MEASURES FOR MACROSCOPIC QUANTUM STATES

FOR SPINS AND PHOTONS

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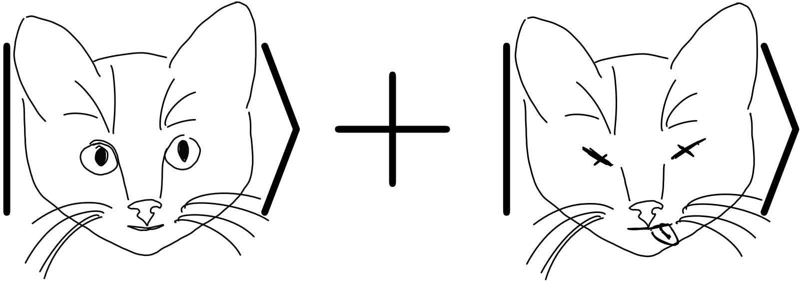
Seefeld, 2 July 2014

“Exploring the limits ...”



Quote from his 1935 paper

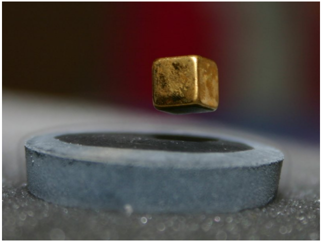
"There is a difference between a shaky or out-of-focus photograph and a snapshot of clouds and fog-banks"

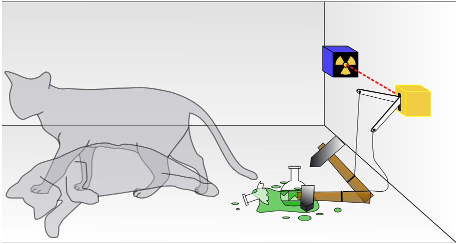
Leggett (1980) noticed that there is a difference between a

accumulated microscopic quantum effect

and a "true"

macroscopic quantum effect.





OVERVIEW

DEFINITION What is a macroscopic quantum state?

MEASURING How to verify macroscopic quantum states?

STABILITY Are macroscopic quantum states stable?

1 Identifying Macroscopic Quantum states

FF and W. Dür, New Journal of Physics **14**, 093039 (2012).
FF, N. Sangouard, and N. Gisin, arXiv:1405.0051 (2014).

What is a measure for macroscopic quantum states?

- Properties of measures
- Comparison of different measures
- “Canonical form”

- Necessary condition: ρ is quantum state of large system:
  Many spins, many photons, large masses, . . .

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Mathematical structure and terminology

$$f : \mathcal{D}(\mathcal{H}) \to \mathbb{R}_+$$
$$\rho \mapsto f(\rho)$$

If $f(\rho)$ is large, the state ρ is *macroscopically quantum*.

f ... *measure for macroscopicity*
$f(\rho)$ ... *effective size* of ρ.

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- Typical normalization: Fix resources, e.g., number of qubits M, mean photon number N, etc.

$$\max_\rho f(\rho) = \text{system size}$$

Further remarks

Form of the state Two possibilities

- $|\psi\rangle \propto |\psi_1\rangle + |\psi_2\rangle$ ... *Schrödinger cat state* (characterizing Leggett's "macroscopically distinct")
- ρ ... *general macroscopic quantum state*

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Scaling vs. absolute number Experimental vs. theoretical needs.

Scaling: Two measures are compatible if the identify the same class of macro-states.

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SCALING VS. ABSOLUTE NUMBER Experimental vs. theoretical needs.

Scaling: Two measures are compatible if the identify the same class of macro-states.

PRELIMINARY STRUCTURE All measures argue with some kind of imposed structure: **"Realistic" or "feasible" Hamiltonians, measurements, etc.**

- Spins: Interaction with fields; two-body interaction; collective measurements. Local operator $H = \sum_i h^{(i)}$
- Photons: $\hat{x}, \hat{p}, \hat{n}$

- A. J. Leggett, Progress of Theoretical Physics Supplement **69**, 80 (1980).
- W. Dür, C. Simon, and J. I. Cirac, Phys. Rev. Lett. **89**, 210402 (2002).
- A. Shimizu and T. Miyadera, Phys. Rev. Lett. **89**, 270403 (2002).
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THE RECENT LITERATURE...

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$\rightarrow$ **Comparison of measures for spins and photons ...**

- Pure states only
- Some assumptions to “fix” some problems of the measures.
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Mapping of photonic state onto spin ensemble

PHYSICALLY A one-mode photonic state with $\langle \hat{n} \rangle = N$ is **fully** absorbed by the ground state of ensemble of M qubits.

MATHEMATICALLY

$$|\psi_{\text{phot}}\rangle \otimes |g\rangle^{\otimes M} \mapsto e^{-iHt} |\psi_{\text{phot}}\rangle \otimes |g\rangle^{\otimes M} = |0\rangle \otimes |\phi_{\text{spin}}\rangle$$

with

$$H \propto a \otimes J_{+} + a^{\dagger} \otimes J_{-}$$

(a spin excitation is created via the annihilation of a photon).

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ASSUMPTION "Macroscopicity" and other properties are conserved via

$$M \gg N$$

Approximation for simpler treatment: $M \gg N$

…state $|\phi_{\text{spin}}\rangle$ lies in the low-energy sector of the Hilbert space.

Algebraically $\langle J_z \rangle_\phi = -M/2 \left[1 - O\left(\frac{N}{M}\right)\right] \Rightarrow \frac{1}{2M} [J_-, J_+] \approx id$

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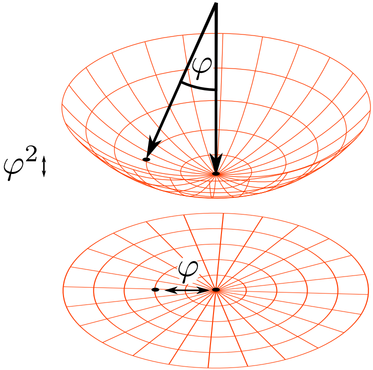
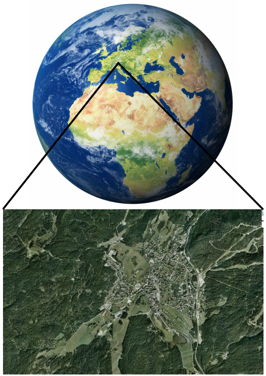
Geometrically Approximate sphere by a plane





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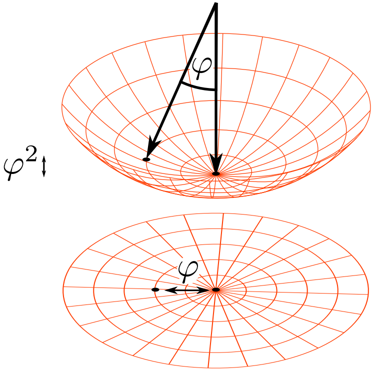
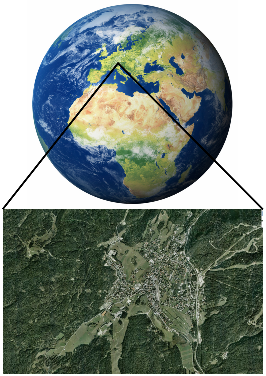
Physically Every spin is “hit” by at most one photon





By this approximation

A Fock state $|k\rangle$ is mapped to a Dicke state $|M,k\rangle$

$$|k\rangle \otimes |M,0\rangle \mapsto |0\rangle \otimes |M,k\rangle$$

where $|M,k\rangle \propto \sum_{\text{permutations}} |e\rangle^{\otimes k} \otimes |g\rangle^{\otimes M-k}$.

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Mapping of operators

$$a \otimes id \mapsto Ua \otimes id U^{\dagger} \approx id \otimes \frac{1}{\sqrt{M}} J_{-}$$

f, g are measures for macroscopic quantum states. Then

- f and g are **compatible**: $\forall\psi : f(\psi) = O(N) \Leftrightarrow g(\psi) = O(N)$
- g **includes** f: $\forall\psi : f(\psi) = O(N) \Rightarrow g(\psi) = O(N)$

Result for the Comparison of Measures

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Schrödinger Cat States $|\psi_1\rangle + |\psi_2\rangle$

$\Downarrow$

(general)

Macroscopic Quantum States

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Distant Superpositions of "Classical States" $|\alpha\rangle + |-\alpha\rangle$

$\Downarrow$

Superpositions of "Macroscopically Distinct States" $|0\rangle + |N\rangle$

$\Downarrow$

(general) Macroscopic Quantum States $|N\rangle$

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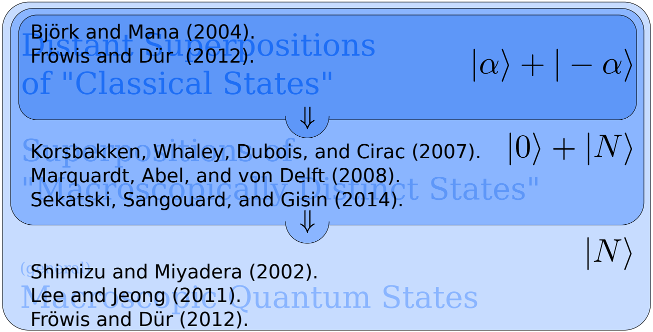
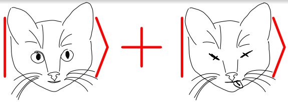
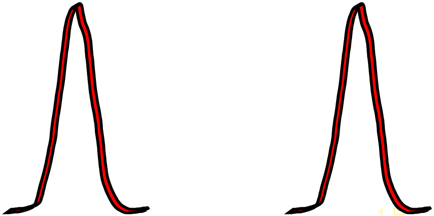


Common feature of all these measures

All **pure macroscopic quantum states** exhibit **large variance** with respect to “realistic” Hamiltonians or measurements.





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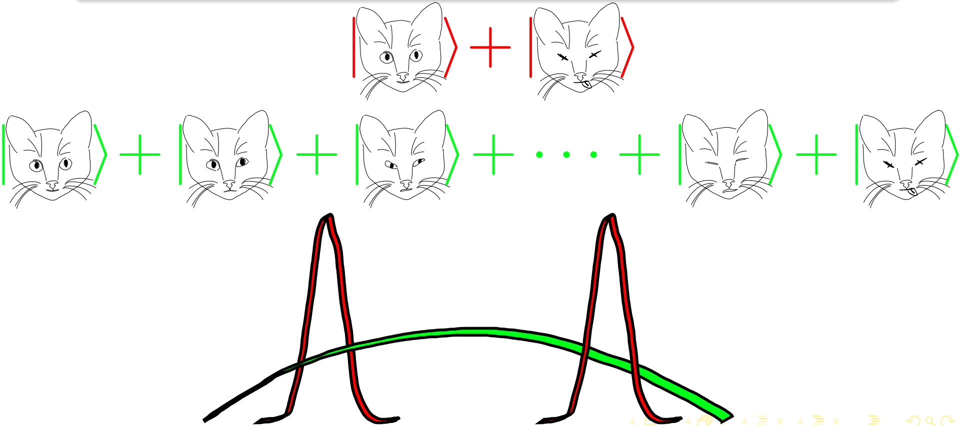
But why should we **insist on two peaks**???



PROPOSAL

Pure states

Look for the maximal variance with respect to “realistic” operators (e.g., local with respect to qubits, modes, etc.). Define

$$f(\psi) = \frac{1}{M} \max_X V_\psi(X)$$

SPINS X . . . local operator, M. . . number of qubits

PHOTONS X . . . sum of quadratures, M. . . number of modes

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Mixed states

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- f is convex in ρ.

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Example: Quantum Fisher information $\mathcal{F}$ (more later):

$$f(\rho) \equiv N_{\text{eff}}(\rho) = \frac{1}{4M} \max_X \mathcal{F}_\rho(X)$$

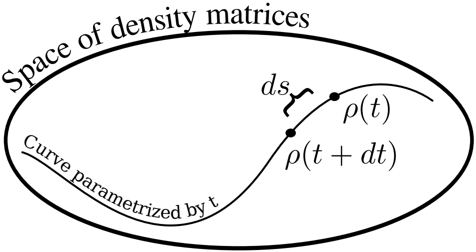
2 Witnessing macroscopic quantumness

FF and W. Dür, New Journal of Physics **14**, 093039 (2012).
Unpublished Results

What signatures do macroscopic quantum states show?

- Verifying fast time evolution
- Improved Heisenberg Uncertainty Relation
- Bound on Quantum Fisher information

Consider a differentiable parametrization through the set of density operators



One has:

$$(ds)^2 = \frac{1}{2}\mathcal{F}(\rho,\rho')(dt)^2$$

$\mathcal{F}(\rho,\rho')$... **Quantum Fisher information.**
Unitary transformation $\rho' = -i[H,\rho] \Rightarrow \mathcal{F}(\rho,\rho') \equiv \mathcal{F}_\rho(H)$

$$\mathcal{F}_\rho(H) \leq 4V_\rho(H); \qquad \mathcal{F}_\psi(H) = 4V_\psi(H); \qquad \text{convex}$$

Large Quantum Fisher information implies Macroscopic Quantum Effect

Bottom Line

Large changes in $\rho(t)$ by altering t implies large Quantum Fisher information.

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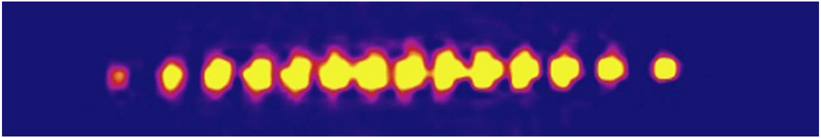
Examles: M qubits.
If Fisher information large [$\mathcal{F} = O(M^2)$]:

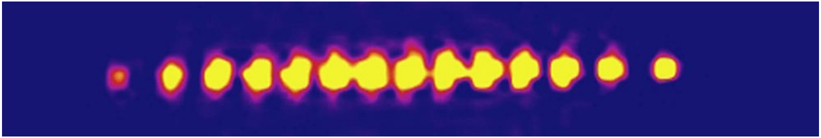
Fast evolution possible that is not explainable by

- separable states (“classical effect”)
- short-range entangled states (“microscopic quantum effect”)


(c) Blatt group

Experimental setup:

- Prepare a GHZ state $\rho = |\mathrm{GHZ}\rangle\langle\mathrm{GHZ}|$ with *M* ions.



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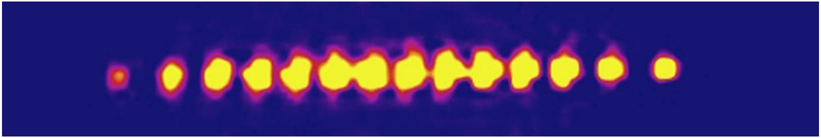

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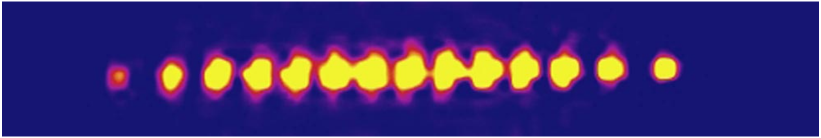
- Measure the parity with $\sigma_z^{\otimes M}$: Is number of ions in excited state even or odd?



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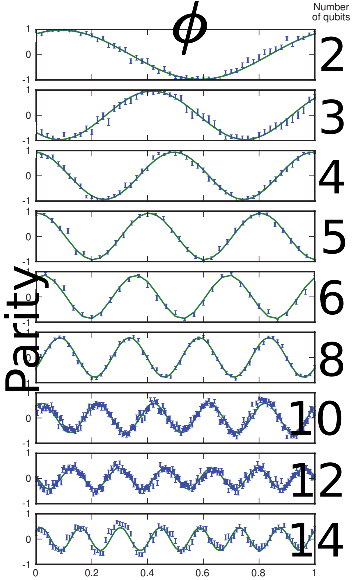
- Measure the parity with $\sigma_z^{\otimes M}$: Is number of ions in excited state even or odd?
- Repeat the experiment with different ϕ and produce statistics.



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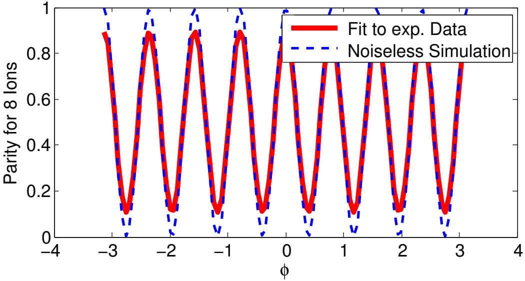
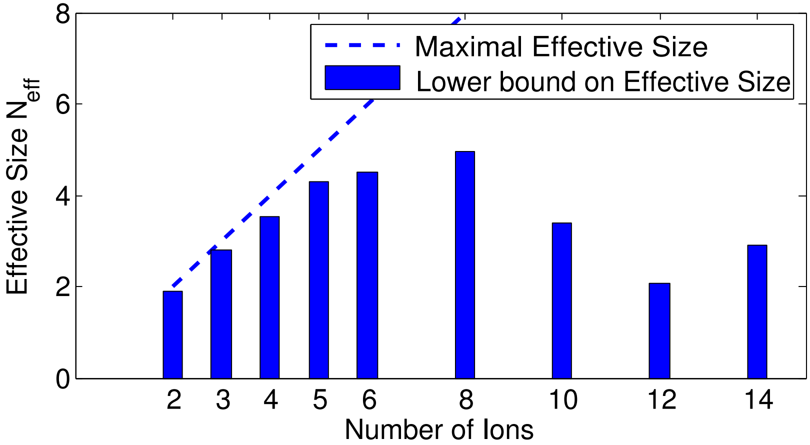


From data, bound Quantum Fisher information

Effective size $N_{\text{eff}} = \mathcal{F}/(4M)$.



Heisenberg uncertainty relation

Given two operators X, Y and define $Z = i[X, Y]$. Then, $\forall \rho$

$$V_\rho(X) V_\rho(Y) \geq \frac{1}{4} \langle Z \rangle_\rho^2$$

Observations:

- Variance V is **concave** under mixing states.
- $\langle Z \rangle^2$ is **convex** under mixing.

$\Rightarrow$ Bound generally less tight for mixed states.

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Improvement

Replace one variance by the Quantum Fisher information

$$\frac{1}{4}\mathcal{F}_\rho(X) V_\rho(Y) \geq \frac{1}{4}\langle Z\rangle_\rho^2$$

Consider following class of functions. For every decomposition $D = \{p_k, |\psi_k\rangle\}_k$ of $\rho = \sum_k p_k |\psi_k\rangle\langle\psi_k|$, we define

$$J_\rho(D, X) = \sum_k p_k V_{\psi_k}(X).$$

IDEA OF THE PROOF (TO BE PUBLISHED)

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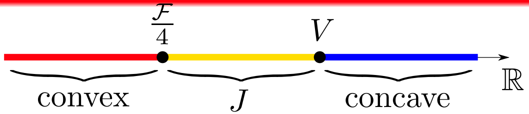
Convex and concave roof

$$\max_D J_\rho(D, X) = V_\rho(X)$$



$$\min_D J_\rho(D, X) = \frac{1}{4}\mathcal{F}_\rho(X)$$

S. Yu, arXiv:1302.5311 (2013).

G. Tóth and D. Petz, Phys. Rev. A **87**, 032324 (2013).

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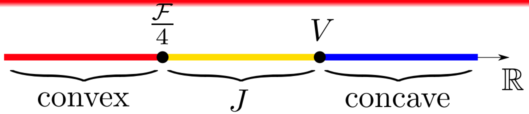

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- Easy to show $\forall D$:

$$J_\rho(D, X) J_\rho(D, Y) \geq \frac{1}{4}\langle Z\rangle_\rho^2$$

- Choose D such that $J_\rho(D, X) = \frac{1}{4}\mathcal{F}_\rho(X)$.
- For the same D : $V_\rho(Y) \geq J_\rho(D, Y)$.

From

$$\mathcal{F}_\rho(X) V_\rho(Y) \geq \langle Z \rangle_\rho^2$$

$$N_{\text{eff}}(\rho) = \frac{1}{4M} \max_X \mathcal{F}_\rho(X)$$

For (spin) squeezed states along x:

Spins:

$$N_{\text{eff}} \geq \frac{\langle J_z \rangle^2}{4MV(J_x)}$$

Photons:

$$N_{\text{eff}} \geq \frac{1}{4V(\hat{x})}$$

Compare to spin squeezing inequalities, e.g., by Mølmer and Sørensen.

3 STABILITY ISSUES OF MACROSCOPIC QUANTUM STATES

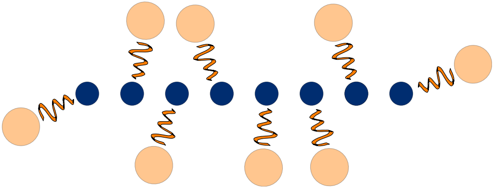
FF, M. VAN DEN NEST, AND W. DÜR, NEW J. PHYS. **15**, 113011 (2013).
Unpublished Results

Are macroscopic quantum states stable?

- Scenario: Spins with local depolarization noise
- Schrödinger cat states are unstable
- Restrictions for the “witness for macroscopicity”

NOISE MODEL Each of the M qubits **locally depolarized.** On qubit i:

$$\mathcal{E}^{(i)}(\rho) = p\rho + (1-p)\mathrm{Tr}_i\rho \otimes \frac{\mathbb{1}^{(i)}}{2}$$



[1] FF, M. van den Nest, and W. Dür, New J. Phys. **15**, 113011 (2013).

Scenario: Spins with local depolarization noise

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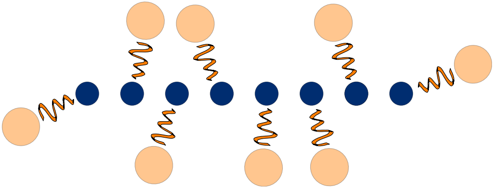

Criterion[1] A state is called $|\psi\rangle \in \mathbb{C}^{2\otimes M}$ **uncertifiable** if $\exists \left|\psi^\perp\right\rangle \perp |\psi\rangle$:

$$\frac{1}{2}\|\mathcal{E}(\psi) - \mathcal{E}(\psi^\perp)\|_1 = \alpha^M$$

with $p < 1, \alpha < 1$; $\|\cdot\|_1$ ... trace norm

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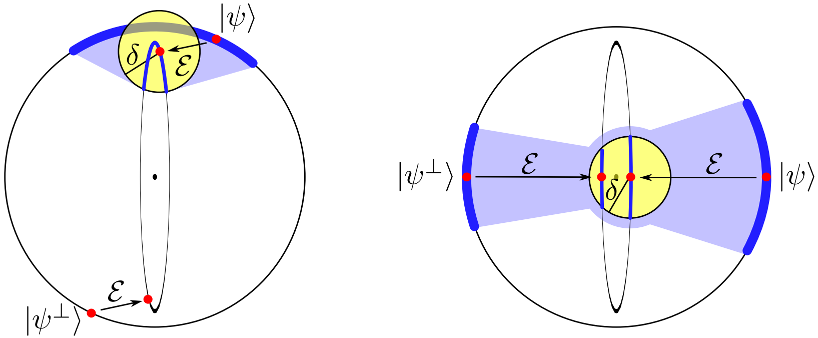
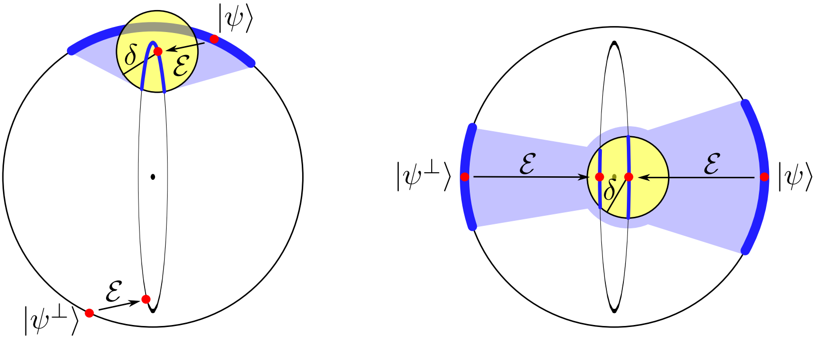
Schrödinger cat states are uncertifiable





- Suppose $|\psi\rangle \propto |\psi_1\rangle + |\psi_2\rangle$ is Schrödinger cat state with $N_{\text{eff}} = O(M)$.
- $|\psi\rangle$ is not certifiable w.r.t. $|\psi_1\rangle - |\psi_2\rangle$ (phase is not detectable)
- $\Rightarrow$ indistinguishable from mixture

$$|\psi_1\rangle\langle\psi_1| + |\psi_2\rangle\langle\psi_2|$$

Schrödinger Cat States $|\psi_1\rangle + |\psi_2\rangle$

$\Downarrow$

(general)
Macroscopic Quantum States

Schrödinger Cat States $|\psi_1\rangle + |\psi_2\rangle$

ARE UNSTABLE

$\Downarrow$

(general)

Macroscopic Quantum States

Measurement precision important for "witness for macroscopicity"

With depolarization noise $\langle X\rangle_{\mathcal{E}(\rho)} = \langle \mathcal{E}(X)\rangle_\rho$

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$$V_{\mathcal{E}(\psi)}(J_x) = \frac{M}{4}(1 - p^2) + p^2 V_\psi(J_x) \geq \frac{M}{4}(1 - p^2)$$

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$$N_{\mathrm{eff}} \geq \frac{\langle J_z \rangle^2_{\mathcal{E}(\psi)}}{M V_{\mathcal{E}(\psi)}(J_x)} = \frac{p^2 \langle J_z \rangle^2_\psi}{M \left[\frac{M}{4}(1 - p^2) + p^2 V_\psi(J_x) \right]}$$

"Ultimate limit" for this witness

Last expression is at most $p^2/(1 - p^2)$. For $p = 0.99$, one is limited to witness only $N_{\mathrm{eff}} \lesssim 50$.

Many thanks to . . .

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Plus ongoing collaborations:
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FWF Der Wissenschaftsfonds.

Take-home messages

Minimal consensus Pure state is **macroscopically quantum** $\Leftrightarrow$ **Large variance** with respect to "realistic operator". Mixed state: Take some convex function (e.g., **Quantum Fisher information**).

Witness Several ways to bound **effective size** from below: Rapid changes in time (or similar); witness with "easy" measurements.

Stability Schrödinger cat states $|\psi_1\rangle + |\psi_2\rangle$ are **unstable**; also difficult for general macroscopic quantum states.