Transfer Matrices and Excitations with Matrix Product States

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Complex Quantum Systems

Abstract

We investigate the relation between **static correlations** and **low energy excitations**. Being a central object in obtaining static correlations we show that the **MPS Transfer Matrix (MPS-TM)** of the ground state already contains important information about the **location and magnitude of the dispersion's minima**. We relate the MPS-TM to the quantum transfer matrix (QTM) at zero temperature and investigate the peculiar structure of its **eigenspectrum.** We also derive a **bound for the decay of momentum filtered correlations** as a function of the dispersion.

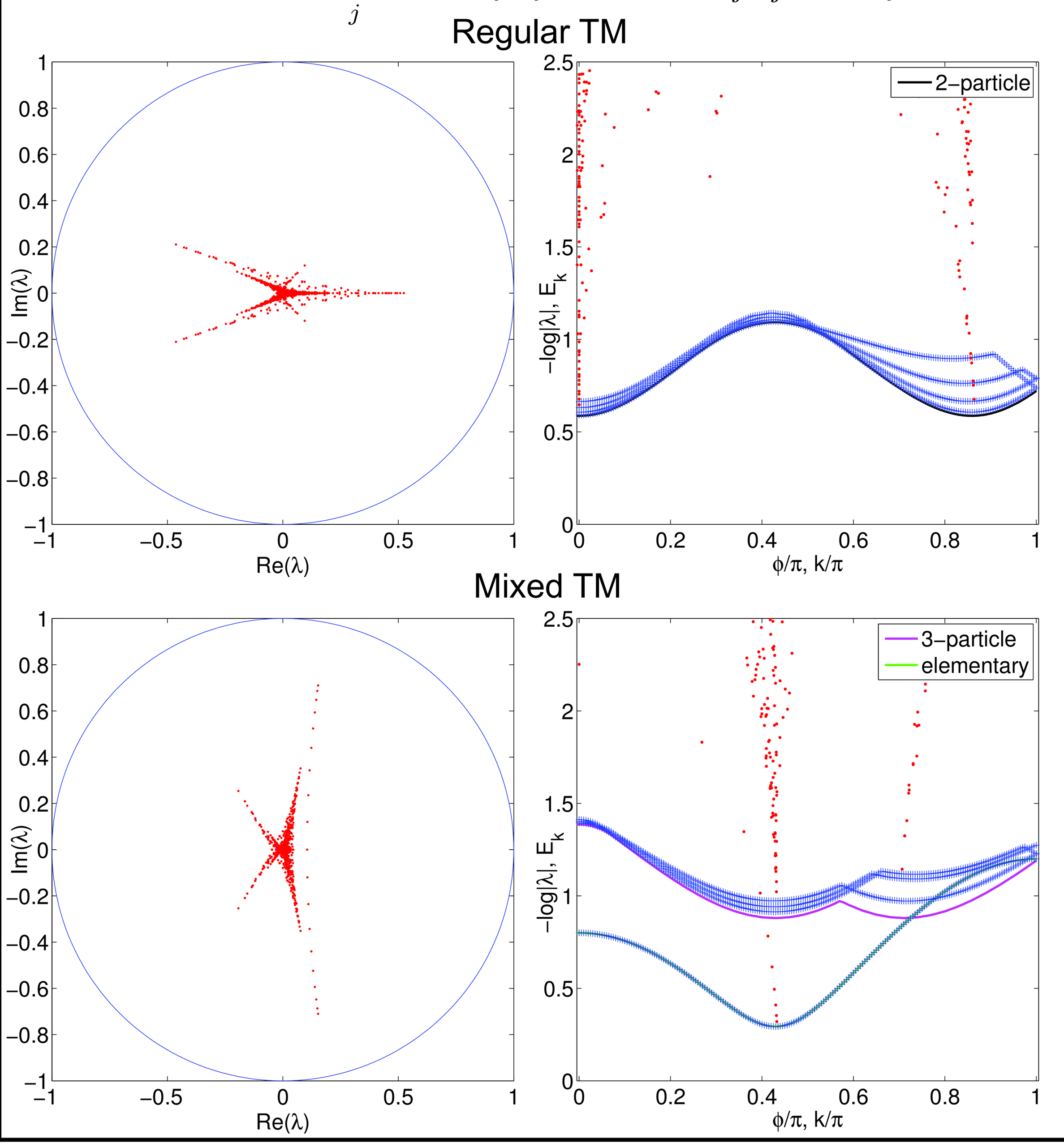
(c)MPS Transfer Matrix

MPS-TM	cMPS-TM generator
$\mathcal{T}_A = \sum_s \bar{A}^s \otimes A^s$	$\mathcal{P}_{Q,R} = \bar{Q}\otimes \mathbb{1} + \mathbb{1}\otimes Q + \bar{R}\otimes R$
mixed MPS-TM	**mixed cMPS-TM generator**
$\mathcal{T}_{A_1}^{A_2} = \sum_s \bar{A}_2^s \otimes A_1^s$	$\mathcal{P}_{Q_1,R_1}^{Q_2,R_2} = \bar{Q}_2\otimes \mathbb{1} + \mathbb{1}\otimes Q_1 + \bar{R}_2\otimes R_1$

Numerical Observations

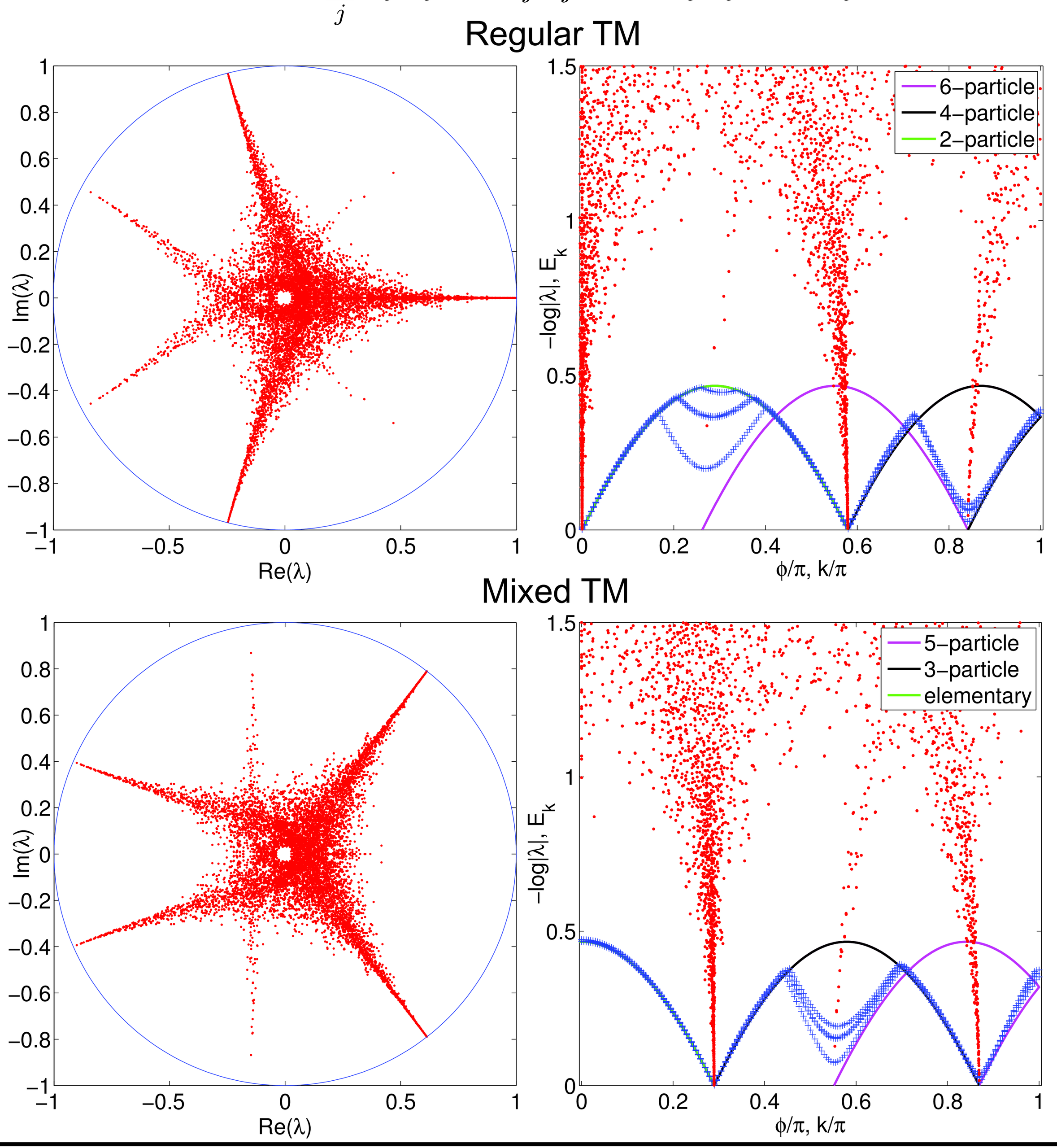
XY Model, γ=0.3, g=0.2, D=40

$$H_{\mathrm{XY}} = -\sum_j (1+\gamma)S^x_j S^x_{j+1} + (1-\gamma)S^y_j S^y_{j+1} + gS^z_j$$



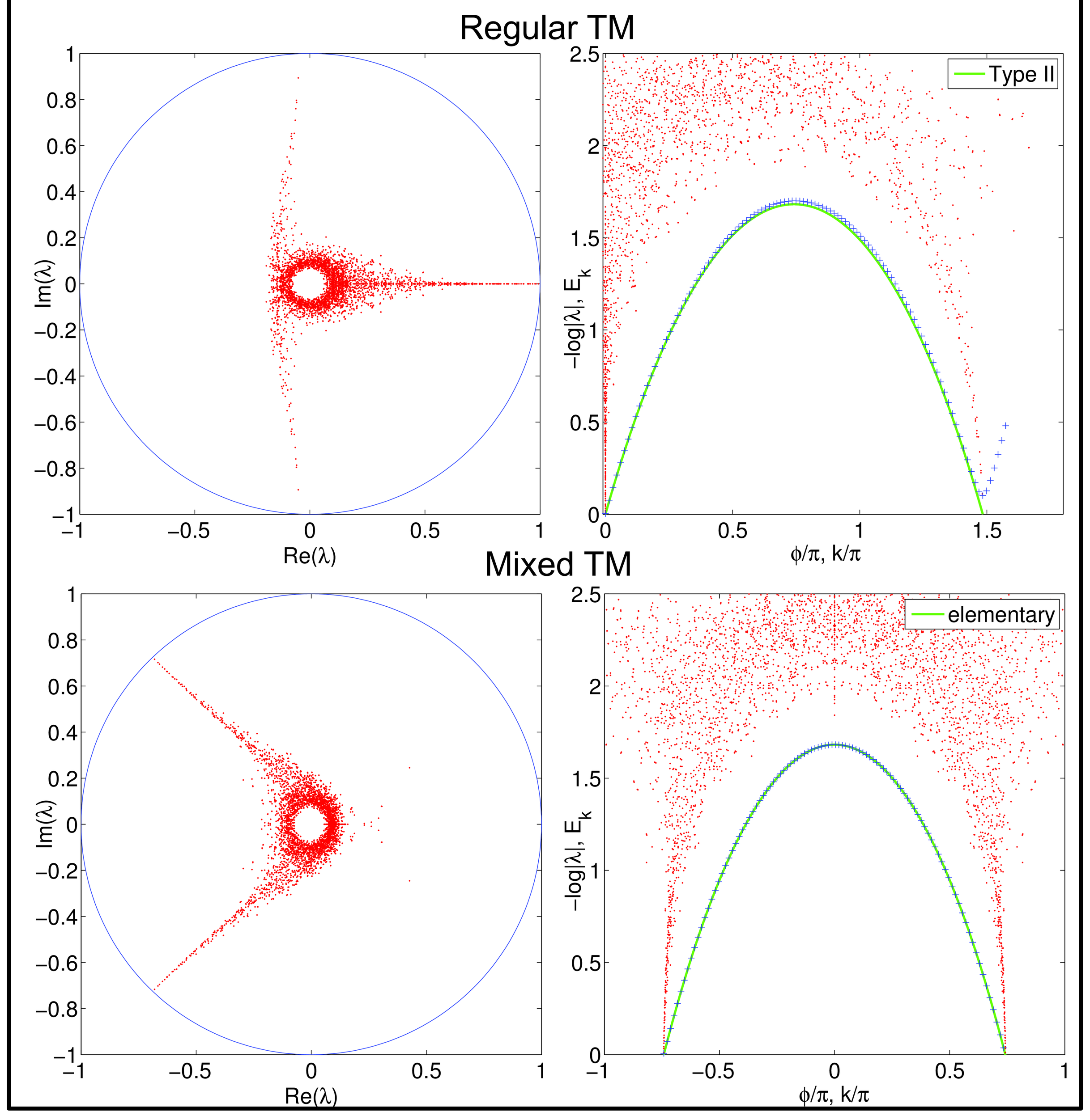
XXZ Model, Δ=-0.5, h=1, D=100

$$H_{\mathrm{XXZ}} = -\sum_j S^x_j S^x_{j+1} + S^y_j S^y_{j+1} + \Delta S^z_j S^z_{j+1} + hS^z_j$$



Lieb-Liniger Model, γ=c/ρ~1.35, D=64

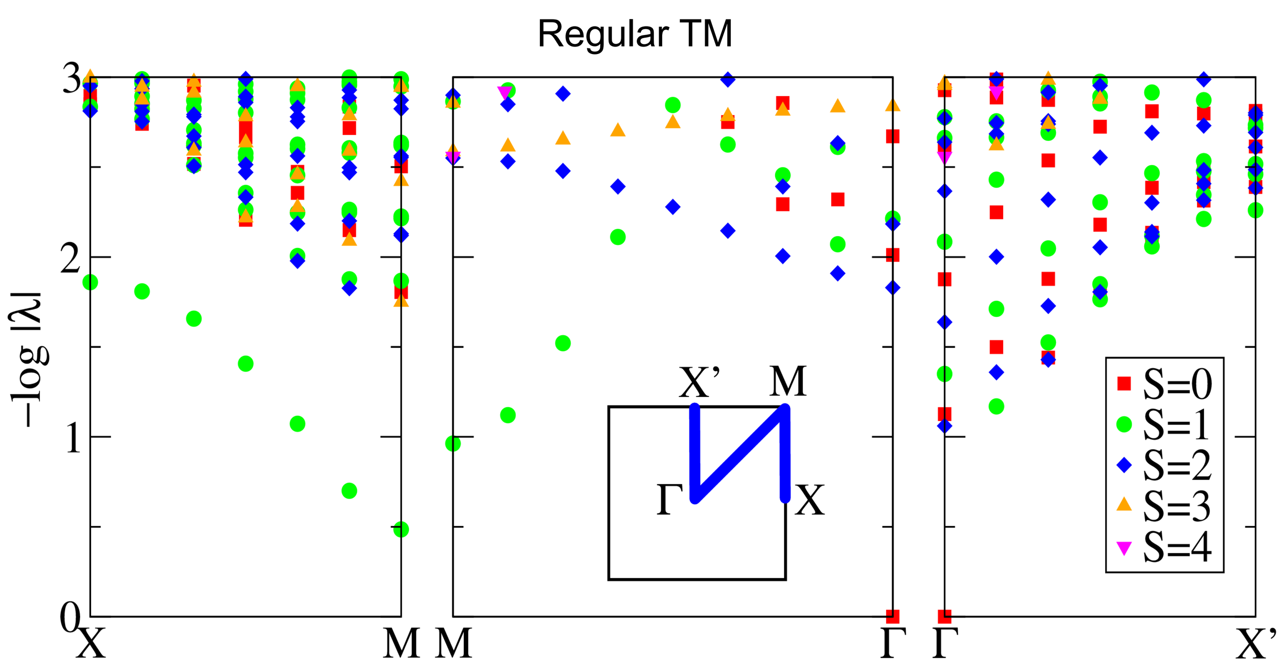
$$H_{\mathrm{LL}} = \int dx \frac{d\psi^\dagger}{dx}\frac{d\psi}{dx} + c\psi^\dagger\psi^\dagger\psi\psi - \mu\psi^\dagger\psi$$



Eigenvalues $\lambda_j = e^{-\varepsilon_j + i\phi_j}$ of regular/mixed (c)MPS-TM along with variational excitation energies from a topologically trivial/non-trivial MPS ansatz [1,2]. If applicable we also plot exact excitation energies.

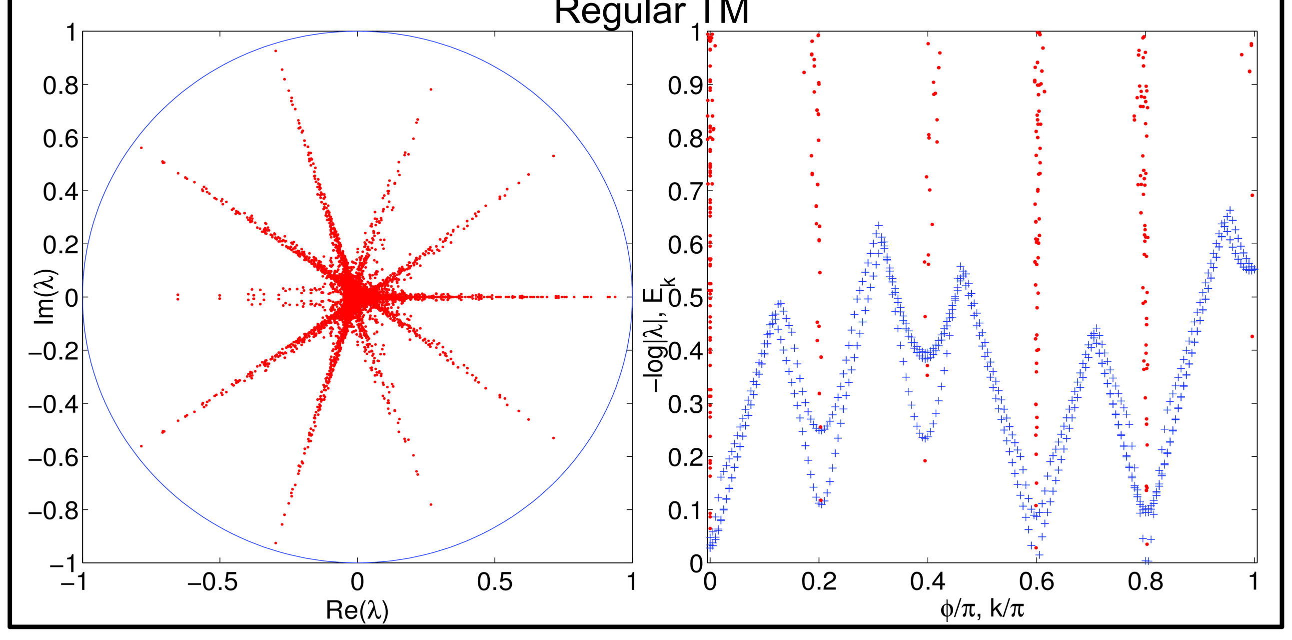
- The eigenspectrum has a very **peculiar shape** (see static correlations)
- Phases ϕ_j of **low lying** ε_j coincide very precisely with **momenta of minima** in dispersion E_k
- Values of **low lying** ε_j serve as first approximations to excitation energies and are related to these energies via characteristic velocities

AKLT on Square Lattice Cylinder (N=12)



Extended Hubbard Model, U=5, V=1, μ=2, D=100

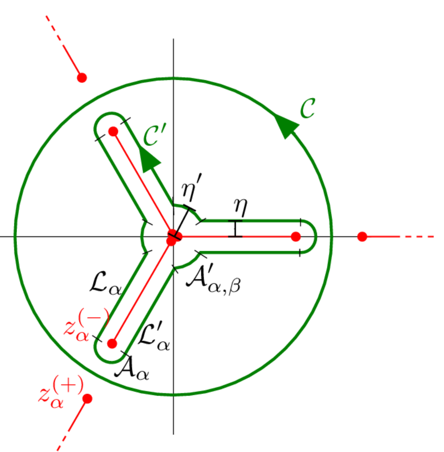
$$H_{\mathrm{HUB}} = -\sum_{\sigma j} c^\dagger_{\sigma j} c_{\sigma j+1} + h.c. + V\sum_j n_j n_{j+1} + U\sum_j n_{\uparrow j} n_{\downarrow j} - \mu\sum_j n_j$$



Static Correlations

- **Shape of Eigenvalue spectrum**

Eigenvalues cluster along **lines with constant complex phase → Ornstein-Zernike Form** is recovered under certain assumptions



$$C(n+1) = \oint_{\mathcal{C}} \frac{dz}{2\pi i} S(z) z^n, \quad z = e^{ik} \qquad \approx \frac{e^{-n/\xi}}{n^\gamma}$$

$$C(n+1) = \sum_j f_j \lambda_j^n = \sum_j f_j e^{(-\varepsilon_j + i\phi_j)n}$$

- **Single Mode Approximation (SMA)**

variational state [3]: $$|\phi_k(O^{(\ell)})\rangle = \frac{1}{\sqrt{V}}\sum_n e^{ikn} O_n^{(\ell)} |\psi_0\rangle$$

variational energy: $$E_k = \frac{1}{2}\frac{\langle\psi_0|[O_{-k},[H,O_k]]|\psi_0\rangle}{\langle\psi_0|O_{-k}O_k|\psi_0\rangle} = \frac{1}{2}\frac{F(k)}{S(k)}$$

structure factor (MPS): $$S(k) \geq 2\Re\,(0|\,\mathcal{J}_O \frac{e^{ik(2\ell+1)}}{\mathbb{1} - e^{ik}\mathcal{T}_A}\mathcal{J}_O\,|0)$$

E_k becomes **small** whenever $\mathcal{T}_A$ has eigenvalues with phase $\phi = \pm k$ and **magnitude close to one.**

- **Momentum Filtered Correlations**

Defined as Fourier transform of static correlation times a gaussian

$$C_k(\ell) = N_r \sum_n e^{-\frac{n^2}{2r}} e^{ikn}(\langle A_0 B_{\ell+n}\rangle - \langle A\rangle\langle B\rangle)$$

The correlation length ξ_k can be bounded (similar to [4]) as

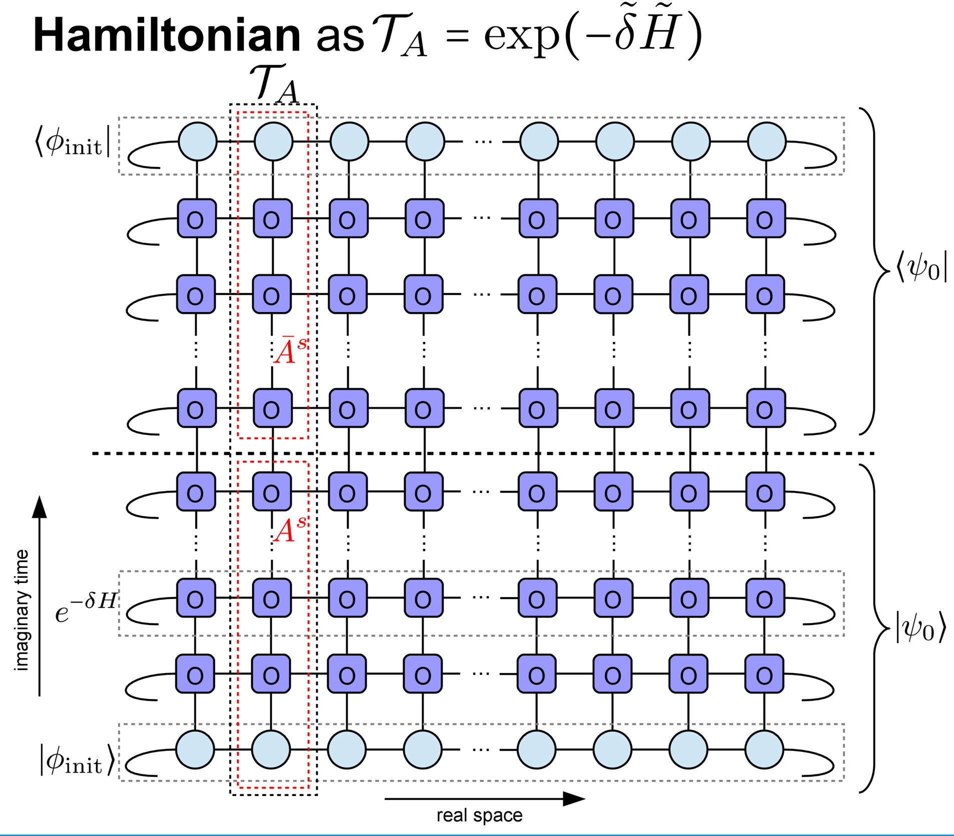
$$\xi_k < \frac{1}{\delta} + \frac{v_{\mathrm{LR}}}{E^*(k,\delta)}, \qquad E^*(k,\delta) = \min_{|k-k'|<\delta} E(k')$$

The Quantum Transfer Matrix

Imaginary Time Evolution

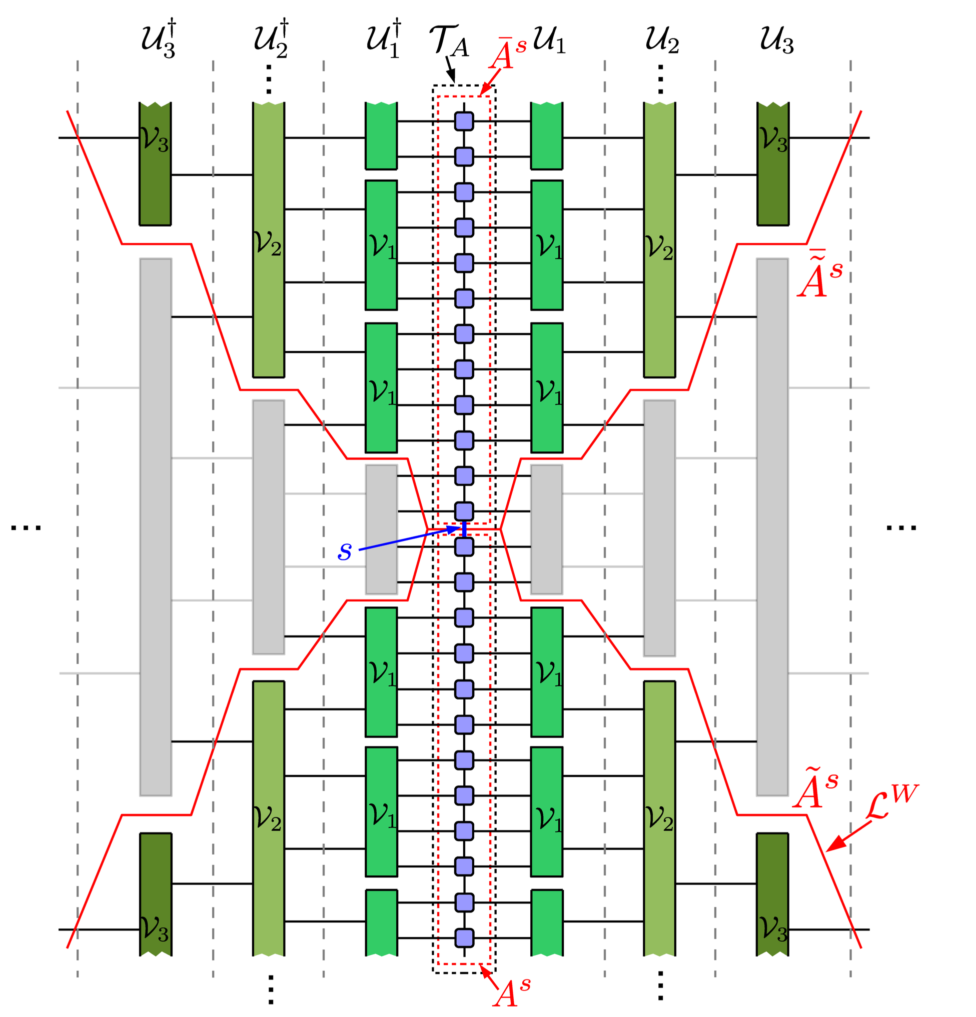
$$|\psi_0\rangle = \lim_{\beta\to\infty} \frac{e^{-\beta H}|\phi_{\mathrm{init}}\rangle}{\|e^{-\beta H}|\phi_{\mathrm{init}}\rangle\|}$$

- Translation invariant MPO decomposition of $\exp(-\delta H)$ → MPS matrix A^s is itself half-infinite MPO
- $\mathcal{T}_A$ corresponds to **quantum transfer matrix** (QTM) at zero temperature
- Can be written in terms of an **effective Hamiltonian** as $\mathcal{T}_A = \exp(-\delta\tilde{H})$



Truncation of Virtual System

- Small D MPS $\tilde{A}^s$ can be understood as obtained from an RG procedure on $\mathcal{T}_A$ on the virtual level → **low energy representation** $\tilde{\mathcal{T}}_A$
- RG network (e.g. MERA [5]) represents dominant eigenstate of $\mathcal{T}_A$



[1] J. Haegeman et al., Phys. Rev. B **85**, 100408 (2012)
[2] D. Draxler et al., Phys. Rev. Lett. **111**, 020402 (2013)
[3] J. Haegeman et al., Phys. Rev. Lett. **111**, 080401 (2013)
[4] M.B. Hastings, Phys. Rev. Lett. **93**, 140402 (2004)
[5] G. Evenbly, G. Vidal, arXiv:1307.0831, arXiv:1312.0303