Chiral spin liquid and emergent anyons in a Kagome lattice Mott insulator

Łukasz Cincio

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

with support from

JOHN TEMPLETON FOUNDATION

L. Cincio, G. Vidal, **Phys. Rev. Lett. 110, 067208 (2013)**

B. Bauer, L. Cincio, B. P. Keller, M. Dolfi, G. Vidal, S. Trebst, A. W. W. Ludwig, **arXiv:1401.3017**

OUTLINE

- Anyon models:
    - ground state degeneracy
    - S and T matrices
    - edge spectrum

- Hubbard model with magnetic field on Kagome lattice
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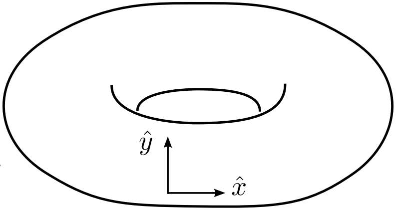
Anyon models

- ground state degeneracy on the torus

X.-G. Wen, 1989

$L_x \times L_y$



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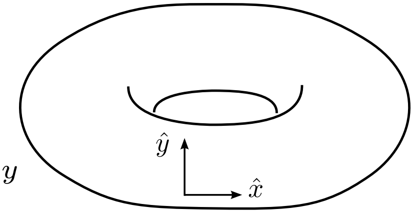

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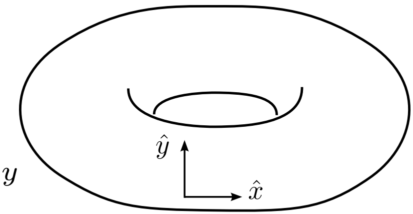

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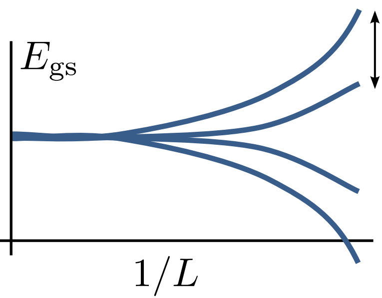

$\Delta \sim \exp(-L/\xi)$

ξ - correlation length

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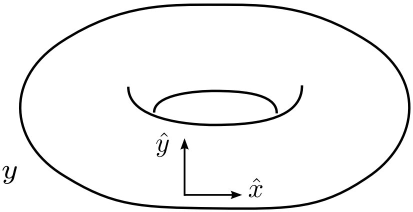

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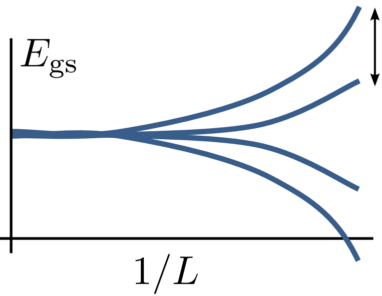

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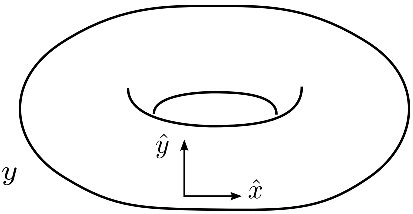

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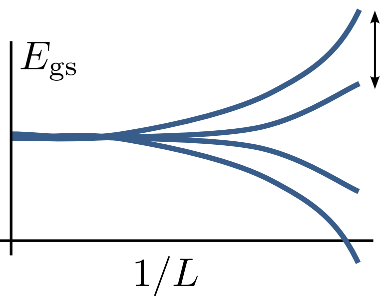

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example: toric code

$i = \mathbb{I}, e, m, \varepsilon$

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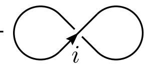
- topological S and T matrices
  - T matrix

$$T_{ii} = \frac{1}{d_i}$$



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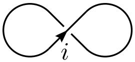
$$T_{ii} = \frac{1}{d_i}$$ 

example: toric code   anyon types: $\mathbb{I}$, e, m, ε

$$T = e^{-i\frac{2\pi}{24}\cdot 0} \begin{array}{c c} \begin{array}{cccc} \mathbb{I} & e & m & \varepsilon \end{array} & \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} & \begin{array}{c} \mathbb{I} \\ e \\ m \\ \varepsilon \end{array} \end{array}$$

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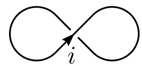
$$T_{ii} = \frac{1}{d_i}$$ 

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topological spins

$$\theta_{\mathbb{I}} = \theta_e = \theta_m = 1$$
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topological central charge

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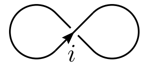
$$T_{ii} = \frac{1}{d_i}$$ 

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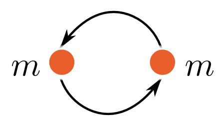

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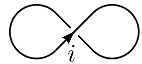
$T_{ii} = \frac{1}{d_i}$ 

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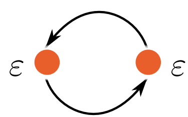

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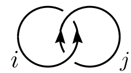
$$S_{ij} = \frac{1}{D}$$ 

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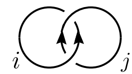
$$S_{ij} = \frac{1}{D}$$ 

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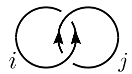
$S_{ij} = \frac{1}{D}$ 

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quantum dimensions
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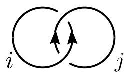
$$S_{ij} = \frac{1}{D}$$ 

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Verlinde formula

fusion rules: $i \times j = \sum_k N_{ij}^k \; k$

$$N_{ij}^k = \sum_m \frac{S_{im} S_{jm} (S_{km})^*}{S_{\mathbb{I}m}}$$

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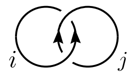

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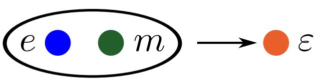

$e \times m = \varepsilon$

$\mathbb{Z}_2 \times \mathbb{Z}_2$ fusion rules

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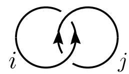

example: toric code

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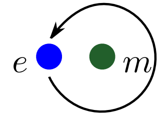
$$2 \cdot S_{em} = -1$$

mutual statistics



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S and T matrices can be extracted from ground states overlaps

Y. Zhang, T. Grover, A. Turner, M. Oshikawa, A. Vishwanath, **PRB 2012**

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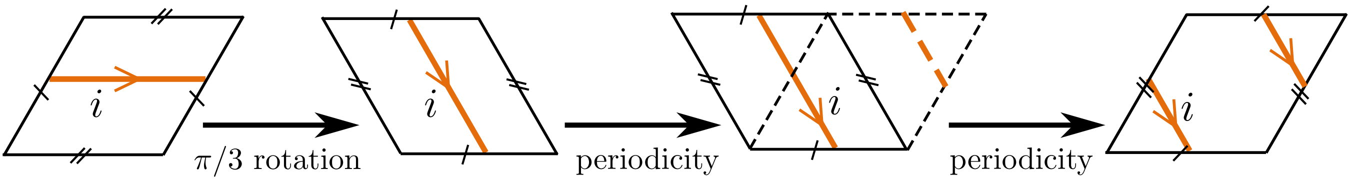

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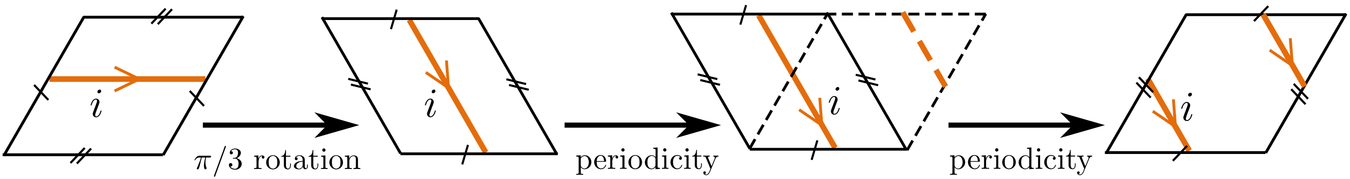

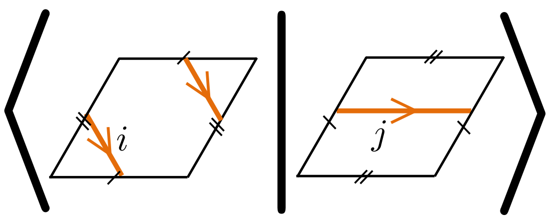

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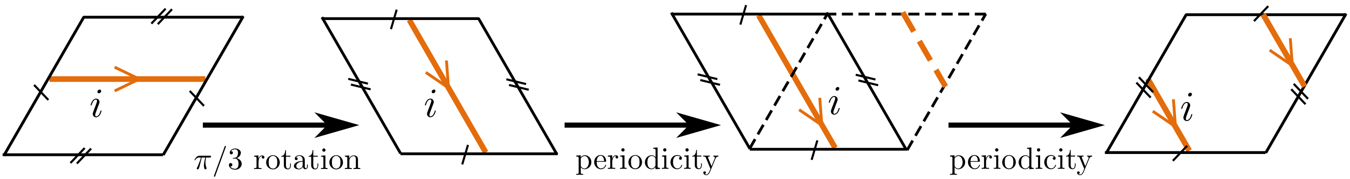

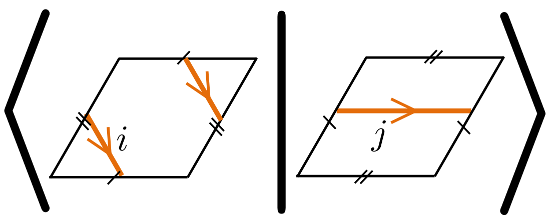

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read S and T

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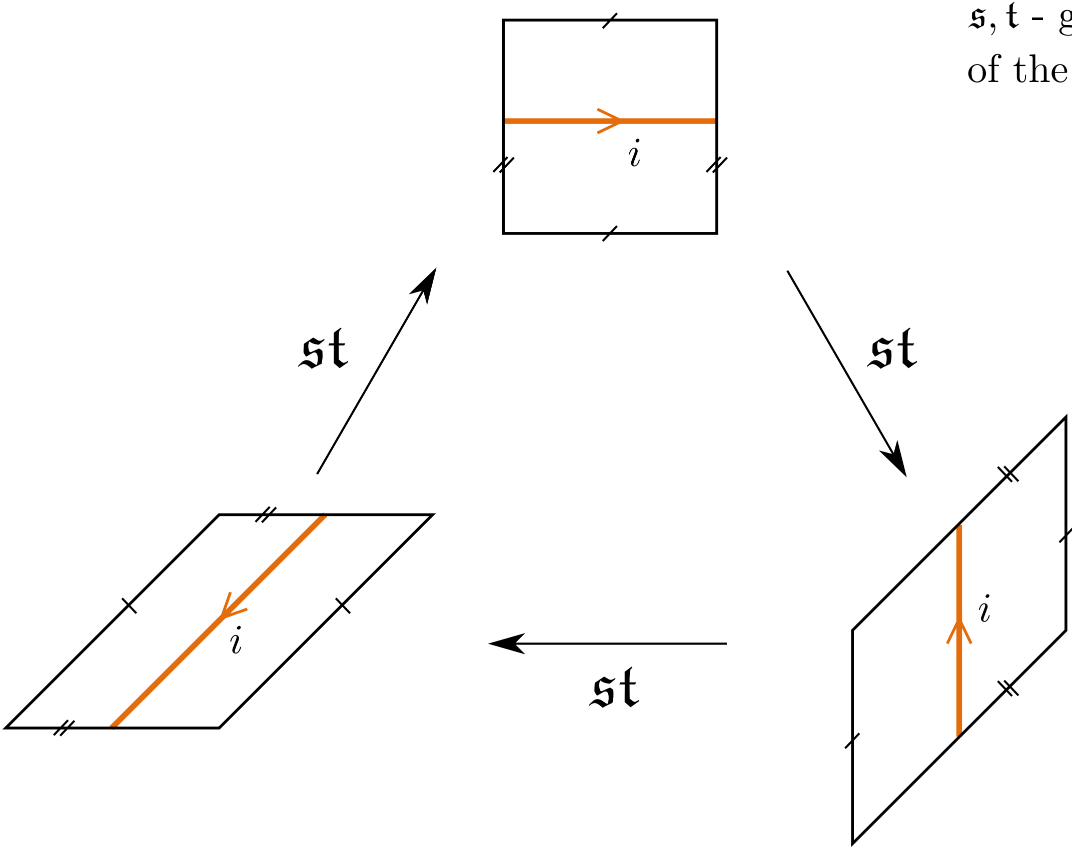
$\mathfrak{s}, \mathfrak{t}$ - generators
of the modular group

$$(\mathfrak{s}\mathfrak{t})^3 = \mathbb{1}$$



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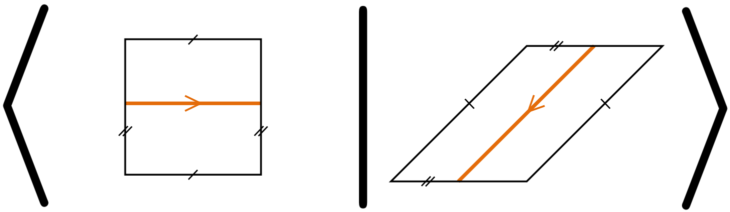
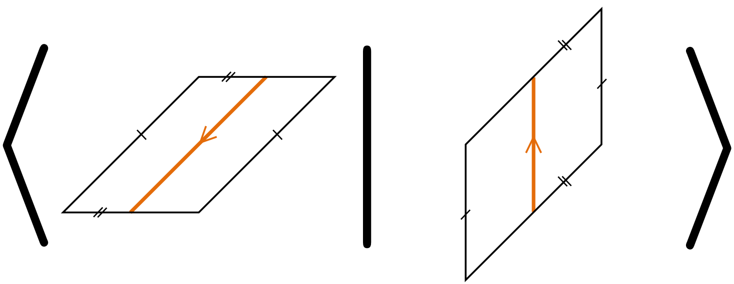
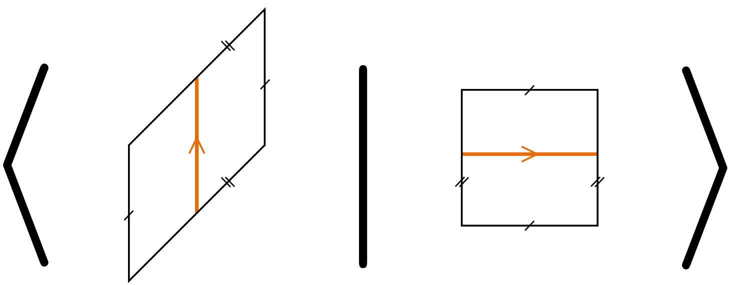


$$= \quad (D^A)^\dagger \; ST \; (D^B P^B)$$



$$= \; (D^B P^B)^\dagger \; ST \; (D^C P^C)$$



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  identify edge CFT by entanglement spectrum

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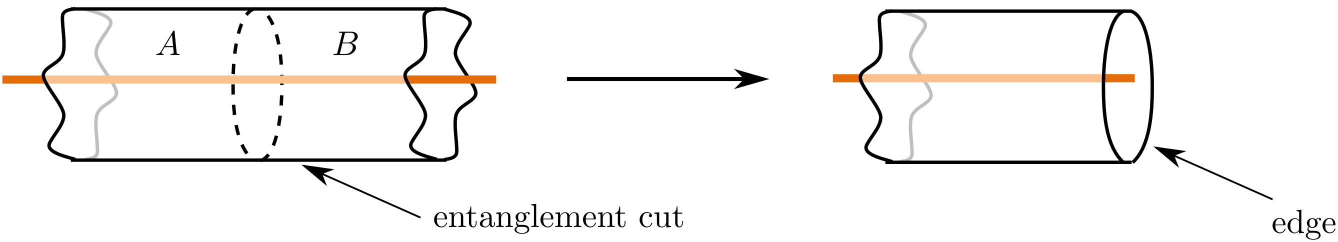

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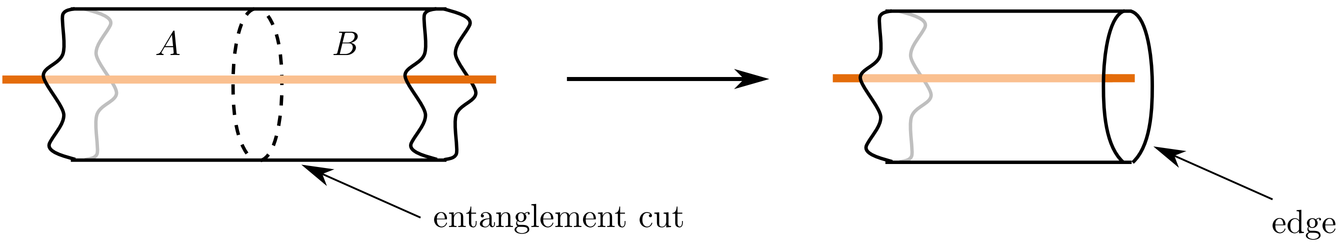

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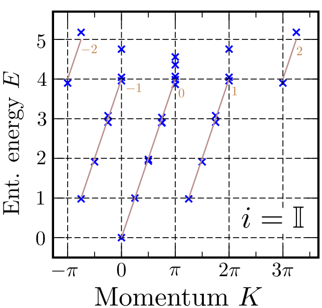
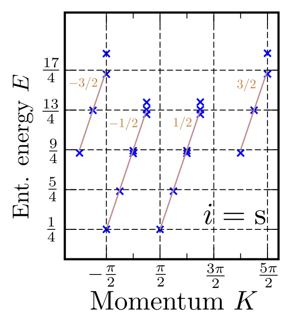
example: hard-core boson Haldane model on Honeycomb $\longrightarrow$ $\mathrm{SU}(2)_1$ WZW

L. Cincio, G. Vidal **PRL 2013**



L_0	m: -2	-1	0	1	2	$su(2)$ decomposition
0			1			(0)
1		1	1	1		(2)
2		1	2	1		(2)+(0)
3		2	3	2		2(2)+(0)
4	1	3	5	3	1	(4)+2(2)+2(0)
5	1	5	7	5	1	(4)+4(2)+2(0)
6	2	7	11	7	2	2(4)+5(2)+4(0)



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$\frac{1}{4}$			1	1			(1)
$\frac{5}{4}$			1	1			(1)
$\frac{9}{4}$		1	2	2	1		(3)+(1)
$\frac{13}{4}$		1	3	3	1		(3)+2(1)
$\frac{17}{4}$		2	5	5	2		2(3)+3(1)
$\frac{21}{4}$		3	7	7	3		3(3)+4(1)
$\frac{25}{4}$	1	5	11	11	5	1	(5)+4(3)+6(1)

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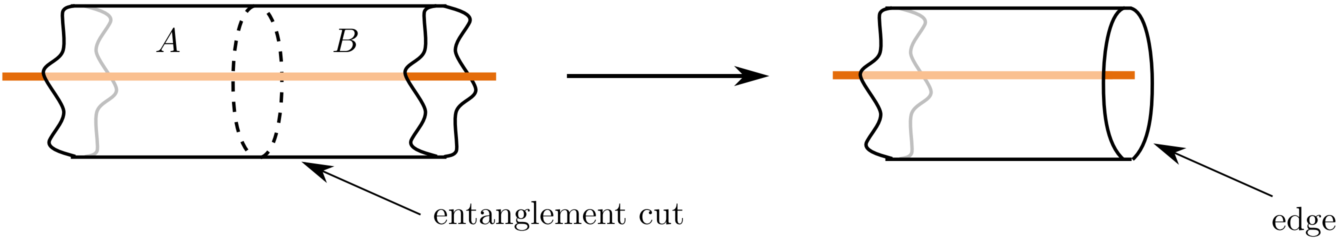
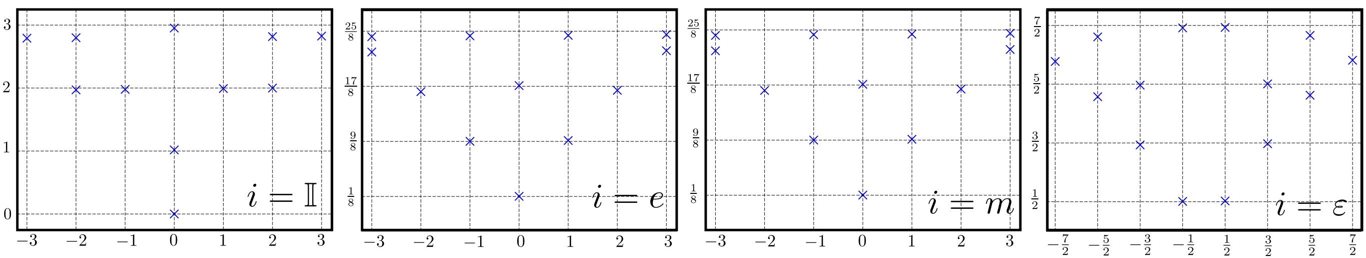
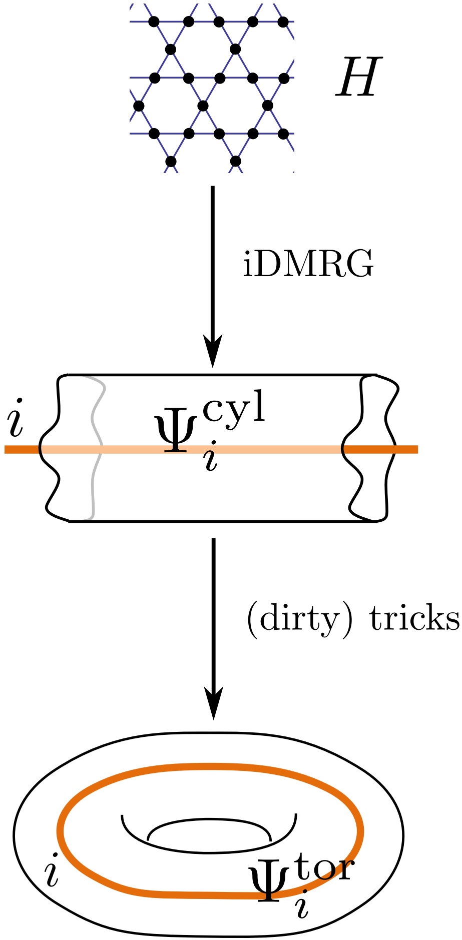


$$\rho_A \equiv e^{-\mathcal{H}_{\text{ent}}} \longrightarrow \{E_\alpha, K_\alpha\}$$

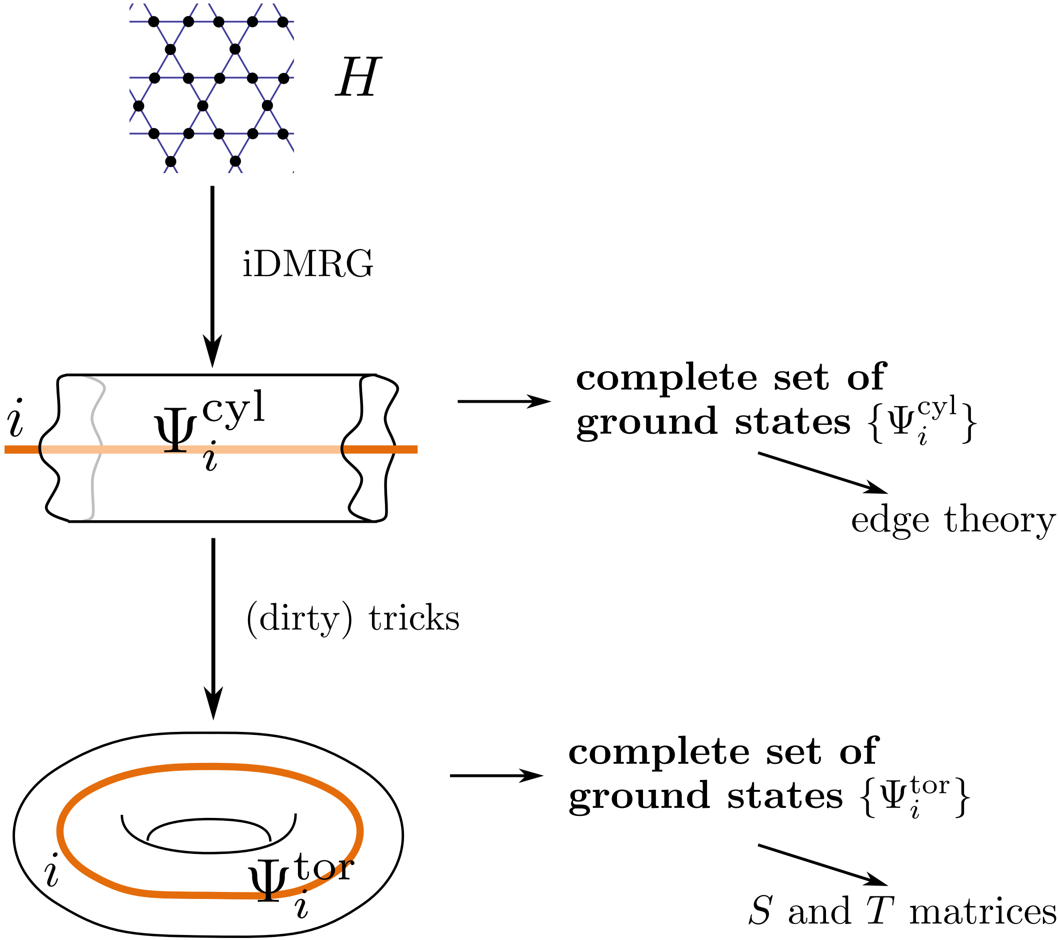
example: Wen-plaquette model $\longrightarrow$ Ising CFT



Numerical method



Numerical method



OUTLINE

- Anyon models:
  - ground state degeneracy
  - S and T matrices
  - edge spectrum

- Hubbard model with magnetic field on Kagome lattice
- Chiral spin liquid and emergent anyons
  - ground state degeneracy
  - S and T matrices
  - edge spectrum

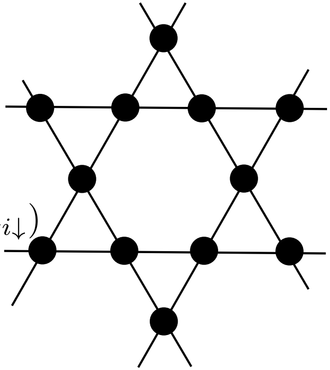
L. Cincio, G. Vidal, **Phys. Rev. Lett. 110, 067208 (2013)**

B. Bauer, L. Cincio, B. P. Keller, M. Dolfi, G. Vidal, S. Trebst, A. W. W. Ludwig, **arXiv:1401.3017**

MODEL

Hubbard model with magnetic field

on Kagome lattice

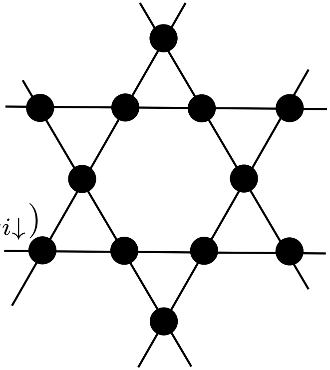


$$H = -\sum_{\langle i,j\rangle,\sigma}(t_{i,j}c^{\dagger}_{i\sigma}c_{j\sigma} + t^{*}_{i,j}c^{\dagger}_{j\sigma}c_{i\sigma}) + U\sum_{i} n_{i\uparrow}n_{i\downarrow} + \frac{h_z}{2}\sum_{i}(n_{i\uparrow} - n_{i\downarrow})$$

MODEL

Hubbard model with magnetic field

on Kagome lattice



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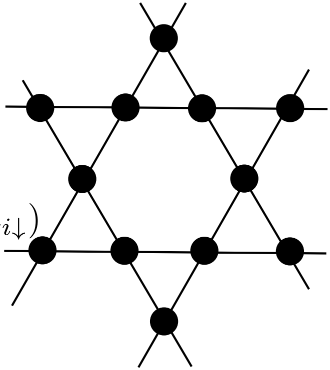
magnetic field induces:

- Zeeman term h_z
- flux Φ through each elementary triangle of the lattice: $t_{i,j} t_{j,k} t_{k,i} = t^3 e^{i\Phi}$

MODEL

Hubbard model with magnetic field

on Kagome lattice



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magnetic field induces:

- Zeeman term h_z
- flux Φ through each elementary triangle of the lattice: $t_{i,j}t_{j,k}t_{k,i} = t^3 e^{i\Phi}$

- Half filling $\langle n\rangle = 1$
- t/U expansion (large U limit) O. I. Motrunich, **PRB 2006**

$$H \; = \; J_{\mathrm{HB}}\sum_{\langle i,j\rangle}\vec{S}_i\cdot\vec{S}_j \; + \; h_z\sum_{i}S^z_i \; + \; J_\chi\sum_{i,j,k\in\triangle}\vec{S}_i\cdot(\vec{S}_j\times\vec{S}_k) \; + \; \cdots$$

- $J_{\mathrm{HB}} \sim \dfrac{t^2}{U}$
- $J_\chi \sim \Phi\dfrac{t^3}{U^2}$

subleading in t/U

MODEL

$$H = J_{\mathrm{HB}} \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j + h_z \sum_i S_i^z + J_\chi \sum_{i,j,k \in \triangle} \vec{S}_i \cdot (\vec{S}_j \times \vec{S}_k) + \cdots$$

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subleading in t/U

- $h_z = 0 \longrightarrow \Phi = 0,\ J_\chi = 0$

$$H = \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j$$

- possible $\mathbb{Z}_2$ spin liquid

S. Yan, D. A. Huse, S. R. White, Science (2011)

H.-C. Jiang, Z. Wang, L. Balents, Nature Physics (2012)

S. Depenbrock, I. P. McCulloch, U. Schollwöck, PRL (2012)

MODEL

$$
\begin{aligned}
H &= \cos\theta \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j \\
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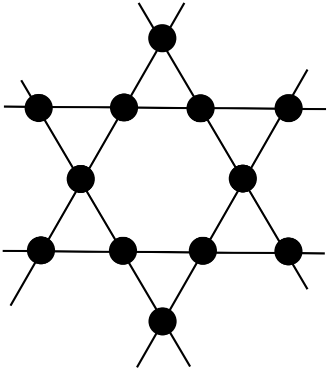
&+ h_z \sum_i S_i^z
\end{aligned}
$$



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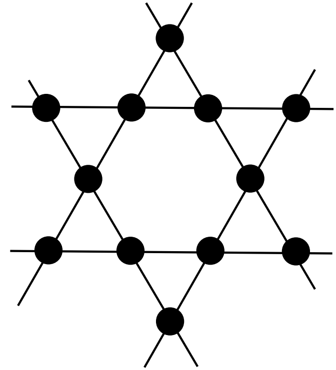
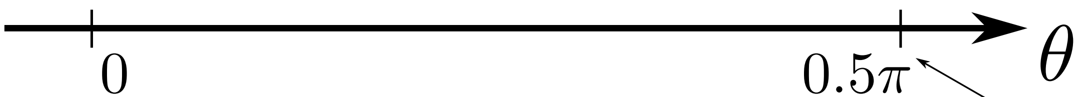
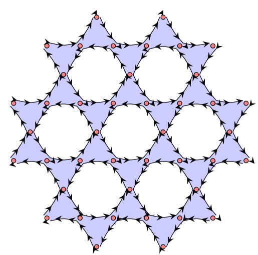


- so far ($h_z = 0$):





pure chiral point

$$H = \sum_{i,j,k\in\triangle} \vec{S}_i \cdot (\vec{S}_j \times \vec{S}_k)$$

speculation:
chiral spin liquid

network model argument

MODEL

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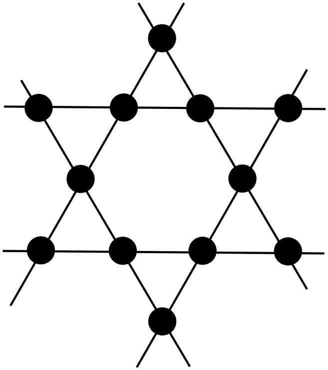

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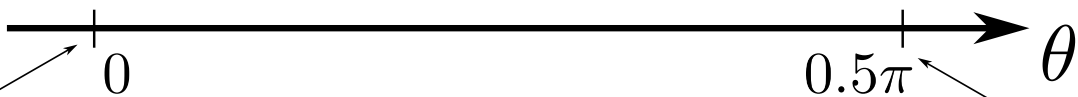

Heisenberg point

$$H = \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j$$

possible $\mathbb{Z}_2$ spin liquid

S. Yan, D. A. Huse, S. R. White, Science (2011)
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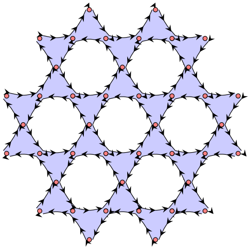

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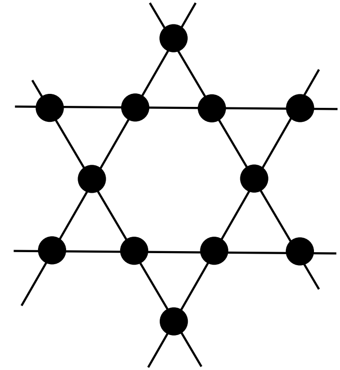

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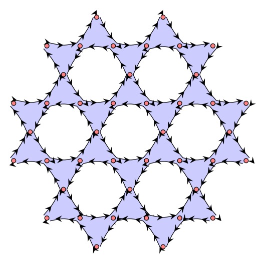

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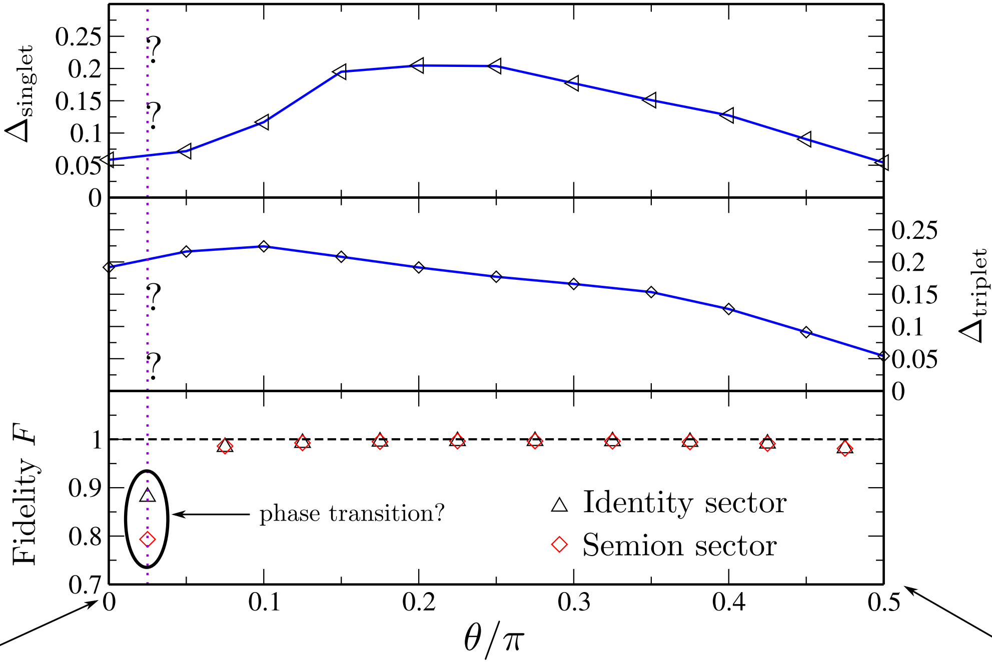
speculation:
chiral spin liquid

network model
argument

GAPS AND FIDELITY



Heisenberg point

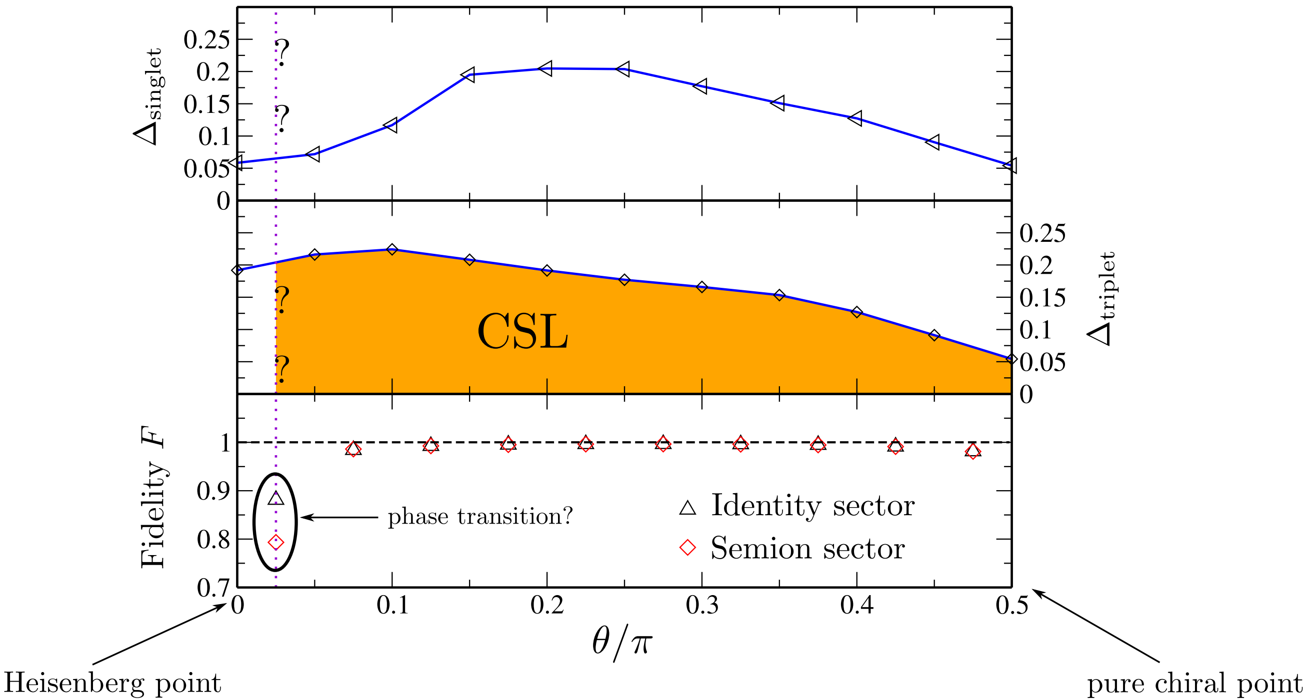
$$H = \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j$$

pure chiral point

$$H = \sum_{i,j,k \in \triangle} \vec{S}_i \cdot (\vec{S}_j \times \vec{S}_k)$$

- Fidelity: $F(\theta) = \langle \Psi_a(\theta - \epsilon) | \Psi_a(\theta + \epsilon) \rangle$

θ–h_z PHASE DIAGRAM



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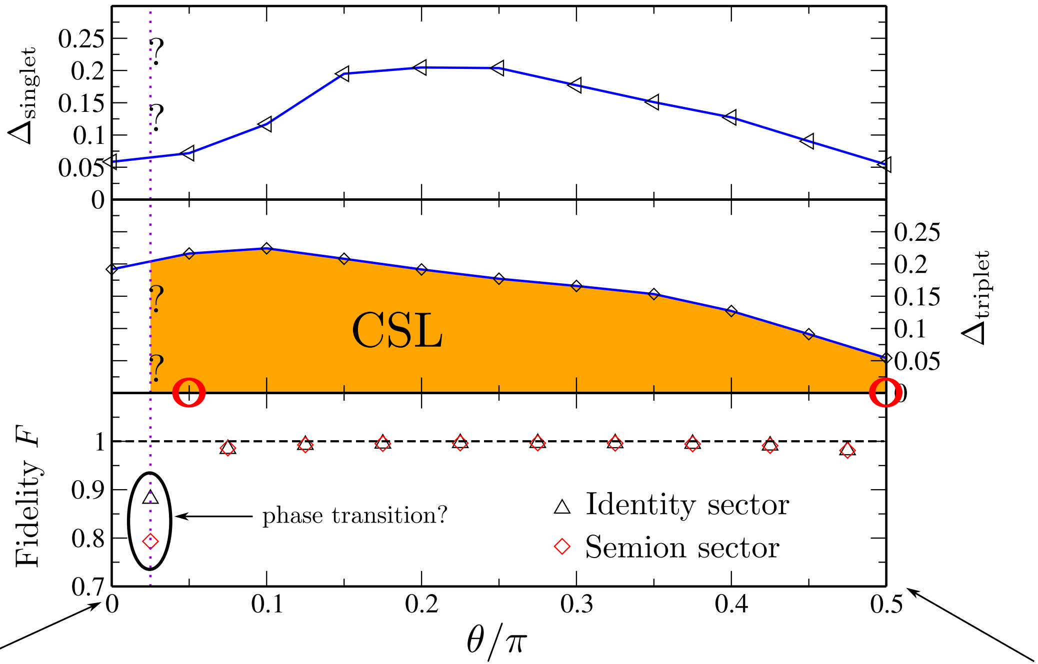

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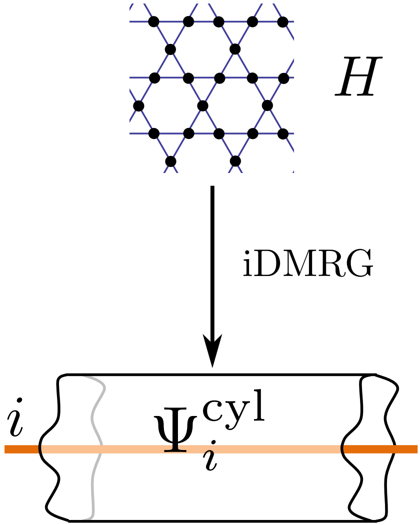
GROUND STATE DEGENERACY

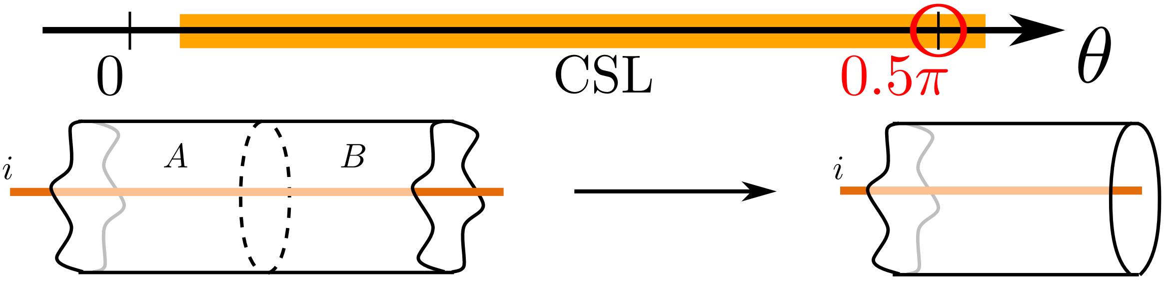


set of ground states

$$\{\Psi_{\mathbb{I}}^{\mathrm{cyl}}, \Psi_{\mathrm{s}}^{\mathrm{cyl}}\}$$

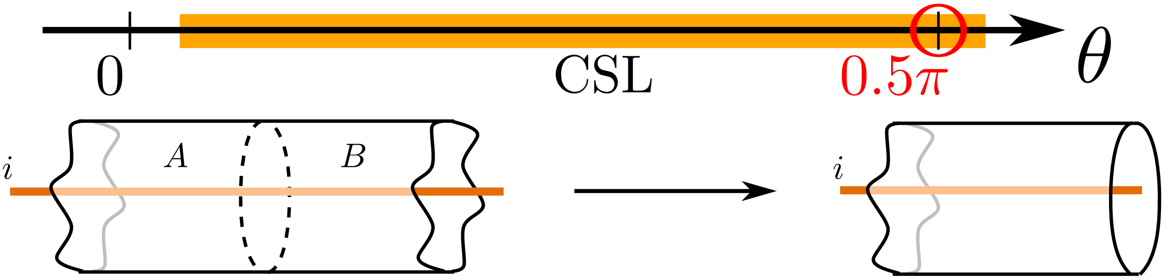
complete?

ENTANGLEMENT SPECTRUM



$$\rho_A^{(i)} = e^{-\mathcal{H}_{\mathrm{ent}}^{(i)}} \longrightarrow \{E_\alpha^{(i)}, K_\alpha^{(i)}\}$$

ENTANGLEMENT SPECTRUM



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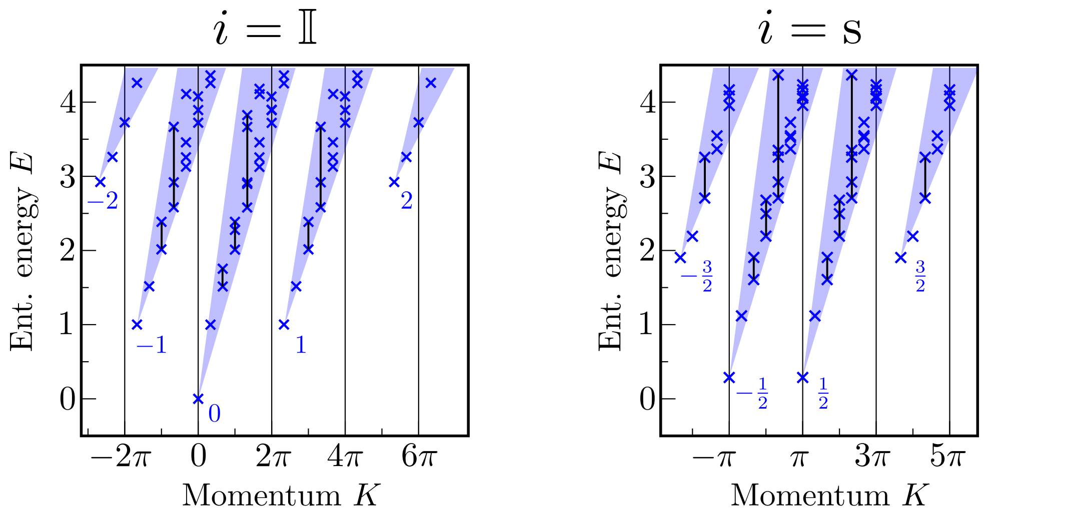
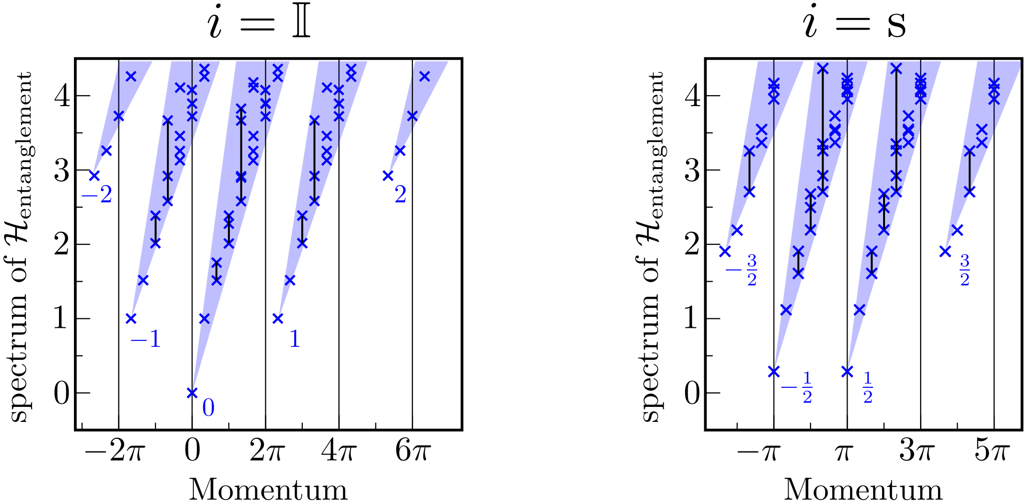


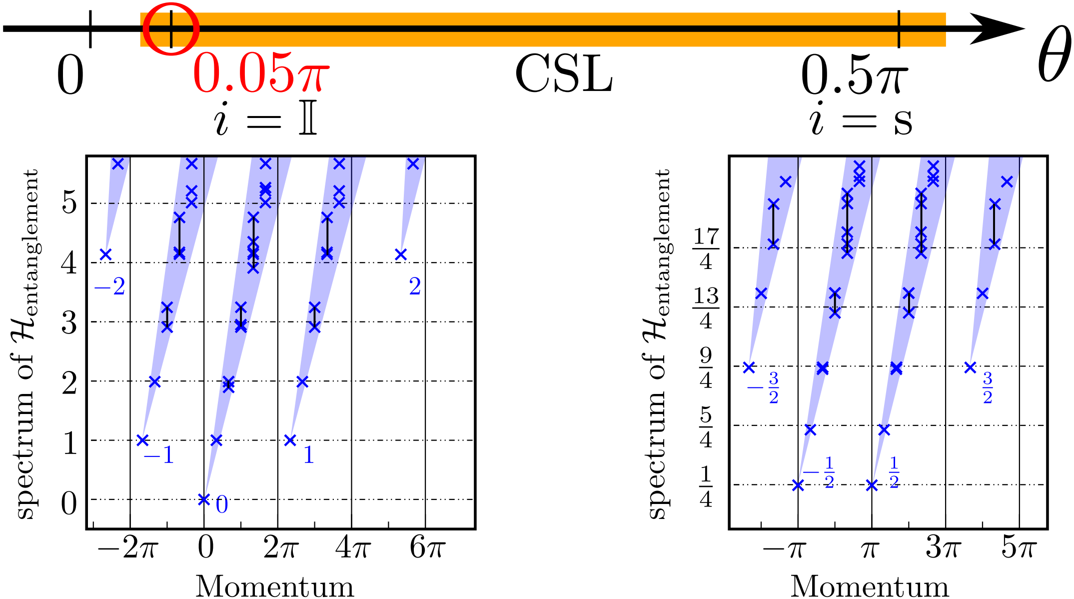
ENTANGLEMENT SPECTRUM



identification: chiral $SU(2)_1$ Wess-Zumino-Witten CFT

- sequence of degeneracies in momentum: 1 - 1 - 2 - 3 - 5 - 7 - ...
- SU(2) multiplets
- **integer** reps. of the spin (identity) and **half-integer** (semion)

ENTANGLEMENT SPECTRUM



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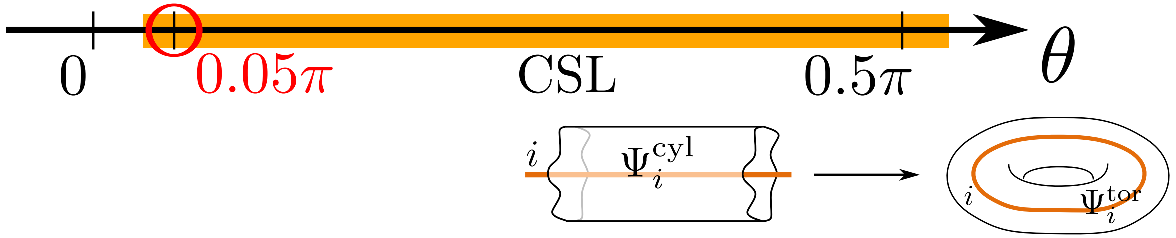
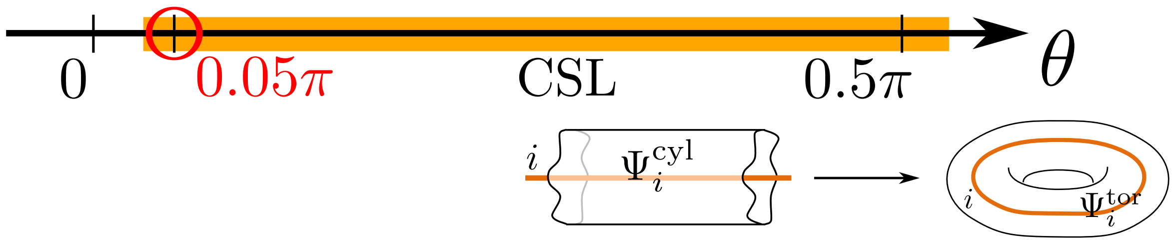
S AND T MATRICES



S and T matrices



$$S = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} + \frac{10^{-3}}{\sqrt{2}}\begin{bmatrix} -4 & -5 \\ -4 & 4+6i \end{bmatrix}$$

$$T = e^{-i\frac{2\pi}{24}\cdot 1}\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \times \left(e^{i\frac{2\pi}{24}0.012}\begin{bmatrix} 1 & 0 \\ 0 & e^{-i0.002\pi} \end{bmatrix} \right)$$

chiral semion
(exact)

deviation

S AND T MATRICES

$$S = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} + \frac{10^{-3}}{\sqrt{2}}\begin{bmatrix} -4 & -5 \\ -4 & 4+6i \end{bmatrix}$$

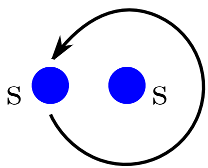
- quantum dimensions

  $d_{\mathbb{I}} = 1,\ d_{\mathrm{s}} = 1$

- total quantum dimension

  $D = \sqrt{2}$

- mutual statistics



$|\Psi\rangle \longrightarrow -|\Psi\rangle$

- fusion rules

  $\mathrm{s} \times \mathrm{s} = \mathbb{I}$

S AND T MATRICES

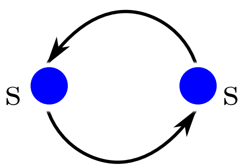
$$T = e^{-i\frac{2\pi}{24}\cdot 1}\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \qquad \times \left(e^{i\frac{2\pi}{24}0.012}\begin{bmatrix} 1 & 0 \\ 0 & e^{-i0.002\pi} \end{bmatrix} \right)$$

- topological central charge

  $c = 1$

- twists

  $\theta_{\mathbb{I}} = 1, \boxed{\theta_{\mathrm{s}} = i}$



$$|\Psi\rangle \longrightarrow i \cdot |\Psi\rangle$$

self statistics

PERTURBATIONS

$$
\begin{aligned}
H &= \cos\theta \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j \\
&+ \sin\theta \sum_{i,j,k \in \triangle} \vec{S}_i \cdot (\vec{S}_j \times \vec{S}_k) \; + \; H_{\text{int}}
\end{aligned}
$$

PERTURBATIONS

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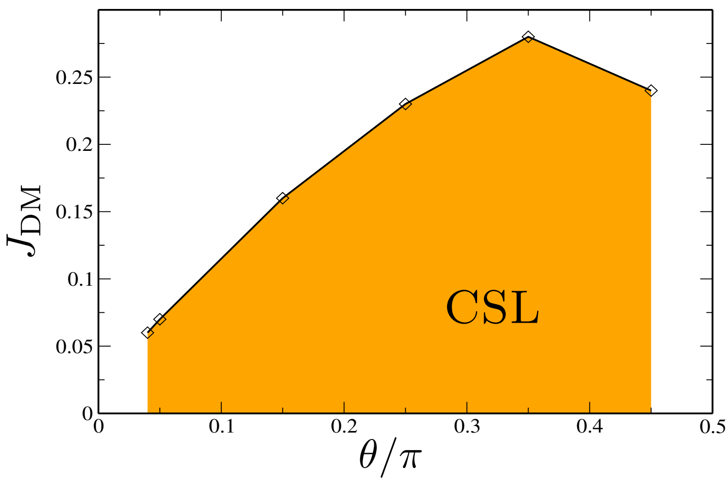
- Dzyaloshinskii-Moriya interaction $H_{\text{int}} = J_{\text{DM}} \sum_{i<j} \hat{z} \cdot (\vec{S}_i \times \vec{S}_j)$



PERTURBATIONS

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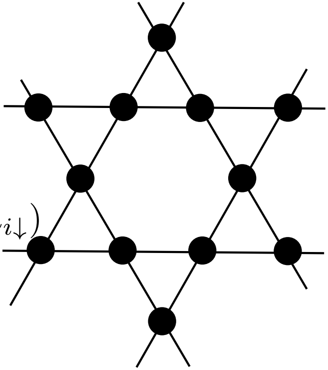
\end{aligned}
$$

- $H_{\mathrm{int}} = J_{\mathrm{NNN}} \sum_{\langle\langle i,j \rangle\rangle} \vec{S}_i \cdot \vec{S}_j$ $\qquad J_{\mathrm{NNN}} \in [-0.1, 0.27]$

- $H_{\mathrm{int}} = J_z \sum_{\langle i,j \rangle} S_i^z S_j^z$ $\qquad J_z \in [-1.2, 0]$

$(\theta = 0.15\pi)$

CONCLUSIONS



- Hubbard model with magnetic field on Kagome lattice

$$H = -\sum_{\langle i,j\rangle,\sigma} (t_{i,j} c^\dagger_{i\sigma} c_{j\sigma} + t^*_{i,j} c^\dagger_{j\sigma} c_{i\sigma}) + U \sum_i n_{i\uparrow} n_{i\downarrow} + \frac{h_z}{2} \sum_i (n_{i\uparrow} - n_{i\downarrow})$$

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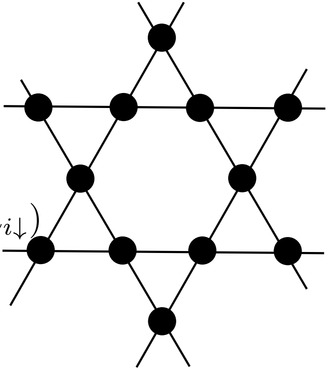
$$H = \cos\theta \sum_{\langle i,j\rangle} \vec{S}_i \cdot \vec{S}_j + \sin\theta \sum_{i,j,k\in\triangle} \vec{S}_i \cdot (\vec{S}_j \times \vec{S}_k) + h_z \sum_i S_i^z$$

CONCLUSIONS



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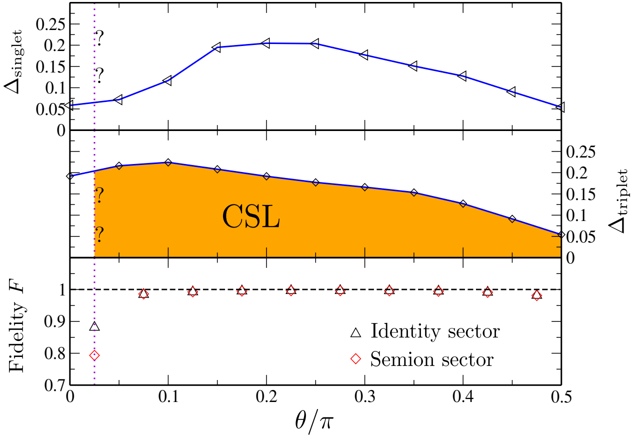
θ–h_z phase diagram



CONCLUSIONS

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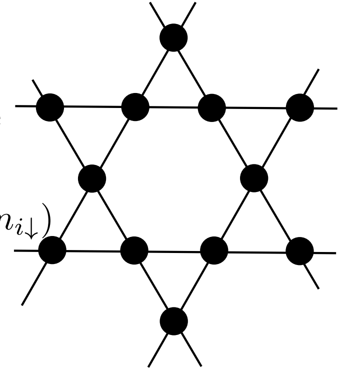

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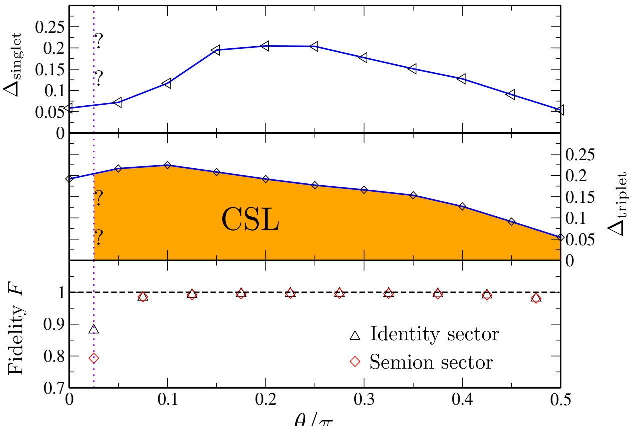
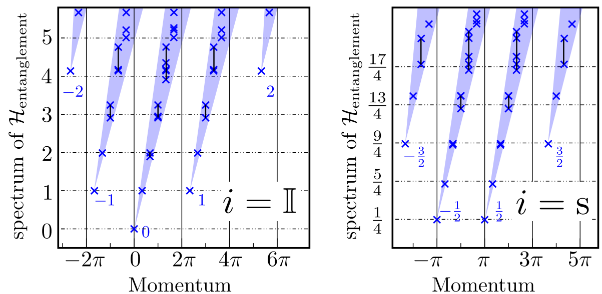
θ–h_z phase diagram



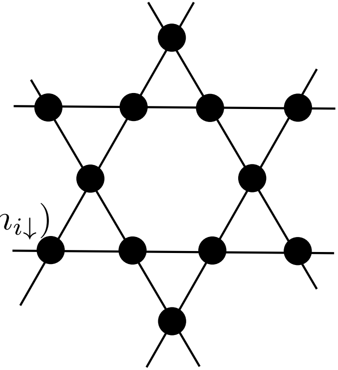
entanglement spectrum



- edge theory: chiral $SU(2)_1$ WZW

CONCLUSIONS

- Hubbard model with magnetic field on Kagome lattice

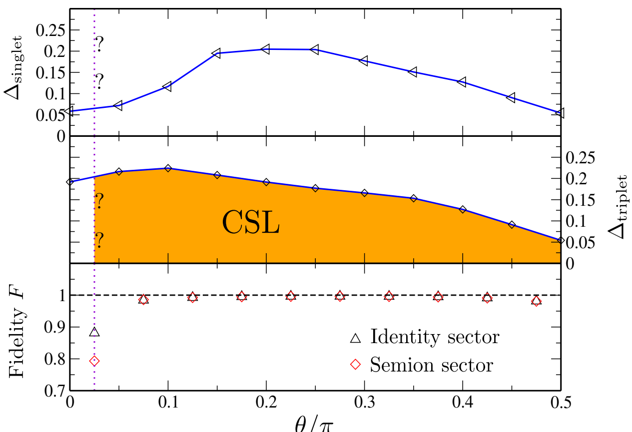


$$H = -\sum_{\langle i,j\rangle,\sigma} (t_{i,j} c^\dagger_{i\sigma} c_{j\sigma} + t^*_{i,j} c^\dagger_{j\sigma} c_{i\sigma}) + U \sum_i n_{i\uparrow} n_{i\downarrow} + \frac{h_z}{2} \sum_i (n_{i\uparrow} - n_{i\downarrow})$$

- t/U expansion

$$H = \cos\theta \sum_{\langle i,j\rangle} \vec{S}_i \cdot \vec{S}_j + \sin\theta \sum_{i,j,k\in\triangle} \vec{S}_i \cdot (\vec{S}_j \times \vec{S}_k) + h_z \sum_i S^z_i$$

θ–h_z phase diagram

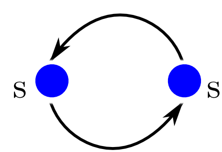


S and T matrices

$$S = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} + \frac{10^{-3}}{\sqrt{2}} \begin{bmatrix} -4 & -5 \\ -4 & 4+6i \end{bmatrix}$$

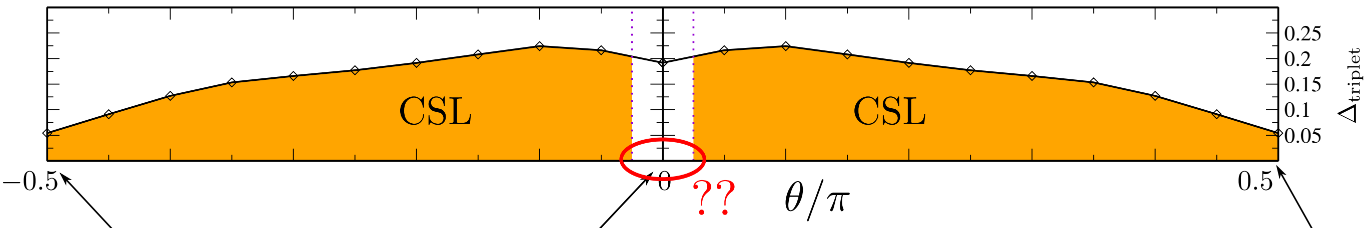
$$T = e^{-i\frac{2\pi}{24}\cdot 1} \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \times \left(e^{i\frac{2\pi}{24}0.012} \begin{bmatrix} 1 & 0 \\ 0 & e^{-i0.002\pi} \end{bmatrix} \right)$$

- anyon model in the bulk: chiral semion



$$|\Psi\rangle \longrightarrow i \cdot |\Psi\rangle$$

WORK IN PROGRESS



pure chiral point

$$H = -\sum_{i,j,k\in\triangle} \vec{S}_i \cdot (\vec{S}_j \times \vec{S}_k)$$

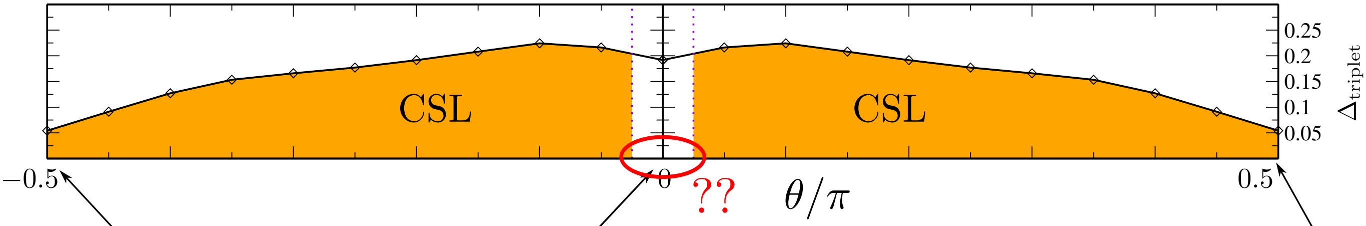
Heisenberg point

$$H = \sum_{\langle i,j\rangle} \vec{S}_i \cdot \vec{S}_j$$

pure chiral point

$$H = \sum_{i,j,k\in\triangle} \vec{S}_i \cdot (\vec{S}_j \times \vec{S}_k)$$

WORK IN PROGRESS



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$$H = -\sum_{i,j,k\in\triangle} \vec{S}_i \cdot (\vec{S}_j \times \vec{S}_k)$$

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pure chiral point

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THANK YOU!