Ising anyons with a string tension

Marc D. Schulz[1,†], S. Dusuel[2], G. Misguich[3], K.P. Schmidt[4], and J. Vidal[5]

[1] University of Minnesota, [2] Lycée Saint-Louis, Paris, [3] IPT, CNRS - CEA, Gif-sur-Yvette, [4] TU Dortmund, [5] LPTMC, UPMC - CNRS, Paris

† mdschulz@umn.edu

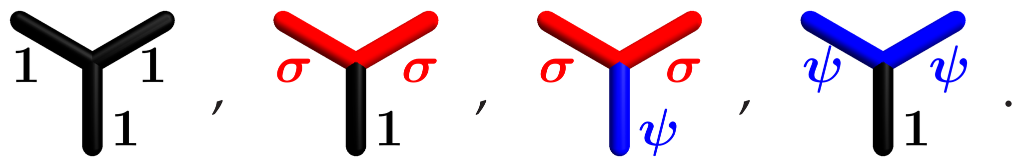
Motivation

- Study robustness of topological order.[1,2]
- Ground state of string-net model [3] is topologically ordered.
- Excitations are (non-) Abelian anyons.
- Simpler (non-) Abelian phases have been studied (e.g. in [4,5,6]).
- Investigate robustness of richer phases.

The Ising string-net model

The unperturbed model [3]:

- Defined on hexagonal lattice
- Microscopic degrees of freedom are defined on links and take values $\mathbf{1}, \boldsymbol{\sigma}, \boldsymbol{\psi}$.
- Branching rules constrain Hilbert space: the allowed vertex configurations for Ising anyons are:



- Hamiltonian $H_{\rm SN}$ is given by

$$H_{\rm SN} = -\sum_{\text{plaquettes } p} B_p,$$

where $B_p = \frac{1}{4}B_p^{\mathbf{1}} + \frac{\sqrt{2}}{4}B_p^{\boldsymbol{\sigma}} + \frac{1}{4}B_p^{\boldsymbol{\psi}}$ measures flux through the plaquette p.

- No flux corresponds to eigenvalue $b_p = 1$.
- Fluxes $\boldsymbol{\sigma}, \boldsymbol{\psi}$ correspond to $b_p = 0$.

General properties of the model

- Eigenvalues b_p of B_p's are conserved quantities: $[B_p, H_{\rm SN}] = 0$.
- Model realizes time-reversal invariant (doubled) phase $\mathbf{D}(\text{Ising})$.
- Topological degeneracy of the ground state $= 3^{2g}$ ($g = 1$ the genus of the torus, $g = 0$ for the plane).
- Fluxes $\boldsymbol{\psi}$ ($\boldsymbol{\sigma}$) are (non-) Abelian anyons.
- Non-local fusion channels of fluxes determined via

$\otimes$	$\boldsymbol{\sigma}$	$\boldsymbol{\psi}$
$\boldsymbol{\sigma}$	$\mathbf{1} \oplus \boldsymbol{\psi}$	$\boldsymbol{\sigma}$
$\boldsymbol{\psi}$	$\boldsymbol{\sigma}$	$\mathbf{1}$

- Local fluxes and their non-local fusion channels characterize eigenstates of $H_{\rm SN}$ completely.
- Eigenstates form tensionless string nets in the bond basis.

Local perturbation

Adding local perturbation [7,8]

$$H_{\rm loc} = -\sum_e \mathcal{P}_e^{\mathbf{1}},$$

where $\mathcal{P}_e^{\mathbf{1}}$ projects onto state $\mathbf{1}$ at link e.

- $H_{\rm loc}$ introduces tension on $\boldsymbol{\sigma}, \boldsymbol{\psi}$-strings.
- Anyonic excitations become dynamic, dressed quasi-particles.
- Condensation of (non-) Abelian anyons yields continuous transition to (non-) topological phase [9].

Perturbed string-net model

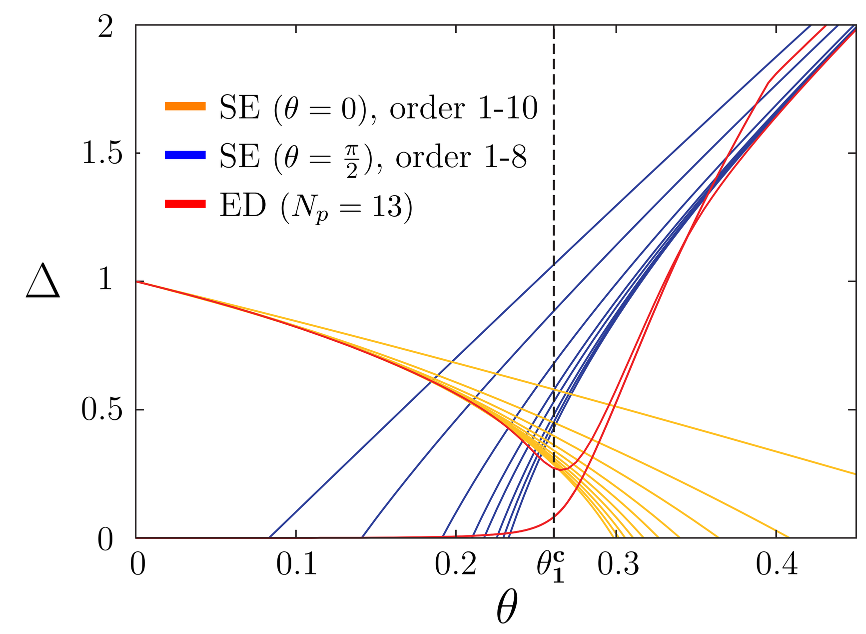
Investigate phase diagram of the Hamiltonian

$$H = \cos(\theta)\, H_{\rm SN} + \sin(\theta)\, H_{\rm loc}.$$

- Topological $\mathbf{D}(\text{Ising})$-phase for $\theta \in [\theta_2^c, \theta_1^c]$
- Polarized $\mathbf{1}$-phase between θ_1^c and $\theta = \pi$
- Star-crystal phase between $\theta = \frac{3}{2}\pi$ and θ_2^c

Phase transition at θ_1^c

- Finite-size and finite-order scaling for critical value yields: $\theta_1^c \approx 0.261$.



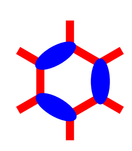
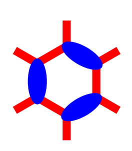
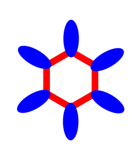
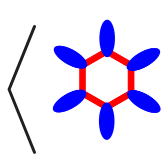
- Critical exponents obtained by fitting the series-expansions and data-collapse of ED-data near θ_1^c yields

$$\Delta \sim (\theta-\theta_c)^{z\nu} \quad \rightarrow \quad z\nu \approx 0.4,$$
$$\omega|_{\theta_c} \sim |\mathbf{k}-\mathbf{k}_c|^{z} \quad \rightarrow \quad z \approx 1,$$
$$\partial_\theta^2 e_0 \sim (\theta-\theta_c)^{-\alpha} \quad \rightarrow \quad \alpha \approx 0.8.$$

- Consistent with hyperscaling relation for 2$^{\rm nd}$-order phase transitions $2-\alpha = \nu(2+z)$.

Star-crystal phase at $\theta = (3\pi/2)^+$

Effective description at $\theta = \left(\frac{3\pi}{2}\right)^+$ yields dimer model:

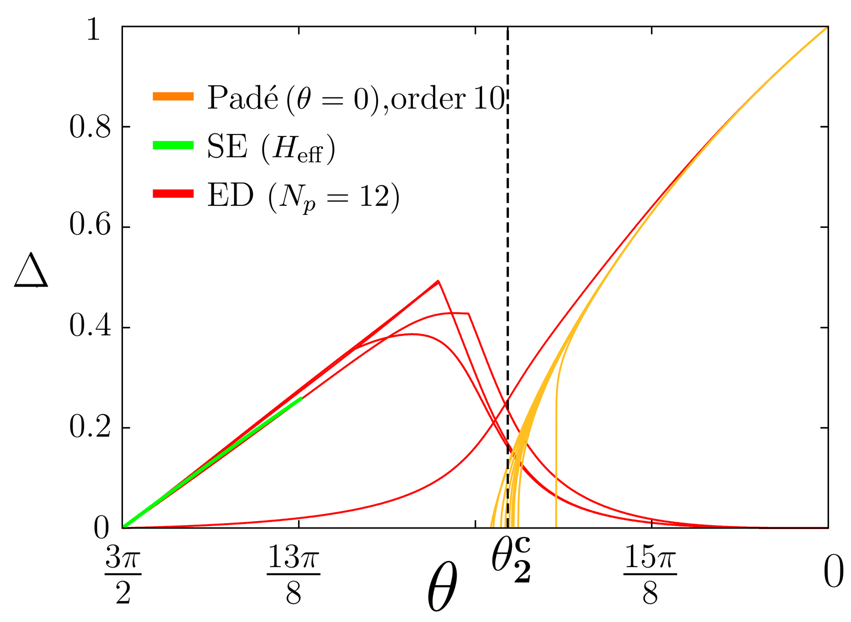
$H_{\rm eff} = -\sum_p \Big[t \Big(\Big|$  $\Big\rangle \Big\langle$  $\Big| + \text{h.c.} \Big) +$
$v \Big|$  $\Big\rangle \Big\langle$  $\Big| + \mathbb{1} \Big],$

with $t = 1/8\cos\theta$ and $v = 1/4\cos\theta$.

- Ground states adiabatically connected to ground states at $t = 0$.
- Translational symmetry-breaking star-crystal order found [10].
- Realizes gapped, non-topological phase.

Phase transition at θ_2^c

- dlog-Padé extrapolation for critical value yields: $\theta_2^c \approx 5.57$.



- Continuous transition between topological and translational symmetry breaking phase.

Method I: pCUT

A Hamiltonian H can be unitarily transformed onto an effective quasi-particle (QP) conserving Hamiltonian $H_{\rm eff}$ using a continuous unitary transformation (CUT)[11].
This transformation can be performed perturbatively [12].

- Results valid directly in the thermodynamic limit
- high-order series expansion (SE)

Application to the perturbed string net:

- The unperturbed model has a discrete spectrum with a gap $\Delta = 1$ between each energy level.
- Build a QP-picture for the non-Abelian fluxes, which captures non-local features like braiding.
- Compute the expansion of the ground-state energy per plaquette e_0 as well as the 1QP-hopping elements.
- Diagonalize $H_{\rm eff}$ analytically to obtain 1QP-gap Δ.

Linked-cluster expansion

- Linked-cluster expansions mandatory for high-order results of the pCUT.
- QP-picture allows linked-cluster expansion also for topologically ordered phases harboring *non-Abelian* excitations.
- Maximal order reached up to 18.

Method II: Exact diagonalization

- Non-perturbative
- Finite-size effects
- Consider typically systems with up to 134 225 920 states (13 plaquettes).
- Lanczos algorithm yields low-energy spectrum.

Outlook

- Characterize phase for $\theta \in \left[\pi, \frac{3\pi}{2}\right]$.
- Consider other string tensions as perturbations (e.g. as in [13]).
- Observables witnessing topological order (e.g. entanglement entropy, S-matrix[14]).
- Continuous phase transitions driven by condensation of chiral excitations?
- Study impact of topology onto criticality.
- Is there universality in topological phase transitions?

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