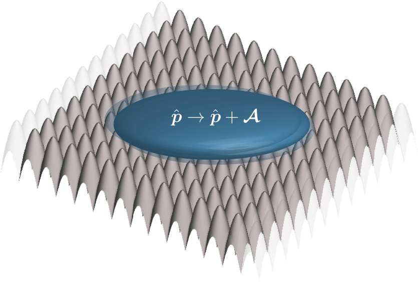
Gauge fields and topological phases with cold atoms...

Baby, you can drive my cloud!

Nathan Goldman



Benasque, June 17th 2014

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Based on the work:

Periodically-driven quantum systems: Effective Hamiltonians and engineered gauge fields

arXiv:1404.4373

NG & **Jean Dalibard**

Discussions: H. Pichler, F. Gerbier, P. Zoller, S. Nascimbène, G. Juzeliunas, M. Ueda, Z. Xu, and N. R. Cooper

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1 Driven quantum matter : effective Hamiltonians, quantum simulation and subtleties

2 Introducing a formalism : presentation of general expressions

3 Application I : Generating a synthetic magnetic field

4 Application II : Generating synthetic spin-orbit couplings

5 Conclusions, last remarks, skipped results, . . .

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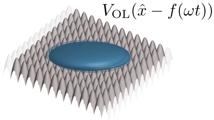
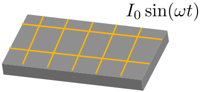
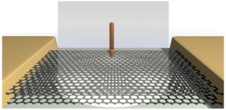
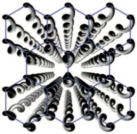
 	Cold atoms in optical lattices	Shaking the lattice, Modulating the hopping (lattice depth), Time-dependent magnetic fields, Additional lasers, ...
 	Cold atoms on the surface of a chip	Modulating the currents,...
 From: Suarez Morrel and Foa Torres, PRB 2012	Electrons in a material (ex: graphene, semiconductors,...) Refs: Cayssol, Dora, Simon and Moessner (Phys. Status Solidi RRL 2013), M. Polini, F. Guinea, M. Lewenstein, H. C. Manoharan and V. Pellegrini (Nat. Nanotech. 2013).	Radiation, mechanical deformation,...
 From: Rechtsman et al., Nature 2013	Light in photonic crystals Ref: I. Carusotto and C. Ciuti (Rev. Mod. Phys. 2013).	Helical waveguides (time=a spatial direction)

The central notion : the effective *time-independent* Hamiltonian(s)

- A static system $\hat{H}_0$ is modulated periodically in time

$$\hat{H}(t) = \hat{H}_0 + \hat{V}(t), \quad \hat{V}(t+T) = \hat{V}(t), \quad T = 2\pi/\omega \text{ : the period}$$

- Generally, one adopts a stroboscopic view $[T \ll t_{\text{charact}}]$: $t = NT$, $N \in \mathbb{N}$

$$|\psi(t = NT)\rangle = \left[\hat{U}(T)\right]^N |\psi_0\rangle = \left[\mathcal{T} e^{-i\int_0^T \hat{H}(\tau)\mathrm{d}\tau}\right]^N |\psi_0\rangle = \left(e^{-iT\hat{\mathcal{H}}_{\text{eff}}}\right)^N |\psi_0\rangle$$

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- Driving is interesting : $\hat{H}_0$ ("normal") $\rightarrow$ $\hat{\mathcal{H}}_{\text{eff}}$ (potentially) **Super !**

- Tuning $\hat{V}(t)$: A versatile tool to engineer gauge fields, exotic band structures, . . .

$$\hat{\mathcal{H}}_{\text{eff}} = \frac{1}{2m}\left[\left(\hat{p}_x + \hat{\mathcal{A}}_x\right)^2 + \left(\hat{p}_y + \hat{\mathcal{A}}_y\right)^2\right] + \dots$$

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- Note : Other forms can be envisaged,

$$\hat{U}(T) = \hat{S}^\dagger e^{-iT\hat{H}_{\text{eff}}}\hat{S}, \quad \hat{S} : \text{an arbitrary unitary matrix}$$

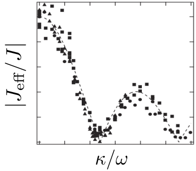
$\hat{H}_{\text{eff}}$ and $\hat{\mathcal{H}}_{\text{eff}}$ share the same effective band structure... which one is *the* "best" ?

Some effective Hamiltonians in the cold-atom world

Tuning the hopping matrix elements in modulated optical lattices

Works by Eckardt, Holthaus and Arimondo

$$\mathcal{J}_0(\kappa/\omega)\left(-J\sum_{\langle j,k\rangle}\hat{a}_k^\dagger\hat{a}_j\right)$$



Lignier, Arimondo et al. 2007

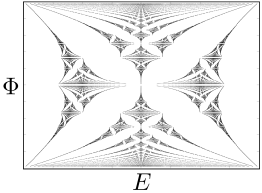
Synthetic magnetic fields in driven optical lattices

Experiments by Bloch, Ketterle and Sengstock

Theory: Sorensen, Demler and Lukin (PRL 2005)
Lim, Morais Smith and Hemmerich (PRL 2008)
Kolovsky (EPL 2011)
Creffield and Sols (EPL 2013)
Bermudez, Schaetz and Porras (PRL 2011)
Hauke et al. (PRL 2012)

See also Jaksch-Zoller, Gerbier-Dalibard, Mazza et al.

$$-J\sum_{\langle j,k\rangle}e^{i\phi_{jk}}\hat{a}_k^\dagger\hat{a}_j$$

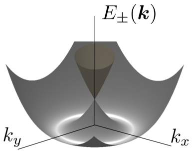


Synthetic spin-orbit couplings for cold atoms in pulsed magnetic fields

Theory: Xu, You and Ueda (PRA 2013)
Anderson, Spielman and Juzeliunas (PRL 2013)

See also Jiang et al. (PRL 2011)

$$\lambda\left(\hat{p}_x\hat{\sigma}_x+\hat{p}_y\hat{\sigma}_y\right)$$



The effective Hamiltonians : How to compute it ?

- In general, the effective Hamiltonian $\hat{\mathcal{H}}_{\text{eff}}$ cannot be derived exactly
- Possible methods :
  - Floquet theory (e.g. Shirley Phys. Rev. 1965, Maricq Phys. Rev. B 1982)
  - Magnus expansion (e.g. Maricq Phys. Rev. B 1982)

$$\hat{U}(T) = \exp\left(-i\left[\int_0^T \hat{H}(\tau)\mathrm{d}\tau - \frac{i}{2}\int_0^T\int_0^t [\hat{H}(t),\hat{H}(\tau)]\mathrm{d}\tau\mathrm{d}t + \dots\right]\right)$$

  - Other perturbative approaches (Avan et al. J. Phys. 1976, Rahav et al. PRB 2003)
  - The Baker-Campbell-Hausdorff (BCH) or Trotter formula

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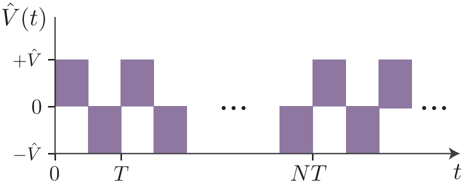
- Illustrating the BCH-Trotter approach : the two-step sequence $\{\hat{H}_0+\hat{V}, \hat{H}_0-\hat{V}\}$

$$\hat{H}(t) = \hat{H}_0 +$$



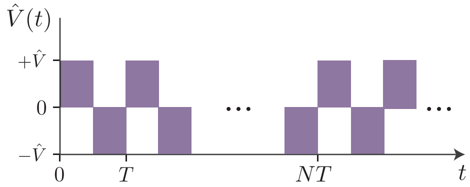
$$\hat{U}(T) = e^{-iT(\hat{H}_0-\hat{V})/2}e^{-iT(\hat{H}_0+\hat{V})/2} = e^{-iT\hat{\mathcal{H}}_{\text{eff}}},$$

$$e^X e^Y = \exp\left(X+Y+\frac{1}{2}[X,Y]+\frac{1}{12}[X-Y,[X,Y]]\dots\right)$$: BCH-Trotter formula

$$\longrightarrow \hat{\mathcal{H}}_{\text{eff}} = \hat{H}_0 - i\frac{T}{4}[\hat{H}_0, \hat{V}] + \mathcal{O}(T^2)$$ The new term could lead to non-trivial effects... ?

The initial phase of the modulation

$$\hat{H}(t) = \hat{H}_0 +$$



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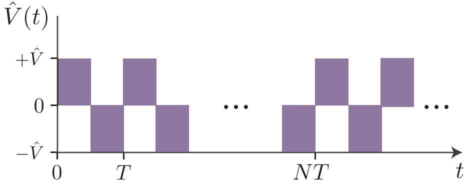

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- Changing the initial phase of the modulation ($t_i \to t_i + T/2$) :

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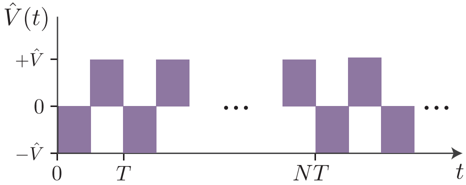

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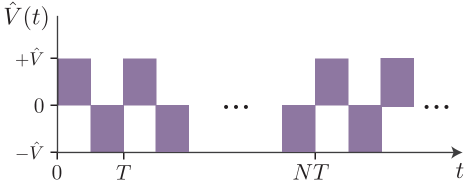

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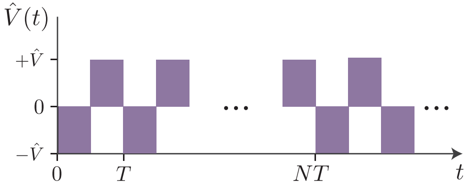

$$\hat{U}(T) = e^{-iT(\hat{H}_0 + \hat{V})/2} e^{-iT(\hat{H}_0 - \hat{V})/2} = e^{-iT\hat{\mathcal{H}}^*_{\text{eff}}},$$

$$\hat{\mathcal{H}}^*_{\text{eff}} = \hat{H}_0 + i\frac{T}{4}[\hat{H}_0, \hat{V}] + \mathcal{O}(T^2).$$

The initial phase of the modulation and unitary transformations

- The first-order terms identified by the BCH-Trotter expansion do not modify the band structure (OK !)

$$\hat{U}(T) = e^{-iT(\hat{H}_0 \mp \hat{V})/2} e^{-iT(\hat{H}_0 \pm \hat{V})/2} = e^{-iT\hat{\mathcal{H}}_{\text{eff}}^{(\pm)}},$$

$$\hat{\mathcal{H}}_{\text{eff}}^{(\pm)} = \hat{H}_0 \mp i\frac{T}{4}[\hat{H}_0, \hat{V}] + \mathcal{O}(T^2) = \hat{S}^{\dagger}\hat{H}_0\hat{S} + \mathcal{O}(T^2), \quad \hat{S} = e^{\mp iT\hat{V}/4}.$$

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- What is $\hat{S}$? It is convenient to re-write the evolution operator at time $t = NT$

$$\begin{aligned} \hat{U}(t = NT)|\psi_0\rangle &= e^{-iNT\hat{\mathcal{H}}_{\text{eff}}^{(\pm)}} |\psi_0\rangle \\ &= \hat{S}^\dagger \left(e^{-iNT\left[\hat{H}_0 + \mathcal{O}(T^2)\right]}\right) \hat{S}|\psi_0\rangle, \end{aligned} \qquad (1)$$

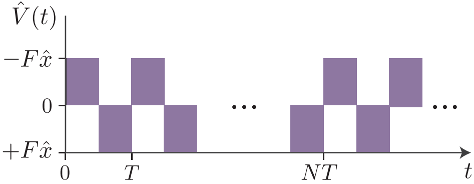
  - The prepared state $|\psi_0\rangle$ undergoes an initial kick $\hat{S}$, which depends on the initial phase of the modulation $(\pm)$!
  - The system evolves for a long time $t = NT$ according to the new $\hat{H}_{\text{eff}} = \hat{H}_0 + \mathcal{O}(T^2)$
  - The system undergoes a final sudden kick $\hat{S}^\dagger$

- Why is this partitionment (1) important ?
  - It highlights the t_i-*independent* **effective Hamiltonian** $\hat{H}_{\text{eff}}$
  - The **initial kick** $\hat{S}$ can have considerable effects on the long-time dynamics !
  - The **micro-motion** $\hat{S}^\dagger$ can be important !

Illustration of the initial kick : the modulated force

- Consider a particle driven by a uniform force F : $\{\hat{H}_0+\hat{V}, \hat{H}_0-\hat{V}\}$ with $\hat{V}=-F\hat{x}$

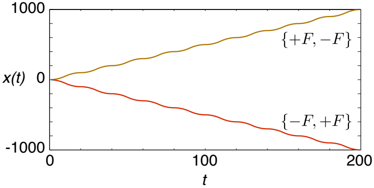
$$\hat{H}(t)=\frac{\hat{p}^2}{2m}+$$



- The BCH-Trotter approach provides the evolution operator $\hat{U}(T)$

$$\hat{U}(T)=\exp\left\{-iT\left[\frac{1}{2m}\left(\hat{p}+\mathcal{A}^{(\pm)}\right)^2+\text{cst}\right]\right\}=\exp\left[-iT\hat{\mathcal{H}}_{\text{eff}}^{(\pm)}\right], \quad \mathcal{A}^{(\pm)}=\pm FT/4$$

- The driving modifies the initial mean velocity : $v(t_i)\to v(t_i)+\mathcal{A}^{(\pm)}/m$



$$|\psi(NT)\rangle=\hat{S}^{\dagger}e^{-iNT\hat{H}_0}\hat{S}|\psi_0\rangle$$

$$\hat{S}=\exp\left(i\mathcal{A}^{(\pm)}\hat{x}\right)$$: initial kick

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Introducing the formalism

- We generalize a formalism proposed by Rahav, Gilary and Fishman [PRA 2003]
- We start with the Schrödinger equation : $i\partial_t \psi(t) = \hat{H}(t)\psi(t)$, $\hat{H}(t) = \hat{H}_0 + \hat{V}(t)$.
- We look for a unitary transformation

$$\phi(t) = e^{i\hat{K}(t)}\psi(t), \quad i\partial_t \phi(t) = \hat{H}_{\text{eff}}\phi(t),$$

$$\hat{H}_{\text{eff}} = e^{i\hat{K}(t)}\hat{H}(t)e^{-i\hat{K}(t)} + i\left(\frac{\partial e^{i\hat{K}(t)}}{\partial t}\right)e^{-i\hat{K}(t)},$$

where we impose the conditions :

- $\hat{H}_{\rm eff}$ is a time-independent operator ;
- $\hat{K}(t)$ is a time-periodic operator, $\hat{K}(t+T) = \hat{K}(t)$, with zero average over one period ;
- $\hat{H}_{\rm eff}$ does not depend on the starting time t_i ; $\hat{K}(t) \to \hat{K}(t; t_i)$

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  - $\hat{H}_{\text{eff}}$ does not depend on the starting time t_i ; $\hat{K}(t) \to \hat{K}(t; t_i)$
- Starting with the state $\psi(t_i)$, the state at an arbitrary final time t_f is given by

$$\psi(t_f) = \hat{U}(t_i \to t_f)\psi(t_i) = e^{-i\hat{K}(t_f)}e^{-i(t_f - t_i)\hat{H}_{\text{eff}}}e^{i\hat{K}(t_i)}\psi(t_i).$$

  - The prepared state $\psi(t_i)$ undergoes an initial kick, which depends on the initial phase of the modulation !
  - The system evolves for a long time $t = t_f - t_i$ according to $\hat{H}_{\text{eff}}$
  - The system undergoes a final sudden kick (i.e. micro-motion). Note : $t_f \neq T \times$ (integer) is arbitrary.

Deriving the effective Hamiltonian : the general formula

- We expand the time-dependent Hamiltonian in terms of the harmonics

$$\hat{H}(t) = \hat{H}_0 + \hat{V}(t),$$
$$\hat{V}(t) = \sum_{j=1}^{\infty} V^{(j)} e^{ij\omega t} + V^{(-j)} e^{-ij\omega t}.$$

- We follow a perturbative treatment to obtain $\hat{H}_{\text{eff}}$ and $\hat{K}(t)$

$$\hat{H}_{\text{eff}} = \sum_{n=0}^{\infty} \frac{1}{\omega^n} \hat{H}_{\text{eff}}^{(n)}, \quad \hat{K}(t) = \sum_{n=1}^{\infty} \frac{1}{\omega^n} \hat{K}^{(n)}$$
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$$\hat{H}_{\text{eff}} = \hat{H}_0 + \frac{1}{\omega} \sum_{j=1}^{\infty} \frac{1}{j} [V^{(j)}, V^{(-j)}] + \frac{1}{2\omega^2} \sum_{j=1}^{\infty} \frac{1}{j^2} \left([[V^{(j)}, \hat{H}_0], V^{(-j)}] + \text{h.c.} \right)$$
$$+ \frac{1}{3\omega^2} \sum_{j,l=1}^{\infty} \frac{1}{jl} \left([V^{(j)}, [V^{(l)}, V^{(-j-l)}]] - 2[V^{(j)}, [V^{(-l)}, V^{(l-j)}]] + \text{h.c.} \right) + \dots,$$
$$\hat{K}(t) = \frac{1}{i\omega} \sum_{j=1}^{\infty} \frac{1}{j} \left(V^{(j)} e^{ij\omega t} - V^{(-j)} e^{-ij\omega t} \right) + \dots$$

Some simple applications

- A single harmonic : a cosine modulation : $\hat{H}(t) = \hat{H}_0 + \hat{V}\cos(\omega t)$

$$\hat{H}_{\text{eff}} = \hat{H}_0 + \frac{1}{4\omega^2}[[\hat{V}, \hat{H}_0], \hat{V}] + \mathcal{O}(1/\omega^3),$$

Some simple applications

- A single harmonic : a cosine modulation : $\hat{H}(t) = \hat{H}_0 + \hat{V}\cos(\omega t)$

$$\hat{H}_{\text{eff}} = \hat{H}_0 + \frac{1}{4\omega^2}[[\hat{V},\hat{H}_0],\hat{V}] + \mathcal{O}(1/\omega^3),$$

- A cosine + sine modulation : $\hat{H}(t) = \hat{H}_0 + \hat{A}\cos(\omega t) + \hat{B}\sin(\omega t)$

$$\hat{H}_{\text{eff}} = \hat{H}_0 + \frac{i}{2\omega}[\hat{A},\hat{B}] + \frac{1}{4\omega^2}\Big([[\hat{A},\hat{H}_0],\hat{A}] + [[\hat{B},\hat{H}_0],\hat{B}]\Big) + \mathcal{O}(1/\omega^3),$$

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- More harmonics... the two-step (square-wave) sequence $\{\hat{H}_0 + \hat{V}, \hat{H}_0 - \hat{V}\}$:

$$\hat{H}_{\text{eff}} = \hat{H}_0 + \frac{\pi^2}{24\omega^2}[[\hat{V}, \hat{H}_0], \hat{V}] + \mathcal{O}(1/\omega^3)$$

$$\hat{K}(t) = -\frac{\pi}{2\omega}\hat{V} + |t|\hat{V} + \mathcal{O}(1/\omega^2) \text{ , for } t \in \left[-\frac{T}{2}, \frac{T}{2}\right].$$

  - Example : the modulated force, $\hat{H}_0 = \hat{p}^2/2m$ and $\hat{V} = -F\hat{x}$

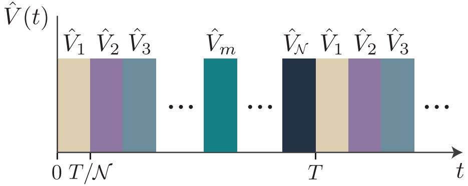
$$\hat{H}_{\text{eff}} = \hat{p}^2/2m + \text{cst},$$

$$\hat{K}(t) = (FT/4)\hat{x} - |t|F\hat{x} + \mathcal{O}(1/\omega^2) \text{ , for } t \in \left[-\frac{T}{2}, \frac{T}{2}\right].$$

The $\mathcal{N}$-step pulse sequence : a general formula

- We now apply the general formulas to the case of $\mathcal{N}$-step sequences

$$\hat{H}(t) = \hat{H}_0 +$$



$$\hat{H}_{\text{eff}} = \hat{H}_0 + \frac{2\pi i}{\mathcal{N}^3\omega} \sum_{m<n=2}^{\mathcal{N}} \mathcal{C}_{m,n} [\hat{V}_m, \hat{V}_n] + \frac{\pi^2(\mathcal{N}-1)^2}{6\mathcal{N}^4\omega^2} \sum_{m=1}^{\mathcal{N}} [[\hat{V}_m, \hat{H}_0], \hat{V}_m]$$

$$+ \frac{\pi^2}{6\mathcal{N}^4\omega^2} \sum_{m<n=2}^{\mathcal{N}} \mathcal{D}_{m,n} \Big([[\hat{V}_m, \hat{H}_0], \hat{V}_n] + [[\hat{V}_n, \hat{H}_0], \hat{V}_m] \Big) + \mathcal{O}(1/\omega^3),$$

$$\hat{K}(0) = \frac{2\pi}{\mathcal{N}^2\omega} \sum_{m=1}^{\mathcal{N}} \hat{V}_m\, m + \mathcal{O}(1/\omega^2).$$

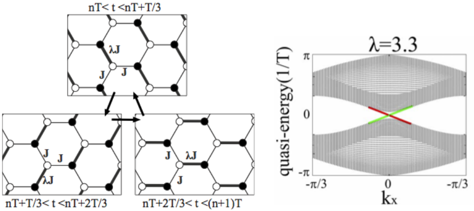
where $\mathcal{C}_{m,n} = \frac{\mathcal{N}}{2} + m - n$ and $\mathcal{D}_{m,n} = 1 + \mathcal{N}^2 - 6\mathcal{N}(n-m) + 6(n-m)^2$.

Applications of the $\mathcal{N}$-step formula : Using first-order terms

- The case $\mathcal{N} = 3$:

$$\hat{H}_{\text{eff}} = \hat{H}_0 + \frac{i\pi}{27\omega}\Big([\hat{V}_1, \hat{V}_2] + [\hat{V}_2, \hat{V}_3] + [\hat{V}_3, \hat{V}_1]\Big) + \mathcal{O}(1/\omega^2)$$

  - Example : the tripod-modulated honeycomb lattice $\rightarrow$ the Haldane-Chern insulator



Kitagawa, Berg, Rudner, Demler (PRB 2010)

- The case $\mathcal{N} = 4$:

$$\hat{H}_{\text{eff}} = \hat{H}_0 + \frac{i\pi}{32\omega}\Big([\hat{V}_1, \hat{V}_2] + [\hat{V}_2, \hat{V}_3] + [\hat{V}_3, \hat{V}_4] + [\hat{V}_4, \hat{V}_1]\Big) + \mathcal{O}(1/\omega^2)$$

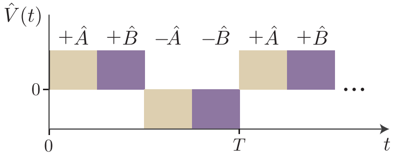
  - Example : Inspired by Sorensen, Demler, Lukin (PRL 2005)
    $\hat{V}_1 = -\hat{V}_3 = (\hat{p}_x^2 - \hat{p}_y^2)/2m$ and $\hat{V}_2 = -\hat{V}_4 = \kappa\hat{x}\hat{y}$

$$\hat{H}_{\text{eff}} = \frac{\hat{p}^2}{2m} - \Omega\hat{L}_z + \mathcal{O}(1/\omega^2), \quad \Omega \sim \kappa/m\omega$$

Two useful four-step sequences

- The α sequence : $\hat{V}_1 = -\hat{V}_3 = \hat{A}$ and $\hat{V}_2 = -\hat{V}_4 = \hat{B}$ $\quad [\approx \hat{A}\cos(\omega t) + \hat{B}\sin(\omega t)]$

$\hat{H}(t) = \hat{H}_0 +$



$$\hat{H}_{\text{eff}} = \hat{H}_0 + \frac{i\pi}{8\omega}[\hat{A},\hat{B}] + \frac{\pi^2}{48\omega^2}\Big([[\hat{A},\hat{H}_0],\hat{A}] + [[\hat{B},\hat{H}_0],\hat{B}]\Big) + \mathcal{O}(1/\omega^3).$$

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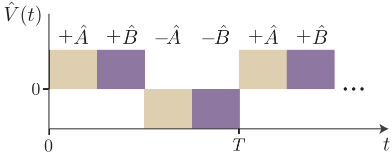
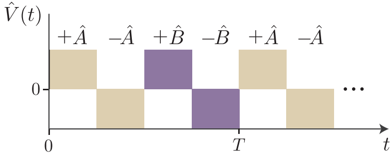


$$\hat{H}_{\text{eff}} = \hat{H}_0 + \frac{i\pi}{8\omega}[\hat{A}, \hat{B}] + \frac{\pi^2}{48\omega^2}\Big([[\hat{A}, \hat{H}_0], \hat{A}] + [[\hat{B}, \hat{H}_0], \hat{B}]\Big) + \mathcal{O}(1/\omega^3).$$

- The β sequence : $\hat{V}_1 = -\hat{V}_2 = \hat{A}$ and $\hat{V}_3 = -\hat{V}_4 = \hat{B}$

$\hat{H}(t) = \hat{H}_0 +$



$$\hat{H}_{\text{eff}} = \hat{H}_0 + \frac{5\pi^2}{384\omega^2}\Big([[\hat{A}, \hat{H}_0], \hat{A}] + [[\hat{B}, \hat{H}_0], \hat{B}]\Big) - \frac{\pi^2}{128\omega^2}\Big([[\hat{A}, \hat{H}_0], \hat{B}] + [[\hat{B}, \hat{H}_0], \hat{A}]\Big) + \mathcal{O}(1/\omega^3).$$

1 Driven quantum matter : effective Hamiltonians, quantum simulation and subtleties

2 Introducing a formalism : presentation of general expressions

3 Application I : Generating a synthetic magnetic field

4 Application II : Generating synthetic spin-orbit couplings

5 Conclusions, last remarks, skipped results, . . .

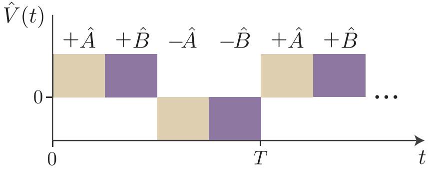
A synthetic magnetic field, and more...

- We consider a spinless system in 2D
- Inspired by Sorensen-Lukin-Demler (PRL 2005), we consider the 4-step sequence

$$\left\{ \frac{\hat{p}_x^2}{m}\,,\ \frac{\hat{p}_x^2+\hat{p}_y^2}{2m}+\kappa\hat{x}\hat{y}\,,\ \frac{\hat{p}_y^2}{m}\,,\ \frac{\hat{p}_x^2+\hat{p}_y^2}{2m}-\kappa\hat{x}\hat{y} \right\}$$

- It corresponds to the α sequence :

$$\hat{H}(t)=\frac{\hat{p}_x^2+\hat{p}_y^2}{2m}+$$



$$\hat{A}=\frac{\hat{p}_x^2-\hat{p}_y^2}{2m}$$

$$\hat{B}=\kappa\hat{x}\hat{y}$$

- The effective Hamiltonian yields

$$\hat{H}_{\text{eff}}=\frac{1}{2m}\Big((\hat{p}_x-\mathcal{A}_x)^2+(\hat{p}_y-\mathcal{A}_y)^2\Big)+\frac{1}{2}m\omega_h^2(\hat{x}^2+\hat{y}^2)+\mathcal{O}(1/\omega^3)$$

$$\boldsymbol{\mathcal{A}}=(-m\Omega\hat{y},m\Omega\hat{x}),\quad \Omega=\frac{\pi\kappa}{8m\omega},\quad \omega_h=\sqrt{\frac{5}{3}}\Omega,$$

- The driving generates simultaneously :
  - a synthetic magnetic field $\boldsymbol{B}=2m\Omega\mathbf{1}_z\sim(\kappa/\omega)$
  - an additional harmonic potential $\sim(\kappa/\omega)^2$

A synthetic magnetic field, and more...

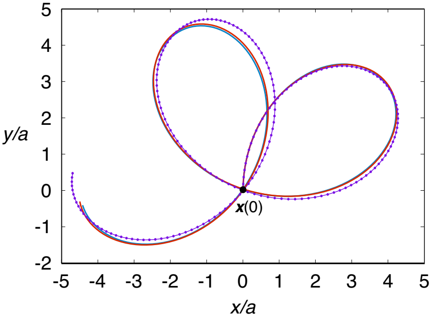
- It is more natural to define the pulsed system on a (optical) lattice : $\hat{H}_0 = \hat{T}_x + \hat{T}_y$

$$\left\{ \frac{\hat{p}_x^2}{m} , \frac{\hat{p}_x^2 + \hat{p}_y^2}{2m} + \kappa \hat{x}\hat{y} , \frac{\hat{p}_y^2}{m} , \frac{\hat{p}_x^2 + \hat{p}_y^2}{2m} - \kappa \hat{x}\hat{y} \right\} = \left\{ 2\hat{T}_x , \hat{H}_0 + \kappa \hat{x}\hat{y} , 2\hat{T}_y , \hat{H}_0 - \kappa \hat{x}\hat{y} \right\} ,$$

- The effective Hamiltonian leads to the Hofstadter model

$$\hat{H}_{\text{eff}} = - J \sum_{m,n} e^{-i\pi\Phi n} \hat{a}^\dagger_{m+1,n} \hat{a}_{m,n} + e^{i\pi\Phi m} \hat{a}^\dagger_{m,n+1} \hat{a}_{m,n} + \text{h.c.}$$
$$+ \frac{1}{2} m^* \omega_h^2 \left(\bar{x}^2 + \bar{y}^2 \right)$$

- We compare the dynamics of the real and effective Hamiltonians :



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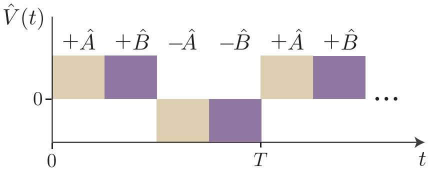
Synthetic spin-orbit coupling : The Quest of the Rashba term $\hat{\boldsymbol{p}} \cdot \hat{\boldsymbol{\sigma}}$

- We consider a spin-1/2 system in 2D [for simplicity]
- Inspired by the previous result, we propose the following sequence

$$\left\{ \frac{\hat{p}_x^2}{m}, \frac{\hat{p}_x^2 + \hat{p}_y^2}{2m} + \kappa(\hat{x}\hat{\sigma}_x - \hat{y}\hat{\sigma}_y), \frac{\hat{p}_y^2}{m}, \frac{\hat{p}_x^2 + \hat{p}_y^2}{2m} - \kappa(\hat{x}\hat{\sigma}_x - \hat{y}\hat{\sigma}_y) \right\}.$$

- It corresponds to the α sequence :

$$\hat{H}(t) = \frac{\hat{p}_x^2 + \hat{p}_y^2}{2m} +$$



$$\hat{A} = \frac{\hat{p}_x^2 - \hat{p}_y^2}{2m}$$

$$\hat{B} = \kappa(\hat{x}\hat{\sigma}_x - \hat{y}\hat{\sigma}_y)$$

- The effective Hamiltonian yields

$$\hat{H}_{\text{eff}} = \hat{H}_0 + \lambda_{\text{R}}\, \hat{\boldsymbol{p}} \cdot \hat{\boldsymbol{\sigma}} - \Omega_{\text{SO}} \hat{L}_z \hat{\sigma}_z + \mathcal{O}(1/\omega^3),$$
$$\lambda_{\text{R}} = \pi\kappa/8m\omega, \quad \Omega_{\text{SO}} = (8m/3)\lambda_{\text{R}}^2,$$

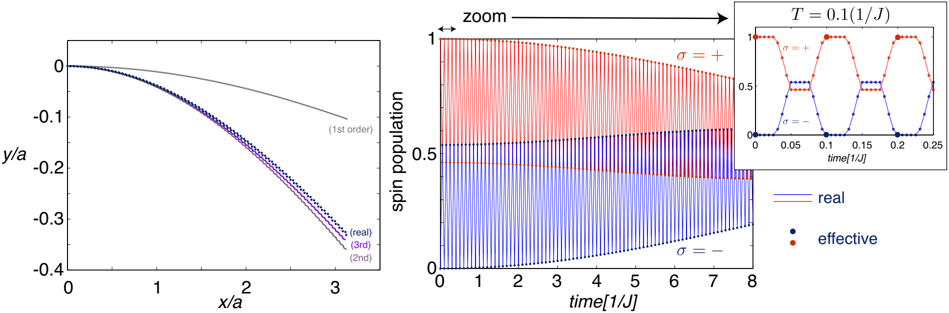
- The driving generates :
  - a Rashba SOC term with $\lambda_{\text{R}} \sim (\kappa/\omega)$
  - a "spin-Hall" SOC term with $\Omega_{\text{SO}} \sim (\kappa/\omega)^2$ [Note : for small λ_{R}, $\hat{L}_z \hat{\sigma}_z \equiv \hat{\boldsymbol{p}} \cdot \hat{\boldsymbol{\sigma}}$]

Synthetic spin-orbit coupling : The Quest of the Rashba term $\hat{\boldsymbol{p}} \cdot \hat{\boldsymbol{\sigma}}$

- Again, it is more natural to define this pulsed system on a lattice : $\hat{H}_0 = \hat{T}_x + \hat{T}_y$

$$\left\{ 2\hat{T}_x \,,\, \hat{H}_0 + \kappa(\hat{x}\hat{\sigma}_x - \hat{y}\hat{\sigma}_y) \,,\, 2\hat{T}_y \,,\, \hat{H}_0 - \kappa(\hat{x}\hat{\sigma}_x - \hat{y}\hat{\sigma}_y) \right\}$$

- We compare the dynamics of the real and effective Hamiltonians :



- The micro-motion captured by $\hat{K}(t)$ is small in real space, but large in spin space !

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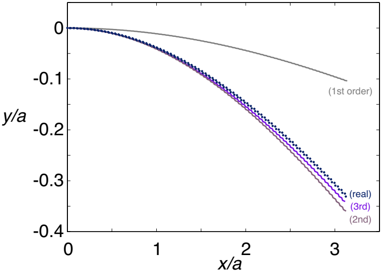
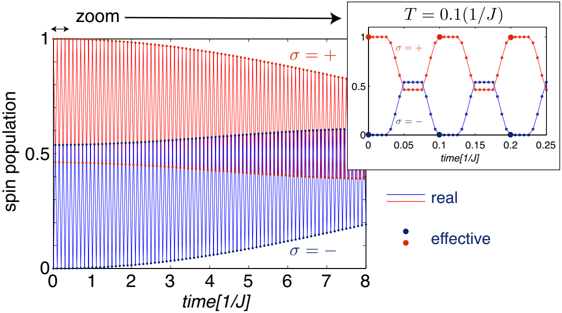




- The micro-motion captured by $\hat{K}(t)$ is small in real space, but large in spin space !
- Combining the Rashba term with a Zeeman term is possible : $\hat{H}_0 \rightarrow \hat{H}_0 + \lambda_{\mathrm{Z}}\hat{\sigma}_z$

$$\hat{H}_{\text{eff}} = \hat{H}_0 + \lambda_{\mathrm{Z}}\hat{\sigma}_z + \lambda_{\mathrm{R}}\,\hat{\boldsymbol{p}} \cdot \hat{\boldsymbol{\sigma}} - \Omega_{\mathrm{SO}}\hat{L}_z\hat{\sigma}_z + \frac{1}{2}m\Omega_{\mathrm{Z}}^2(x^2+y^2)\hat{\sigma}_z, \quad \Omega_{\mathrm{Z}} = 2\sqrt{\lambda_{\mathrm{Z}}\Omega_{\mathrm{SO}}}$$

$\longrightarrow$ topological superfluidity ?

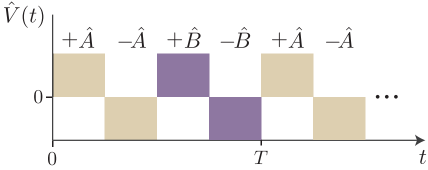
The XA scheme

- Xu-You-Ueda (PRA 2013) and Anderson-Spielman-Juzeliunas (PRL 2013) proposed a scheme to realize a Rashba SOC Hamiltonian

$$\left\{ \hat{H}_0 + \kappa \hat{x}\hat{\sigma}_x, \hat{H}_0 - \kappa \hat{x}\hat{\sigma}_x, \hat{H}_0 + \kappa \hat{y}\hat{\sigma}_y, \hat{H}_0 - \kappa \hat{y}\hat{\sigma}_y \right\}, \quad \hat{H}_0 = \frac{\hat{p}_x^2 + \hat{p}_y^2}{2m}.$$

- It corresponds to the β sequence :

$$\hat{H}(t) = \frac{\hat{p}_x^2 + \hat{p}_y^2}{2m} +$$



$$\hat{A} = \kappa \hat{x}\hat{\sigma}_x$$
$$\hat{B} = \kappa \hat{y}\hat{\sigma}_y$$

- Using the β-sequence formula, we obtain

$$\hat{H}_{\text{eff}} = \hat{H}_0 - \Omega_{\text{SO}} \hat{L}_z \hat{\sigma}_z + \mathcal{O}(1/\omega^3), \quad \Omega_{\text{SO}} = m\lambda_{\text{R}}^2,$$
$$\hat{K}(0) = -m\lambda_{\text{R}}\, \hat{\boldsymbol{x}} \cdot \hat{\boldsymbol{\sigma}} + \mathcal{O}(1/\omega^2), \quad \lambda_{\text{R}} = \pi\kappa/8m\omega, \quad \text{[for a sequence starting with } +\hat{A}\text{]}$$

- The evolution operator after one period reads

$$\hat{U}(T) = e^{-i\hat{K}(T)}\, e^{-iT\hat{H}_{\text{eff}}}\, e^{i\hat{K}(0)} = e^{-iT\hat{\mathcal{H}}_{\text{eff}}},$$
$$\hat{\mathcal{H}}_{\text{eff}} = \hat{H}_0 - \lambda_{\text{R}}\, \hat{\boldsymbol{p}} \cdot \hat{\boldsymbol{\sigma}} + \mathcal{O}(1/\omega^3) \text{ : Rashba SOC Hamiltonian}$$

The XA scheme : an alternative almost-exact treatment

- Following Xu-You-Ueda and Anderson-Spielman-Juzeliunas

$$\left\{\hat{H}_0 + \kappa\hat{x}\hat{\sigma}_x, \hat{H}_0 - \kappa\hat{x}\hat{\sigma}_x, \hat{H}_0 + \kappa\hat{y}\hat{\sigma}_y, \hat{H}_0 - \kappa\hat{y}\hat{\sigma}_y\right\}, \quad \hat{H}_0 = \frac{\hat{p}_x^2 + \hat{p}_y^2}{2m}.$$

- Each half-sequence can be treated exactly : for example the first half-sequence is

$$e^{-i(\hat{H}_0 - \kappa\hat{x}\hat{\sigma}_x)T/4} e^{-i(\hat{H}_0 + \kappa\hat{x}\hat{\sigma}_x)T/4} = e^{-iT\left[\hat{H}_0/2 - \lambda_{\mathrm{R}}\hat{p}_x\hat{\sigma}_x\right]},$$

- The evolution operator over one period is obtained using the Trotter expansion to minimal order $\exp A \exp B \approx \exp(A+B)$:

$$\hat{U}(T) = e^{-iT\left[\hat{H}_0/2 - \lambda_{\mathrm{R}}\hat{p}_y\hat{\sigma}_y\right]} e^{-iT\left[\hat{H}_0/2 - \lambda_{\mathrm{R}}\hat{p}_x\hat{\sigma}_x\right]} = \exp\left(-i\hat{\mathcal{H}}_{\mathrm{eff}}T\right),$$

$\hat{\mathcal{H}}_{\mathrm{eff}} = \hat{H}_0 - \lambda_{\mathrm{R}}\,\hat{\boldsymbol{p}}\cdot\hat{\boldsymbol{\sigma}} + \mathcal{O}\left(1/\omega^2\right)\ldots$ in agreement with our result !

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- Using this treatment, we obtained a similar expression for the lattice analogue

$$\hat{\mathcal{H}}_{\text{eff}} = \hat{H}_0\left\{\frac{1}{2} + \frac{1}{2}\text{sinc}\left(4am^*\lambda_{\mathrm{R}}\right)\right\} + \hat{\boldsymbol{p}}\cdot\hat{\boldsymbol{\sigma}}\left\{\frac{\cos\left(4am^*\lambda_{\mathrm{R}}\right) - 1}{8(am^*)^2\lambda_{\mathrm{R}}}\right\},$$

- Note 1 : for $\lambda_{\mathrm{R}} \ll aJ$, we recover the lattice-free result.
- Note 2 : the strength of the Rashba SOC is limited to $\lambda_{\mathrm{R}}^* = (2/\pi)aJ$ on the lattice !

Back to the initial kick issue ! Ouch !

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- The system will first undergo a sudden kick !

$$|\psi(t)\rangle = e^{-it[\hat{H}_0 - \lambda_{\mathsf{R}}\, \hat{\boldsymbol{p}}\cdot\hat{\boldsymbol{\sigma}}]}\, e^{i\,\delta p\, \hat{y}\hat{\sigma}_y}\, |\psi_0\rangle, \quad \delta p = 4m\lambda_{\mathsf{R}}$$

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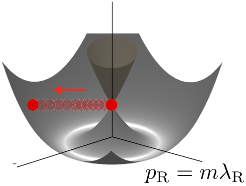
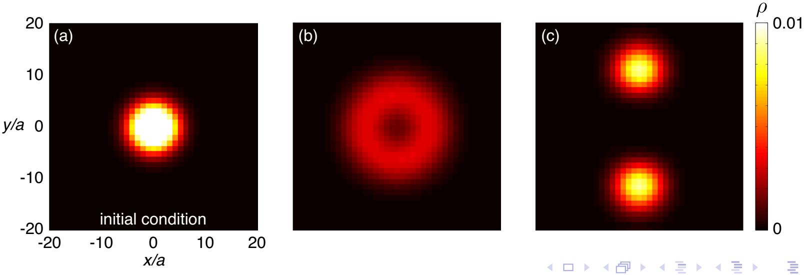




The xy scheme : an almost-exact treatment

- We also found a sequence α, treatable almost-exactly :

$$\left\{\hat{T}_y - \kappa\hat{x}\hat{\sigma}_x, \hat{T}_y - \kappa\hat{y}\hat{\sigma}_y, \hat{T}_x + \kappa\hat{x}\hat{\sigma}_x, \hat{T}_x + \kappa\hat{y}\hat{\sigma}_y\right\},$$

- Our perturbative approach yields

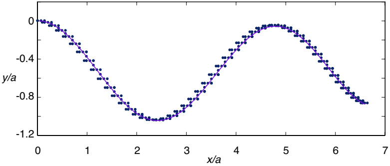
$$\hat{H}_{\text{eff}} = \hat{H}_0 + \lambda_{\text{R}}\,\hat{\boldsymbol{p}}\cdot\hat{\boldsymbol{\sigma}} - \gamma\,\hat{x}\hat{y}\,\hat{\sigma}_z + \text{cst} + \mathcal{O}(1/\omega^3), \quad \gamma = \pi\kappa^2/4\omega$$

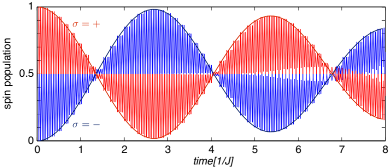
- There is a regime where each subsequence can be treated exactly on the lattice !

$$\begin{aligned}
\hat{U}(T) &= \hat{U}_{-B}\hat{U}_{-A}\hat{U}_{+B}\hat{U}_{+A},\\
\hat{U}_{+A} &= e^{-iT\hat{p}_y^2/4m^*}\,e^{i\pi\hat{x}/a}, \quad \hat{U}_{-A} = e^{i\pi\hat{x}/a}e^{iT\lambda_{\text{R}}^*\hat{p}_x\hat{\sigma}_x},\\
\hat{U}_{+B} &= e^{i\pi\hat{y}/a}e^{-iT\lambda_{\text{R}}^*\hat{p}_y\hat{\sigma}_y}, \quad \hat{U}_{-B} = e^{-iT\hat{p}_x^2/4m^*}\,e^{i\pi\hat{y}/a},
\end{aligned}$$

- The effective Hamiltonian yields a maximized Rashba SOC [start with pulse $+\hat{A}$]

$$\hat{U}(T) = \exp\left(-i\hat{\mathcal{H}}_{\text{eff}}\right), \; \hat{\mathcal{H}}_{\text{eff}} = \frac{1}{2}\hat{H}_0 + \lambda_{\text{R}}^*\,\hat{\boldsymbol{p}}\cdot\hat{\boldsymbol{\sigma}} + \mathcal{O}\left(1/\omega^2\right) \qquad \text{[no } xy \text{ term !].}$$





1 Driven quantum matter : effective Hamiltonians, quantum simulation and subtleties

2 Introducing a formalism : presentation of general expressions

3 Application I : Generating a synthetic magnetic field

4 Application II : Generating synthetic spin-orbit couplings

5 Conclusions, last remarks, skipped results, . . .

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- We obtained general formulas and identified “interesting/useful” driving schemes

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- The micro-motion is small in real space, but it is generally large in momentum and spin space ! Might be tricky for measurements...
- Collisions due to [spin-dependent] micro-motion could be important... Heating ?

More details in NG and J. Dalibard, arXiv :1404.4373 !