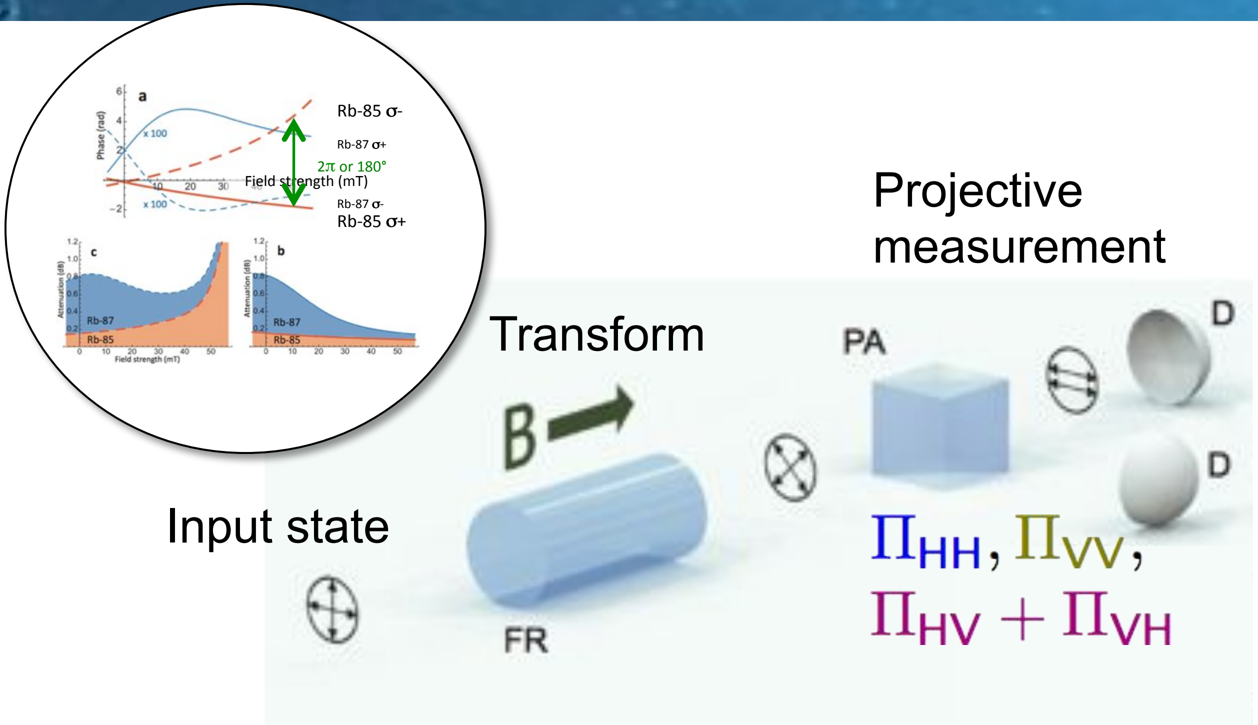


A. Cere, et al. Opt. Lett. (2009)

Atom-resonant heralded single photons by
interaction-free measurement
F. Wolfgramm, et al. PRL 106, 053602 (2011)

TOMOGRAPHY OF ATOM-RESONANT NOON STATES

State tomography by Faraday rotation



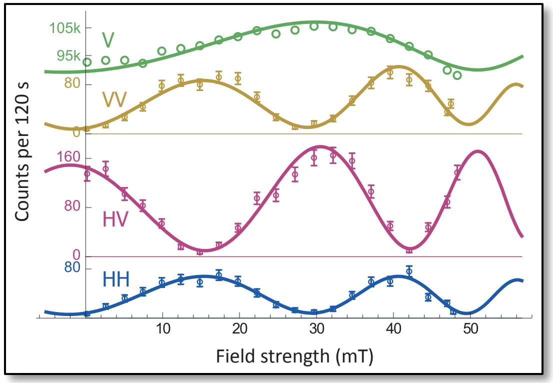
Input state

Transform

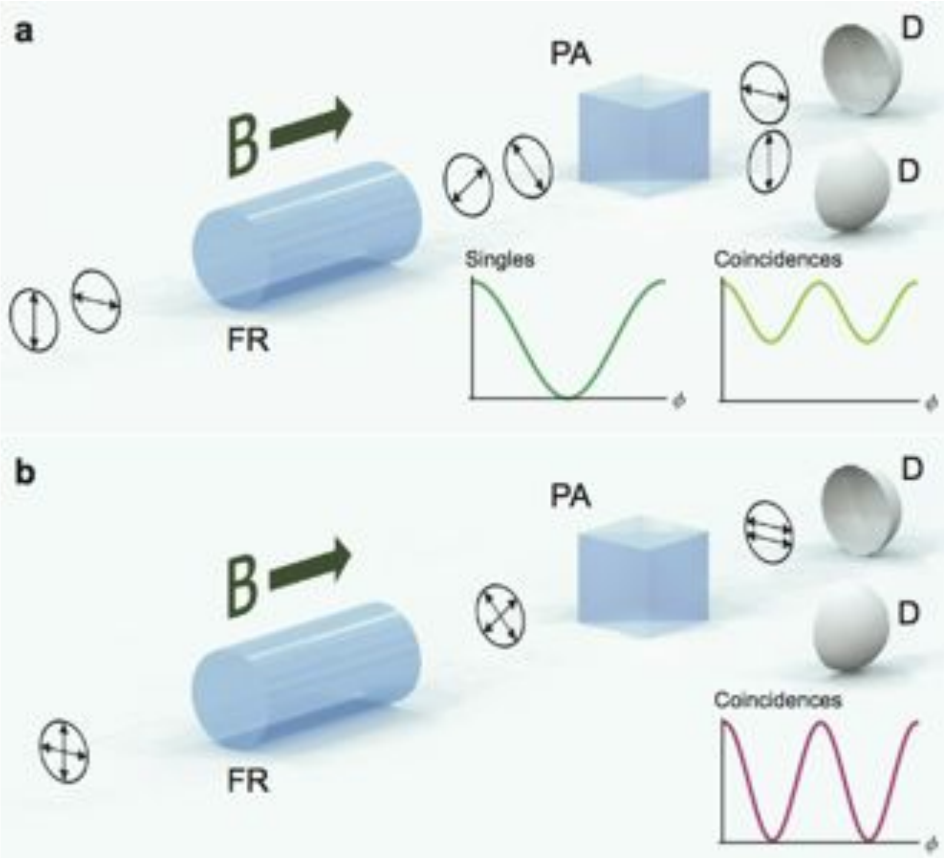
Projective measurement

$\Pi_{HH}, \Pi_{VV},$
$\Pi_{HV} + \Pi_{VH}$

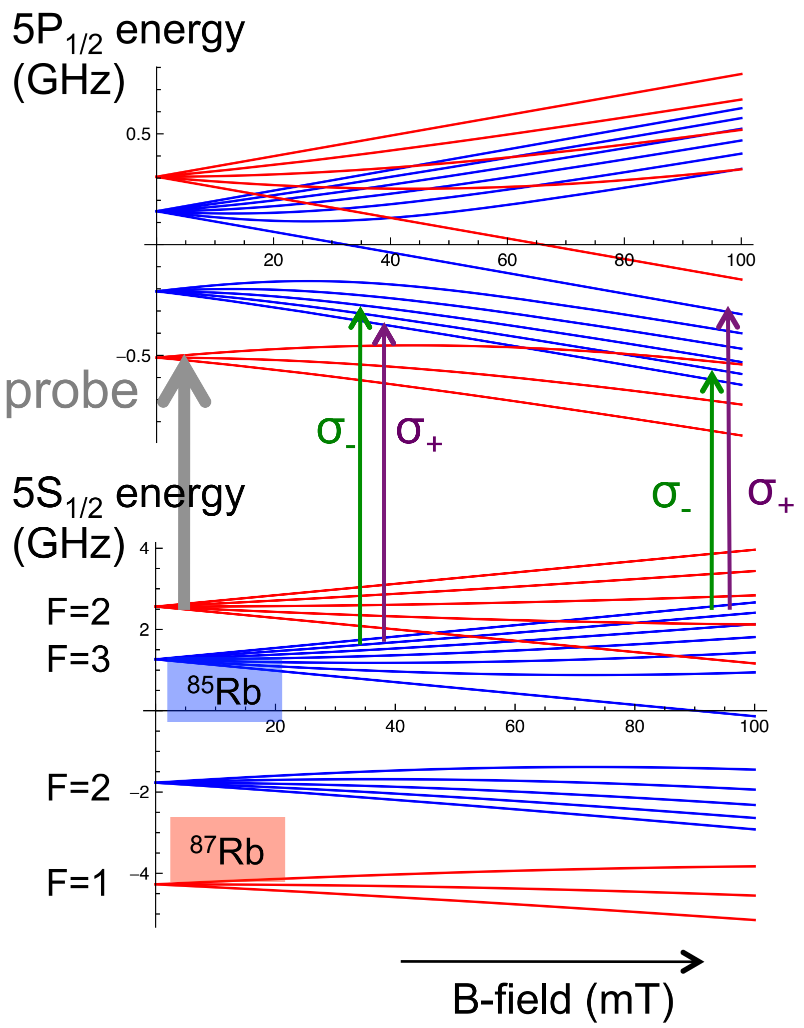
Detection events



Atomic vapor as a B-field-dependent wave-plate

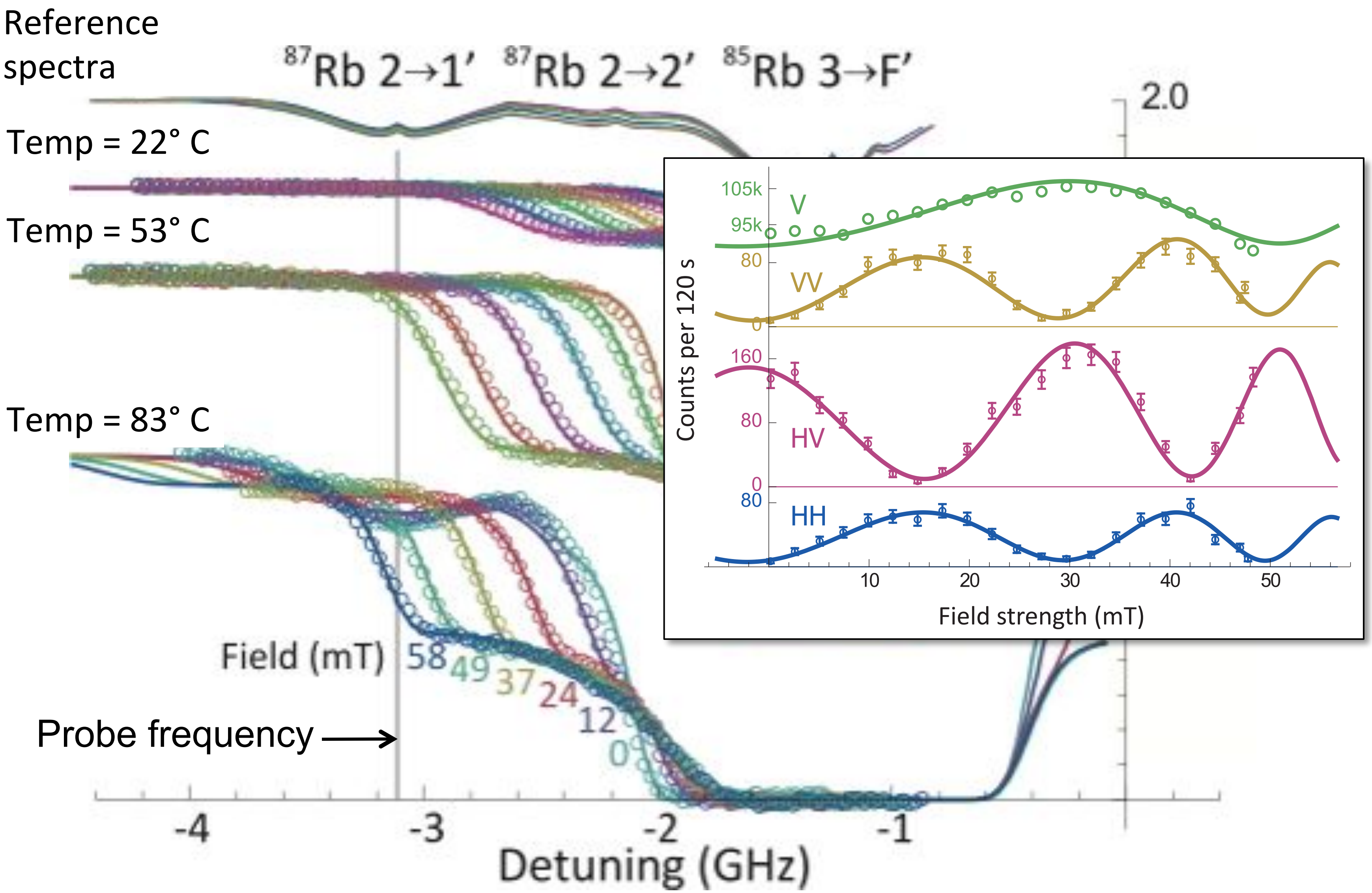


Atomic energies vs. field



First-principles model
+ measured: Temp, Isotope fraction, Field calibration

Spectroscopic characterization



Reference spectra

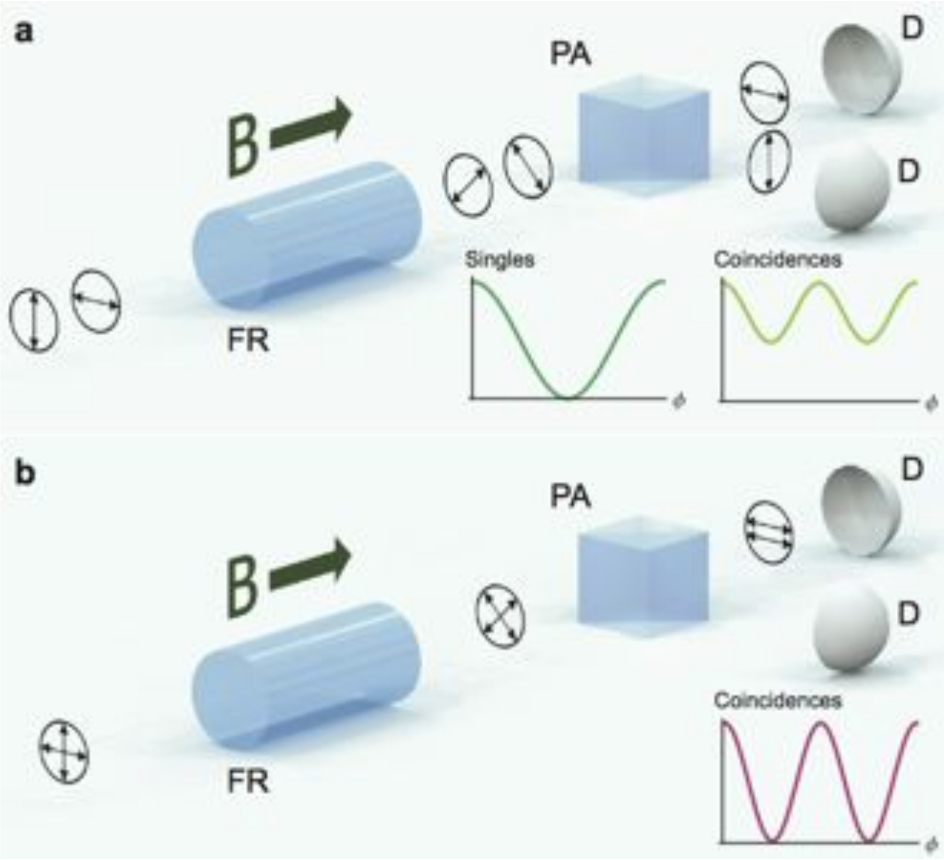
Temp = 22° C

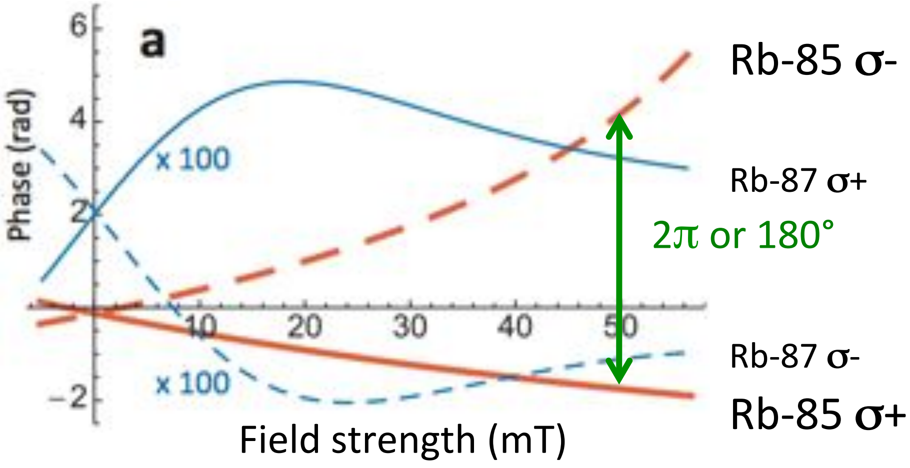
Temp = 53° C

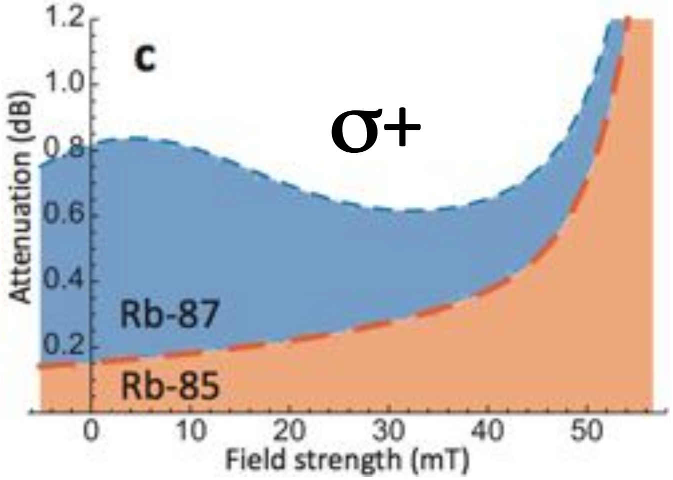
Temp = 83° C

Probe frequency →

Atomic vapor as a B-field-dependent wave-plate

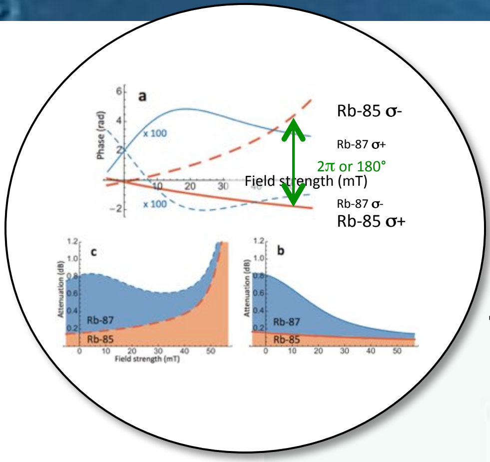




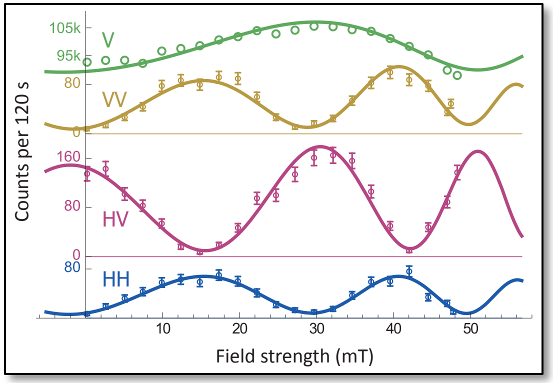


optical transformation
performed by atoms

State tomography by Faraday rotation



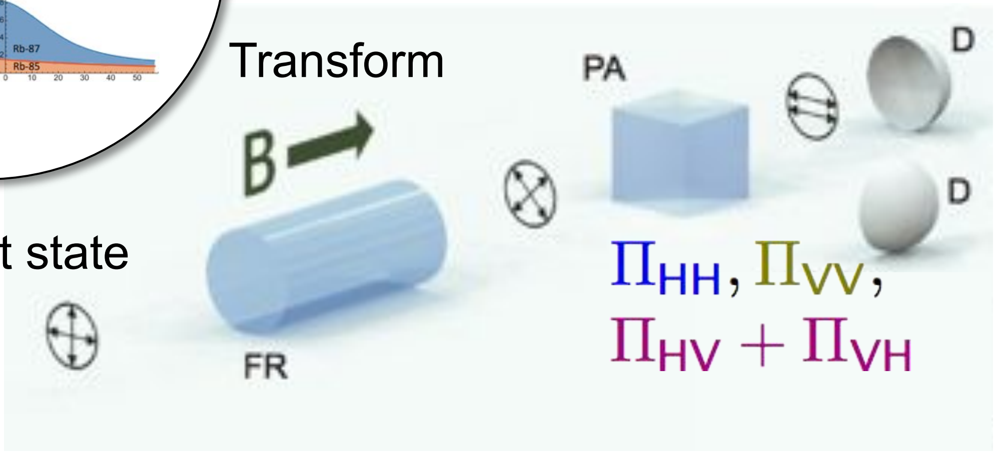
Detection events



Projective measurement

Transform

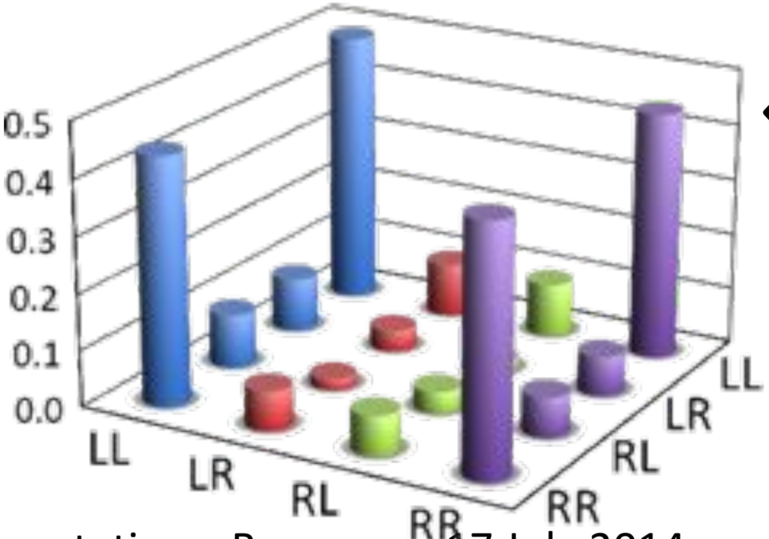
Input state



$\Pi_{HH}, \Pi_{VV},$
$\Pi_{HV} + \Pi_{VH}$

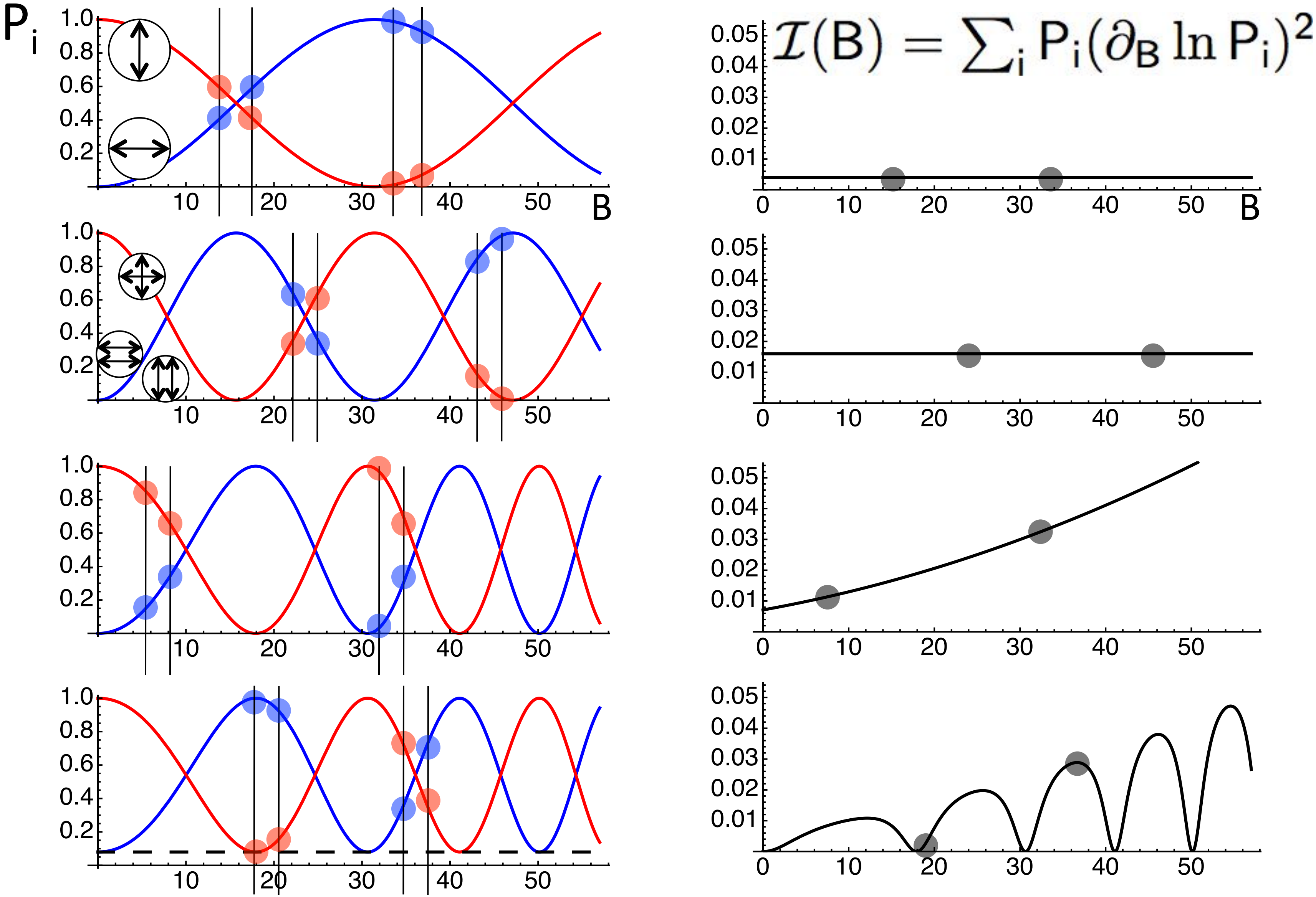
state tomography

90% fidelity NooN
88% purity
98% triplet



FISHER INFORMATION

Fisher information

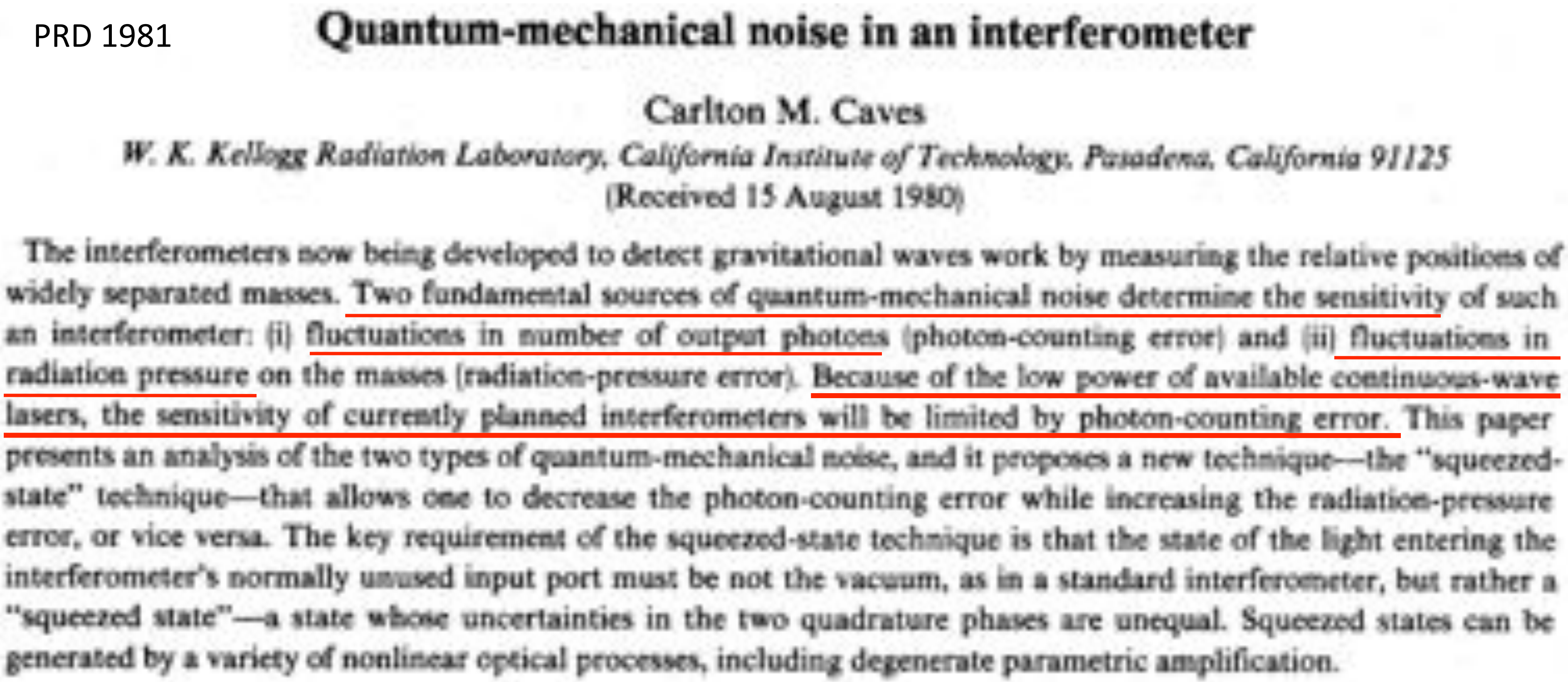


$$\mathcal{I}(B) = \sum_i P_i(\partial_B \ln P_i)^2$$

DELICATE MEASUREMENTS

Origins of quantum metrology

PRD 1981



Quantum-mechanical noise in an interferometer

Carlton M. Caves
W. K. Kellogg Radiation Laboratory, California Institute of Technology, Pasadena, California 91125

(Received 15 August 1980)

The interferometers now being developed to detect gravitational waves work by measuring the relative positions of widely separated masses. Two fundamental sources of quantum-mechanical noise determine the sensitivity of such an interferometer: (i) fluctuations in number of output photons (photon-counting error) and (ii) fluctuations in radiation pressure on the masses (radiation-pressure error). Because of the low power of available continuous-wave lasers, the sensitivity of currently planned interferometers will be limited by photon-counting error. This paper presents an analysis of the two types of quantum-mechanical noise, and it proposes a new technique—the "squeezed-state" technique—that allows one to decrease the photon-counting error while increasing the radiation-pressure error, or vice versa. The key requirement of the squeezed-state technique is that the state of the light entering the interferometer's normally unused input port must be not the vacuum, as in a standard interferometer, but rather a "squeezed state"—a state whose uncertainties in the two quadrature phases are unequal. Squeezed states can be generated by a variety of nonlinear optical processes, including degenerate parametric amplification.

in 1980:
photons are expensive

→

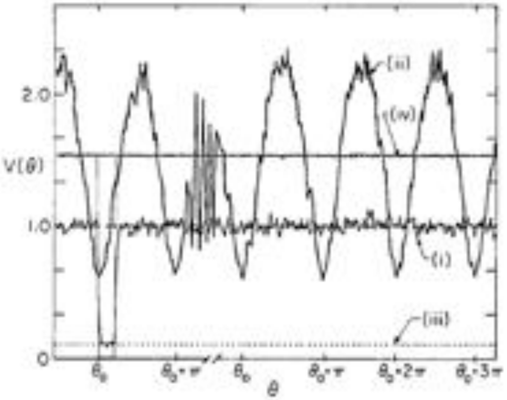
"standard model" of Q. metrology
number as limiting resource

Path of technology development in Q. Metrology

1981

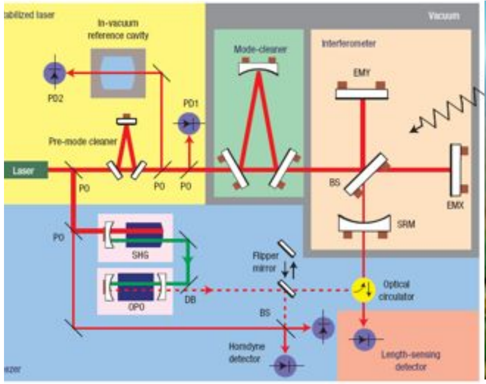
Caves proposes squeezing for GW interferometry

1985



Slusher, Kimble first squeezed light

2000s



Prototype squeezed-light GW detectors

2011-2015

GEO600
Advanced LIGO
Advanced VIRGO

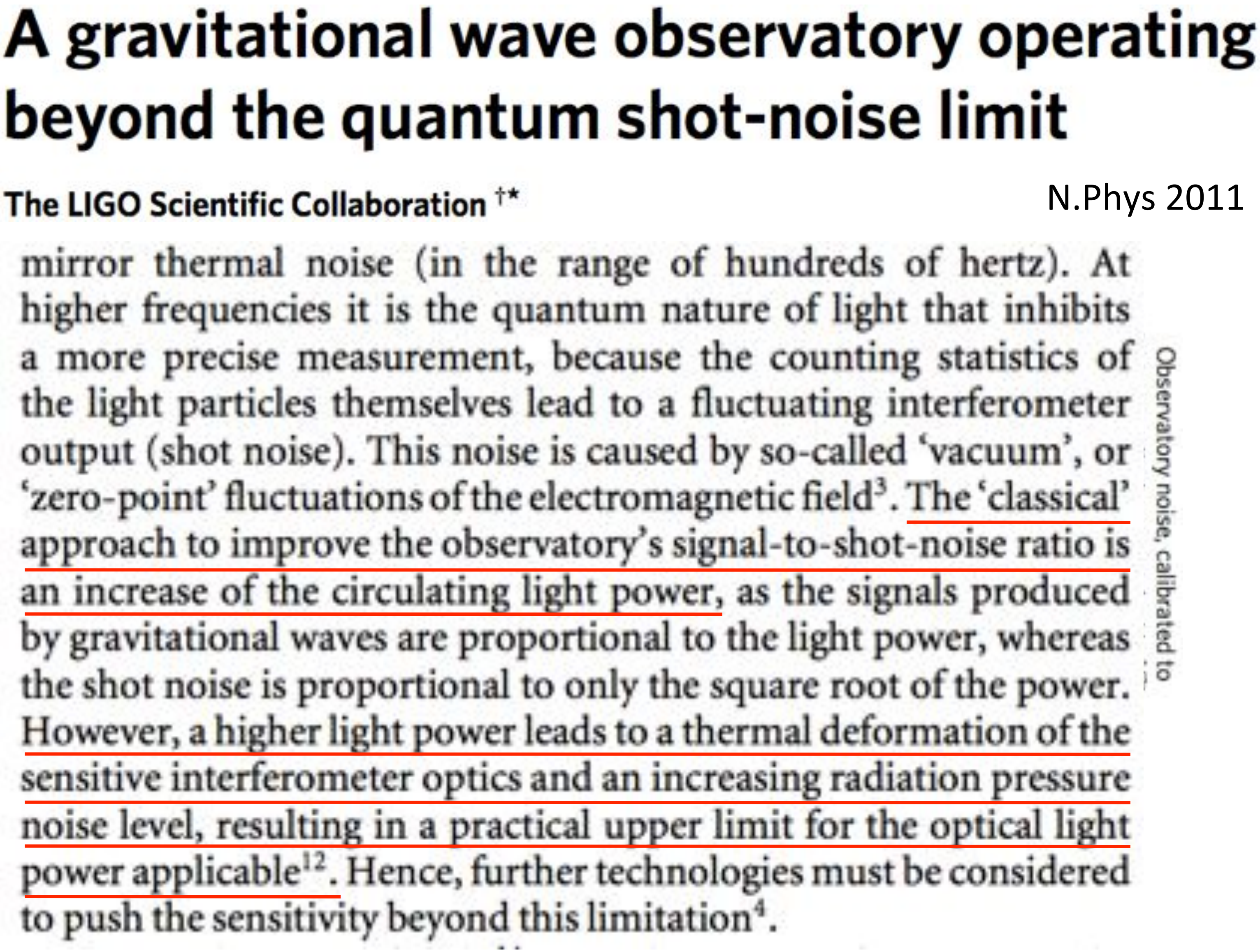
30 years

2x laser power ⟷ 3dB of squeezing

Quantum Science Implementations, Benasque, 17 July 2014

Morgan W. Mitchell, ICFO

GEO 600 sensitivity boost



A gravitational wave observatory operating beyond the quantum shot-noise limit

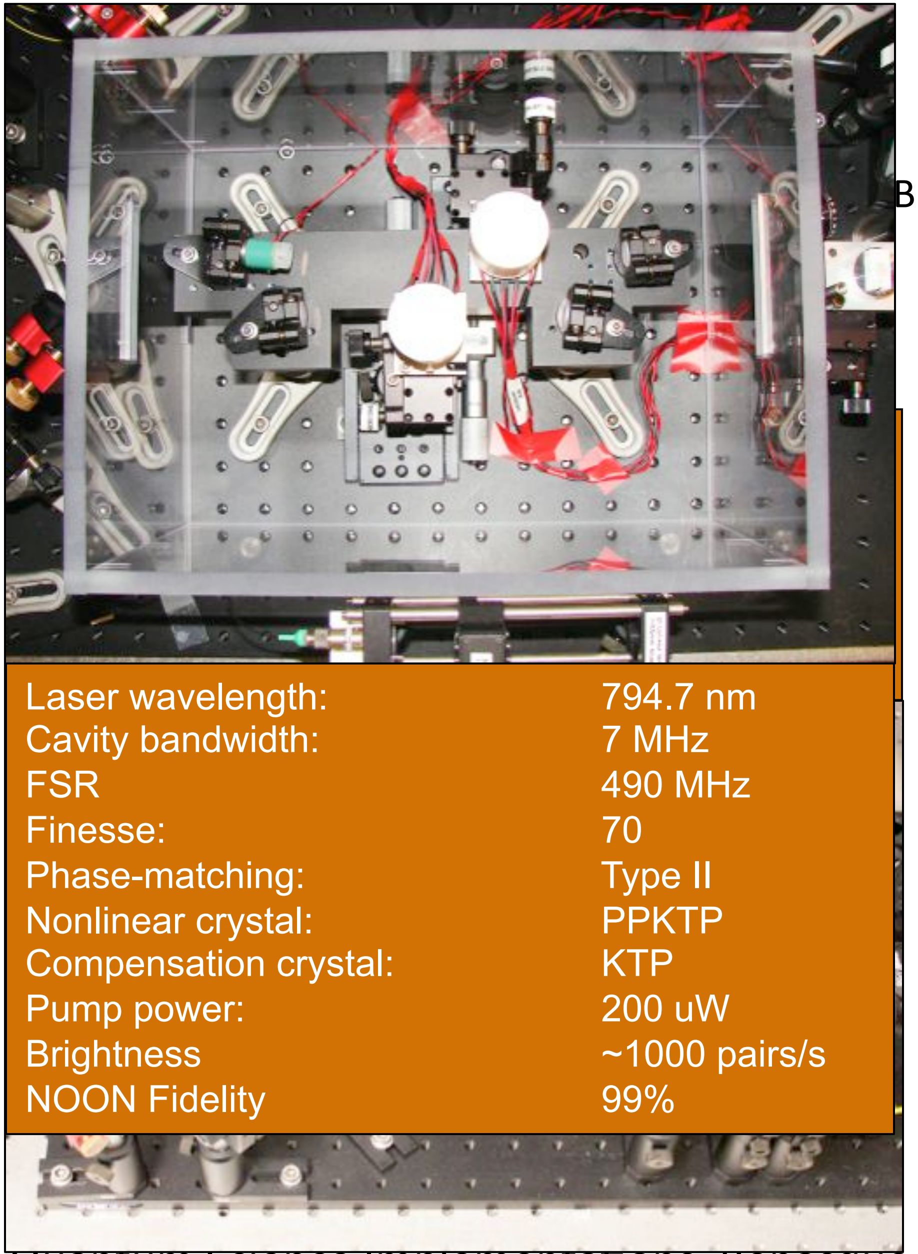
The LIGO Scientific Collaboration †*

mirror thermal noise (in the range of hundreds of hertz). At higher frequencies it is the quantum nature of light that inhibits a more precise measurement, because the counting statistics of the light particles themselves lead to a fluctuating interferometer output (shot noise). This noise is caused by so-called 'vacuum', or 'zero-point' fluctuations of the electromagnetic field[3]. The 'classical' approach to improve the observatory's signal-to-shot-noise ratio is an increase of the circulating light power, as the signals produced by gravitational waves are proportional to the light power, whereas the shot noise is proportional to only the square root of the power. However, a higher light power leads to a thermal deformation of the sensitive interferometer optics and an increasing radiation pressure noise level, resulting in a practical upper limit for the optical light power applicable[12]. Hence, further technologies must be considered to push the sensitivity beyond this limitation[4].

N.Phys 2011

Comment by R. Schmied: see also Ockeloen, et al. PRL 2013

Narrowband entangled-photon technology



Laser wavelength:	794.7 nm
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Finesse:	70
Phase-matching:	Type II
Nonlinear crystal:	PPKTP
Compensation crystal:	KTP
Pump power:	200 uW
Brightness	~1000 pairs/s
NOON Fidelity	99%

Cavity-enhanced Type-II SPDC source

Bright filter-free source of indistinguishable photon pairs
F. Wolfgramm, et al. Opt. Express (2008)

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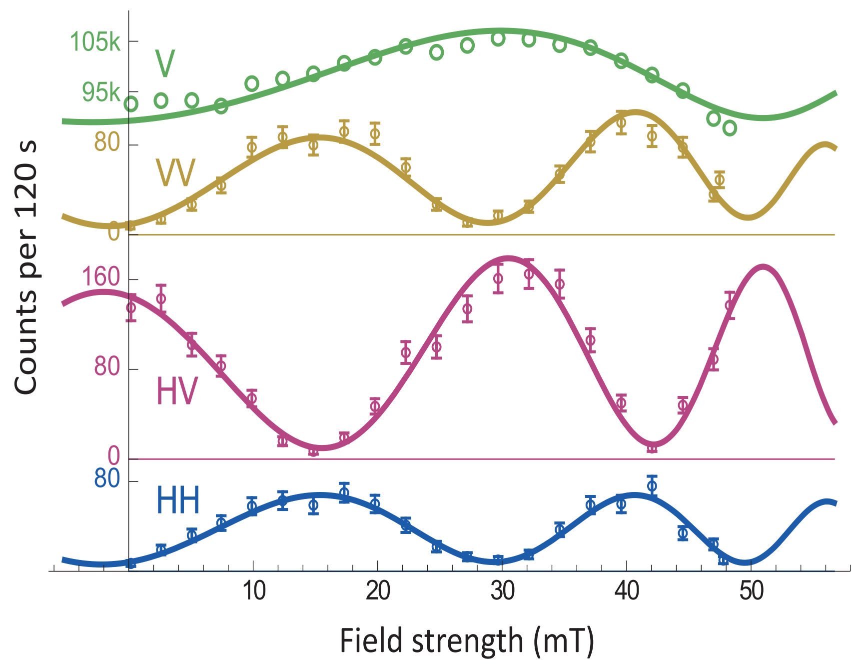

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A. Cere, et al. Opt. Lett. (2009)

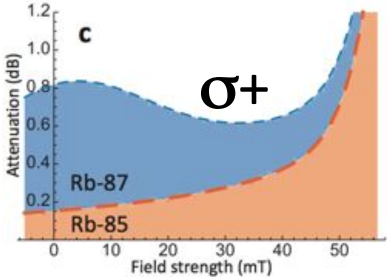
Atom-resonant heralded single photons by
interaction-free measurement
F. Wolfgramm, et al. PRL 106, 053602 (2011)

Quantifying performance

Fisher information : information gained

$$\mathcal{I}(B) = \sum_i P_i (\partial_B \ln P_i)^2$$

Number of photons scattered : damage



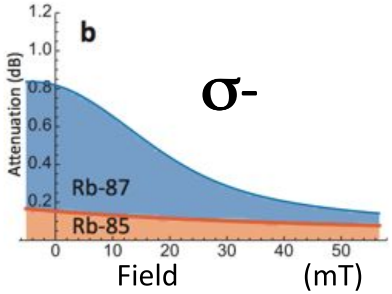


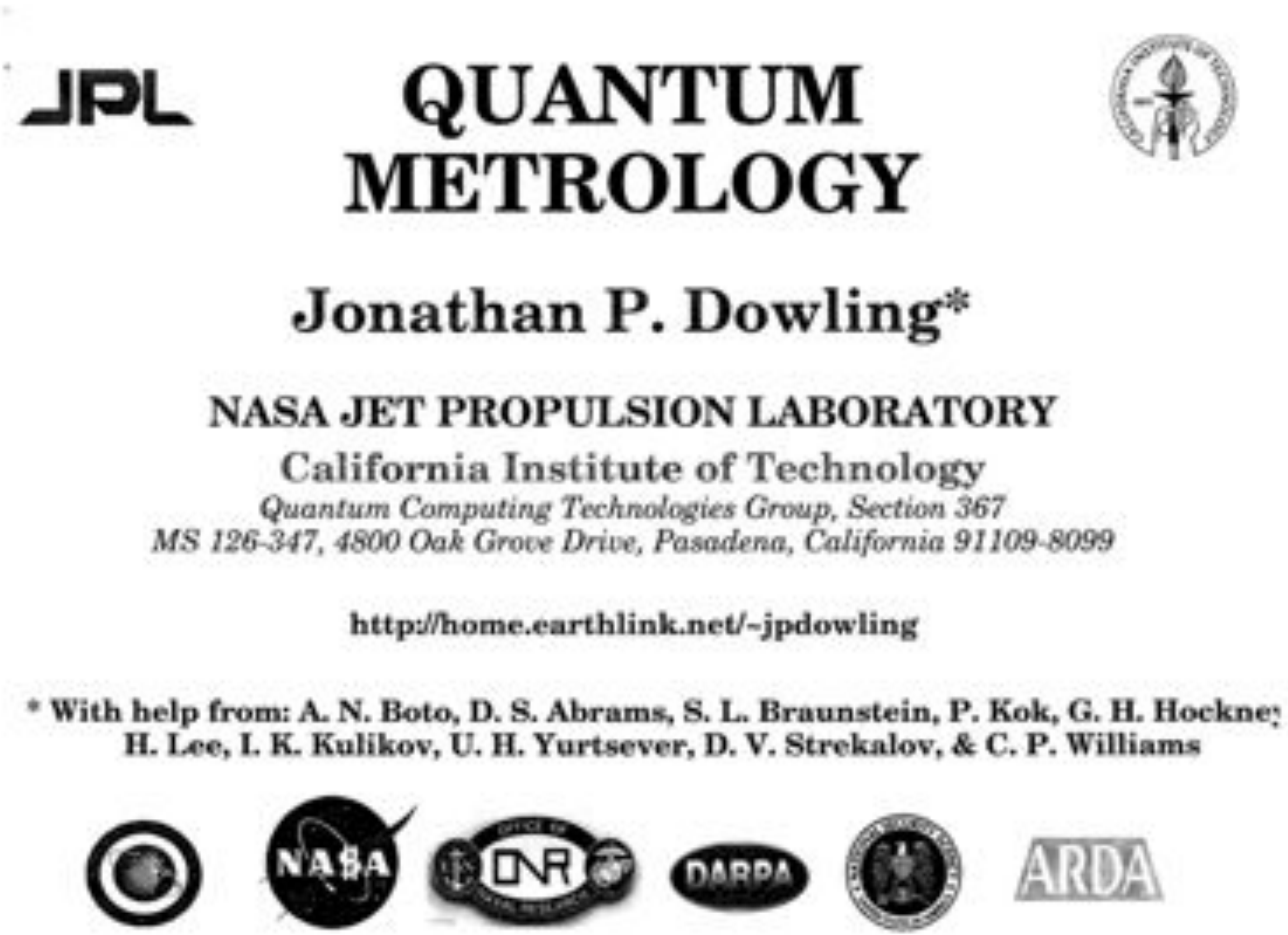
Figure of merit : information / damage

Quantum-limited non-destructive probing

Entanglement-enhanced probing of a delicate material system

Florian Wolfgramm[1†], Chiara Vitelli[2], Federica A. Beduini[1], Nicolas Godbout[3] and Morgan W. Mitchell[1,4*]

What are NooN states good for ? (2002)



JPL

QUANTUM METROLOGY

Jonathan P. Dowling*

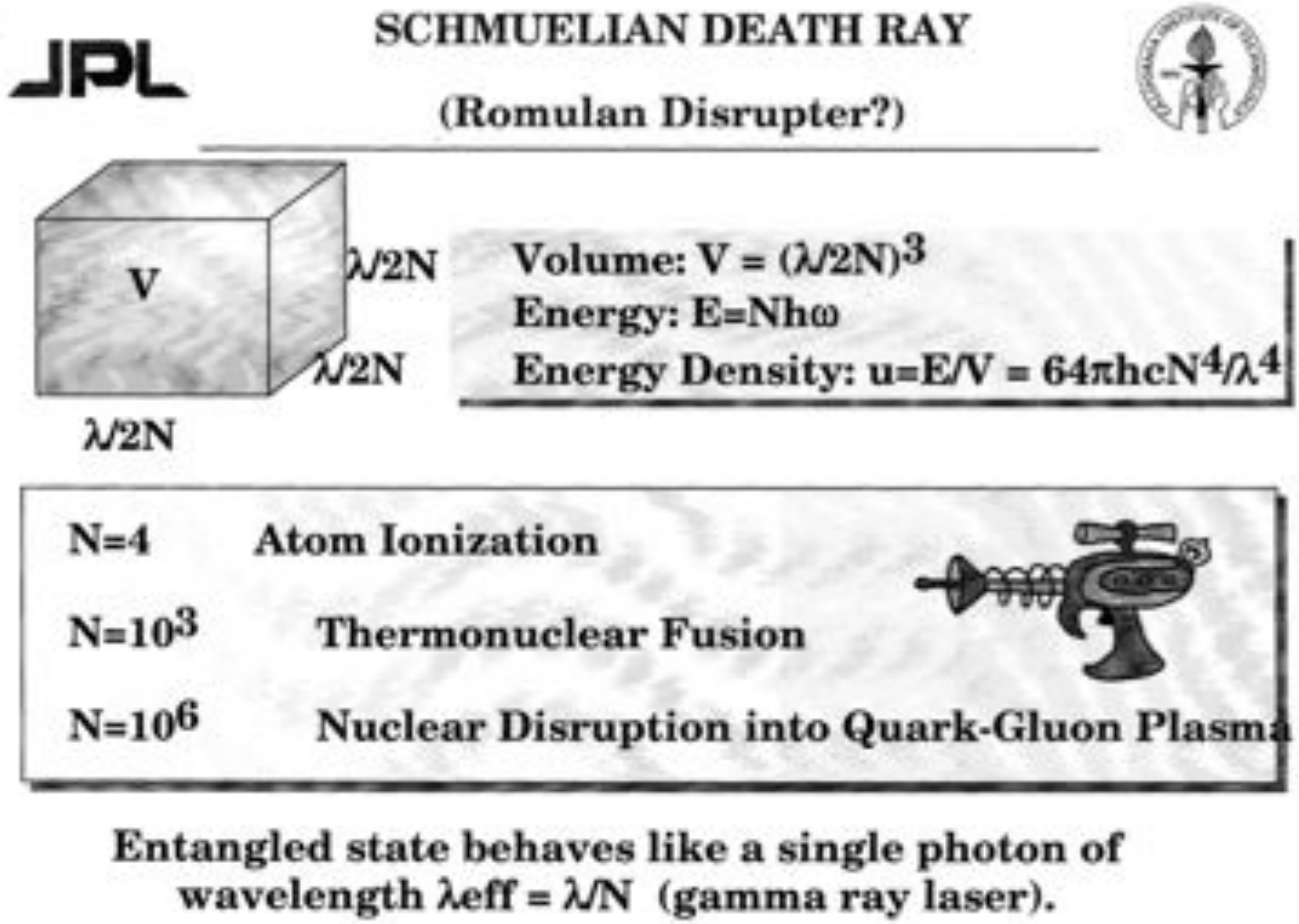
NASA JET PROPULSION LABORATORY
California Institute of Technology
Quantum Computing Technologies Group, Section 367
MS 126-347, 4800 Oak Grove Drive, Pasadena, California 91109-8099

http://home.earthlink.net/~jpdowling

* With help from: A. N. Boto, D. S. Abrams, S. L. Braunstein, P. Kok, G. H. Hockne
H. Lee, I. K. Kulikov, U. H. Yurtsever, D. V. Strekalov, & C. P. Williams

trs-new.jpl.nasa.gov/dspace/bitstream/2014/13607/1/01-2819.pdf

What are NooN states good for ?



JPL

SCHMUELIAN DEATH RAY

(Romulan Disrupter?)

Volume: $V = (\lambda/2N)^3$
Energy: $E = N\hbar\omega$
Energy Density: $u = E/V = 64\pi\hbar c N^4/\lambda^4$

$N=4$	**Atom Ionization**
$N=10^3$	**Thermonuclear Fusion**
$N=10^6$	**Nuclear Disruption into Quark-Gluon Plasma**

Entangled state behaves like a single photon of wavelength $\lambda_{eff} = \lambda/N$ (gamma ray laser).

Quantum-limited non-destructive probing

Entanglement-enhanced probing of a delicate material system

Florian Wolfgramm[1†], Chiara Vitelli[2], Federica A. Beduini[1], Nicolas Godbout[3] and Morgan W. Mitchell[1,4*]

NATURE PHOTONICS | NEWS AND VIEWS

Schrodinger's cat has a light touc

Researchers in Barcelona beat the standard quantum limit for gentle measurements

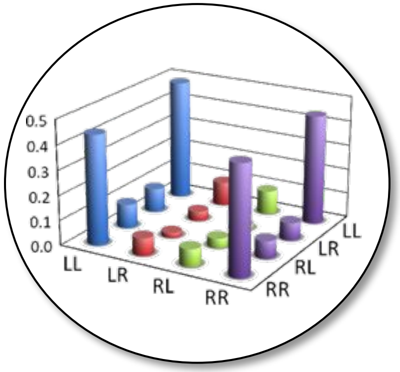
Nature Photonics 7, 8–9 (2013) | doi:10.1038/nphoton.2012.333

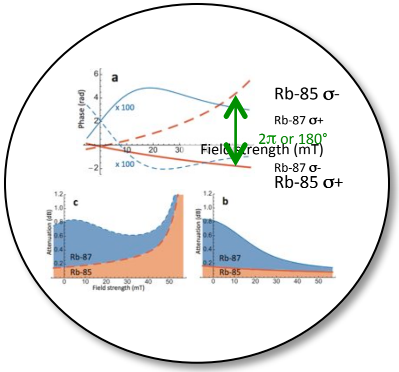
Published online 27 December 2012

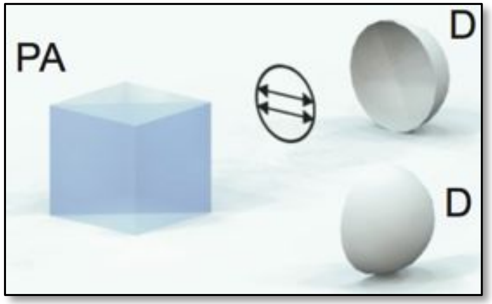
In this week's Nature Photonics, researchers at the Institute of Photonic Sciences (ICFO) report the first use of quantum entanglement to make an ultra-gentle measurement. Florian Wolfgramm and co-workers measured the magnetic field inside a diffuse cloud of rubidium atoms by probing the cloud with pairs of polarization-entangled photons.

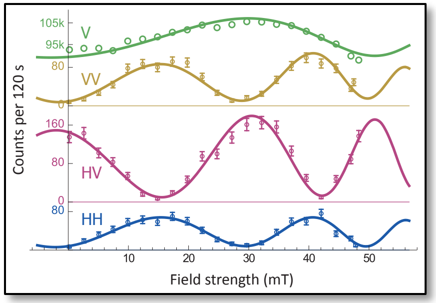
that destroys Princess Leia's … guys, this scene captures the … powerful laser to destroy a … researchers get to have this kind of fun while doing serious science, it's not such a good idea if you're performing interferometry — using the phase coherence of light to measure an object's size, movement or material properties.

Metrological properties



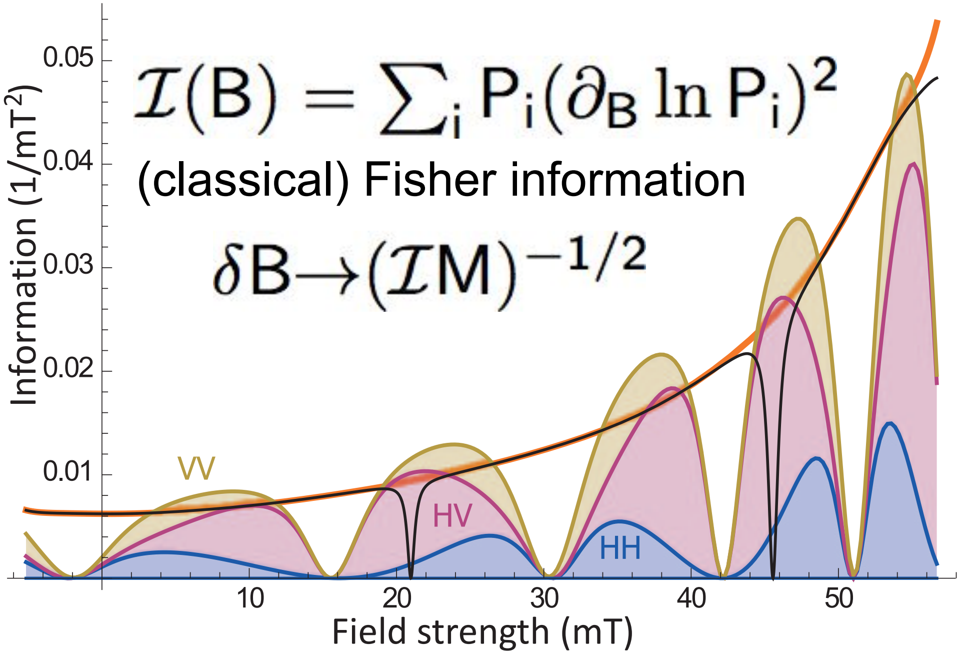






input state + transformation + detection → probabilities

$$P_i(B)$$

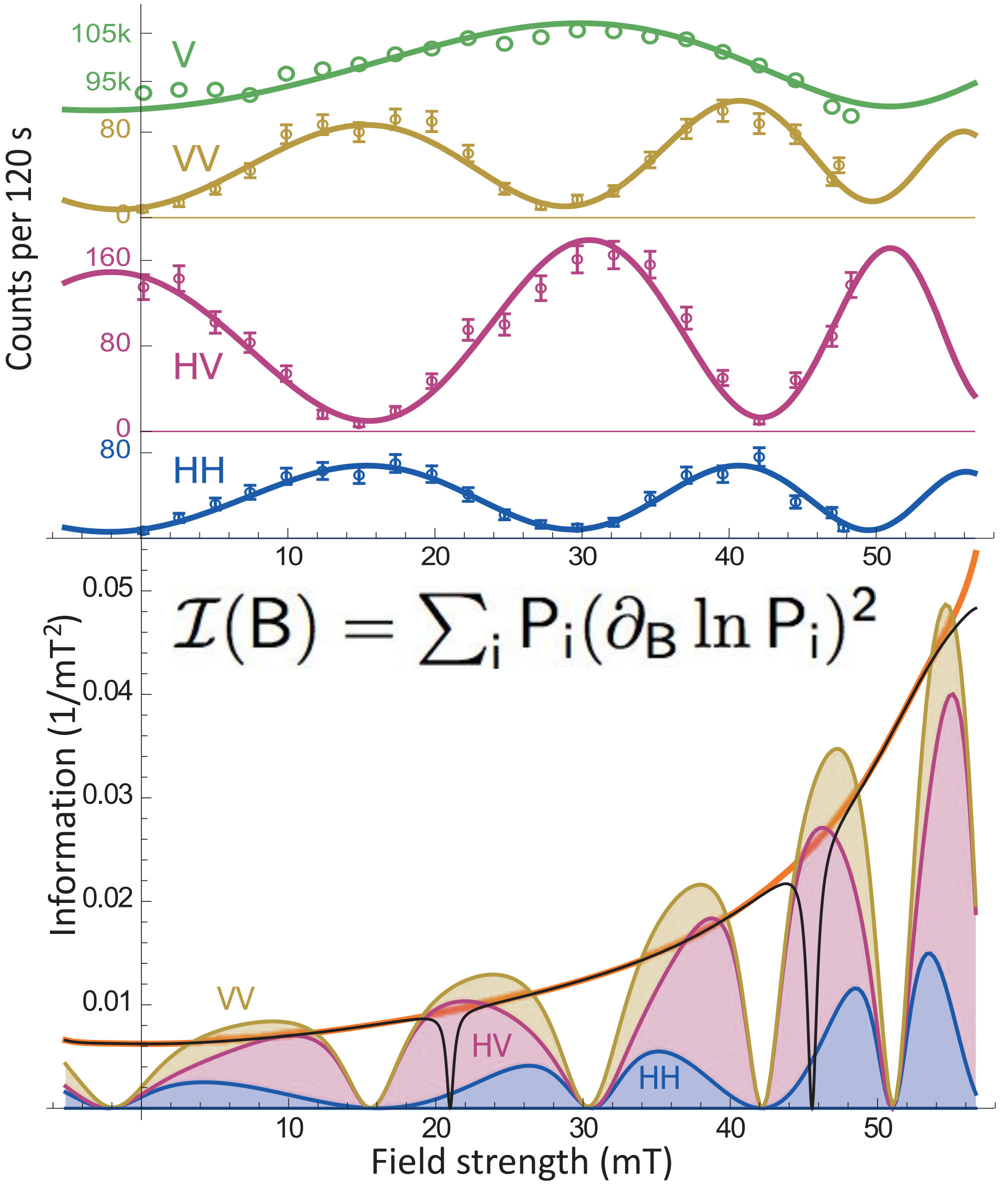


$$\mathcal{I}(B) = \sum_i P_i (\partial_B \ln P_i)^2$$

(classical) Fisher information

$$\delta B \rightarrow (\mathcal{I} M)^{-1/2}$$

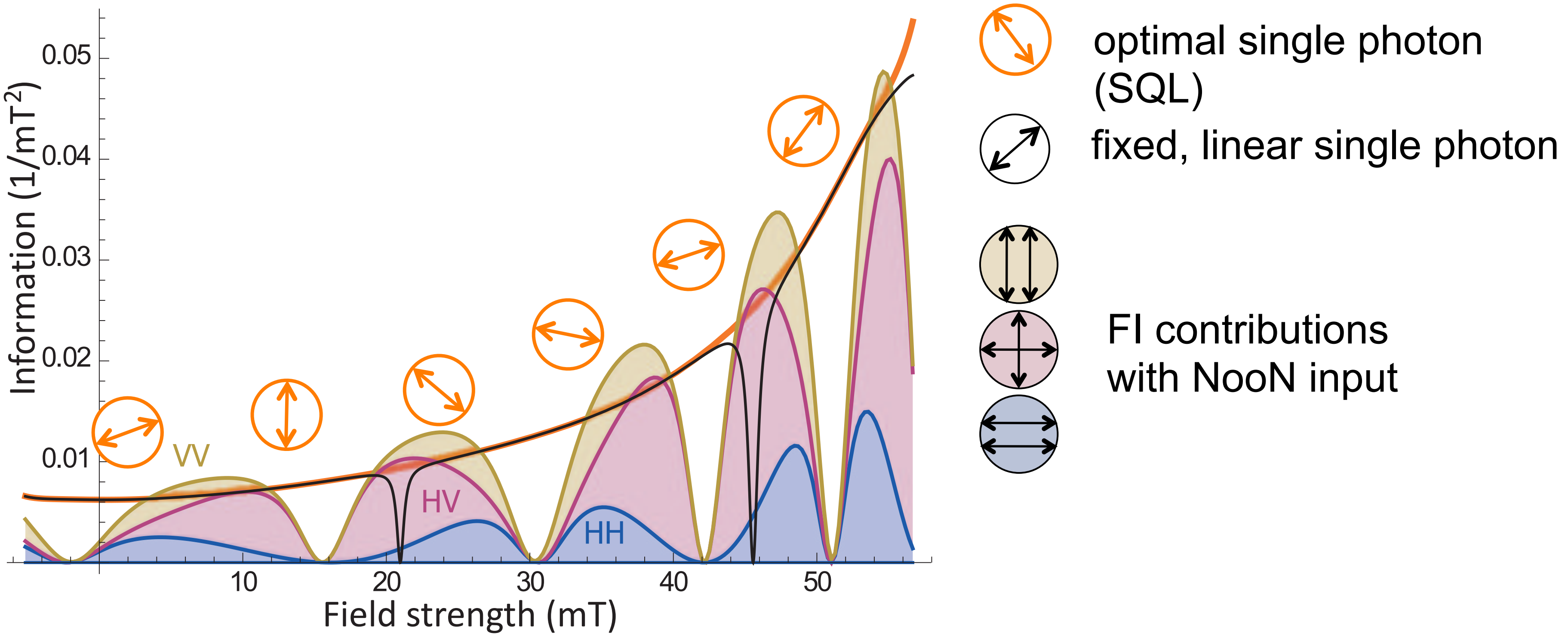
Metrological properties



$$\mathcal{I}(B) = \sum_i P_i(\partial_B \ln P_i)^2$$

Metrological properties

Fisher Information per input photon



SQL: best possible with classical resources (optimal single photon input + optimal POVM detection).

SQL includes field-dependent absorption effects.

NooN state beats SQL by 30 ± 5%

Metrological properties

Fisher Information per scattered photon

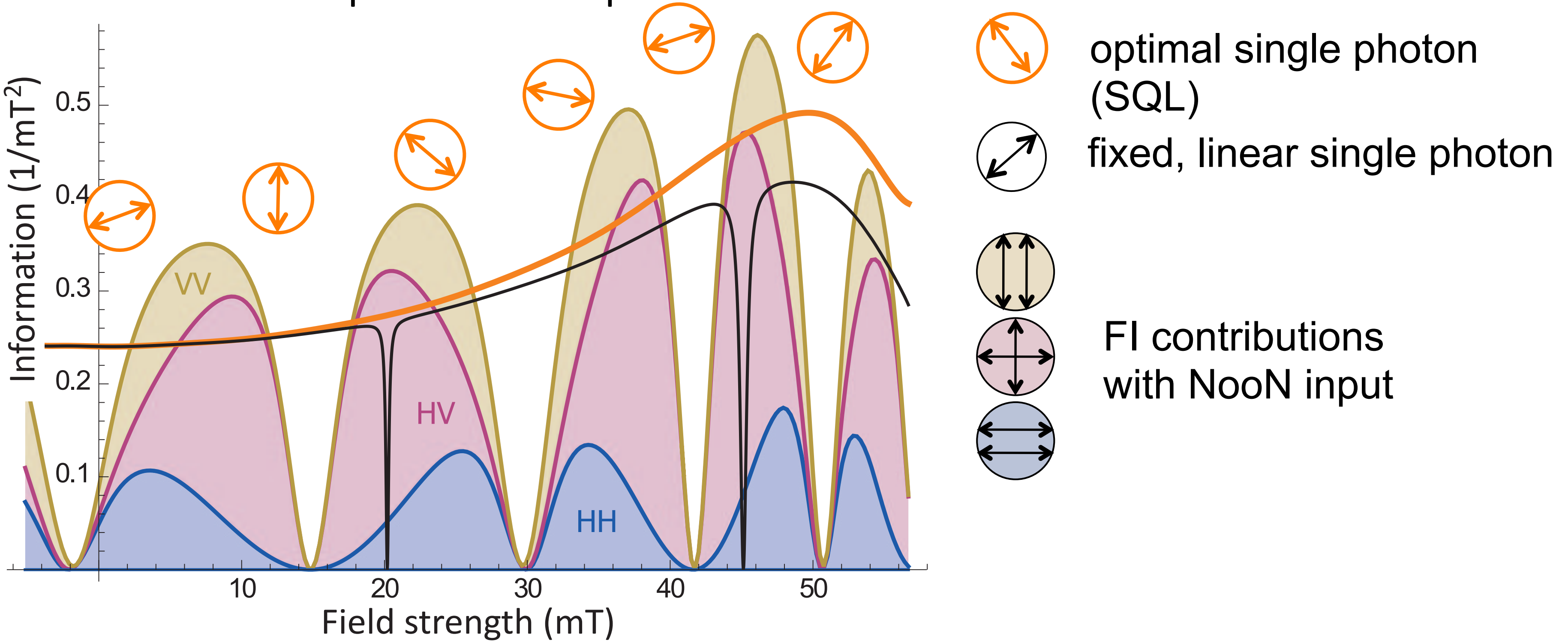
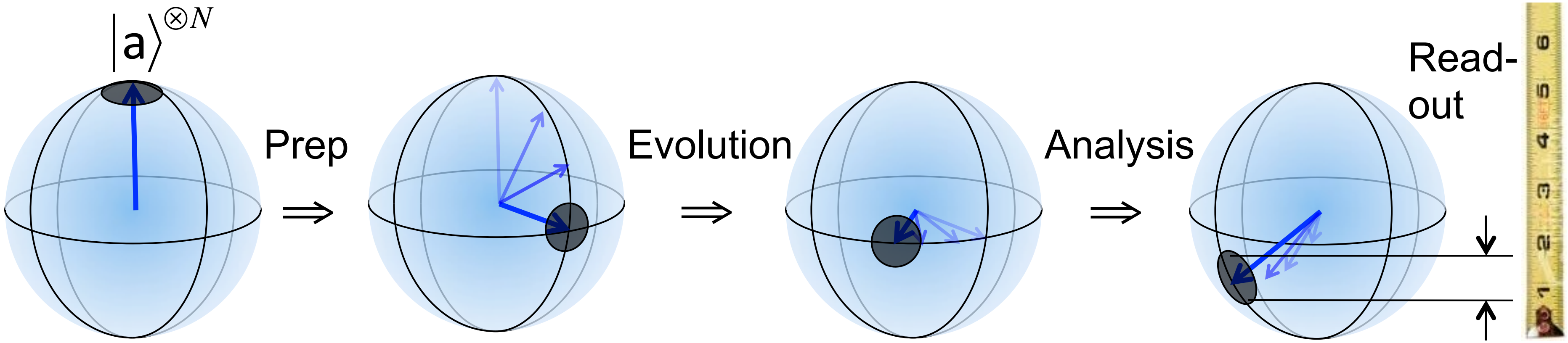


Figure of merit: Fisher information per damage to the ^{85}Rb spin ensemble.

NooN state beats SQL by 23 ± 4%

NONLINEAR SENSING

Standard Quantum Limit



$|\mathrm{a}\rangle^{\otimes N}$

Prep $\Rightarrow$ Evolution $\Rightarrow$ Analysis $\Rightarrow$ Read-out

$|\mathrm{b}\rangle^{\otimes N}$

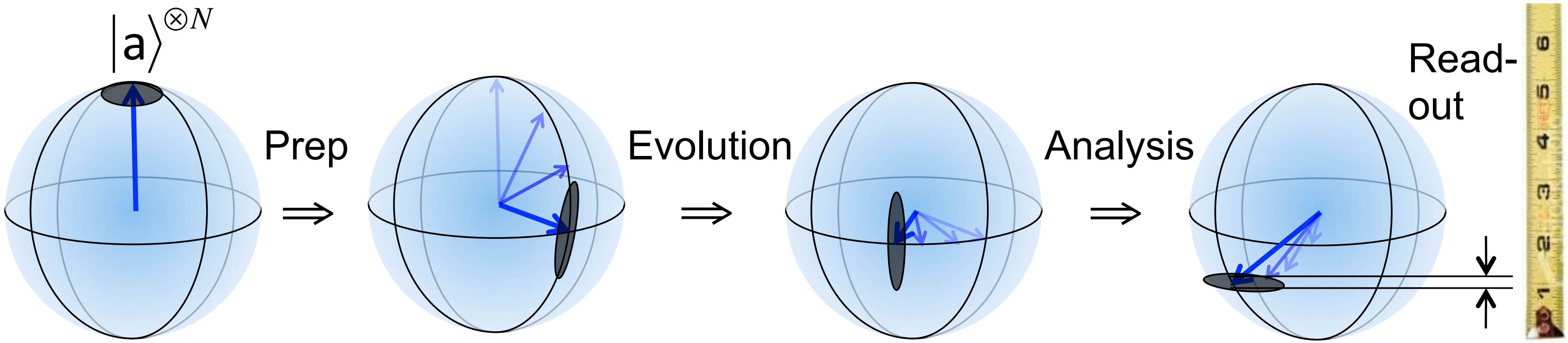
$$H = \hbar \chi S_Z \qquad \phi = \chi t$$

$$\boxed{\delta\chi \geq N^{-1/2} t^{-1}}$$ SQL

$$\delta S_X \delta S_Y \geq \frac{1}{2} \left| \langle S_Z \rangle \right|$$

$$[S_X, S_Y] = i S_Z$$

Squeezed states



$|b\rangle^{\otimes N}$

$$\delta S_X \delta S_Y \geq \frac{1}{2} \left| \langle S_Z \rangle \right|$$

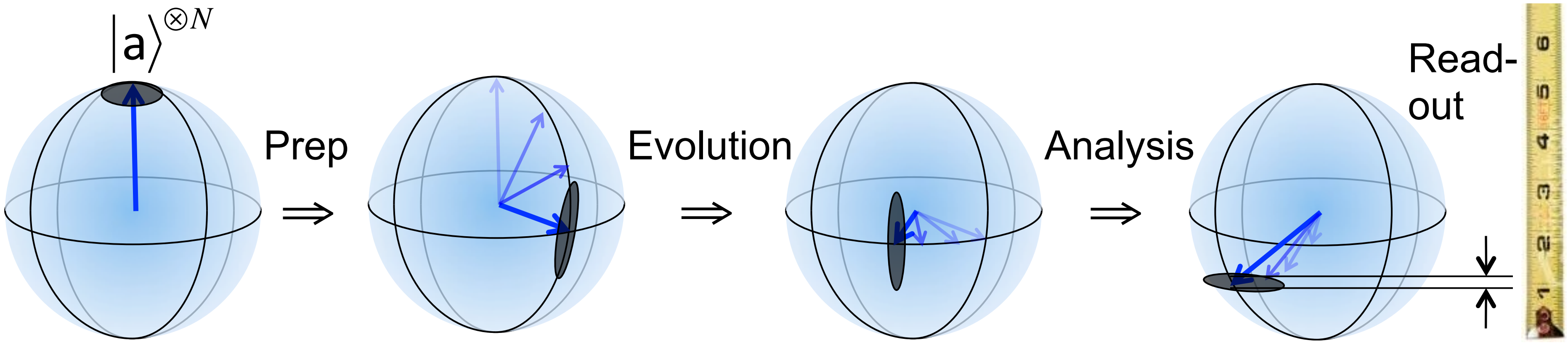
$$H = \hbar \chi S_Z \qquad \phi = \chi t$$

$$\boxed{\delta \chi \geq N^{-1/2} t^{-1}}$$ SQL

$$\delta \chi < N^{-1/2} t^{-1}$$

Squeezed states

$$|\mathrm{a}\rangle^{\otimes N}$$



$$|\mathrm{b}\rangle^{\otimes N}$$

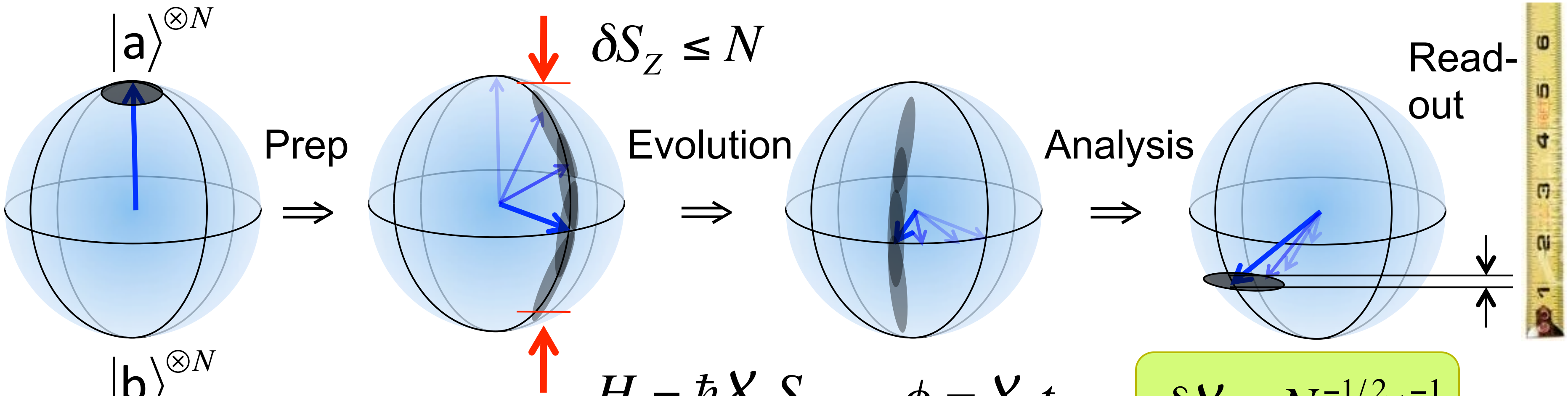
$$H = \hbar \chi S_Z \qquad \phi = \chi t$$

$$\boxed{\delta\chi \geq N^{-1/2} t^{-1}}$$ SQL

$$\delta S_X \delta S_Y \geq \frac{1}{2} \left| \langle S_Z \rangle \right|$$

$$\delta\chi < N^{-1/2} t^{-1}$$

Heisenberg limit



$\delta S_Z \leq N$

$H = \hbar \chi S_Z \qquad \phi = \chi t$

$$\delta S_X \delta S_Y \geq \frac{1}{2} \left| \langle S_Z \rangle \right|$$

$\boxed{\delta \chi \geq N^{-1/2} t^{-1}}$ SQL

$\delta \chi < N^{-1/2} t^{-1}$

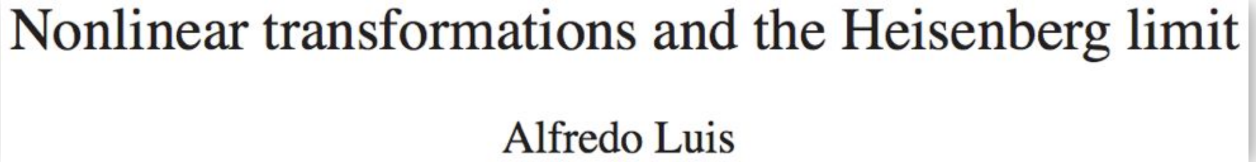
$\boxed{\delta \chi \geq N^{-1} t^{-1}}$ HL

PRL 2006 **Quantum Metrology**

Vittorio Giovannetti,[1] Seth Lloyd,[2] and Lorenzo Maccone[3]

Nonlinear metrology versus the Heisenberg limit

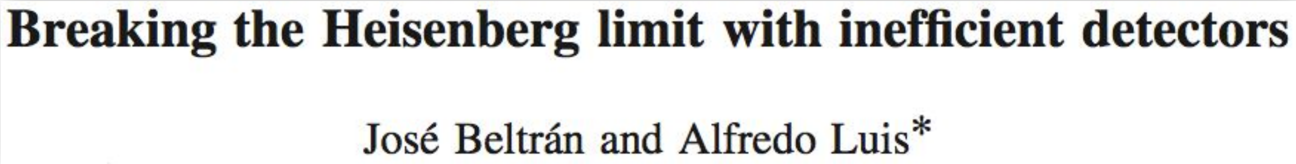
PLA 2004



Nonlinear transformations and the Heisenberg limit

Alfredo Luis

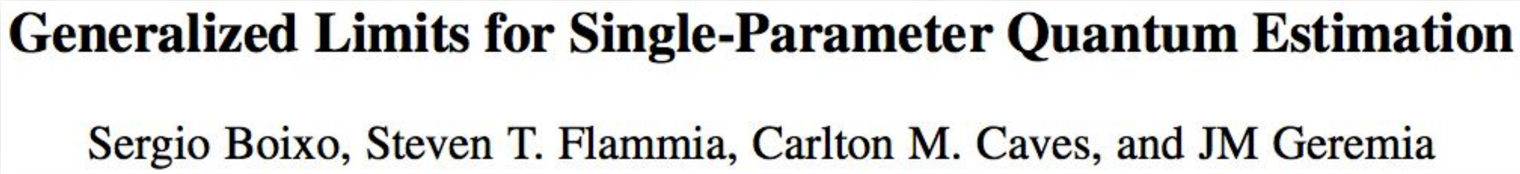
PRA 2005



Breaking the Heisenberg limit with inefficient detectors

José Beltrán and Alfredo Luis[*]

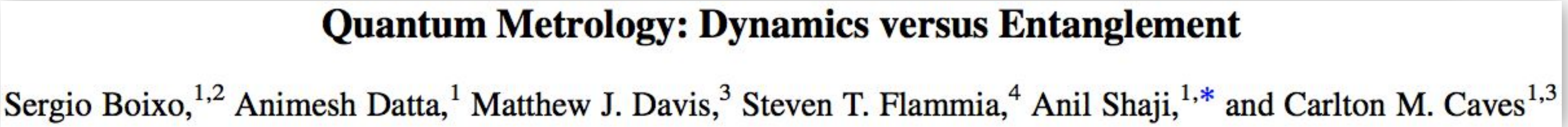
PRL 2007



Generalized Limits for Single-Parameter Quantum Estimation

Sergio Boixo, Steven T. Flammia, Carlton M. Caves, and JM Geremia

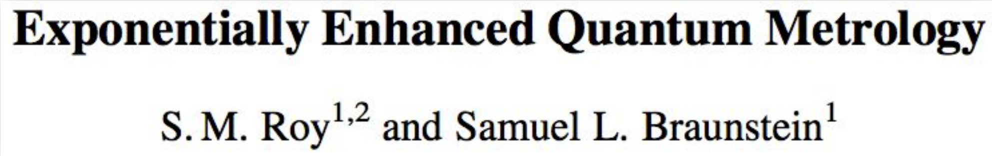
PRL 2008



Quantum Metrology: Dynamics versus Entanglement

Sergio Boixo,[1,2] Animesh Datta,[1] Matthew J. Davis,[3] Steven T. Flammia,[4] Anil Shaji,[1,*] and Carlton M. Caves[1,3]

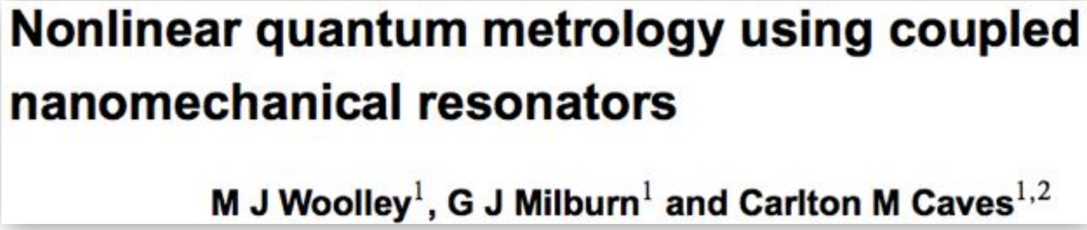
PRL 2008



Exponentially Enhanced Quantum Metrology

S. M. Roy[1,2] and Samuel L. Braunstein[1]

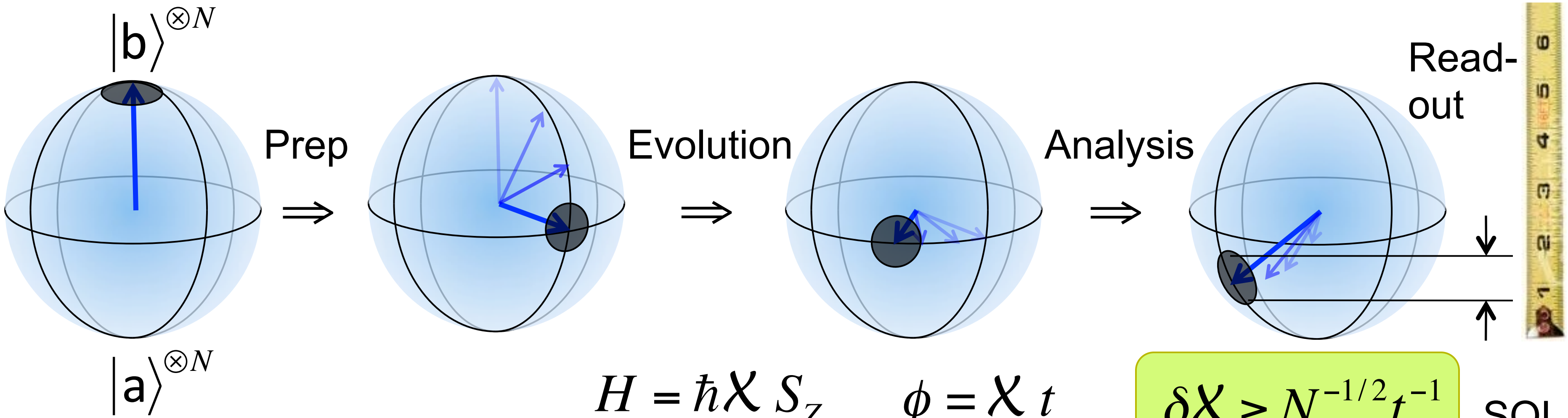
NJP 2008



Nonlinear quantum metrology using coupled nanomechanical resonators

M J Woolley[1], G J Milburn[1] and Carlton M Caves[1,2]

Scaling envy



$H = \hbar \chi S_z \qquad \phi = \chi t$

$\delta\chi \ge N^{-1/2} t^{-1}$ SQL

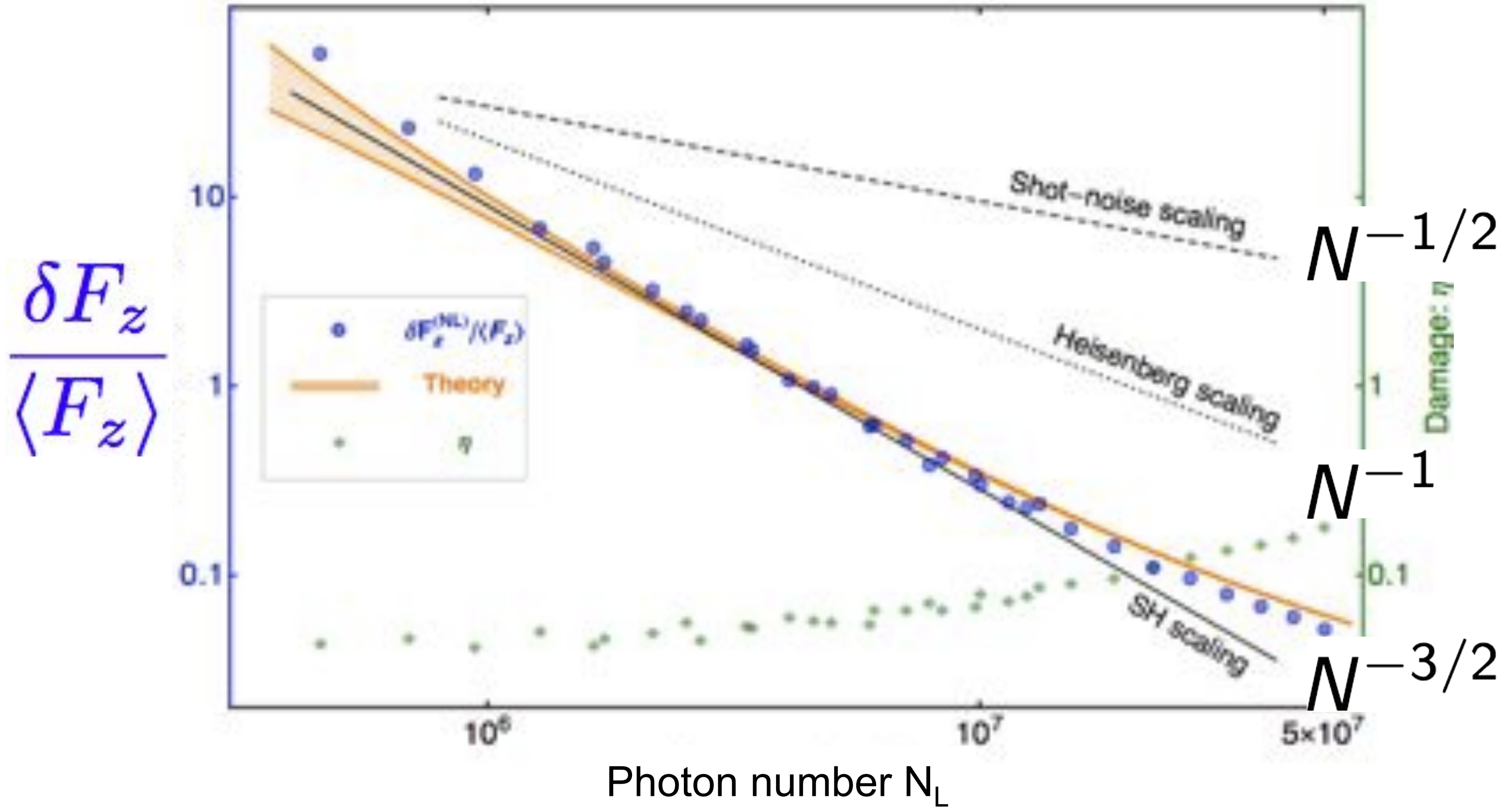
$\delta\chi < N^{-1/2} t^{-1}$

$\delta\chi \ge N^{-1} t^{-1}$ HL

$H = \hbar \Upsilon N S_z \qquad \phi = \Upsilon N t$

$\delta\Upsilon \ge N^{-3/2} t^{-1}$

Boixo *et al.* model confirmed



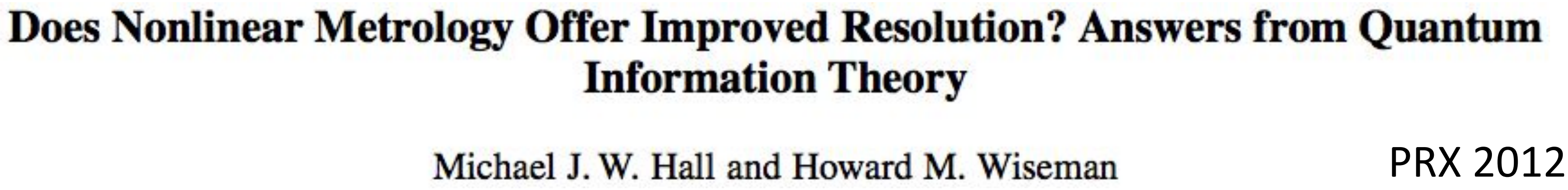
"Interaction-based quantum metrology showing scaling beyond the Heisenberg limit." Napolitano et al. Nature 2011

The resulting chaos

General Optimality of the Heisenberg Limit for Quantum Metrology

Marcin Zwierz,[1] Carlos A. Pérez-Delgado,[1,2] and Pieter Kok[1,*]

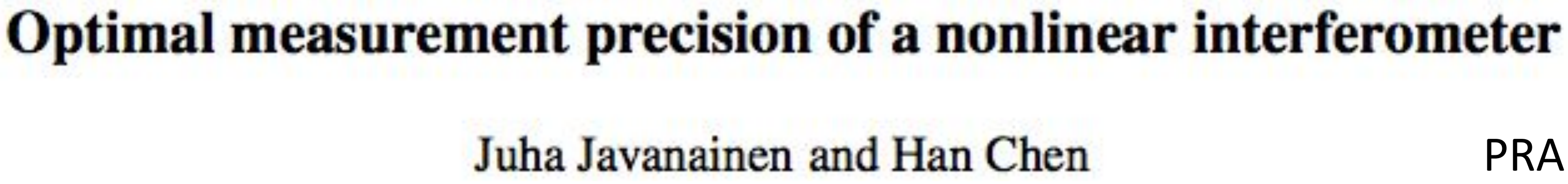
PRL 2010



Does Nonlinear Metrology Offer Improved Resolution? Answers from Quantum Information Theory

Michael J. W. Hall and Howard M. Wiseman

PRX 2012

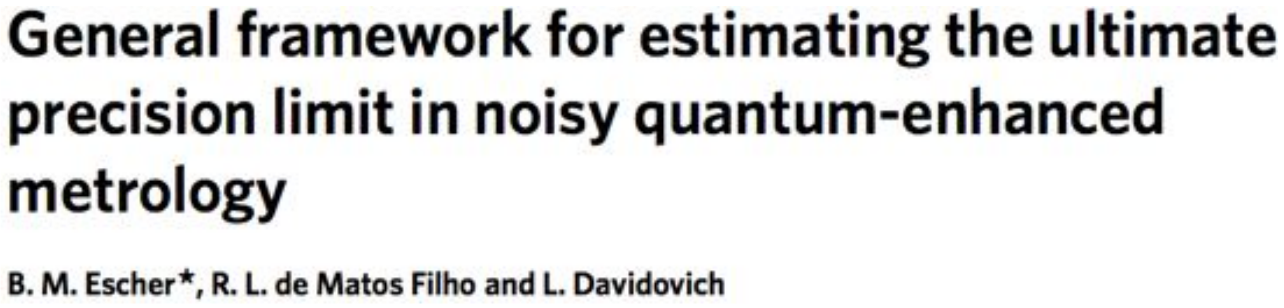


Optimal measurement precision of a nonlinear interferometer

Juha Javanainen and Han Chen

PRA 2012

Revising our ambitions


General framework for estimating the ultimate precision limit in noisy quantum-enhanced metrology

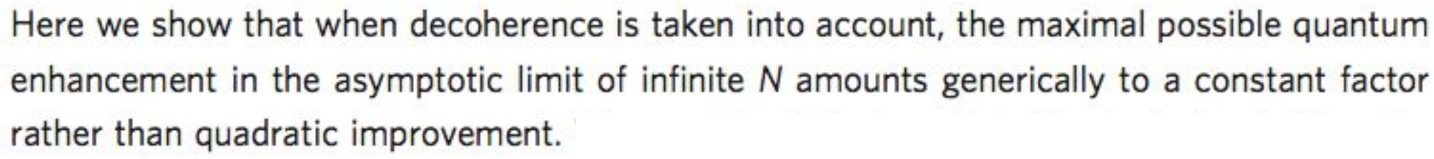
B. M. Escher*, R. L. de Matos Filho and L. Davidovich

N. Phys 2011


The elusive Heisenberg limit in quantum-enhanced metrology

Rafał Demkowicz-Dobrzański[1], Jan Kołodyński[1] & Mădălin Guţă[2]

N. Comms 2012

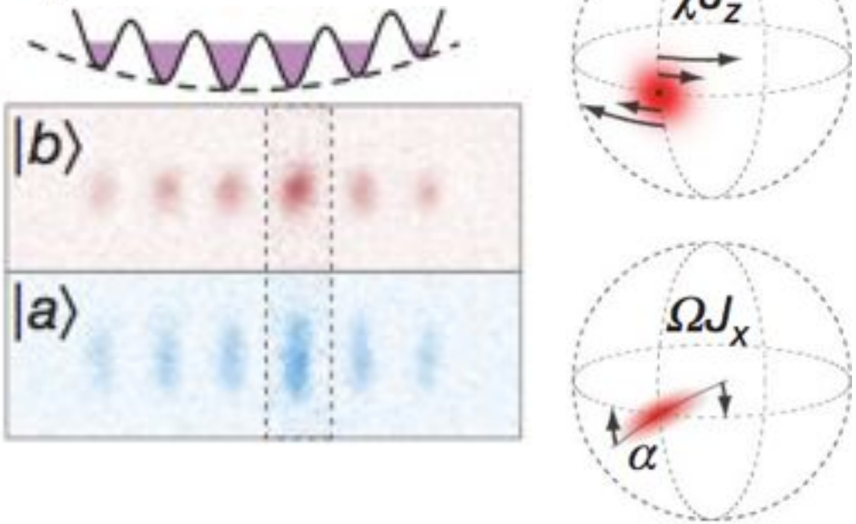

Here we show that when decoherence is taken into account, the maximal possible quantum enhancement in the asymptotic limit of infinite N amounts generically to a constant factor rather than quadratic improvement.

Real-world non-linear interferometers

Nonlinear atom interferometer surpasses classical precision limit

C. Gross[1], T. Zibold[1], E. Nicklas[1], J. Estève[1]† & M. K. Oberthaler[1]

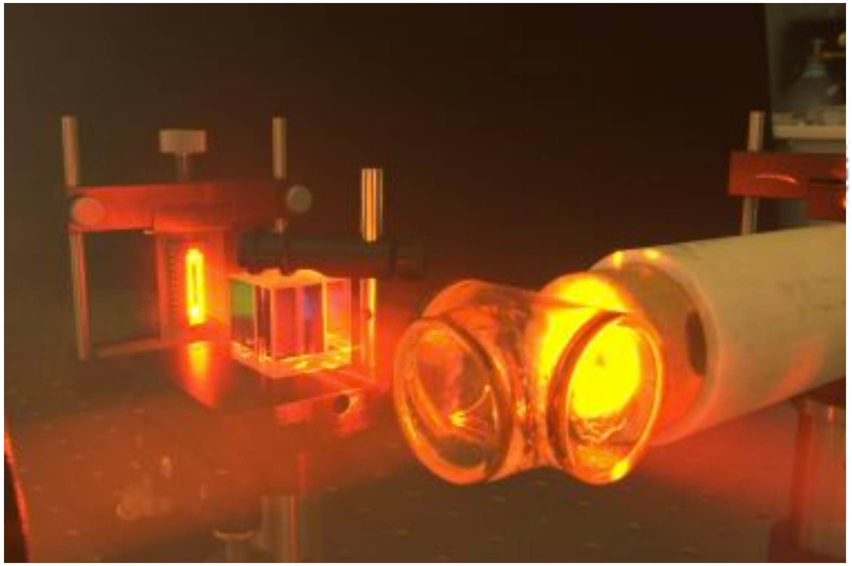
Nature, 2010

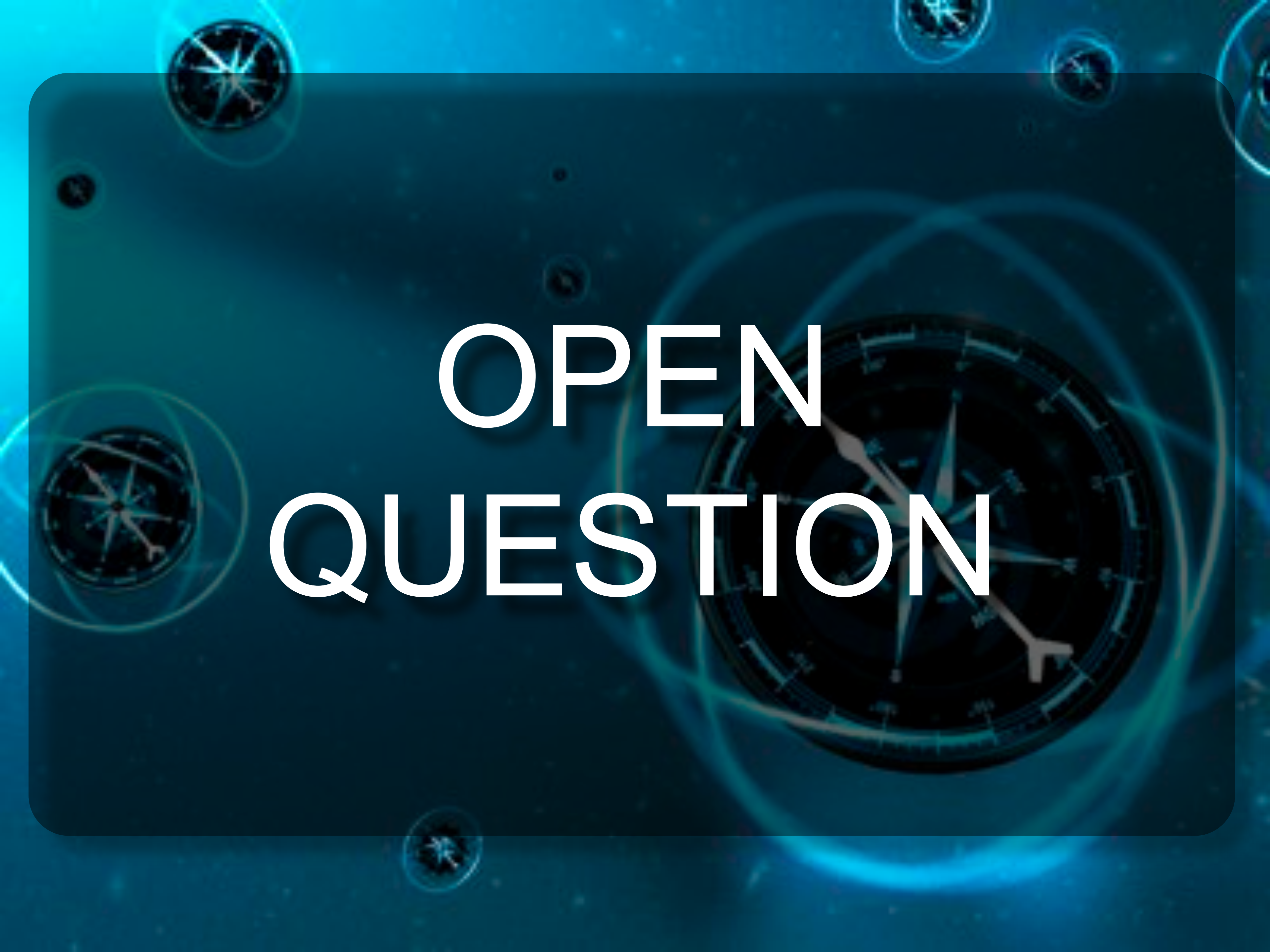


A subfemtotesla multichannel atomic magnetometer

I. K. Kominis*†, T. W. Kornack*, J. C. Allred‡ & M. V. Romalis*

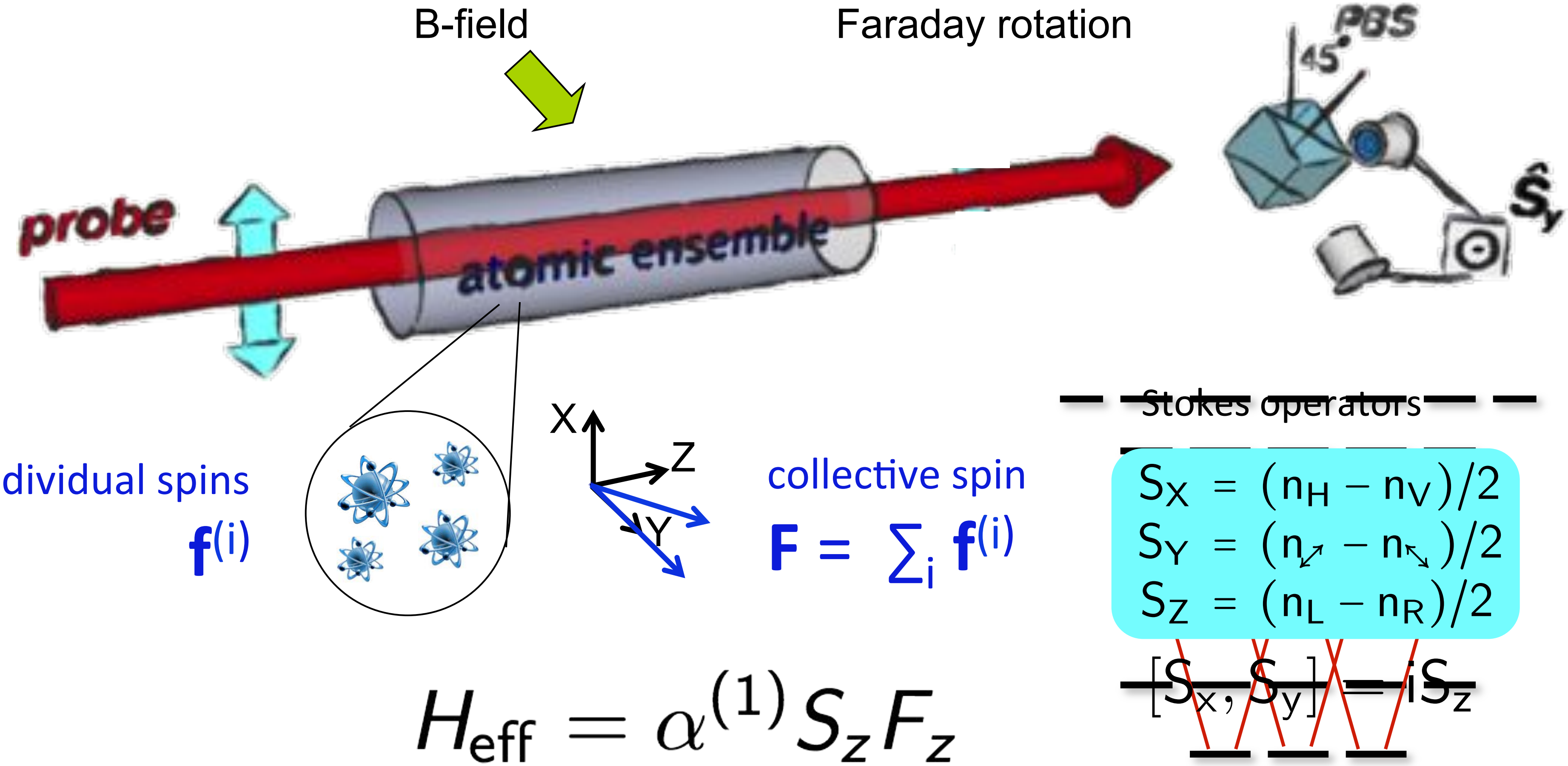
Nature, 2003





OPEN QUESTION

Optical magnetometer



Individual spins

$\mathbf{f}^{(i)}$

collective spin

$\mathbf{F} = \sum_i \mathbf{f}^{(i)}$

Stokes operators

$S_X = (n_H - n_V)/2$

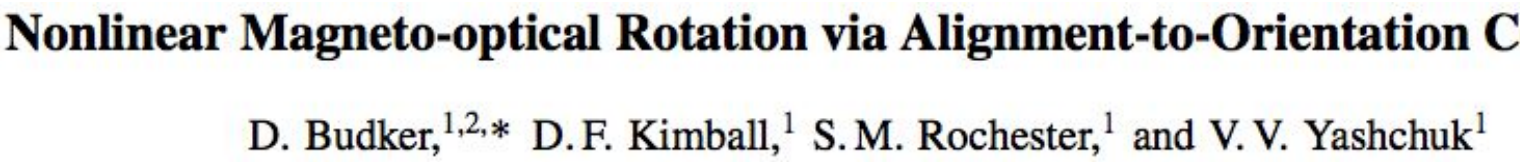
$S_Y = (n_{\nearrow} - n_{\nwarrow})/2$

$S_Z = (n_L - n_R)/2$

$[S_x, S_y] = iS_z$

$$H_{\text{eff}} = \alpha^{(1)} S_z F_z$$

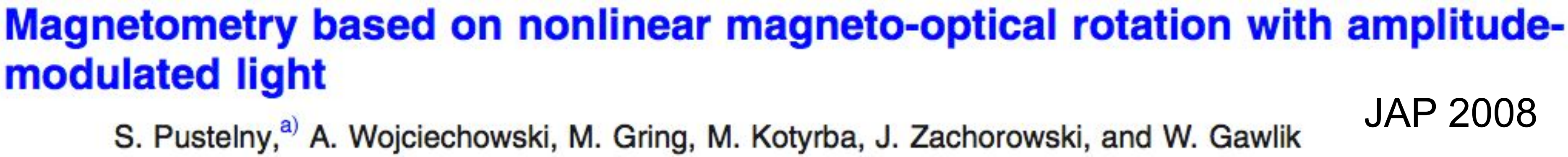
Nonlinear magneto-optic rotation



Nonlinear Magneto-optical Rotation via Alignment-to-Orientation Conversion

D. Budker,[1,2,*] D. F. Kimball,[1] S. M. Rochester,[1] and V. V. Yashchuk[1]

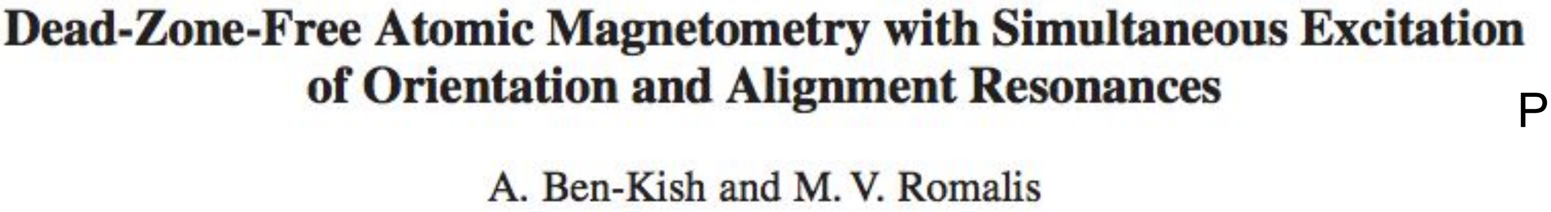
PRL 2000



Magnetometry based on nonlinear magneto-optical rotation with amplitude-modulated light

S. Pustelny,[a)] A. Wojciechowski, M. Gring, M. Kotyrba, J. Zachorowski, and W. Gawlik

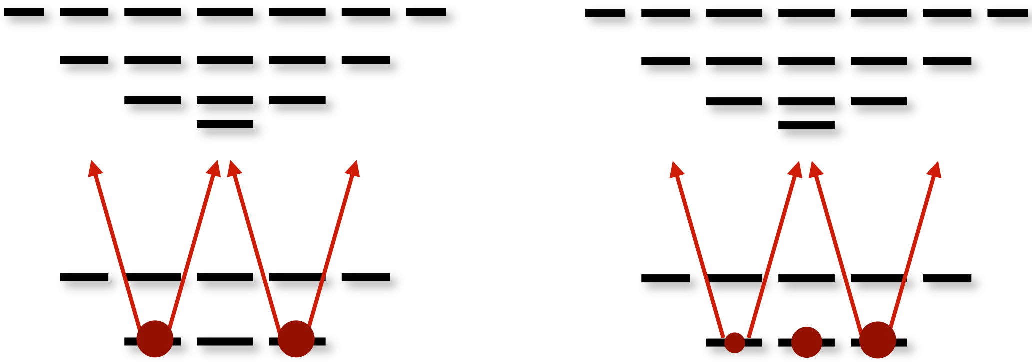
JAP 2008



Dead-Zone-Free Atomic Magnetometry with Simultaneous Excitation of Orientation and Alignment Resonances

A. Ben-Kish and M. V. Romalis

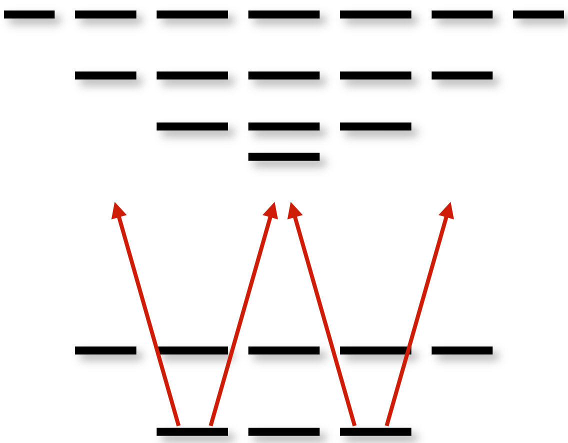
PRL 2010



alignment j_x

orientation f_z

Light-atom interactions



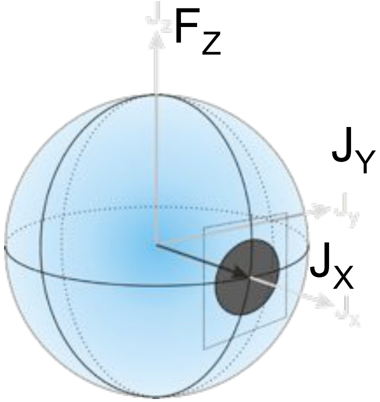
f_z

$j_x \equiv |-1\rangle\langle+1| + \text{h.c.}$

$j_y \equiv i|-1\rangle\langle+1| + \text{h.c.}$

$$\mathbf{J} \equiv \sum_i \mathbf{j}^{(i)}, \mathbf{F} \equiv \sum_i \mathbf{f}^{(i)}$$

$$[J_x, J_y] = iF_z$$



$$H_{\text{eff}}^{(2)} = \alpha^{(1)} S_Z F_Z + \alpha^{(2)} (S_X J_X + S_Y J_Y) - g\mu_B \mathbf{F} \cdot \mathbf{B}$$

vector

tensor

magnetic

Faraday rotation

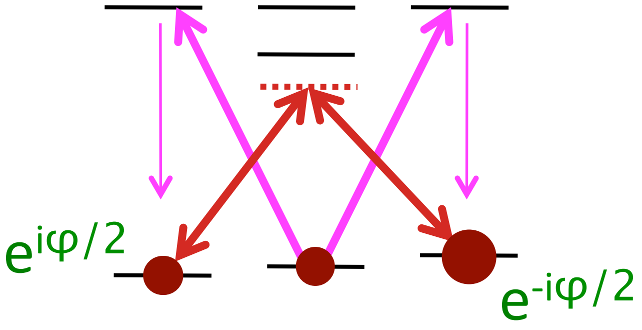
stimulated Raman

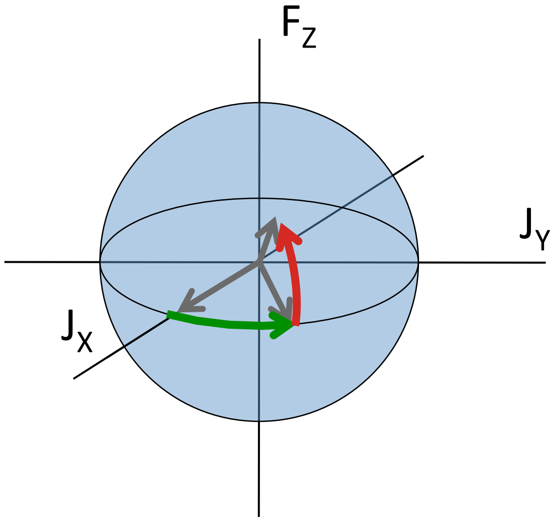
precession

Alignment-to-orientation conversion

$$H_{\rm eff}^{(2)} = \alpha^{(1)} S_Z F_Z + \alpha^{(2)} (S_X J_X + S_Y J_Y) - g\mu_B \mathbf{F}\cdot\mathbf{B}$$

optical pumping	pump
precession	$F \cdot B$
stimulated Raman aka AOC	$S_X J_X$
Faraday rotation	$S_Z F_Z$





$J_Y \propto B_Z$

$F_Z \propto J_Y N_{\rm probe}$

$\varphi \propto F_Z \propto J_Y N_{\rm probe}$

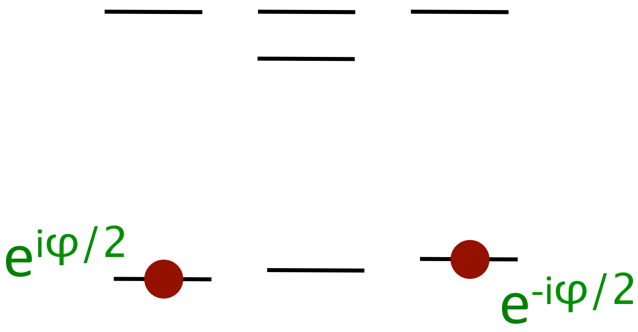
Linear-to-Elliptical (LTE) readout

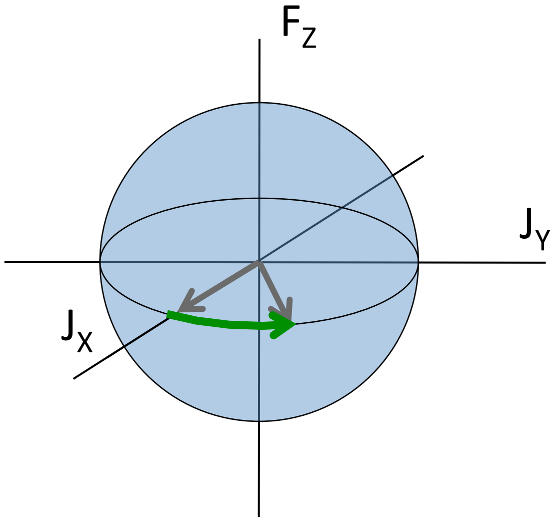
$$H_{\mathrm{eff}}^{(2)} = \alpha^{(1)} S_Z F_Z + \alpha^{(2)} (S_X J_X + S_Y J_Y) - g\mu_B \mathbf{F} \cdot \mathbf{B}$$

optical pumping **pump**

precession $F \cdot B$

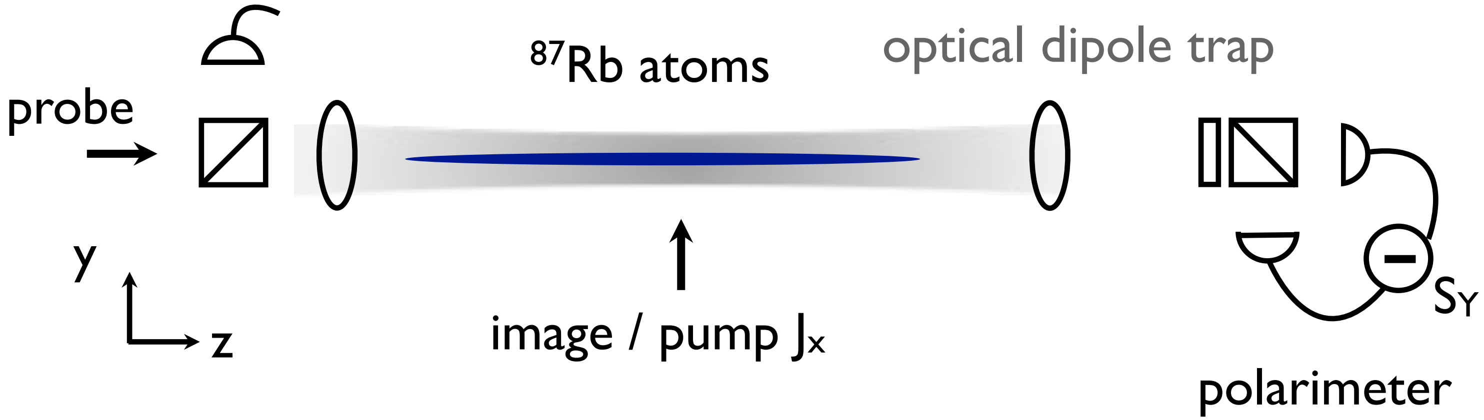
Faraday rotation $S_Y J_Y$





$$\varphi \propto J_Y \propto B_Z$$

Quantum interface with cold ^{87}Rb ensemble



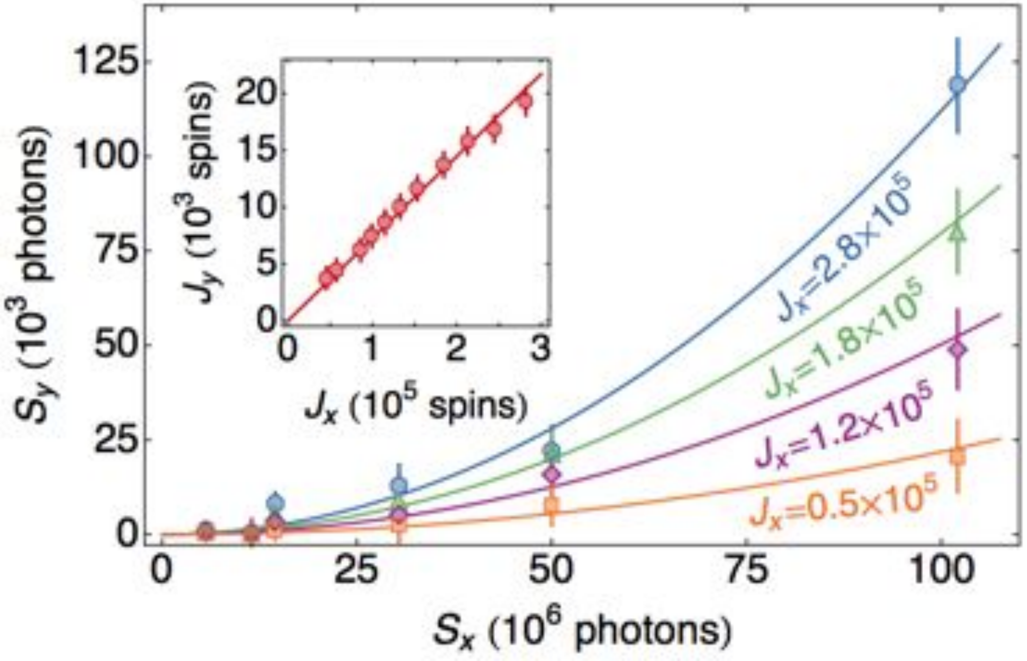
1 μs long pulses
linearly polarized
"mode matched" to atoms
0.7 GHz from D_2 line

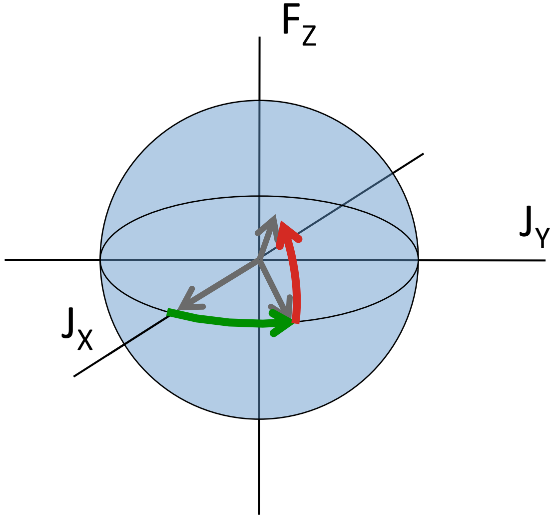
$\sim 10^6$ ^{87}Rb atoms at 25μK
f=1 ground-state

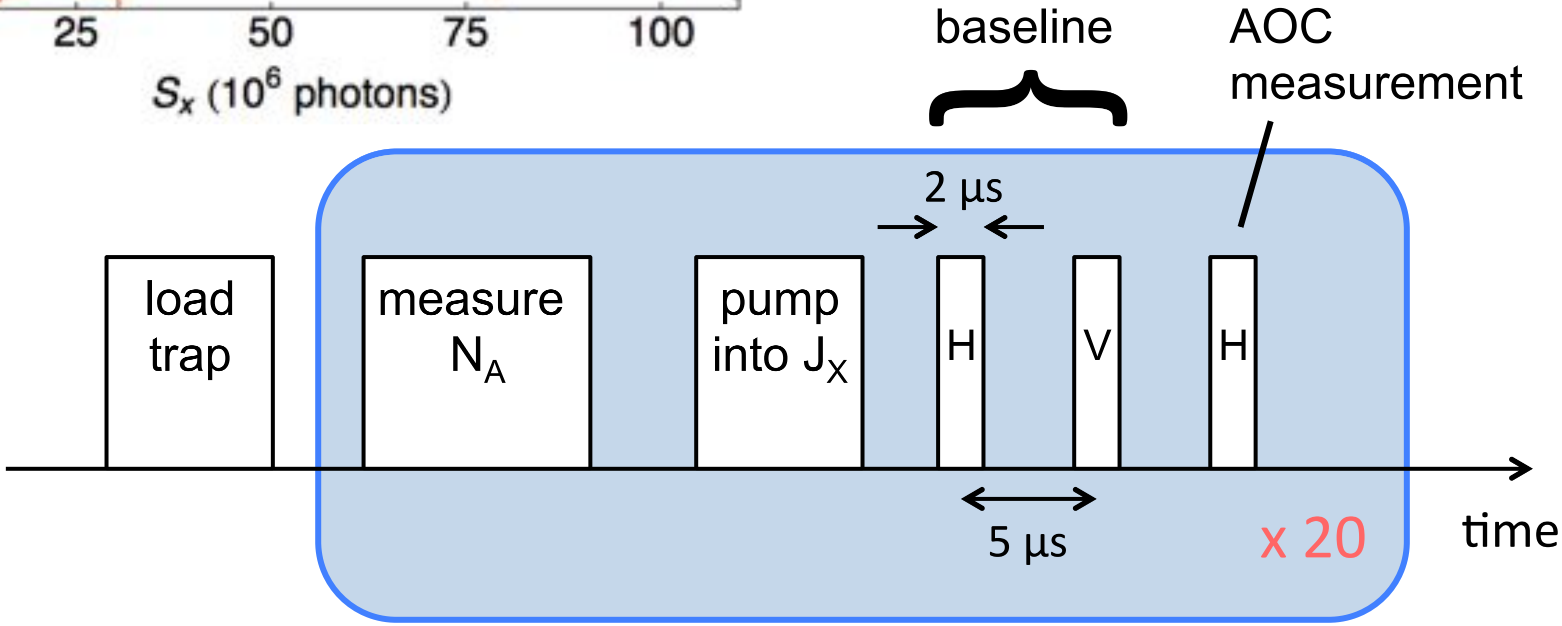
[1] effective OD > 50
[2] Sensitivity 512 spins, < SQL
[3] QND measurement
[4] spin squeezing

1 Kubasik, et al. PRA 79, 043815 (2009)
2 Koschorreck, et al. PRL (2010)
3 Koschorreck, et al. PRL (2010), Sewell, et al. N. Phot. (2013)
4 Sewell, et al. arXiv (2011)

Experimental sequence and signal







baseline

AOC measurement

load trap

measure N_A

pump into J_X

H

V

H

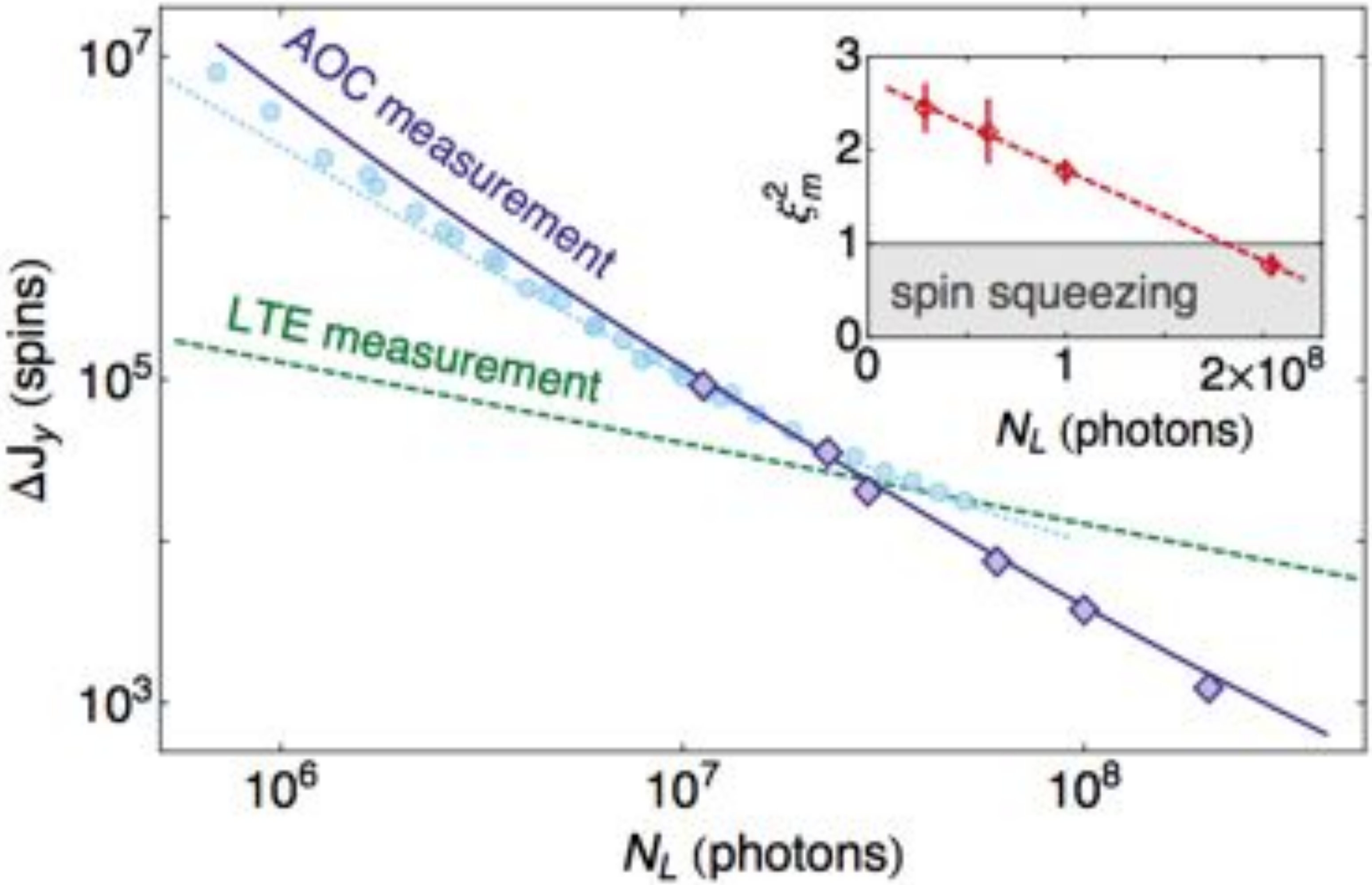
2 µs

5 µs

x 20

time

AOC beats the best linear measurement



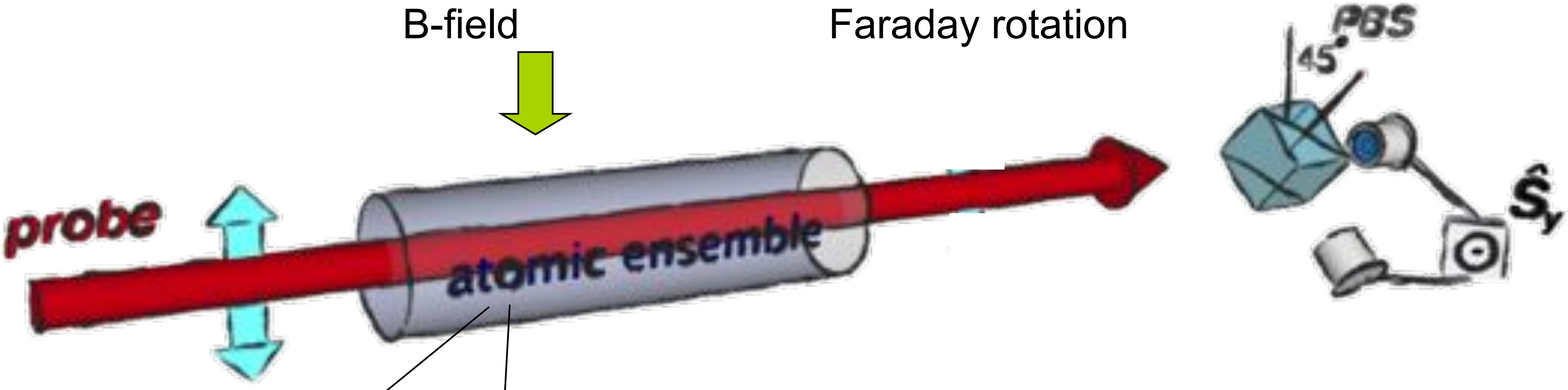
Sewell et al. arXiv:1310.5889 to appear in PRX

SQUEEZED-ATOM MAGNETOMETER

Faraday rotation optical magnetometer

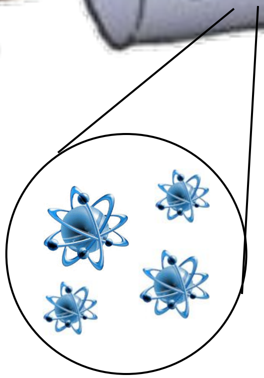
B-field

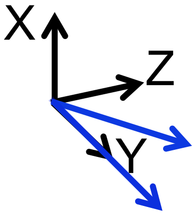
Faraday rotation



Individual spins

$\mathbf{f}^{(i)}$





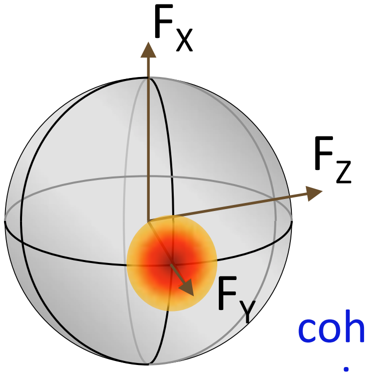
collective spin

$\mathbf{F} = \sum_i \mathbf{f}^{(i)}$

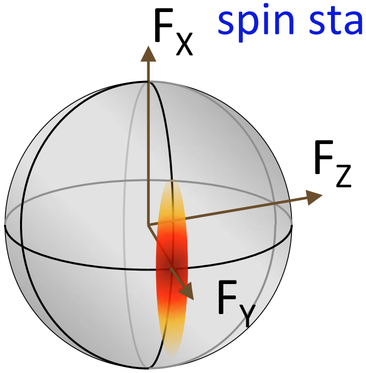
$[F_x, F_y] = i\, F_z$

and cycl. permutations

$\delta F_x\, \delta F_z \geq \tfrac{1}{2}\, |\langle F_y \rangle|$

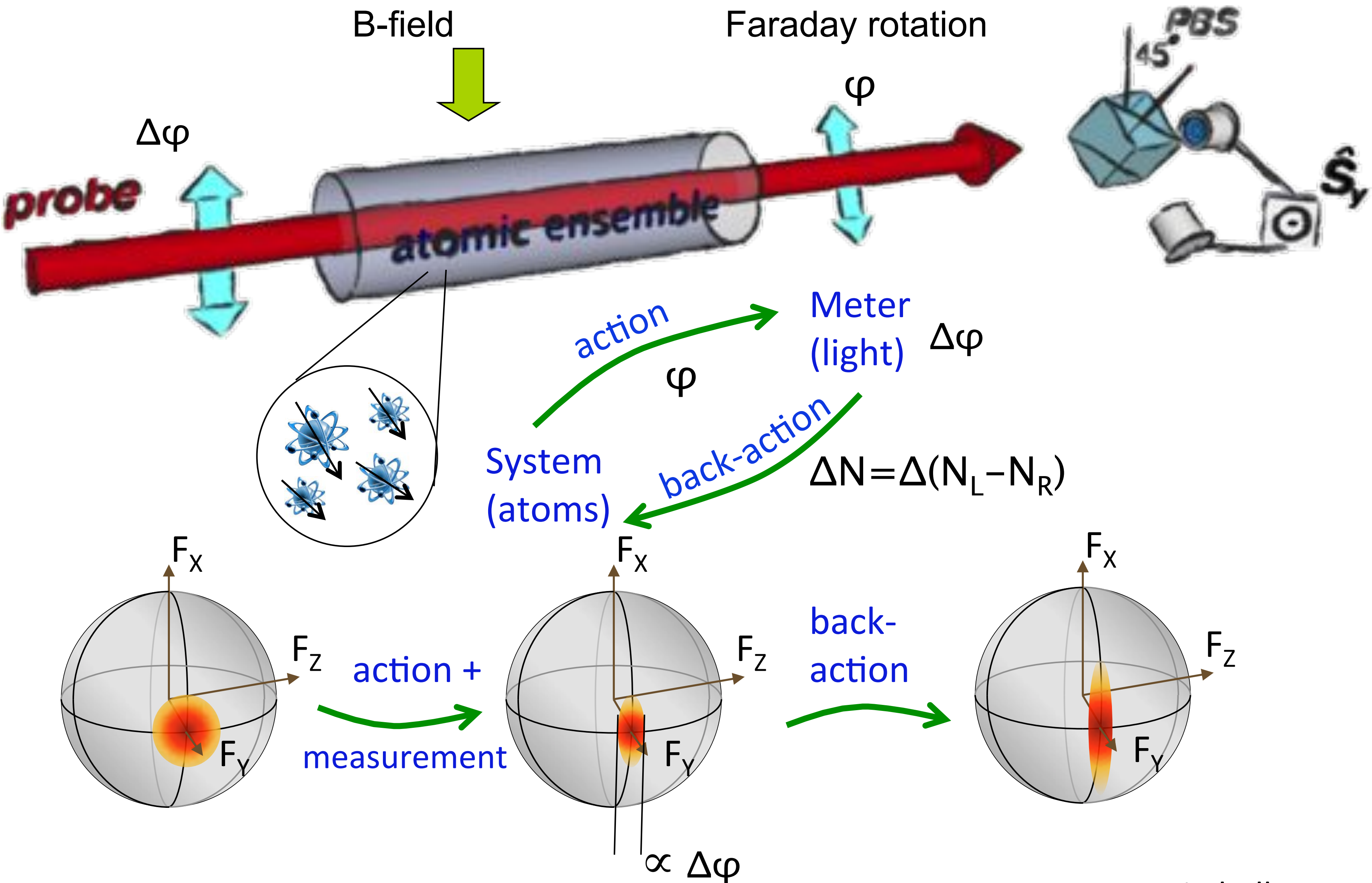


coherent spin state



squeezed spin state

QND in optical magnetometer



B-field

Faraday rotation

$\Delta\varphi$

φ

Meter (light) $\Delta\varphi$

action

φ

System (atoms)

back-action

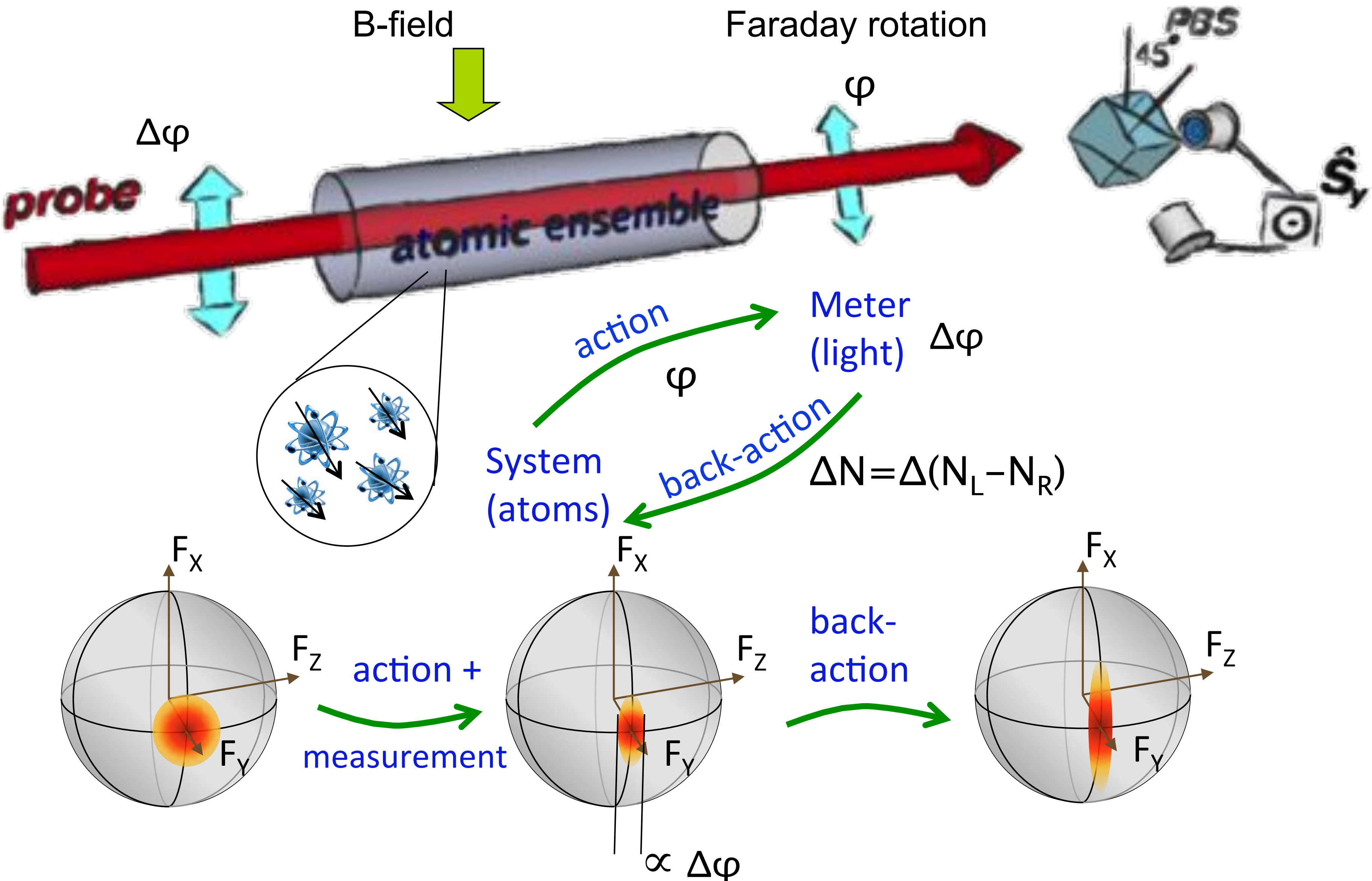
$\Delta N = \Delta(N_L - N_R)$

action + measurement

back-action

$\propto \Delta\varphi$

QND in optical magnetometer



B-field

Faraday rotation

φ

$\Delta\varphi$

action

φ

Meter (light)

$\Delta\varphi$

System (atoms)

back-action

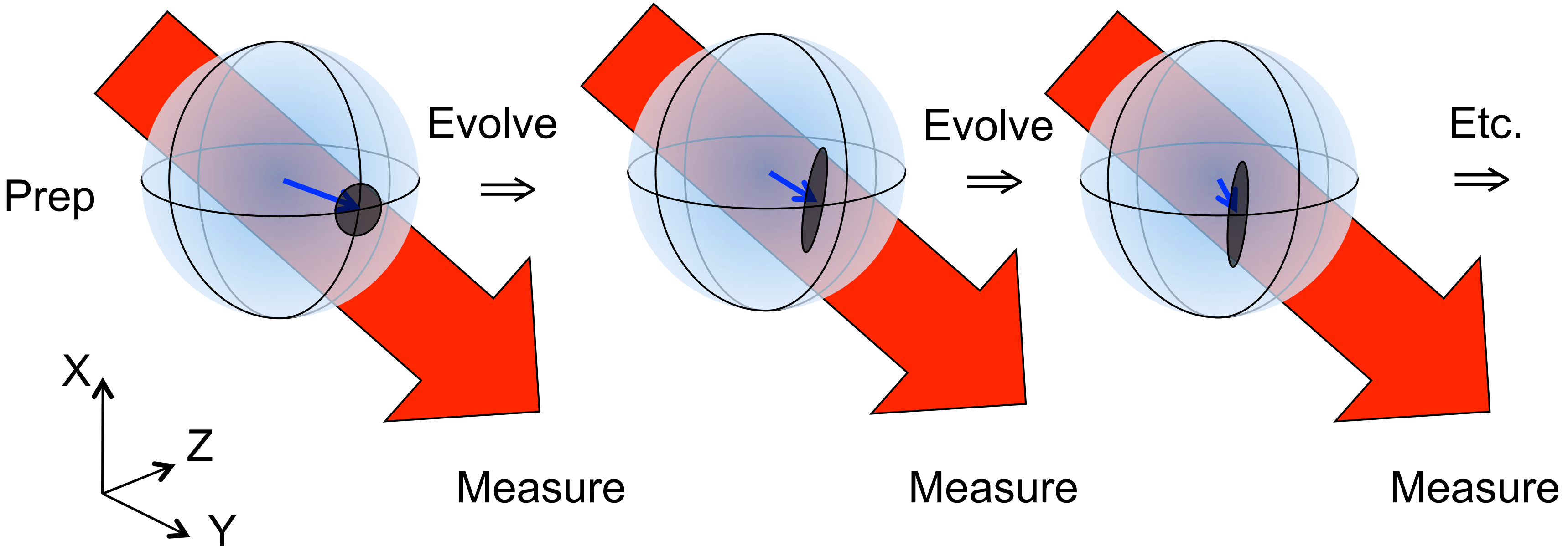
$\Delta N = \Delta(N_L - N_R)$

action + measurement

back-action

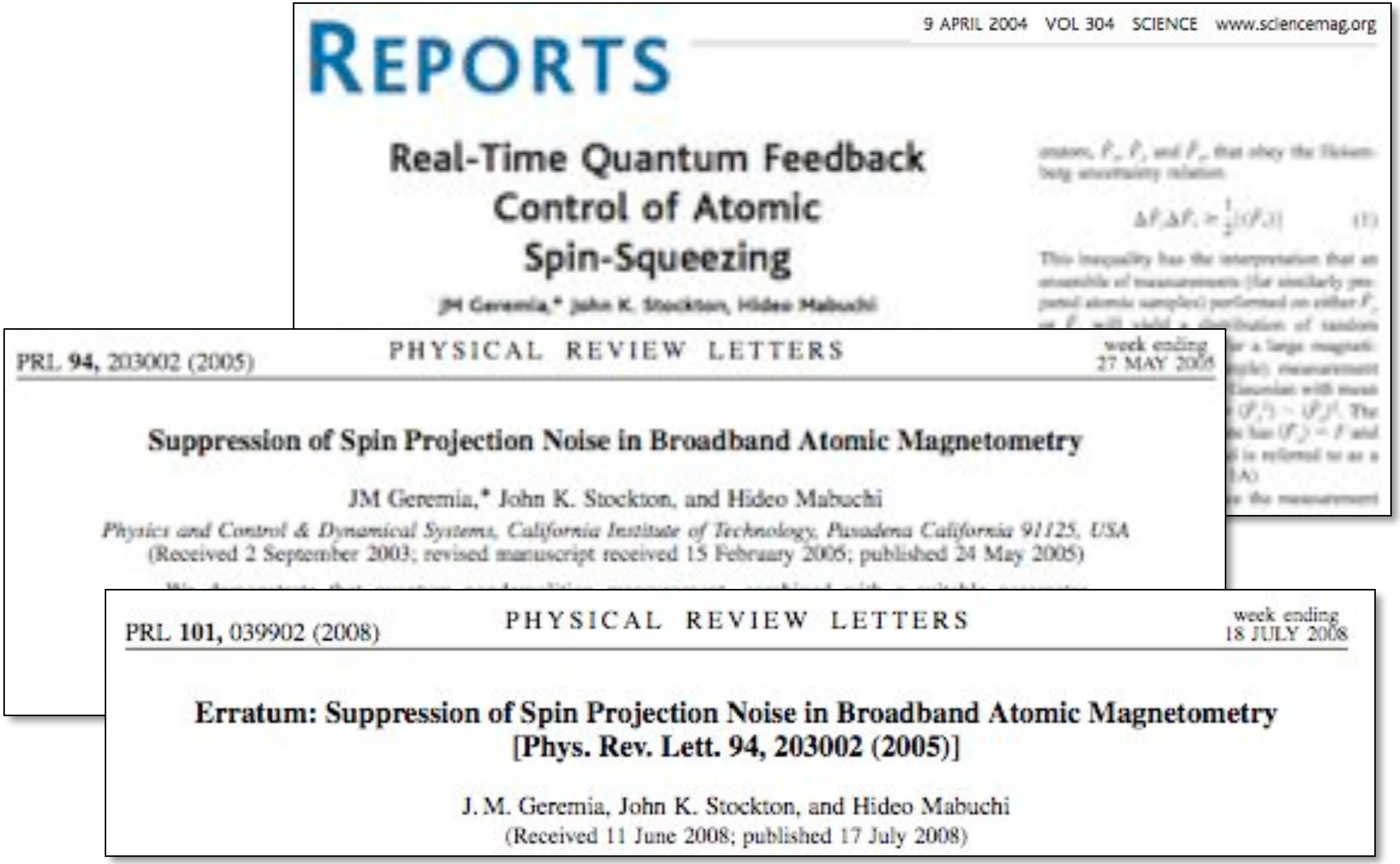
$\propto \Delta\varphi$

Measurement-induced squeezing



	Kuzmich	Mabuchi	Polzik	Vuletic	Takahashi	Thompson
Proposal	F=1/2					
Clocks			J=1/2	J=1/2		J=1/2
Magnetometer		F=4 ^{133}Cs				
Other					I=1/2 ^{171}Yb	

To boldly go where others have gone before



9 APRIL 2004 VOL 304 SCIENCE www.sciencemag.org

REPORTS

Real-Time Quantum Feedback Control of Atomic Spin-Squeezing

JM Geremia,* John K. Stockton, Hideo Mabuchi

PRL 94, 203002 (2005) PHYSICAL REVIEW LETTERS week ending 27 MAY 2005

Suppression of Spin Projection Noise in Broadband Atomic Magnetometry

JM Geremia,* John K. Stockton, and Hideo Mabuchi

Physics and Control & Dynamical Systems, California Institute of Technology, Pasadena California 91125, USA

(Received 2 September 2003; revised manuscript received 15 February 2005; published 24 May 2005)

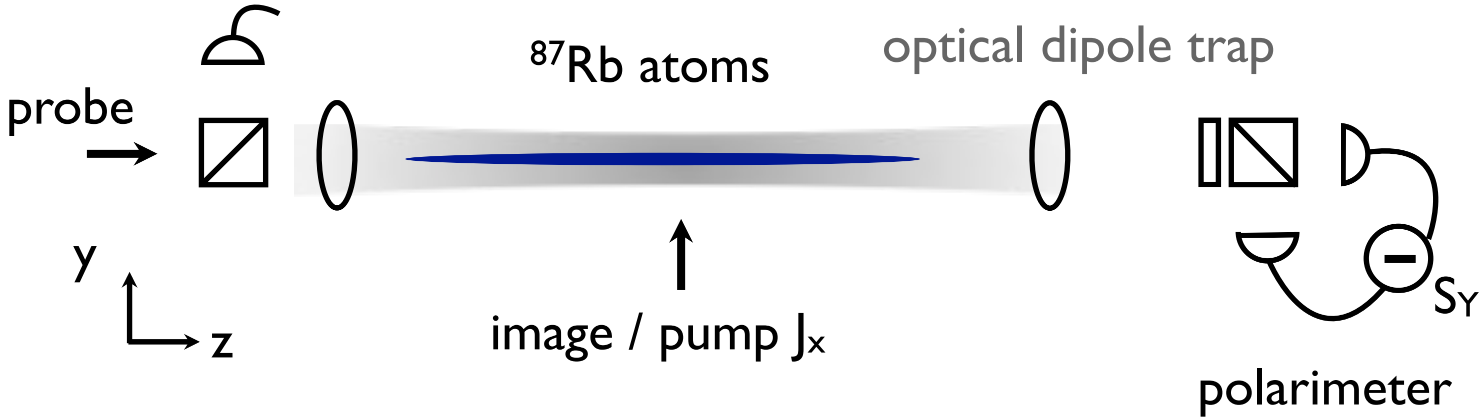
PRL 101, 039902 (2008) PHYSICAL REVIEW LETTERS week ending 18 JULY 2008

Erratum: Suppression of Spin Projection Noise in Broadband Atomic Magnetometry [Phys. Rev. Lett. 94, 203002 (2005)]

J. M. Geremia, John K. Stockton, and Hideo Mabuchi

(Received 11 June 2008; published 17 July 2008)

Quantum interface with cold ^{87}Rb ensemble



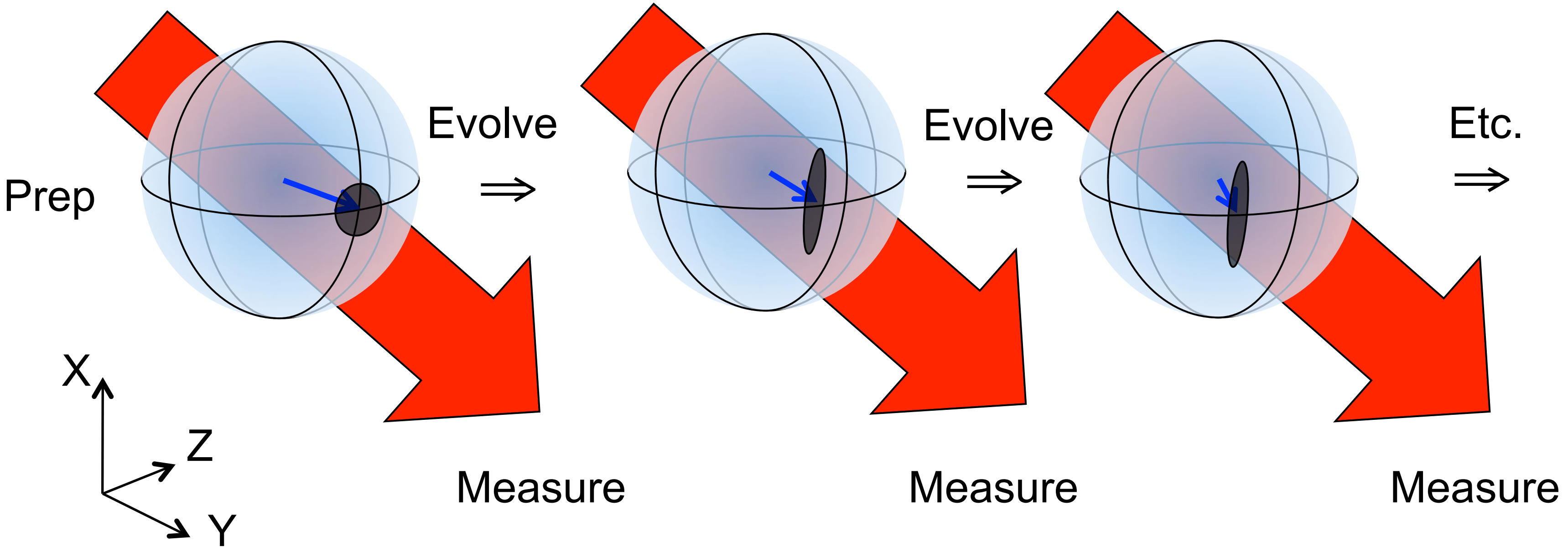
1μs long pulses
linearly polarized
"mode matched" to atoms
0.7 GHz from D_2 line

[1] effective OD > 50
[2] Sensitivity 512 spins, < SQL
[3] QND measurement
[4] spin squeezing

~10^6 ^{87}Rb atoms at 25μK
f=1 ground-state

1 Kubasik, et al. PRA 79, 043815 (2009)
2 Koschorreck, et al. PRL (2010)
3 Koschorreck, et al. PRL (2010),
+ Sewell, et al. N. Phot. (2013)
4 Sewell, et al. PRL (2012)

Measurement-induced squeezing



Prep

Evolve ⇒

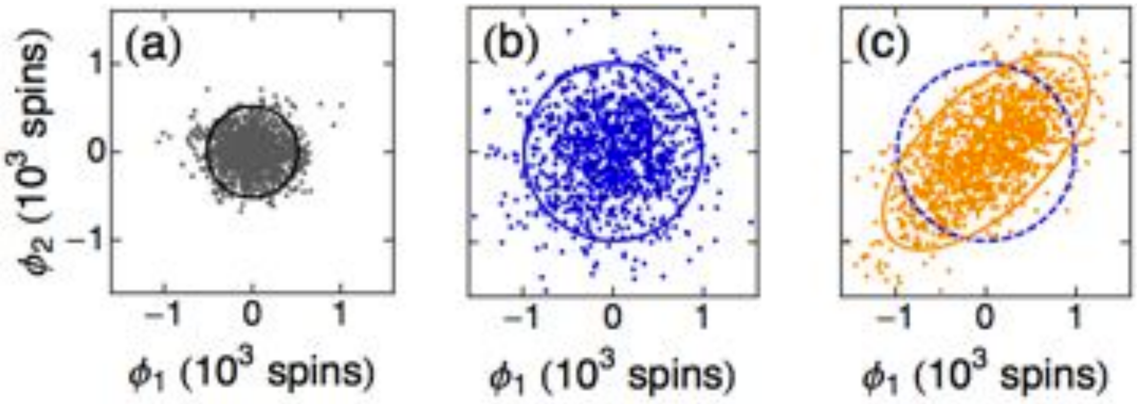
Measure

Evolve ⇒

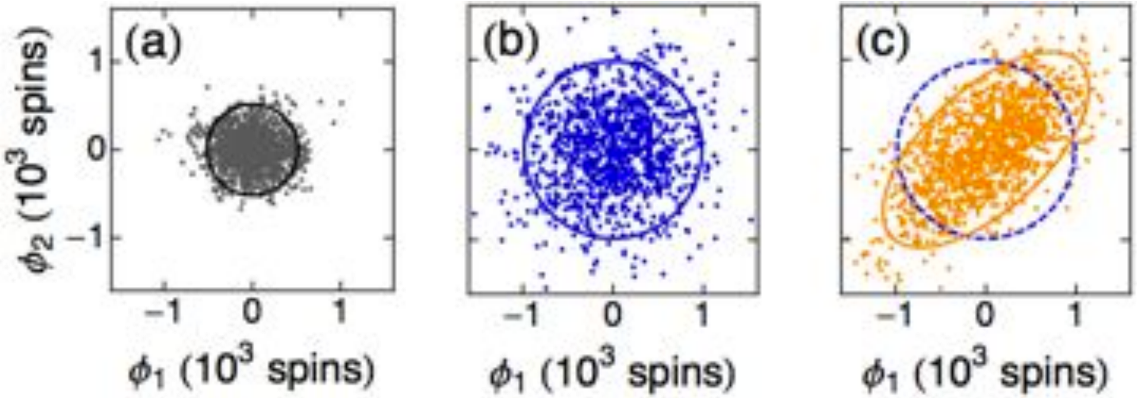
Measure

Etc. ⇒

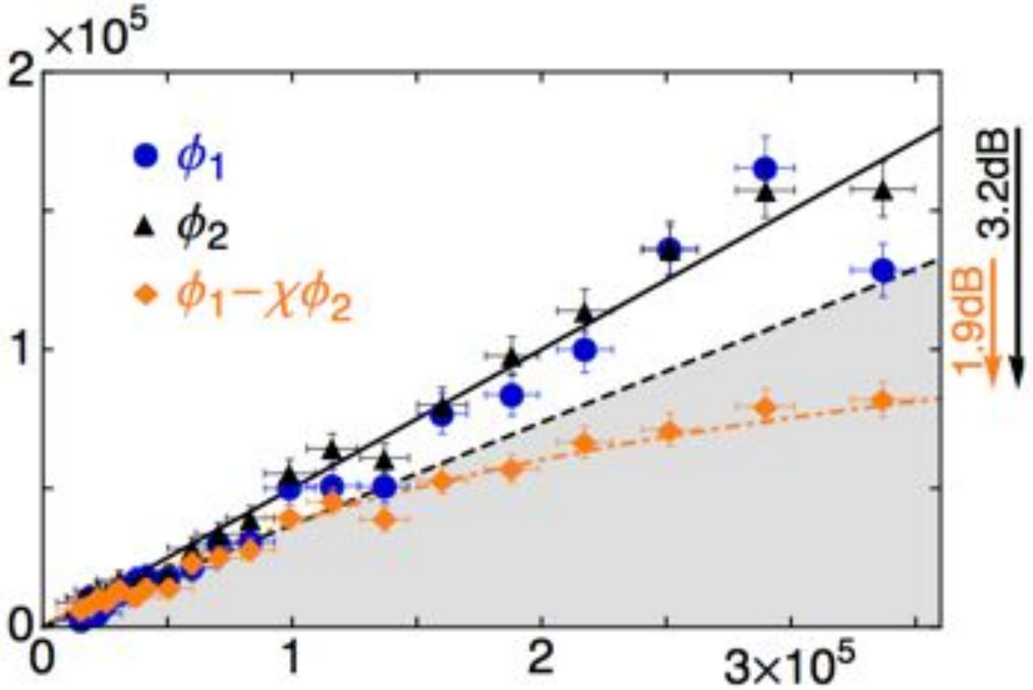
Measure



Squeezing of spin alignment-orientation



var K_θ

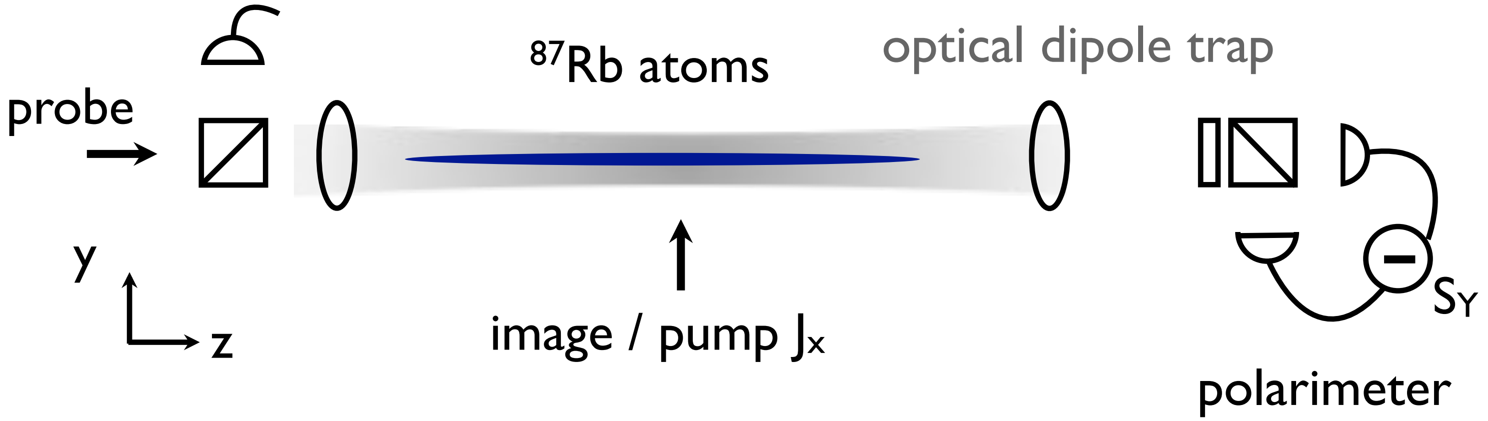


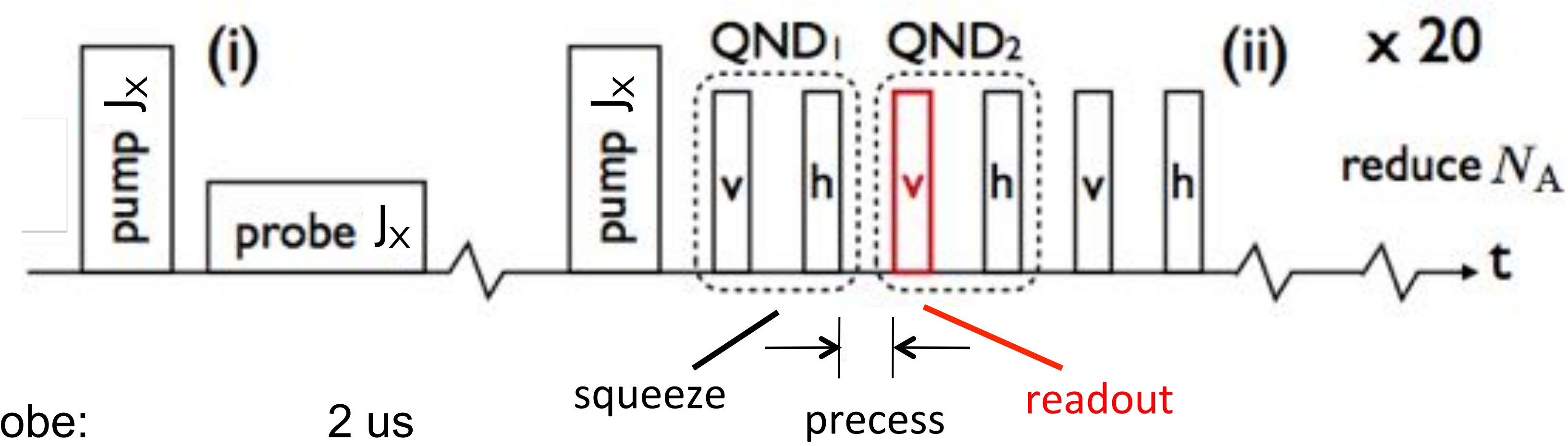
J_X (spins)

squeezing of alignment-orientation variable K_θ

Sewell et al. PRL **109**, 253605 (2012)

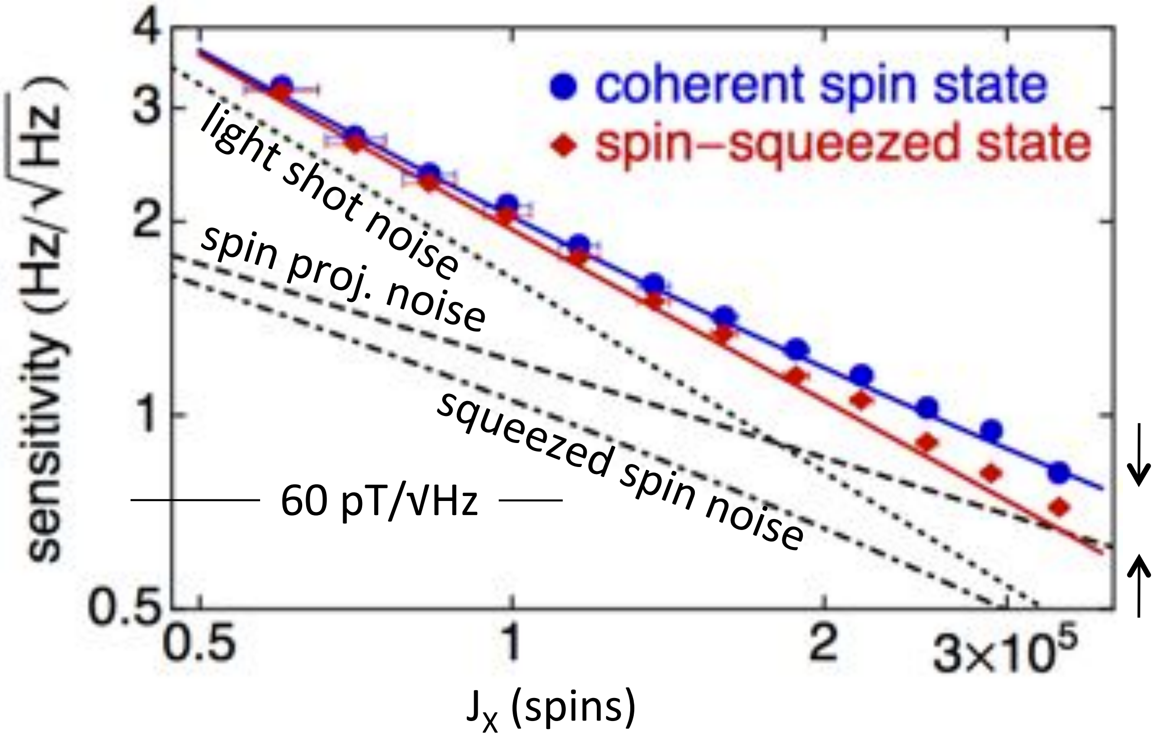
Measurement sequence

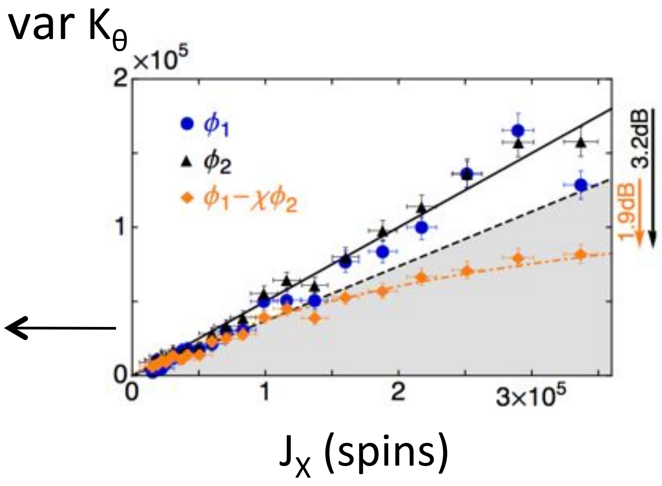




probe:	2 us
precession:	5 us
detuning:	600 MHz

Squeezed-atom magnetometry





Sewell et al. PRL **109**, 253605 (2012)