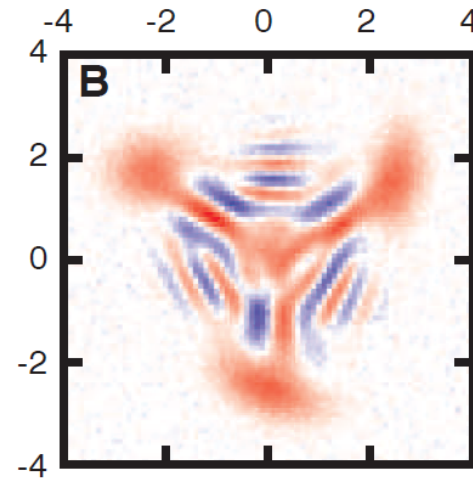
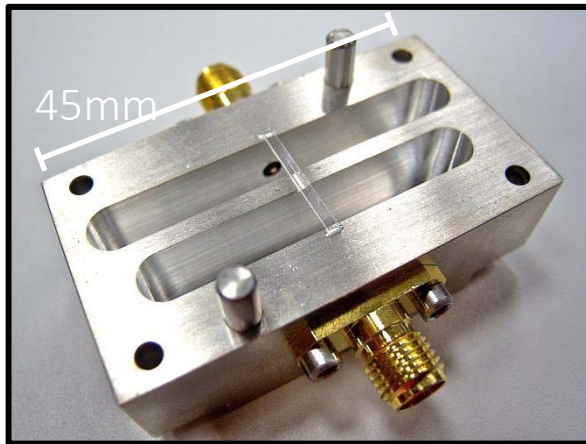


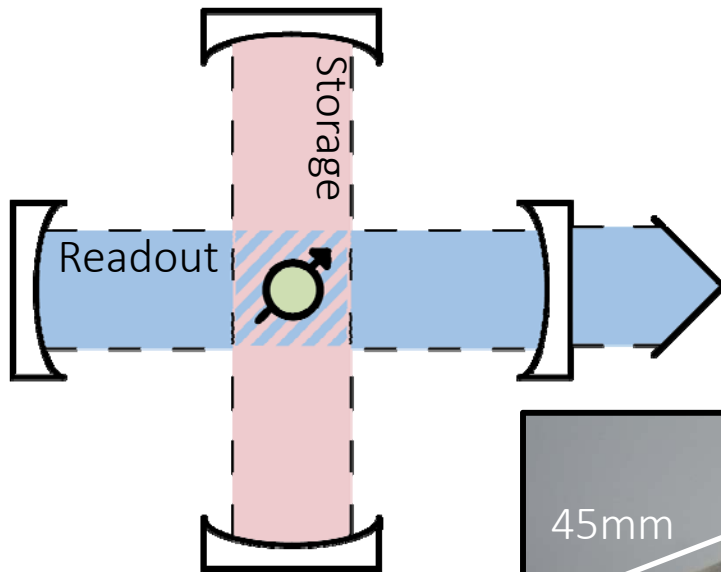
Quantum Control of Superconducting Circuits



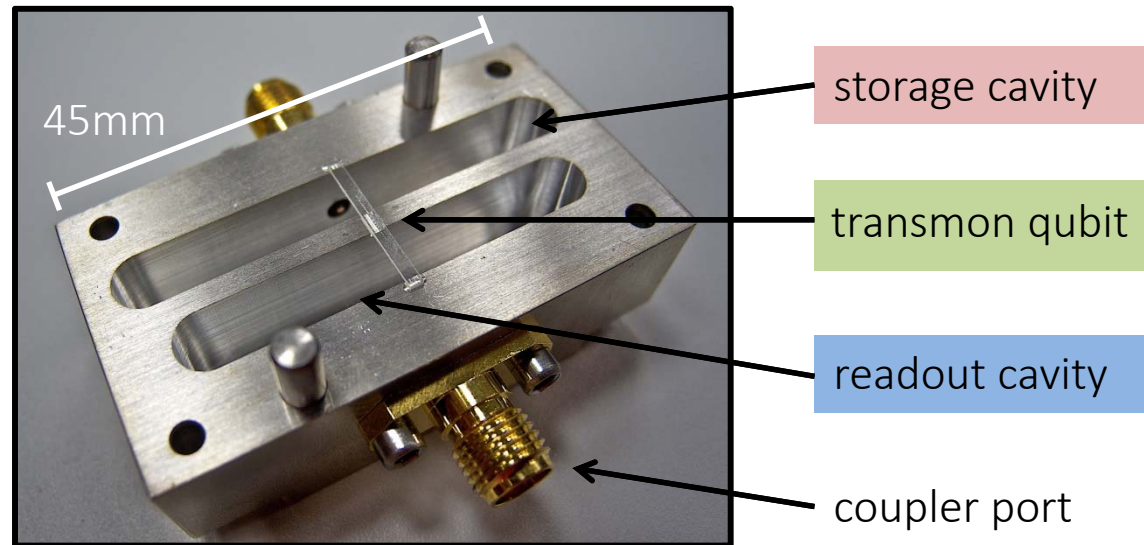
Liang Jiang
Yale University, Applied Physics
Benasque Workshop
2014.7.8

Two-cavity architecture

Cavity QED equivalent:



One fast probe cavity for qubit state detection and one long-lived cavity for photon manipulation/storage



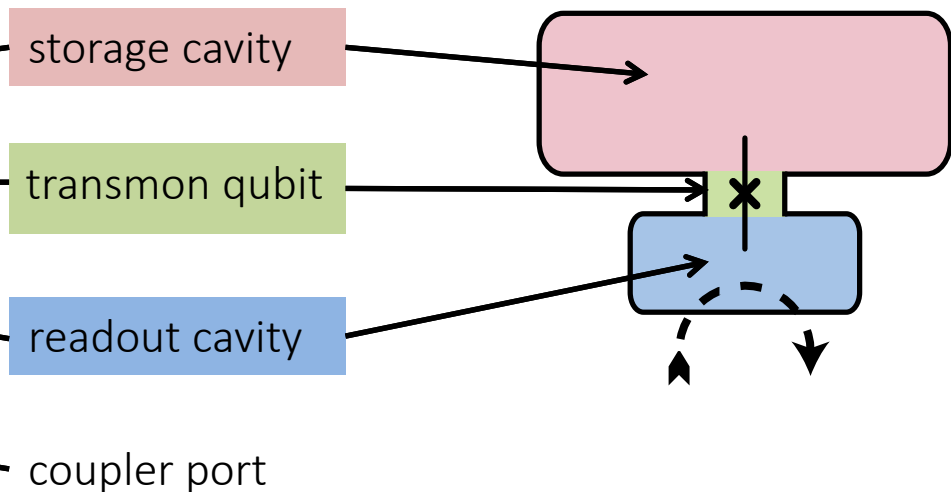
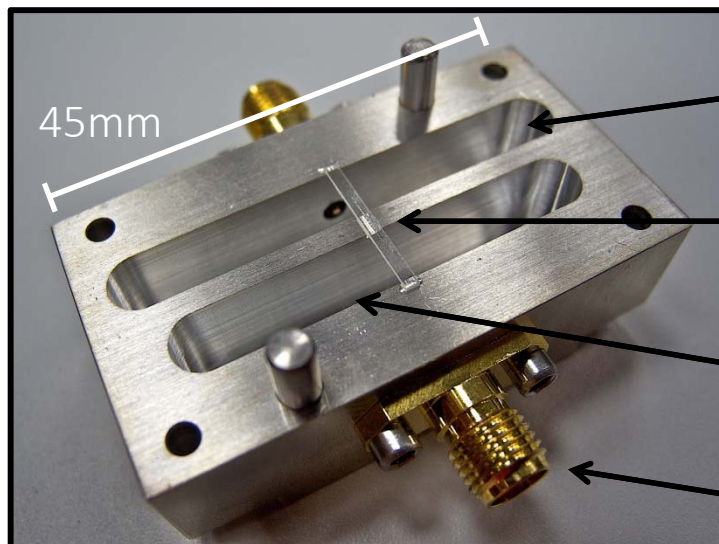
Two-cavity architecture

Effective Hamiltonian:

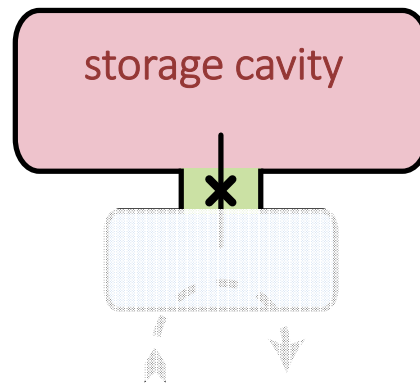
$$H / \hbar = \omega_s a^\dagger a + \omega_r b^\dagger b + \omega_q |e\rangle\langle e| - \chi_s a^\dagger a |e\rangle\langle e| - \chi_r b^\dagger b |e\rangle\langle e|$$

Interaction strength dominates photon loss: $\chi_s / \kappa_s > 300$

κ_s : storage photon decay rate



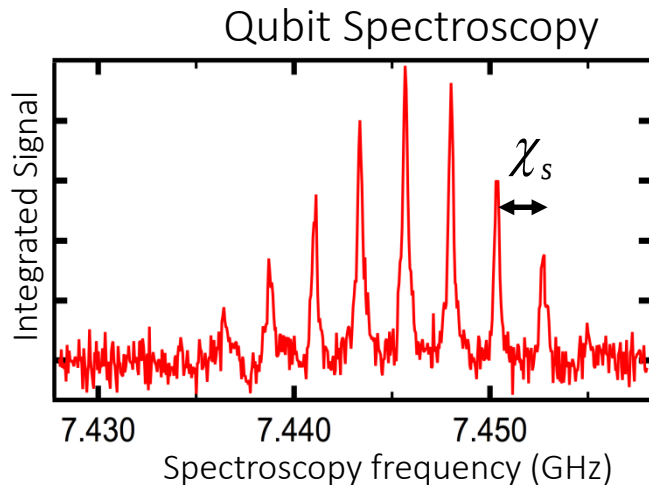
Coherent Operations



- Qubit Rotation (conditional & unconditional)
- Cavity displacement (conditional & unconditional)

Manipulating the qubit

$$H / \hbar = \omega_s a^\dagger a + (\omega_q - \chi_s a^\dagger a) |e\rangle\langle e|$$



Strong Drive:

Unconditional qubit rotation:

$$Y_\pi |n, g\rangle \Rightarrow |n, g\rangle$$

for all n

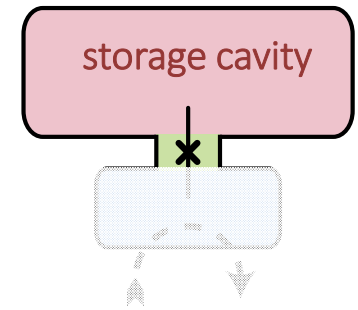
Weak Drive:

Conditional qubit rotation:

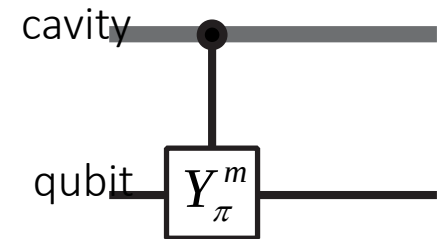
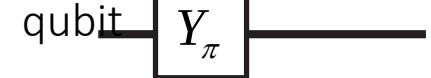
$$Y_\pi^m |m, g\rangle \Rightarrow |m, e\rangle$$

$$Y_\pi^m |n, g\rangle \Rightarrow |n, g\rangle$$

$n \neq m$



Quantum circuit



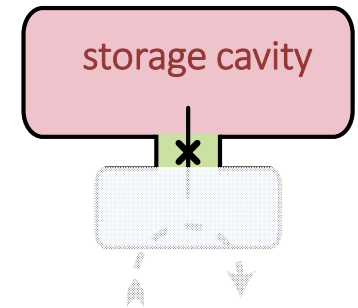
Query the qubit: ‘Are there m photons in the cavity?’

Johnson B.R. et al. Nature Phys. 6, 663-667 2010

Kirchmair G. et al. Nature 495 205-209 2013

Manipulating the cavity

$$H / = (\omega_s - \chi_s |e\rangle\langle e|) a^\dagger a + \omega_q |e\rangle\langle e|$$



Strong Drive:

Unconditional cavity displacement

$$D_\beta |0\rangle \Rightarrow |\beta\rangle$$

indep. of qubit state

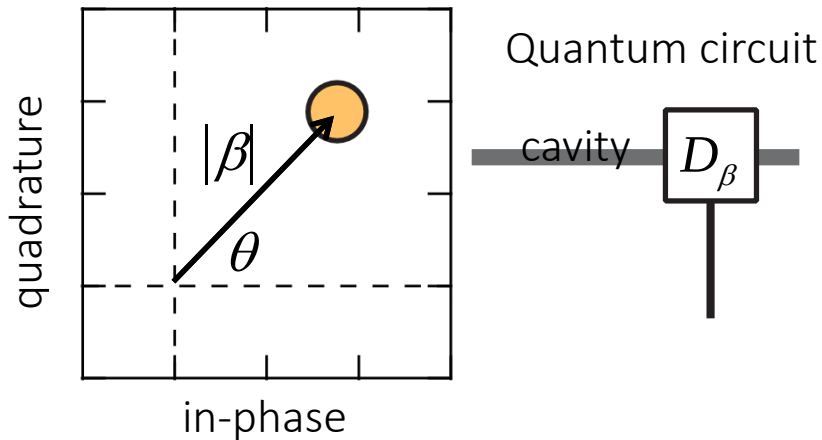
Weak Drive:

Conditional cavity displacement

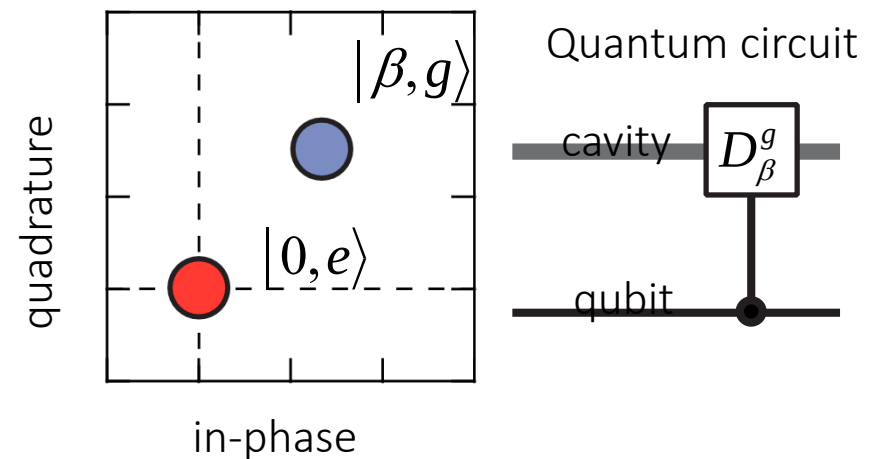
$$D_\beta^g |0, g\rangle \Rightarrow |\beta, g\rangle$$

$$D_\beta^g |0, e\rangle \Rightarrow |0, e\rangle$$

Phase-space diagram

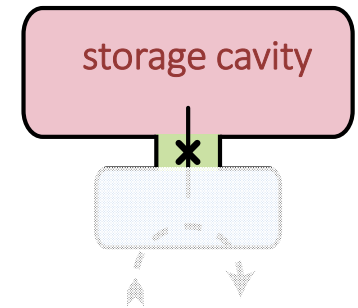


Phase-space diagram

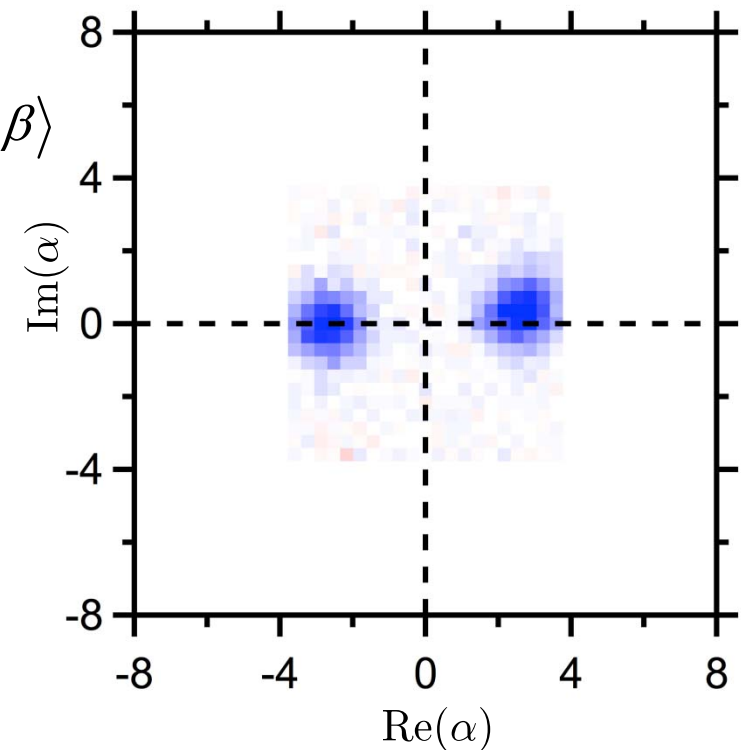
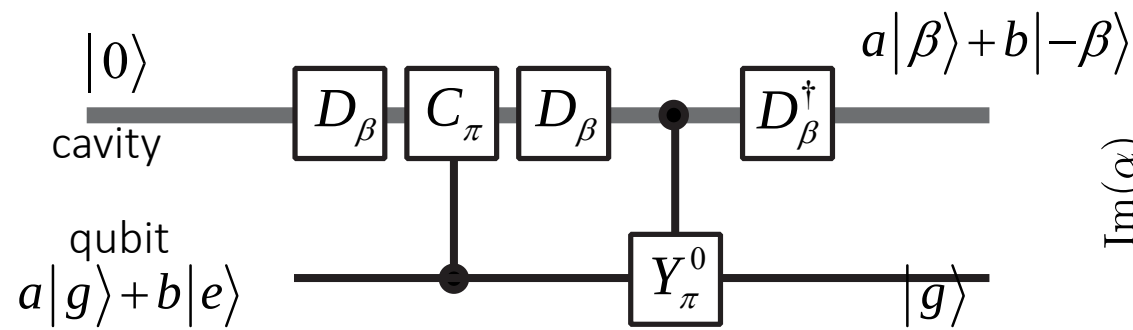


Create *superposition* of cavity state depending on the qubit.

Deterministic qubit-cavity mapping



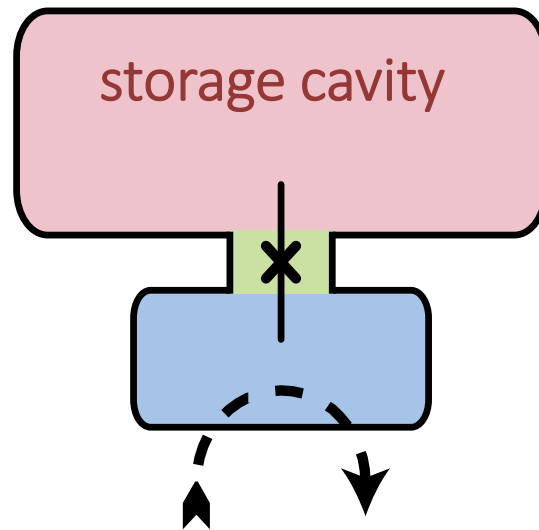
Transfer arbitrary state from qubit to cavity



Theory:
Leghtas et. al PRA 87, 042315 (2013)

Experiment:
Vlastakis et. al Science 342, 6158 (2013)

Measurement of Qubit & Cavity



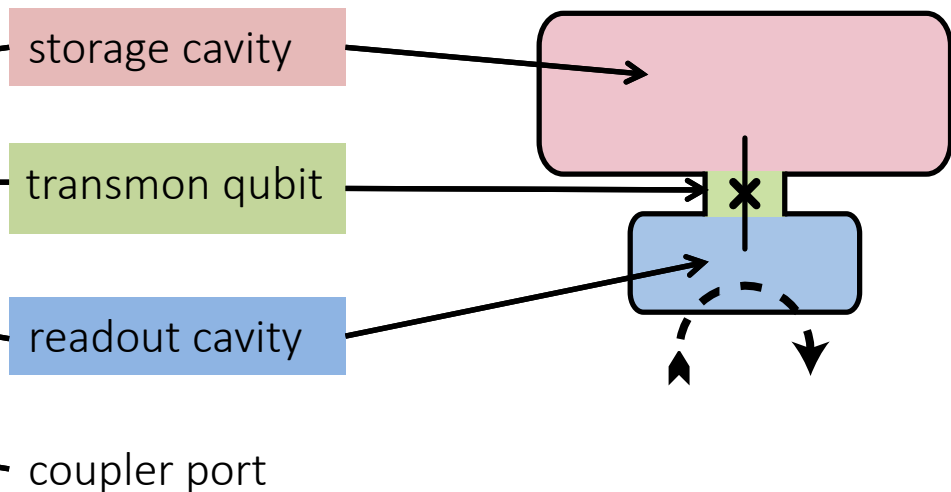
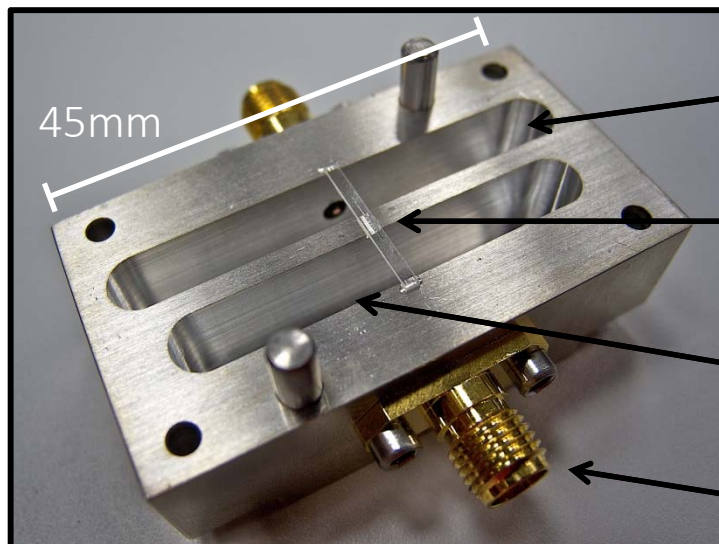
Two-cavity architecture

Effective Hamiltonian:

$$H / \hbar = \omega_s a^\dagger a + \omega_r b^\dagger b + \omega_q |e\rangle\langle e| - \chi_s a^\dagger a |e\rangle\langle e| - \chi_r b^\dagger b |e\rangle\langle e|$$

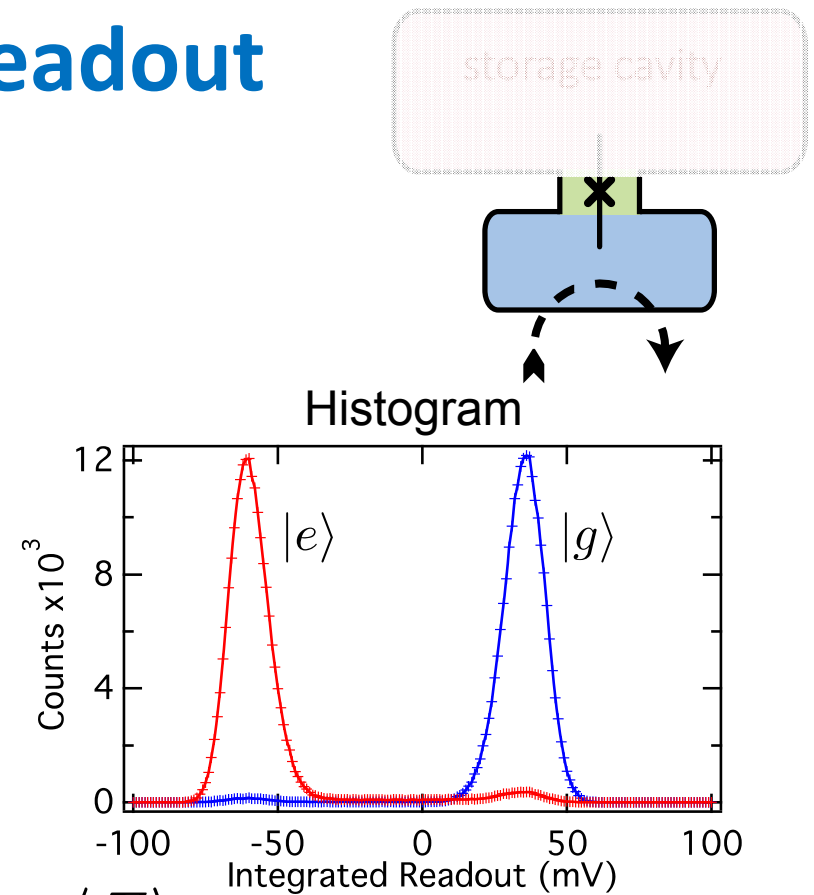
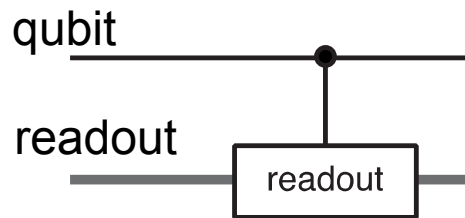
Interaction strength dominates photon loss: $\chi_s / \kappa_s > 300$

κ_s : storage photon decay rate



Dispersive qubit state readout

$$H/\hbar = (\omega_r - \chi_r |e\rangle\langle e|) b^\dagger b$$



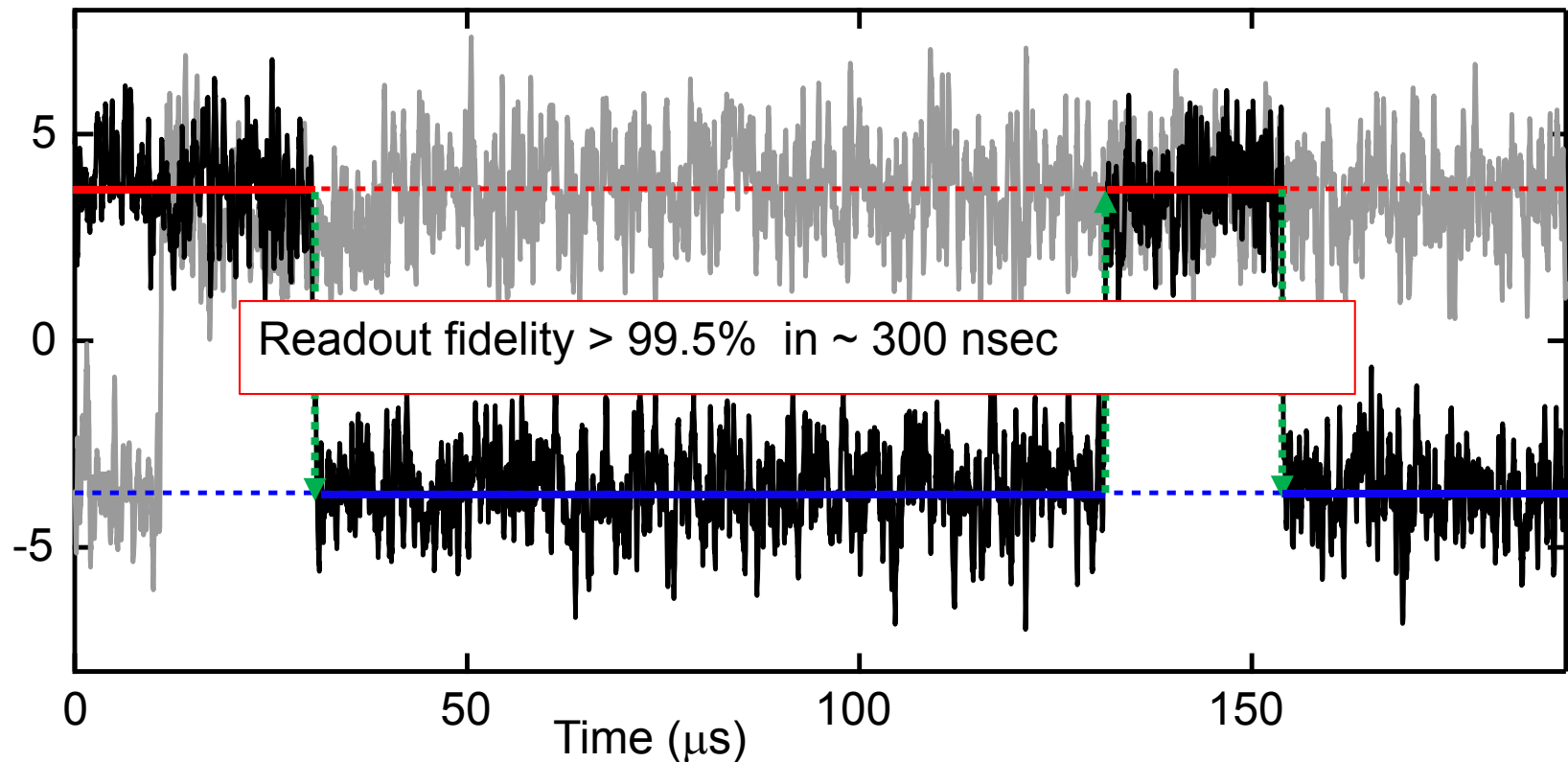
Single-shot qubit state detection, $\langle Z \rangle$

- Readout fidelity: 95%
- Readout repetition rate: 600ns

QND Measurement of Qubit!

Results from Devoret group, Yale: Hatridge et al., Science 2013*

dispersive circuit QED readout + JJ paramp



Many groups now working with JJ paramps & feedback, including:
Berkeley, Delft, JILA, ENS/Paris, IBM, Wisc., Saclay, UCSB, ...

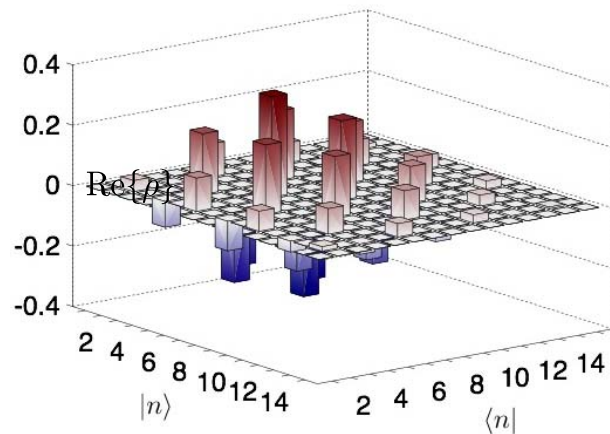
*First jumps: R. Vijay et al., (UCB)

Measurement of Cavity

- Density Matrix:
(in discrete Fock basis)

$$\langle \hat{O} \rangle = \text{Tr} [\hat{O} \rho]$$

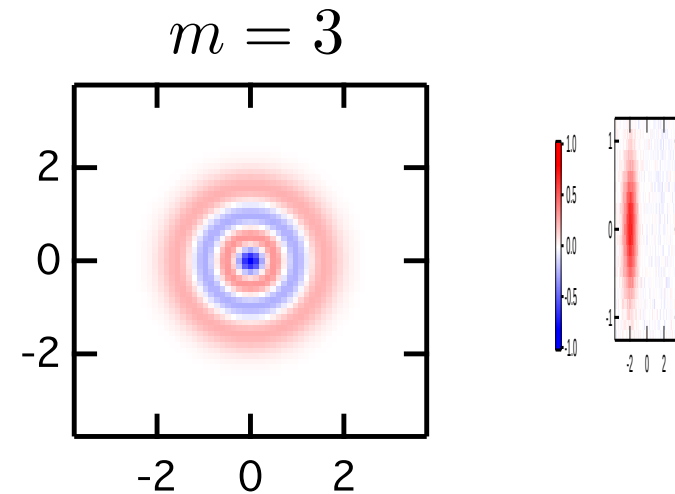
An example cavity density matrix:



- “Distribution” function
(continuous variable)

$$\langle \hat{O} \rangle = \int O(\alpha) P(\alpha) d^2 \alpha$$

An example of “distribution” functions



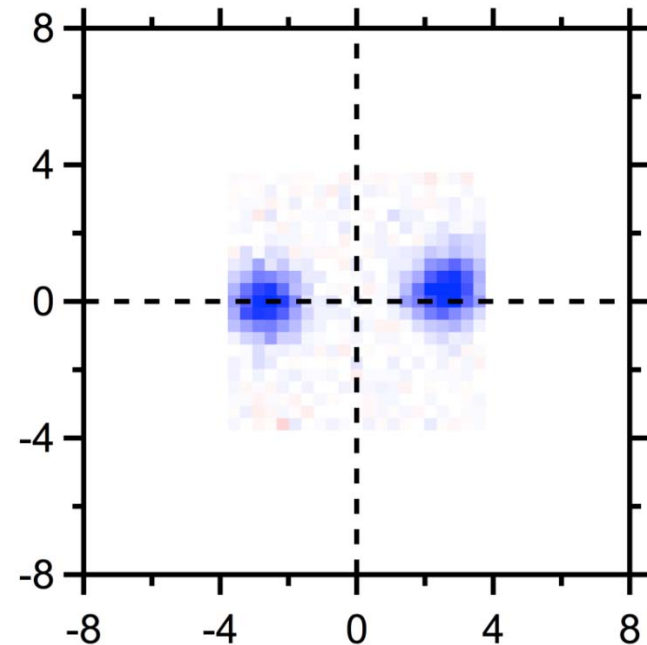
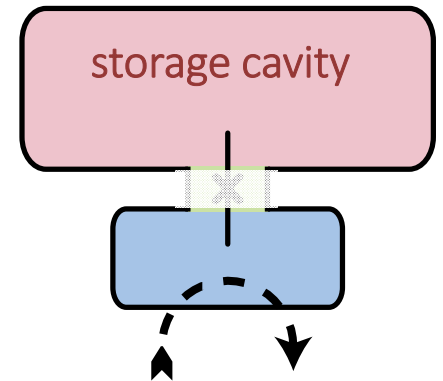
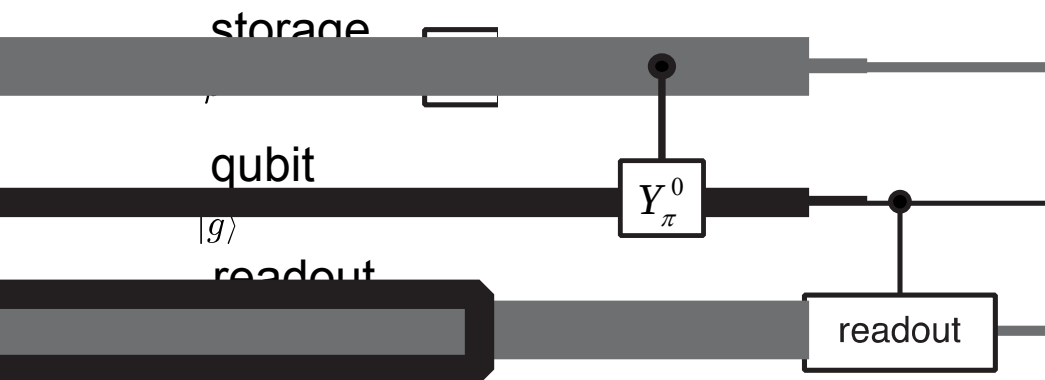
Cavity Q-function

Husimi-Q distribution:

$$Q(\alpha) \propto \langle \alpha | \rho | \alpha \rangle$$

$$= \text{Tr} \left[|0\rangle \langle 0| D(\alpha)^\dagger \rho D(\alpha) \right]$$

Overlap of a state ρ with
a coherent state $|\alpha\rangle$

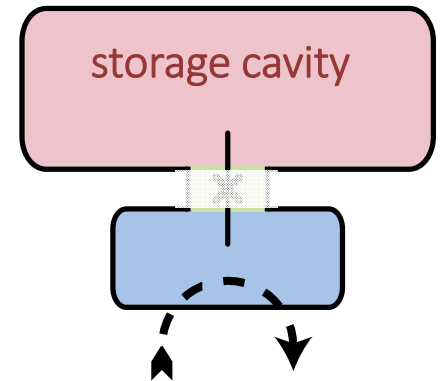
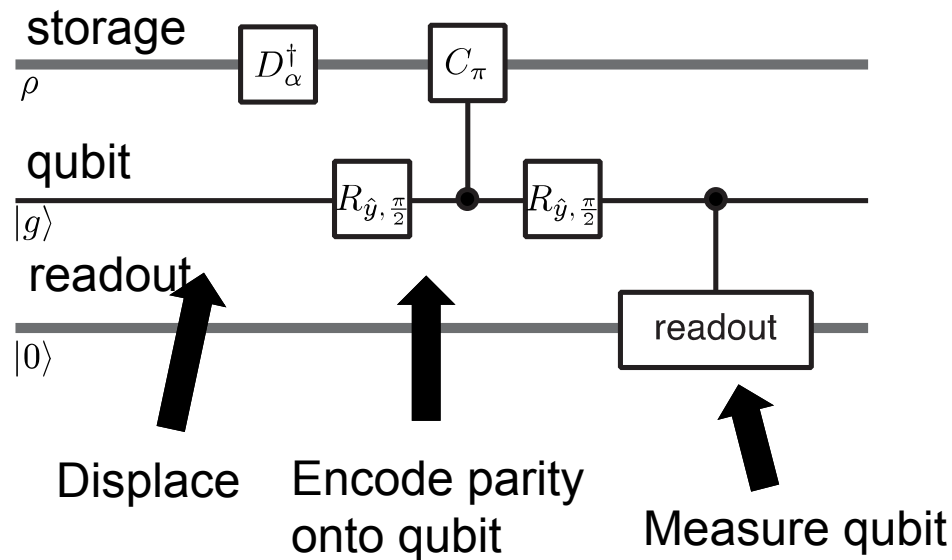


Cavity Wigner function

$$W(\alpha) = \text{Tr} \left[(-1)^{a^\dagger a} D(\alpha)^\dagger \rho D(\alpha) \right]$$

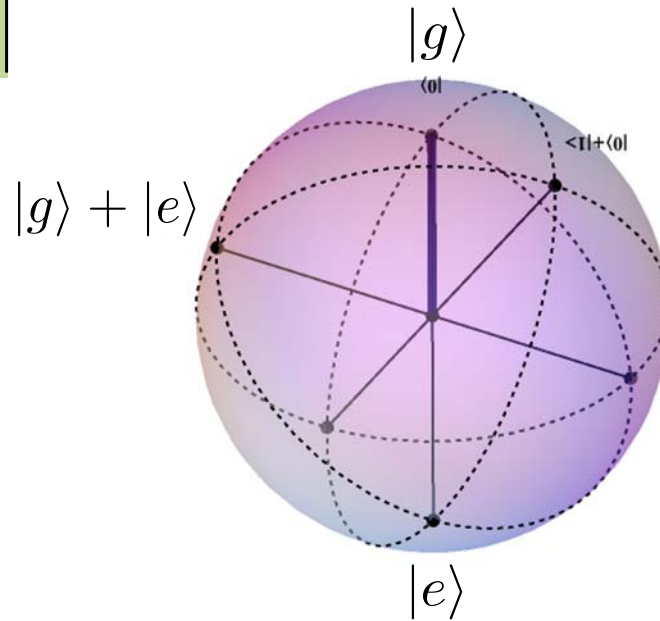
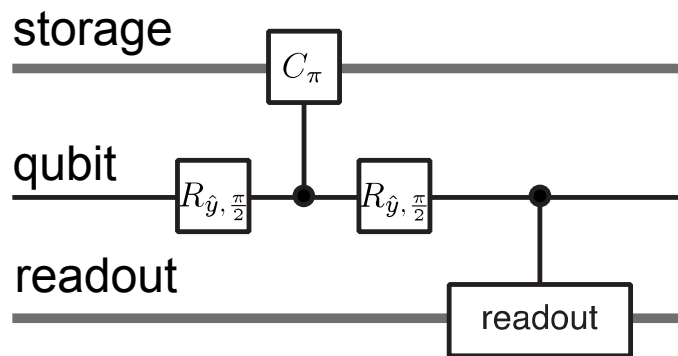
Mean value of photon parity for a displaced cavity state

Q-Circuit for measuring $W(\alpha)$



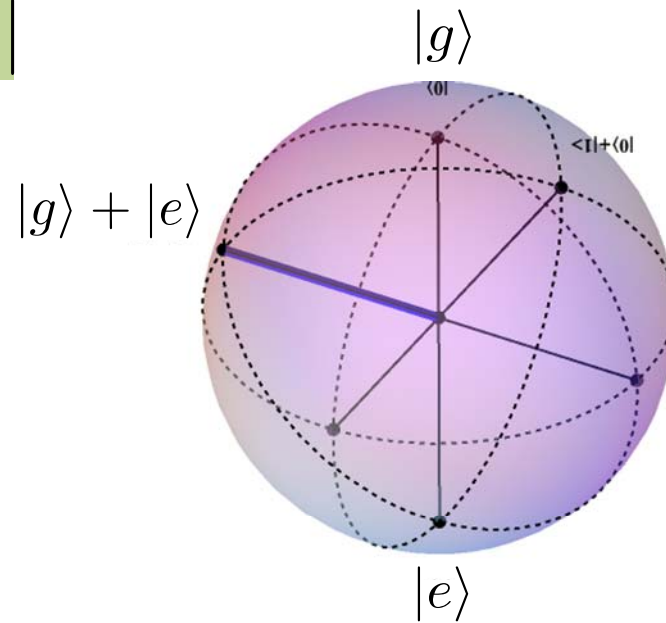
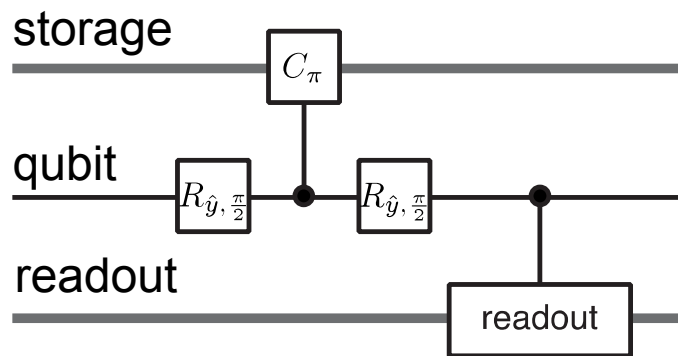
Photon number parity detection

$$H/\hbar = (\omega_q - \chi_s a^\dagger a) |e\rangle \langle e|$$



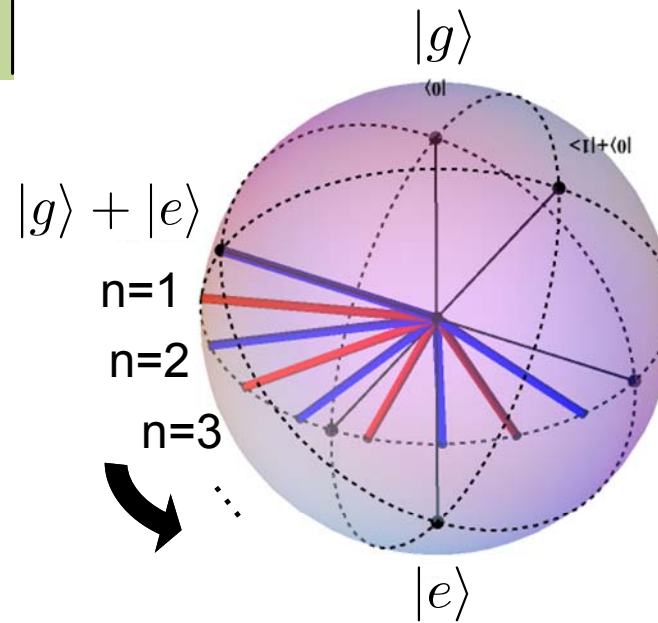
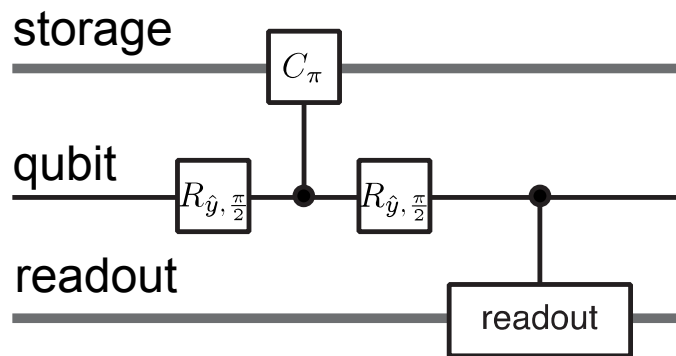
Photon number parity detection

$$H/\hbar = (\omega_q - \chi_s a^\dagger a) |e\rangle \langle e|$$



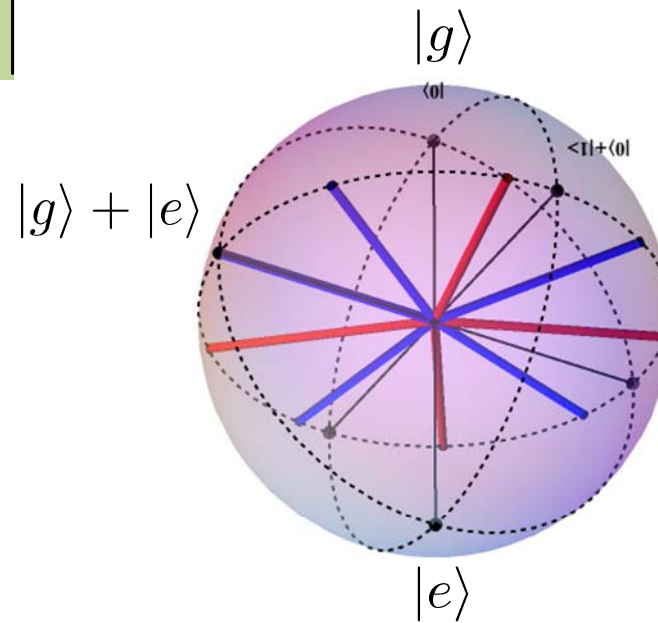
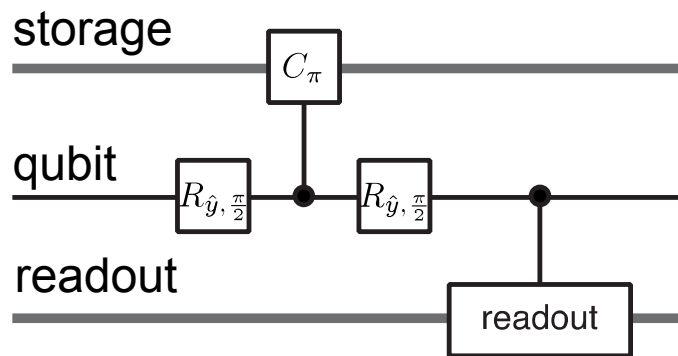
Photon number parity detection

$$H/\hbar = (\omega_q - \chi_s a^\dagger a) |e\rangle \langle e|$$



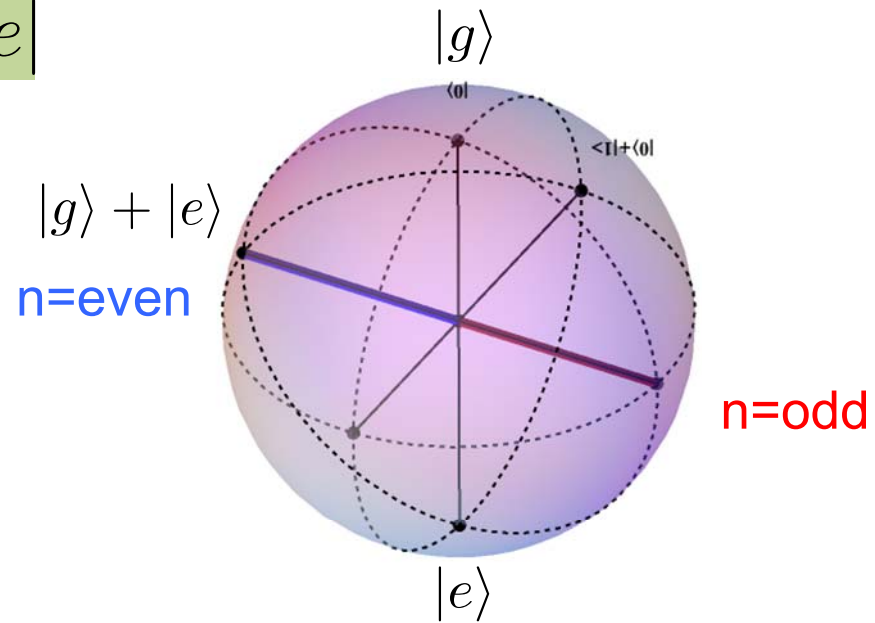
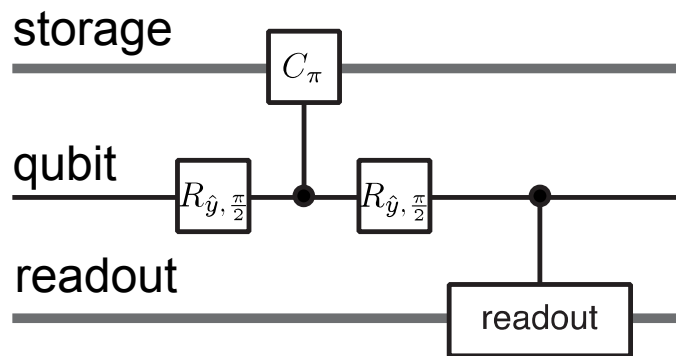
Photon number parity detection

$$H/\hbar = (\omega_q - \chi_s a^\dagger a) |e\rangle \langle e|$$



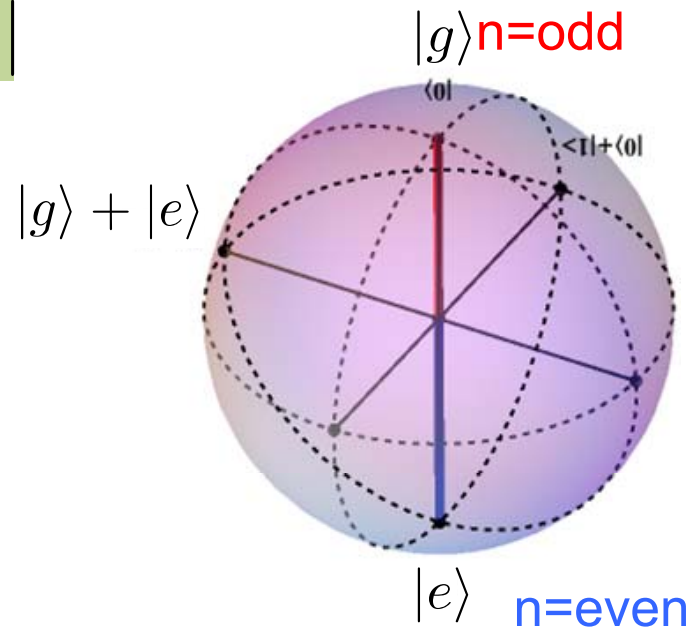
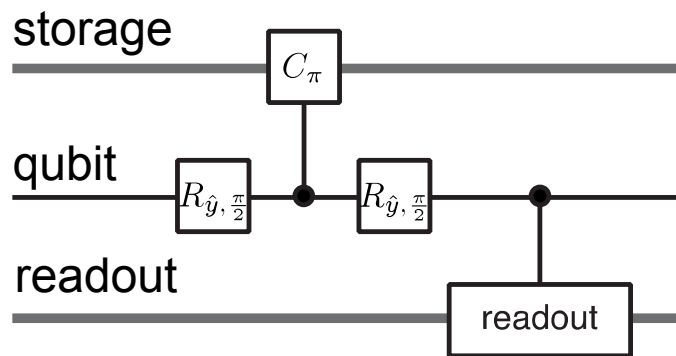
Photon number parity detection

$$H/\hbar = (\omega_q - \chi_s a^\dagger a) |e\rangle \langle e|$$



Photon number parity detection

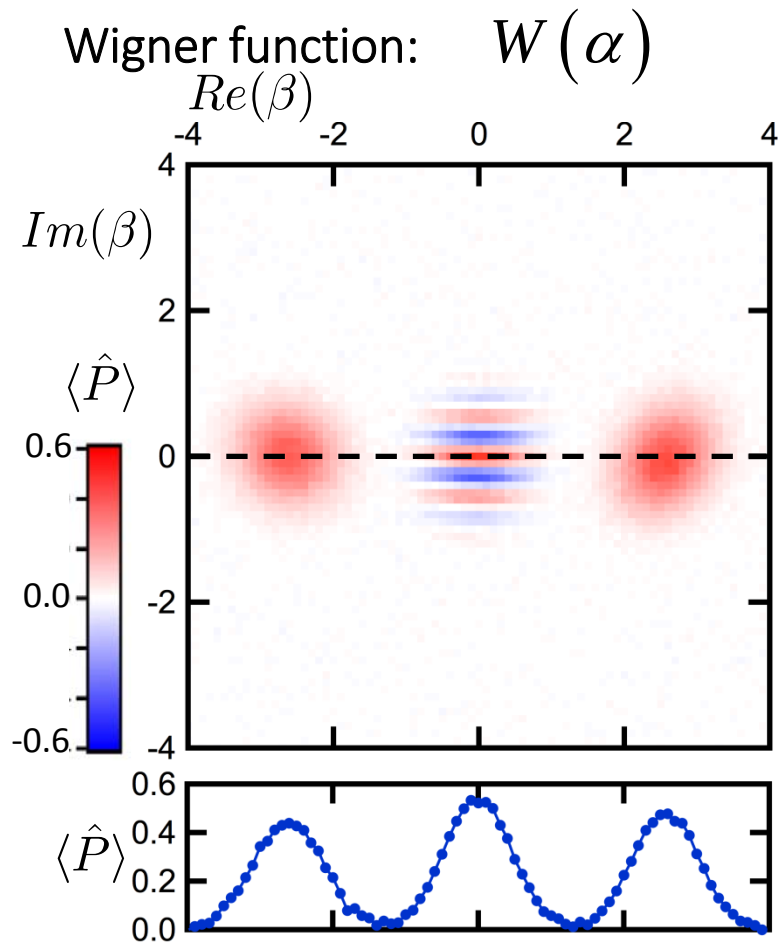
$$H/\hbar = (\omega_q - \chi_s a^\dagger a) |e\rangle \langle e|$$



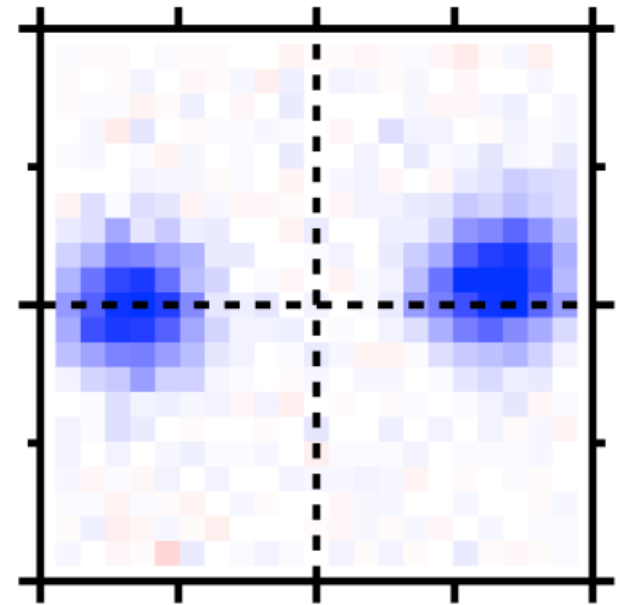
- Detect photon number parity, $\langle \Pi = e^{i\pi a^\dagger a} \rangle$
 - Ideally no direct cavity detection
 - Readout fidelity: ~80%

Wigner tomography of a cat state

Cavity state: $\cos(\frac{\theta}{2}) |\alpha\rangle + \sin(\frac{\theta}{2}) e^{i\phi} |-\alpha\rangle$



Q function



Expt'l. Wigner tomography:
Monroe, et al., 1996 ion traps (NIST)
Haroche/Raimond, 2008 Rydberg (ENS)
Hofheinz et al., 2009 in circuits (UCSB)