CHERN INSULATORS

Part 2

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Lectures on
"Numerical and analytical methods for strongly correlated systems"
Benasque, Spain, September 2 & 5, 2014

Two lectures: rough plan

- The quantum Hall effect
  - Crash course on integer and fractional effects
  - Why look for alternative realizations?
- Integer Chern insulators ~ lattice quantum Hall systems at zero field
  - Example lattice models
  - General properties, comparison with continuum Landau levels
  - Experiments!
- Fractional Chern insulators
  - Brief comments on challenge and methods
  - Relation to FQH states (adiabatic continuity, entanglement spectra, edge states, etc.)
  - Which FQH analogues to expect, when and why?
  - Competing instabilities
- Higher Chern numbers
  - Various constructions and why only some host FCIs
  - Topology + frustration: novel FCIs in surface bands of Weyl semi-metals
  - Experiments?

Done!

Skipped, ask me if interested...

Topics of todays lecture

Fractional Chern insulators

Continued

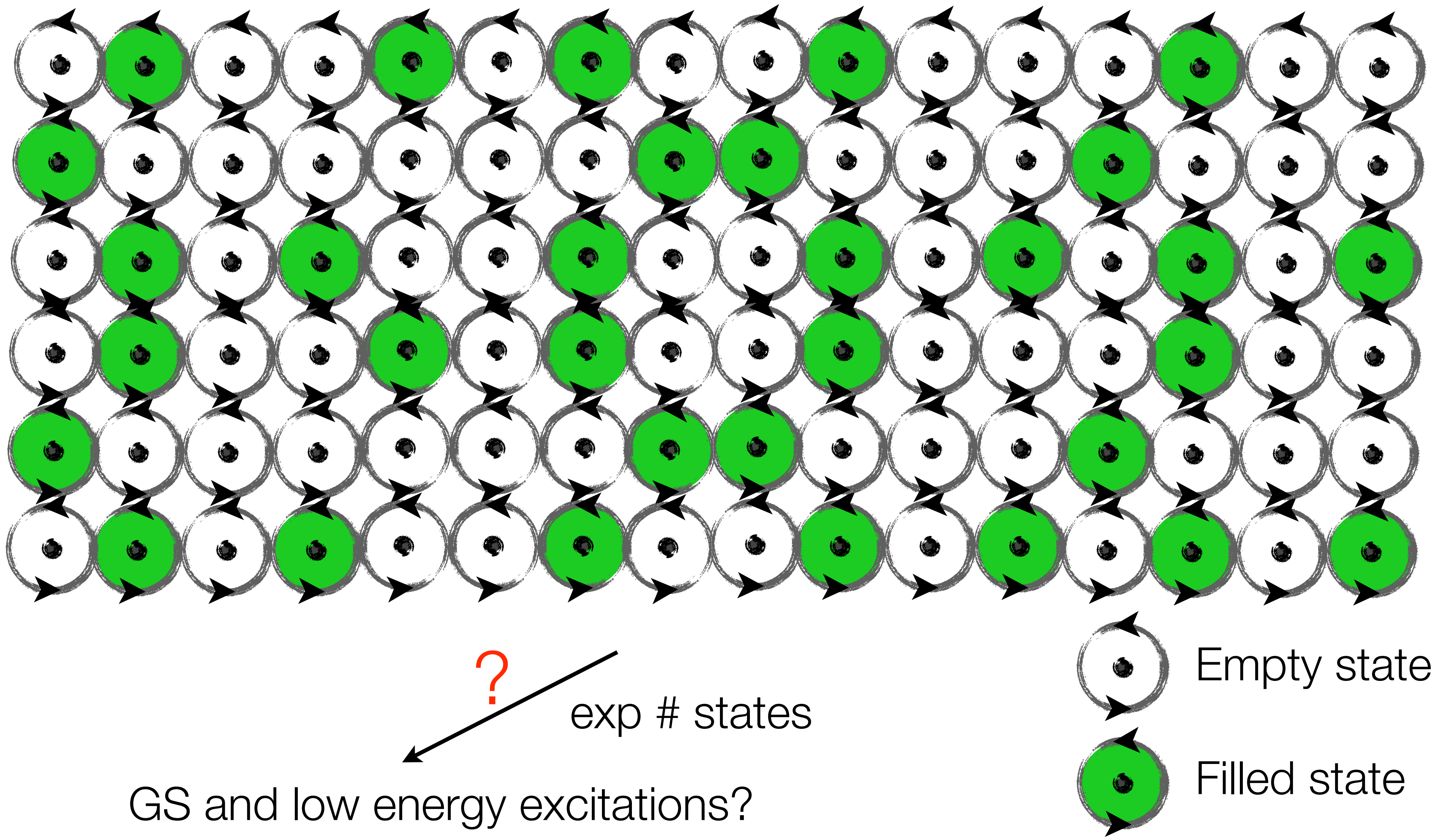
Useful references:

S. A. Parameswaran, R. Roy & S. L. Sondhi
Fractional Quantum Hall Physics in Topological Flat Bands
C. R. Physique 14, 816 (2013) [arXiv:1302.6606]

E. J. Bergholtz & Z. Liu
Topological Flat Band Models and Fractional Chern Insulators
Int. J. Mod. Phys. B 27, 1330017 (2013) [arXiv:1308.0343]

Recap: the problem

- Interactions projected to a flat band with non-zero Chern number



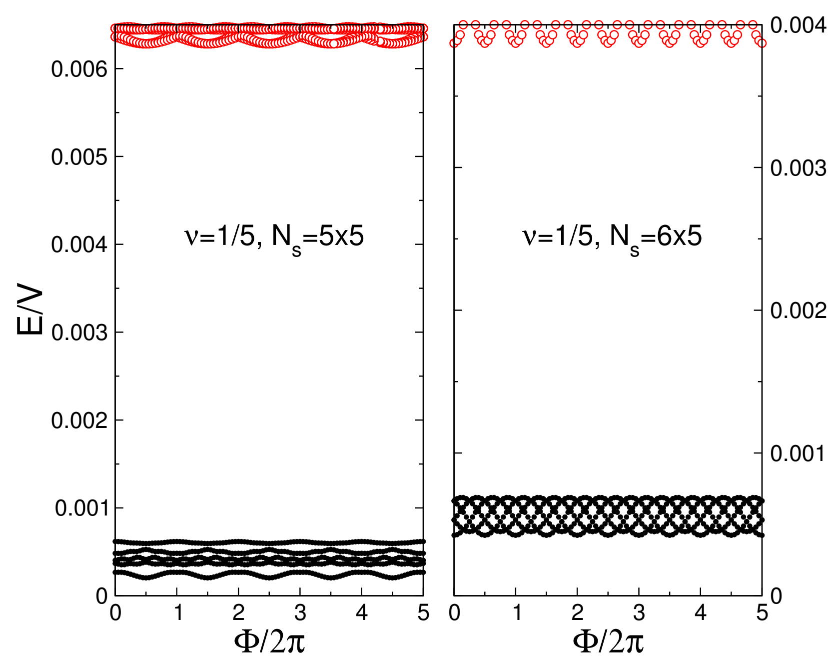
- Extremely hard, non-perturbative problem with no generically applicable cure.
  - complicated by the (unavoidable) algebraic tails of the Wannier functions

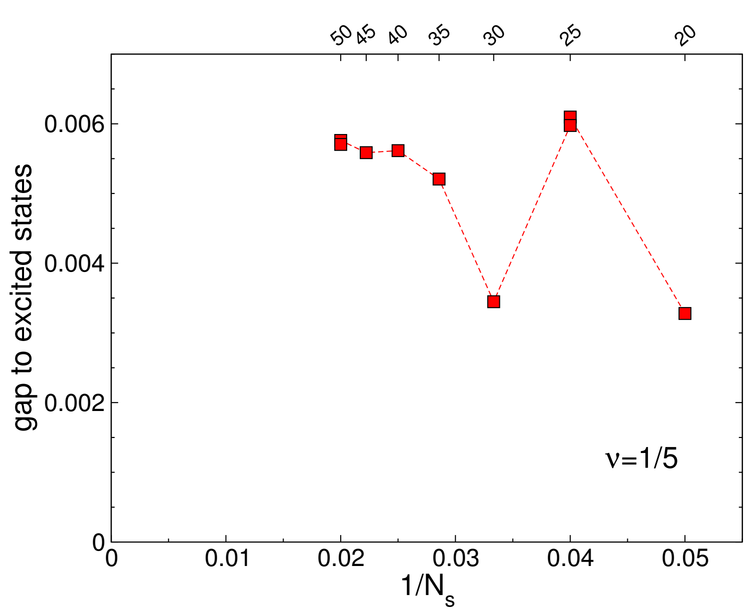
Early numerical evidence for FCIs in flat C=1 bands

- Laughlin-like states well established
- Non-abelian states (Moore-Read, Read-Rezayi) are found (for multi-particle interactions)

See arXiv:1308.0343 for original references...

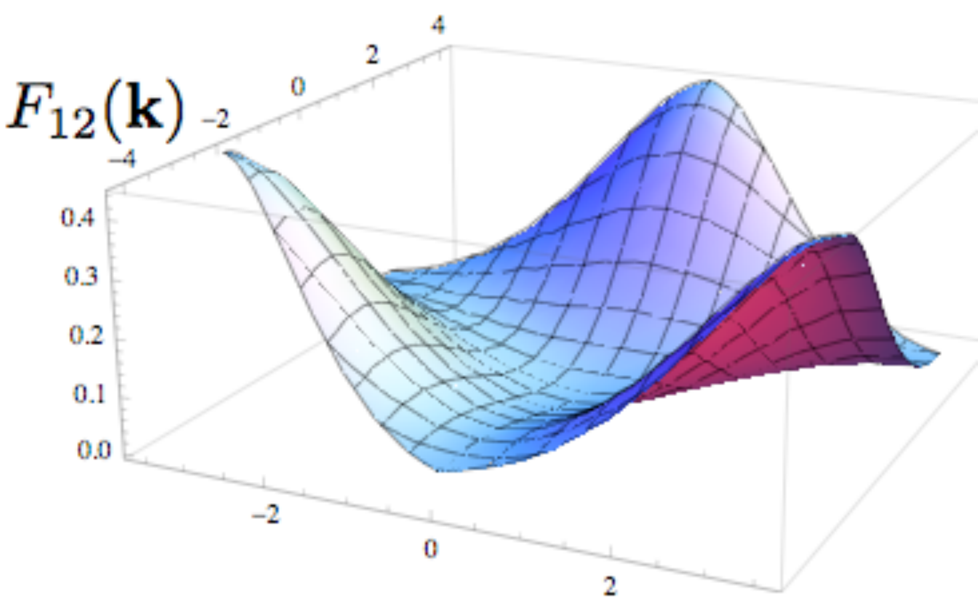
- Some hierarchy / composite fermion states
- Evidence from numerical diagonalization: gaps, topological degeneracies, spectral flow, particle entanglement spectra etc.
- Later: orbital entanglement, adiabatic continuation, edge states, modular matrices,...





Are flat C=1 bands identical to Landau levels?

- No, there is a “memory” of the underlying lattice, hence less “universal” physics.
- The Berry curvature *is not completely flat* in models with a finite number of bands.



- In the continuum there are infinitely many Landau bands and the Berry curvature is constant

$$A_j(\mathbf{k}) = -i\langle n_{\mathbf{k}}|\partial_{k_j}|n_{\mathbf{k}}\rangle, \quad F_{ij} = \partial_{k_i}A_j(\mathbf{k}) - \partial_{k_j}A_i(\mathbf{k})$$

$$C = \frac{1}{2\pi}\int_{BZ} F_{12}(\mathbf{k})d^2k = 1$$

Quantized

Sensitive to details, never uniform

- Less symmetry -- no translation invariance (in reciprocal space)
  - Reminiscent of a spatially varying magnetic field
  - May cause deviations from Landau level physics
  - Generally no nice (analytic) wave functions

See e.g. T.S. Jackson, G. Möller & R. Roy, arXiv:1408.0843

(Kapit-Mueller is a striking exception)

- Exception: certain small finite size systems at peculiar filling fractions

T. Scaffidi and S. Simon arXiv:1407.1321

$$\Psi = \cancel{\prod_{i<j}(z_i - z_j)^3 e^{-\sum_i |z_i|^2/4}}$$

Why just some states and what about their relative stability?

A.M. Läuchli, Z. Liu, E.J. Bergholtz
and R. Moessner, PRL, 111, 126802 (2013)

- Pseudopotential analogy
  - Extremely useful in Landau levels   F. D. M. Haldane, Phys. Rev. Lett. **51**, 605 (1983).

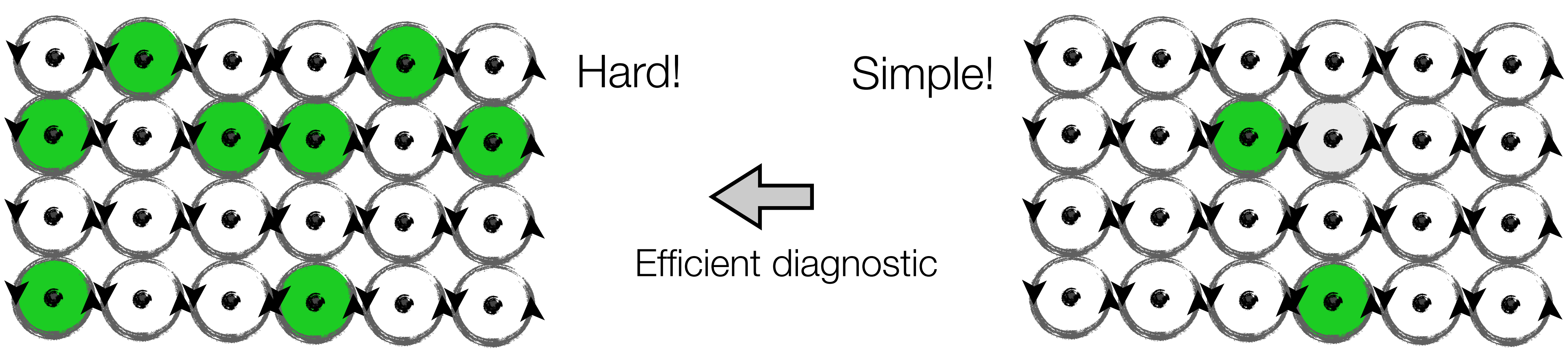
$$H = \sum_{i<j} \sum_{m=0}^{\infty} \mathcal{V}_m P_m(M_{ij}), \qquad H = \sum_{i<j} \sum_{m=0}^{\infty} \mathcal{V}_m L_m(-\nabla_i^2)\delta(\mathbf{r}_i - \mathbf{r}_j) \qquad \mathcal{V}_m = \int_0^{\infty} q\tilde{V}(q) L_m(q^2) e^{-q^2} dq,$$

$$f_{\mathrm{rel}}^{(q)}(z_1 - z_2) = \vartheta_1((z_1 - z_2)/L_1|\tau)^q \sim (z_1 - z_2)^q$$

  - Given the pseudopotential parameters we essentialy know the phase diagram
  - But there are typically no nice wave functions in the Chern bands. What to do?

Insight: the pseudopotential parameters, $\mathcal{V}_m$, can also be extracted from the spectrum of two interacting particles in a Landau level

  - Can be directly generalized to the Chern band case



Pseudopotential analogy

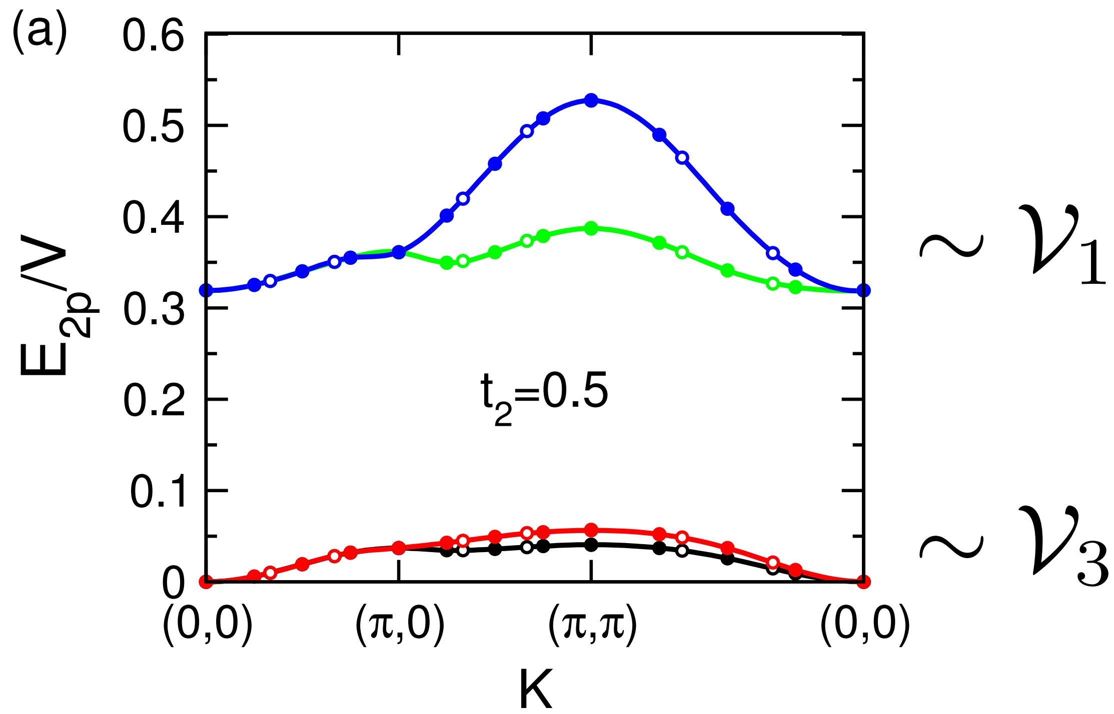
A.M. Läuchli, Z. Liu, E.J. Bergholtz and R. Moessner, PRL, 111, 126802 (2013)

- Example of two-particles interacting within the Chern band:

$$H_{\mathrm{int}} = \sum_{\langle i,j \rangle} n_i n_j$$ **projected** to the flat C=1 band.

- For a Checkerboard model this gives 4 finite energy levels per CM momenta (on kagome it gives 6 etc)

- FQH analogy suggests 2 energy levels per pseudopotential (due to the lower number of CM sectors in the Chern band)



- Numerics essentially agree with pseudopotential predictions

- We find FCI phases at filling

$\nu = 1/3, 2/5, 3/7, 4/9, 5/9, 4/7, 3/5$ Stabilized by $\mathcal{V}_1$

$\nu = 1/5, 2/7$ Stabilized by $\mathcal{V}_3$

- Often efficient as a *diagnostic* for a given flat band model

Example: “engineering” of complex FCIs

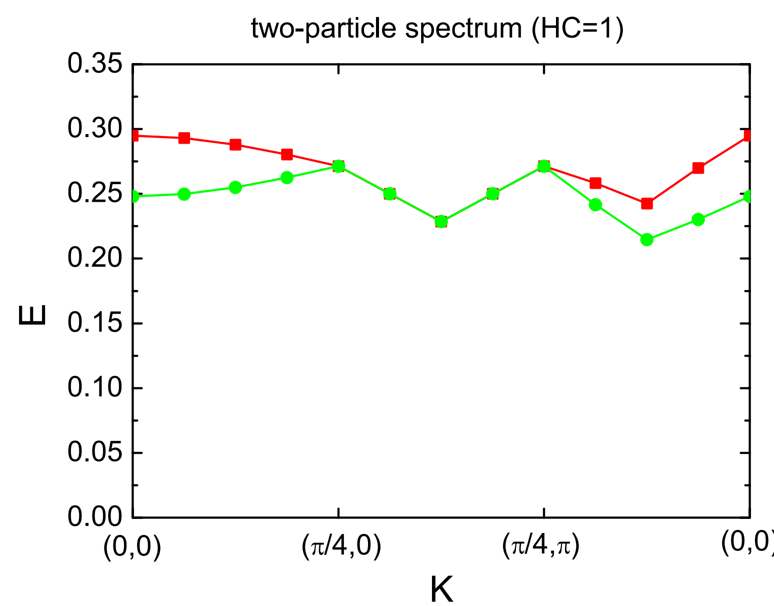
Z. Liu, E.J. Bergholtz
and Eliot Kapit,
PRB 88, 205101 (2013)

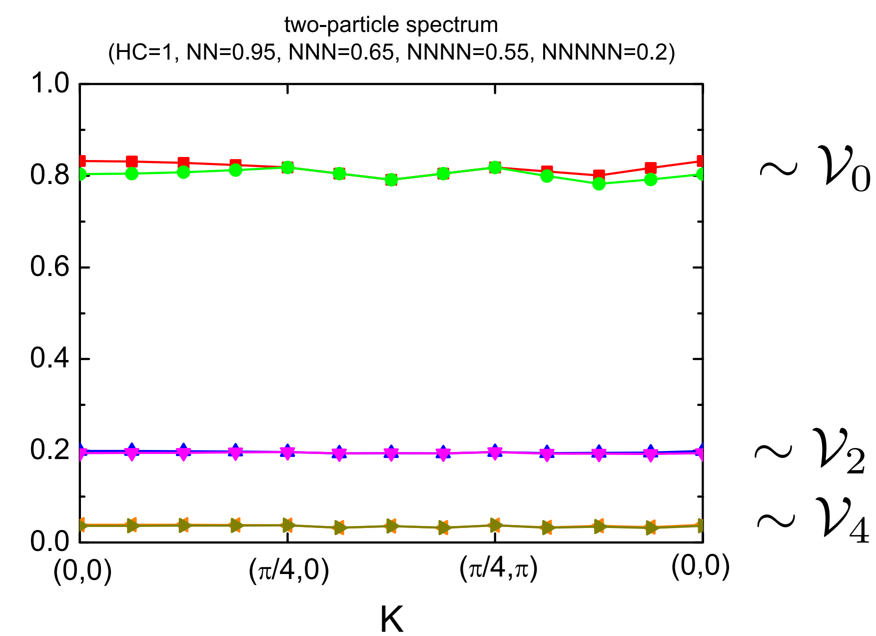
- Choose a good workhorse: the Kapit-Mueller model

E. Kapit and E. Mueller, Phys. Rev. Lett. **105**, 215303 (2010).

- Consider the two-particle problem (here bosons)

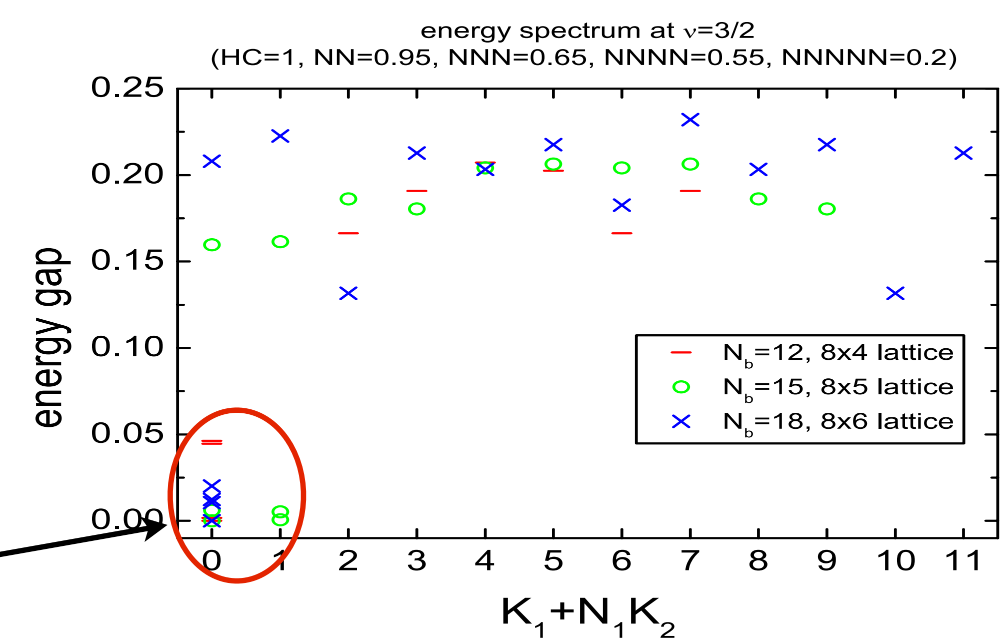
  - Look for an interaction that mimics the desired pseudo-potential spectra





- Back to the many-body problem

  - We find exotic non-abelian FCI states (Moore-Read and Read Rezayi) for reasonable two-body interactions!



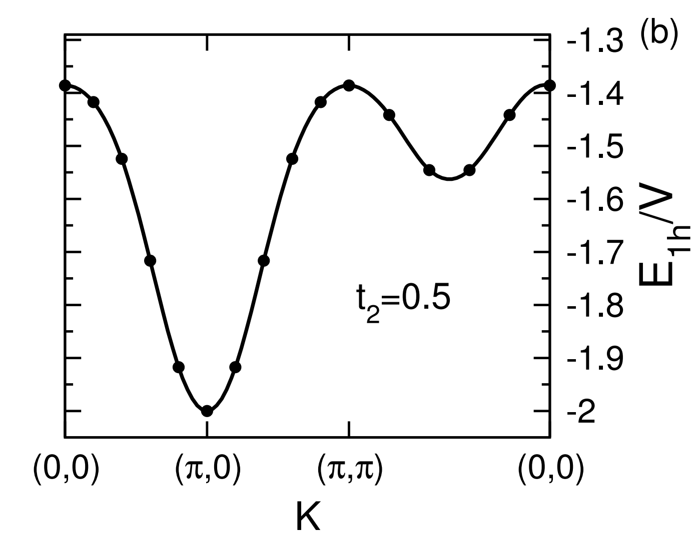
4-fold topological degeneracy of the k=3 RR state (Fibonacci anyons)

Are there new competing instabilities in the lattice case?

A.M. Läuchli, Z. Liu, E.J. Bergholtz
and R. Moessner, PRL, 111, 126802 (2013)

- Lack of translation invariance in reciprocal space spoils particle-hole symmetry

- Particle-hole transformation generates a dispersive single-hole term



$$c_{\mathbf{k}} \to c^\dagger_{\mathbf{k}} \quad : \quad H = \sum_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3\mathbf{k}_4} V_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3\mathbf{k}_4} c^\dagger_{\mathbf{k}_1} c^\dagger_{\mathbf{k}_2} c_{\mathbf{k}_3} c_{\mathbf{k}_4},$$

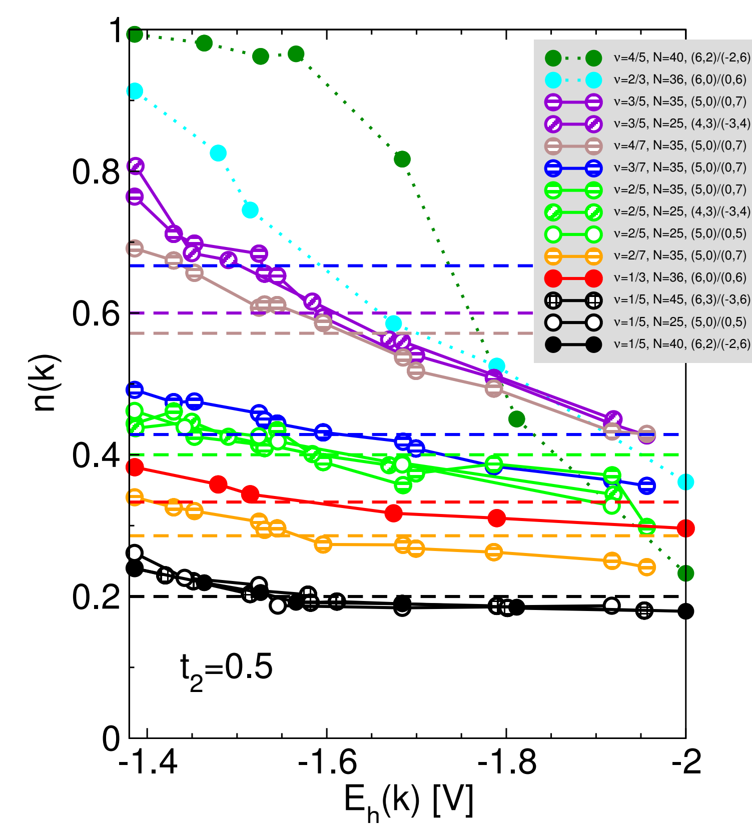
$$\to \sum_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3\mathbf{k}_4} V^*_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3\mathbf{k}_4} c^\dagger_{\mathbf{k}_1} c^\dagger_{\mathbf{k}_2} c_{\mathbf{k}_3} c_{\mathbf{k}_4} + \sum_{\mathbf{k}} E_h(\mathbf{k}) c^\dagger_{\mathbf{k}} c_{\mathbf{k}}$$

- Non-negligible dispersion ($E_h(\mathbf{k})$ is a constant in a Landau level)

- Generically gives a non-constant n(k)

- Wins over the conventional two-body terms at high filling fractions!

Interaction-only induced gapless states



Summary on fractional Chern insulators in C=1 bands

Huge potential:

- Interaction scale set by lattice spacing $\Rightarrow \Delta E \sim 500K$!?
  (very optimistic estimate due to Tang, Mei & Wen, PRL '11)

- No need for strong magnetic fields

<u>FQH/FCI states survive despite strong lattice effects</u>

- Many interesting differences compared to the continuum, but fairly well understood problem by now

More details in the reviews:

S. A. Parameswaran, R. Roy & S. L. Sondhi
Fractional Quantum Hall Physics in Topological Flat Bands
C. R. Physique 14, 816 (2013) [arXiv:1302.6606]

E. J. Bergholtz & Z. Liu
Topological Flat Band Models and Fractional Chern Insulators
Int. J. Mod. Phys. B 27, 1330017 (2013) [arXiv:1308.0343]

Higher Chern numbers

Useful references:

M. Udagawa, & E. J. Bergholtz
Correlations and entanglement in flat band models with variable Chern numbers
arXiv:1407.0329

M. Barkeshli, X.-L. Qi
Topological Nematic States and Non-Abelian Lattice Dislocations
Phys. Rev. X 2, 031013 (2012)

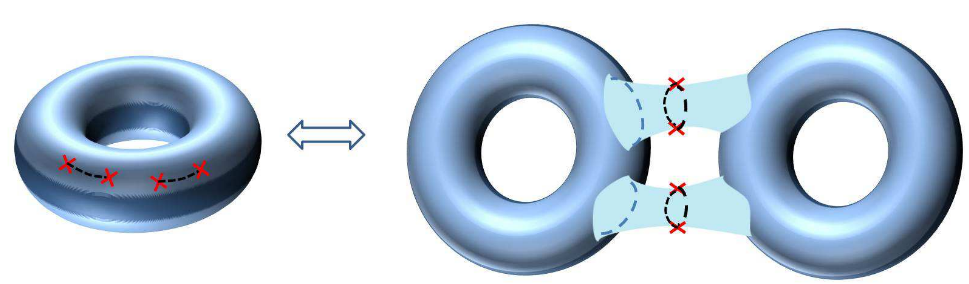
E.J. Bergholtz, Z. Liu, M.Trescher, R. Moessner & M. Udagawa
Topology and interactions in a frustrated slab: tuning from Weyl semi-metal to C>1 fractional Chern insulators
arXiv:1408.3669

Flat C>1 bands

- Can be constructed in many ways
  - Longer-range hopping
  - Larger unit-cells
- Example, Dirac model: $\mathcal{H}(\mathbf{k}) = \mathbf{d}(\mathbf{k}) \cdot \boldsymbol{\sigma}$

$$\begin{cases} d_x(\mathbf{k}) = \sin(N_x k_x) \\ d_y(\mathbf{k}) = \sin(N_y k_y) \\ d_z(\mathbf{k}) = m + \cos(N_x k_x) + \cos(N_y k_y) \end{cases} \Rightarrow C = \begin{cases} N_x N_y & \text{for } 0 < m < 2 \\ -N_x N_y & \text{for } -2 < m < 0 \\ 0 & \text{otherwise} \end{cases}$$

  - Multi-layer and long-range interpretation a matter of taste (think of a square lattice)
  - Somewhat trivial, but...
- Boundary conditions are important for topologically ordered states
  - related to the ground state degeneracy (Wen '90, Haldane '85)
- Non-trivial boundary conditions could hence lead to qualitatively new phenomena.
- Dislocations as non-Abelian "wormholes" aka "genons"!?
  - would correspond to a change topology and thereby the enhance the degeneracy



M. Barkeshli and X.-L. Qi, Phys. Rev. X **2**, 031013 (2012).

A lot of related activity....

- FCIs and/or flat bands with C>1 (far from a complete list)

F. Wang and Y. Ran, *Nearly flat band with Chern number $C = 2$ on the dice lattice*, Phys. Rev. B **84**, 241103(R) (2011).

M. Trescher and E. J. Bergholtz, *Flat bands with higher Chern number in pyrochlore slabs*, Phys. Rev. B **86**, 241111(R) (2012).

Z. Liu, E.J. Bergholtz, H. Fan, and A.M. Läuchli, *Fractional Chern Insulators in Topological Flat bands with Higher Chern Number*, Phys. Rev. Lett. **109**, 186805 (2012).

A. Sterdyniak, C. Repellin, B. Andrei Bernevig, and N. Regnault, *Series of Abelian and non-Abelian states in $C > 1$ fractional Chern insulators*, Phys. Rev. B **87**, 205137 (2013).

S. Yang, Z.-C. Gu, K. Sun, and S. Das Sarma, *Topological flat band models with arbitrary Chern numbers*, Phys. Rev. B **86**, 241112(R) (2012).

L. Hormozi, G. Moller, and S. H. Simon, *Fractional Quantum Hall Effect of Lattice Bosons Near Commensurate Flux*, Phys. Rev. Lett. **108**, 256809 (2012).

A. G. Grushin, T. Neupert, C. Chamon, and C. Mudry, *Enhancing the stability of fractional Chern insulators against competing phases*, Phys. Rev. B **86**, 205125 (2012).

Y.-L Wu, N. Regnault, and B. A. Bernevig, *Bloch Model Wave Functions and Pseudopotentials for All Fractional Chern Insulators*, Phys. Rev. Lett. **110**, 106802 (2013).

D. Wang, Z. Liu, J. Cao, and H. Fan, *Tunable Band Topology Reflected by Fractional Quantum Hall States in Two-Dimensional Lattices*, Phys. Rev. Lett. **111**, 186804 (2013).

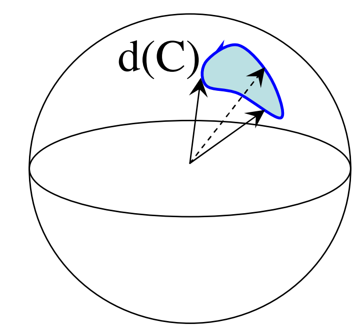
Y.-H. Wu, J. K. Jain, and K. Sun, *Fractional Topological Phases in Generalized Hofstadter Bands with Arbitrary Chern Numbers*, arXiv:1309.1698.

Related early work: G. Moller and N.R. Cooper, *Composite Fermion Theory for Bosonic Atoms in Optical Lattices*, Phys. Rev. Lett. **103**, 105303 (2009).

A more “homogenous” example

- Change the wrapping of the sphere directly

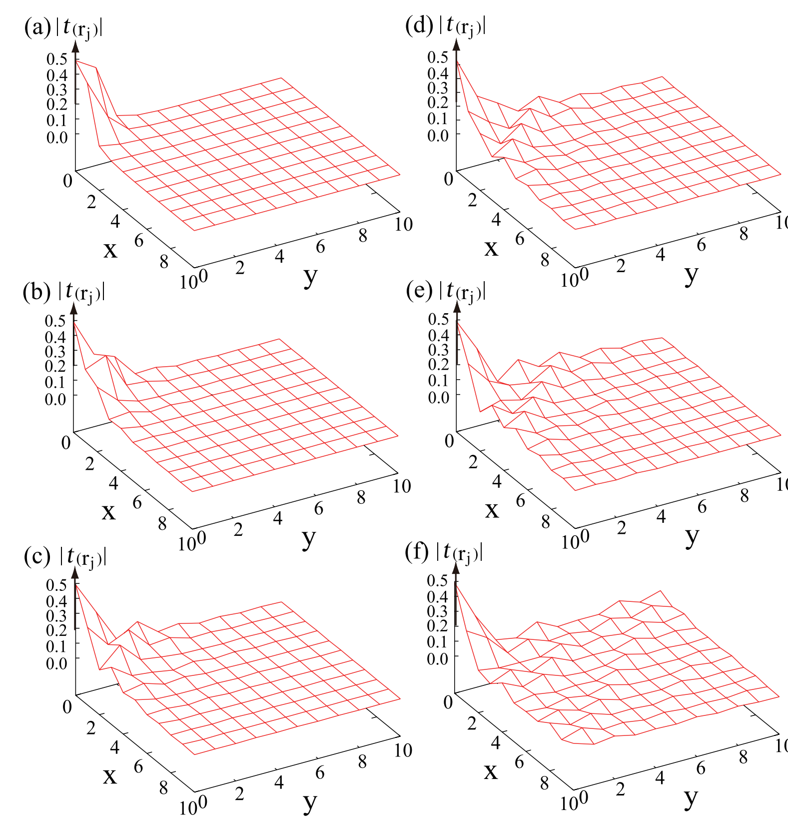
$$C = \frac{1}{4\pi}\int dk_x \int dk_y\ \hat{\mathbf{d}} \cdot \left(\frac{\partial \hat{\mathbf{d}}}{\partial k_x} \times \frac{\partial \hat{\mathbf{d}}}{\partial k_y}\right)$$



S$_2$ of $\hat{\mathrm{d}}(\mathrm{k})$

$$\hat{\mathbf{d}}(\mathbf{k}) \to \hat{\mathbf{d}}^{(N)}(\mathbf{k}) = \Big(\sin\theta_k \cos(N\phi_k), \sin\theta_k \sin(N\phi_k), \cos\theta_k\Big)$$

- Also gives C=N, but with more “homogenous” hopping



(with a flat band model as starting point)

- Chern number lost if hopping truncated

- In this case hopping at distance d>C needed

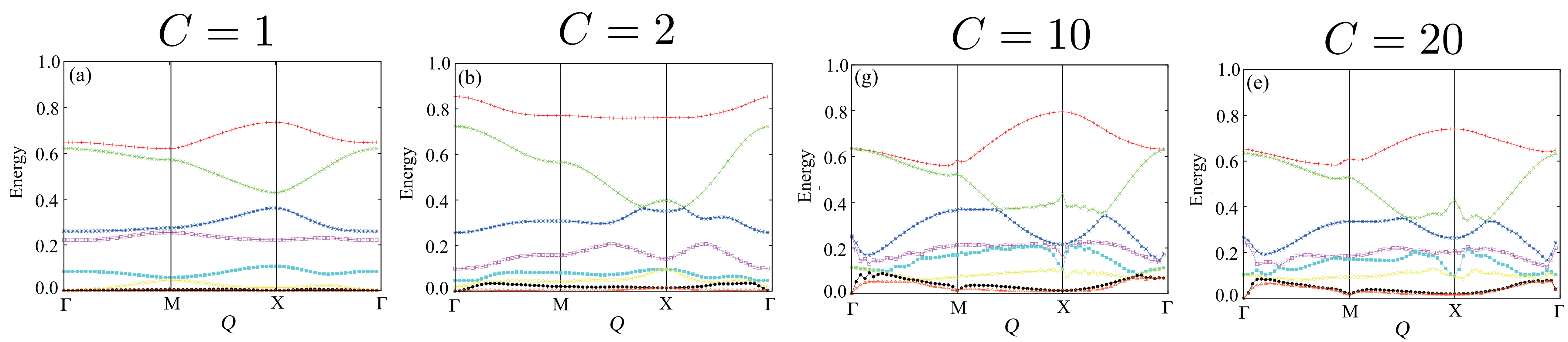
- Generally, for fixed number of bands hopping with range $d \sim \sqrt{C}$ needed

Two-particle problem

M. Udagawa, & E. J. Bergholtz
arXiv:1407.0329

- Two-band Dirac model defined above for various Chern numbers

(otherwise fixed parameters)



Quite smooth spectrum, somewhat separated spectra

Jagged spectrum, no clear separation of scales

- Gives a rationale for why C>1 FCIs were hard to find initially

  - We tried something else...

Flat C>1 bands from geometrical frustration: novel FCIs, Weyl semimetals and Fermi arcs

- We asked: Is it possible to make N C=1 bands hybridize so that one band absorbs all the topology (C=N) while the others become trivial (C=0)?

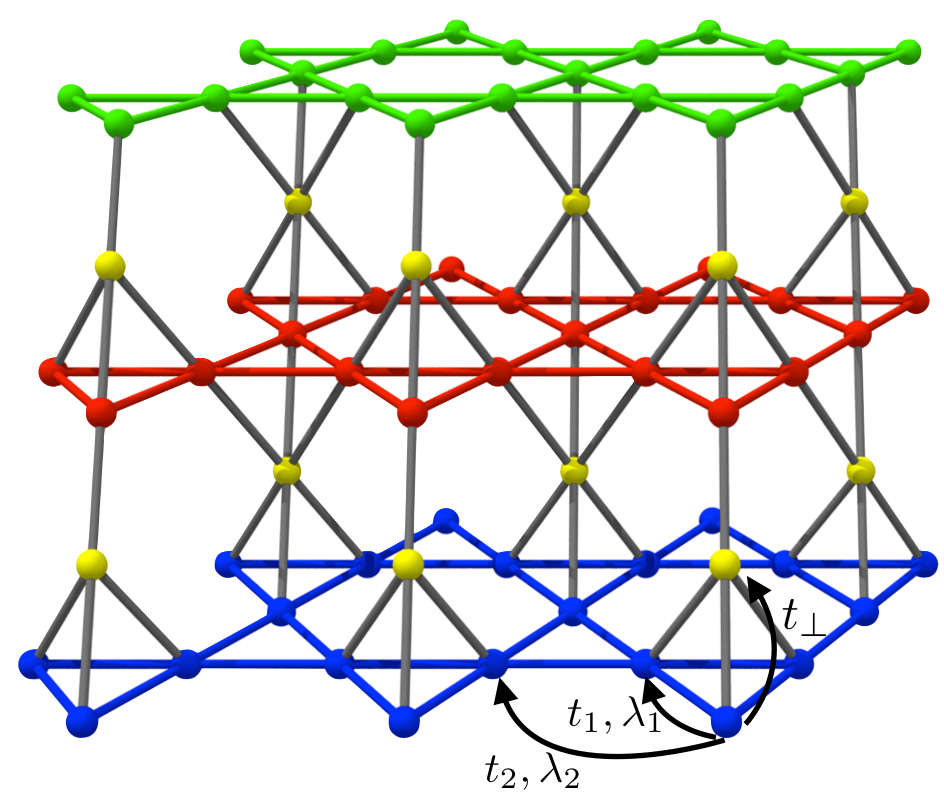
$$1 + 1 \rightarrow 2 + 0?$$

- ...and answer: yes, using
  - Multilayer systems
  - Geometrical frustration

M. Trescher and E.J. Bergholtz,
Phys. Rev. B 86, 241111(R) (2012)

- We implement our idea on pyrochlore slabs inspired by pyrochlore based spin-orbit coupled materials, e.g. iridates.

$$H = \sum_{i,j,\sigma} t_{ij} c^\dagger_{i\sigma} c_{j\sigma} + i \sum_{i,j,\alpha,\beta} \lambda_{ij} (\mathbf{E}_{ij} \times \mathbf{R}_{ij}) \cdot \boldsymbol{\sigma}_{\alpha\beta} c^\dagger_{i\alpha} c_{j\beta}$$

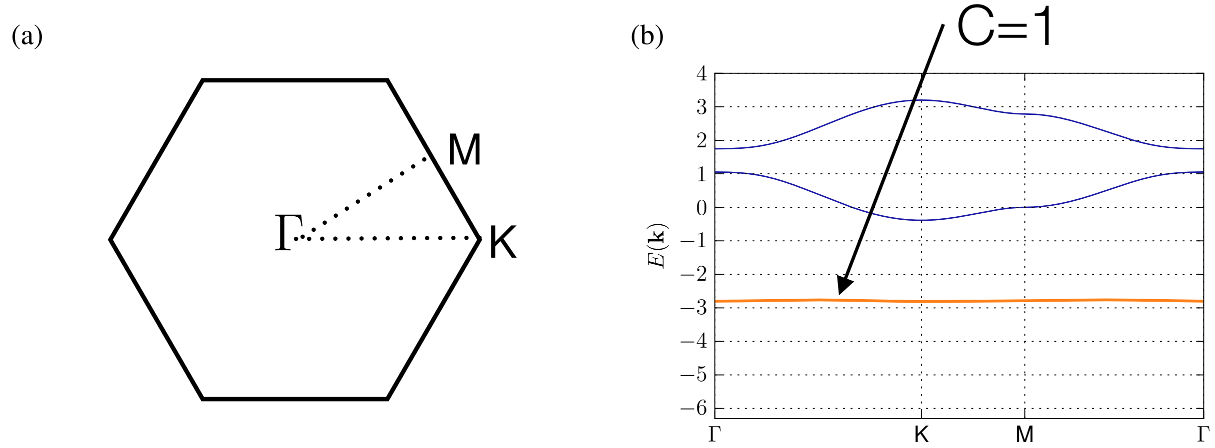


- Note: no interlayer tunneling give N degenerate C=1 bands -- this is _not_ what we wanted...

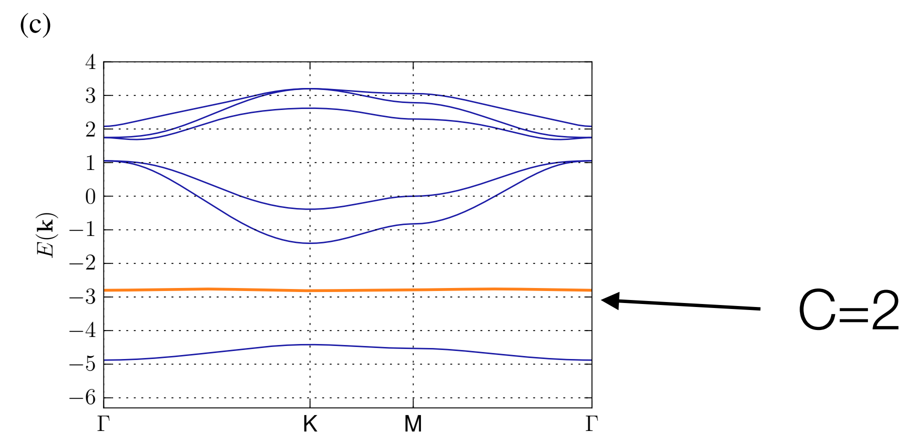
Results: 2d bulk dispersion

M. Trescher and E.J. Bergholtz,
Phys. Rev. B 86, 241111(R) (2012)

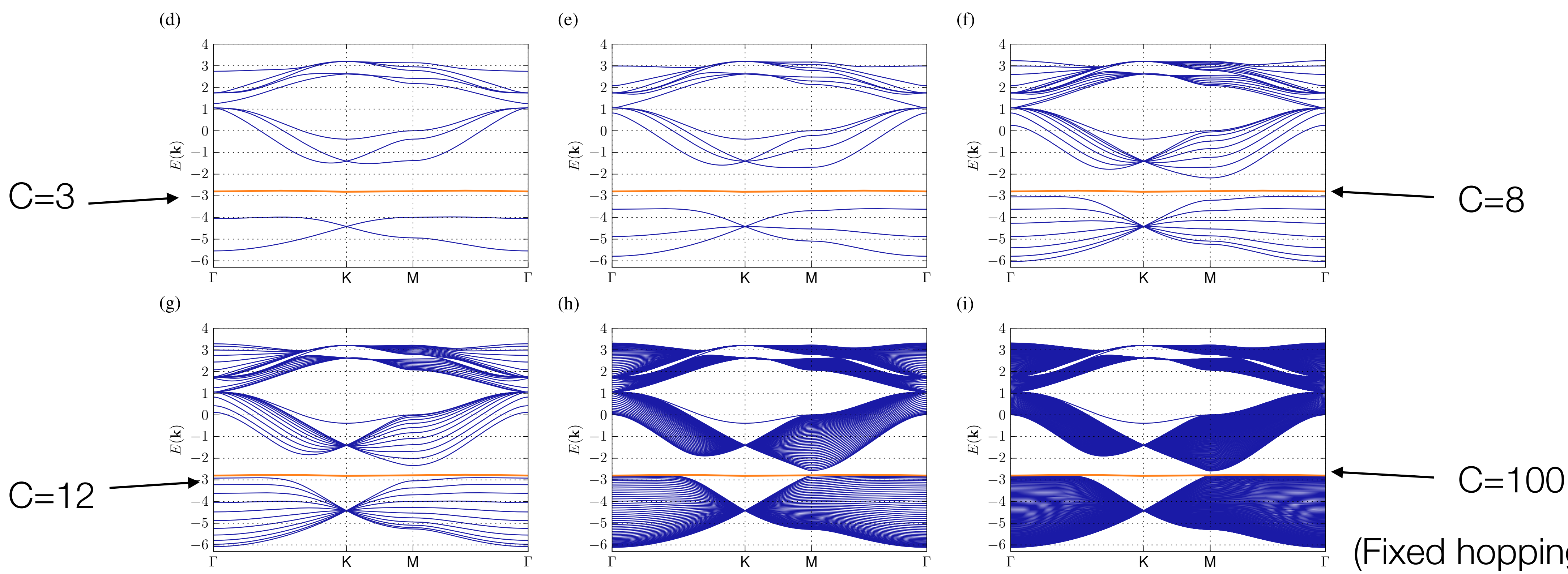
- Dispersion for one layer



- Dispersion with two layers



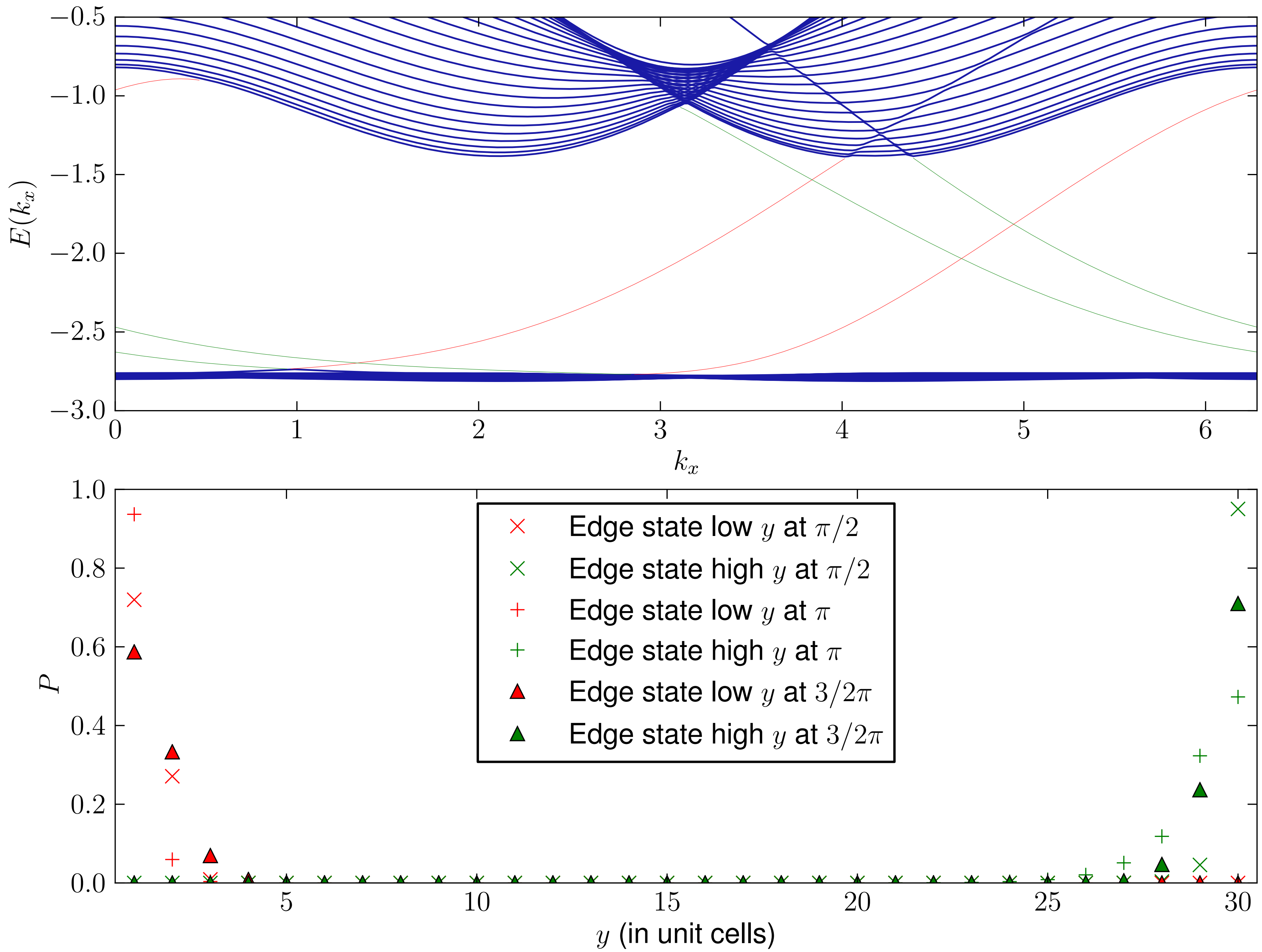
- For N kagome layers we find a flat band with C=N!



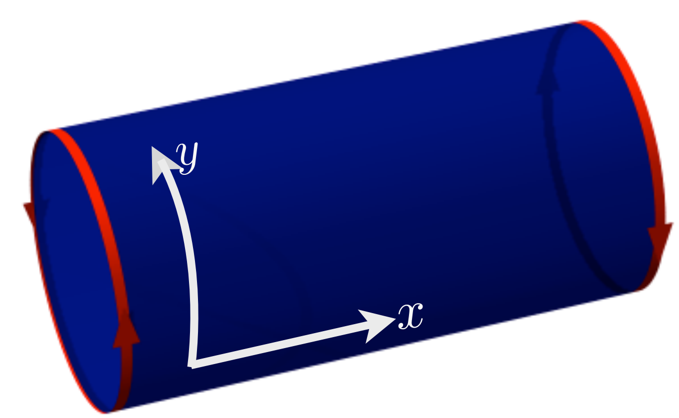
(Fixed hopping parameters here, just changing N)

1d edge states: revealed in cylinder geometry

- Example: the C=2 band has 2 gapless chiral edge states at each end



(Microscopically different edges to avoid deceiving degeneracies)

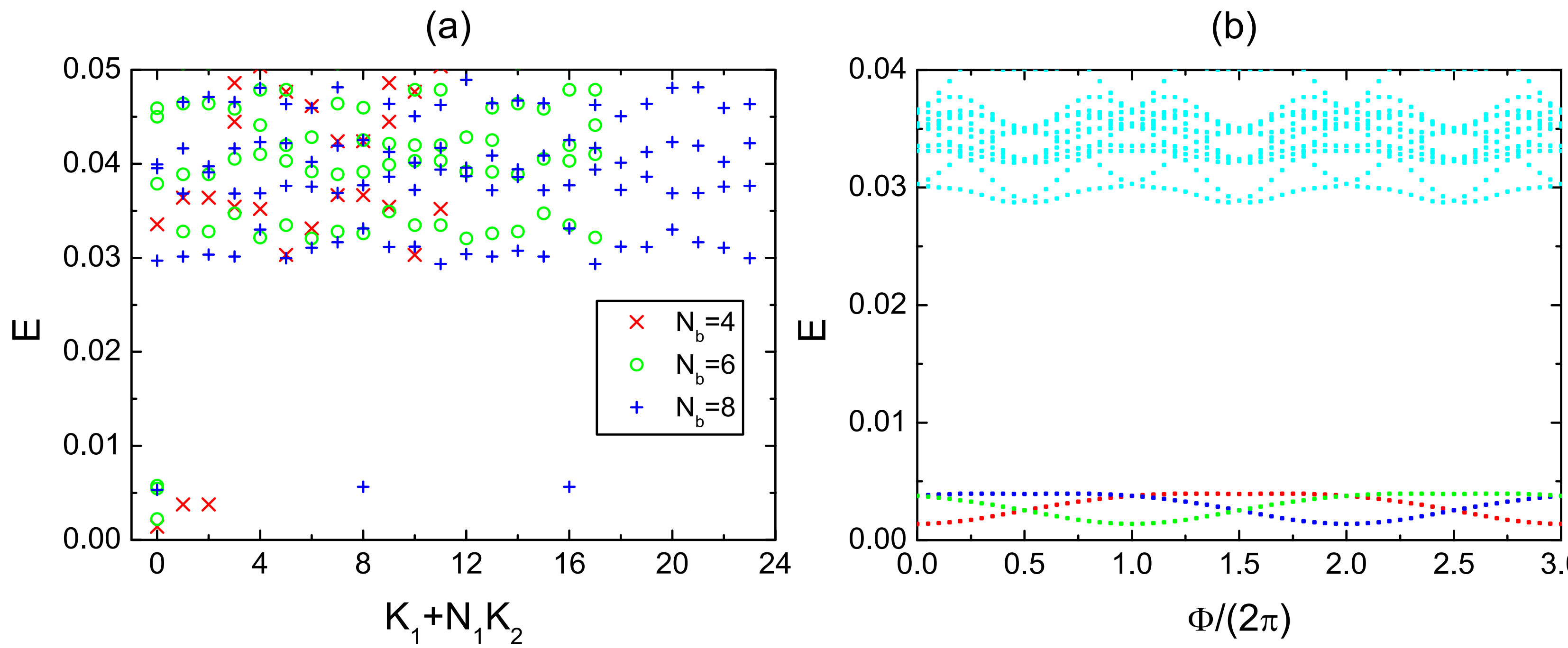


Can qualitatively new FCI phases form within C>1 bands?

Z. Liu, E.J. Bergholtz, H. Fan, A. M. Läuchli
Phys. Rev. Lett. 109, 186805 (2012)

- Yes, we find convincing evidence for a series of bosonic FCI states at $\nu_b = 1/(C+1)$

$C = 2, \nu_b = 1/3:$



- Fermionic states at $\nu_f = 1/(2C+1)$ (likely absent at higher filling fractions!)

- Strong evidence also for C>1 generalizations of non-Abelian FQH states found in this model!

E.J. Bergholtz, Z. Liu, M. Trescher, R. Moessner, and M. Udagawa, arXiv:1408.3669

A. Sterdyniak, C. Repellin, B.A. Bernevig, and N. Regnault, Phys. Rev. B 87, 205137 (2013)

- Different from standard multi-layer systems...

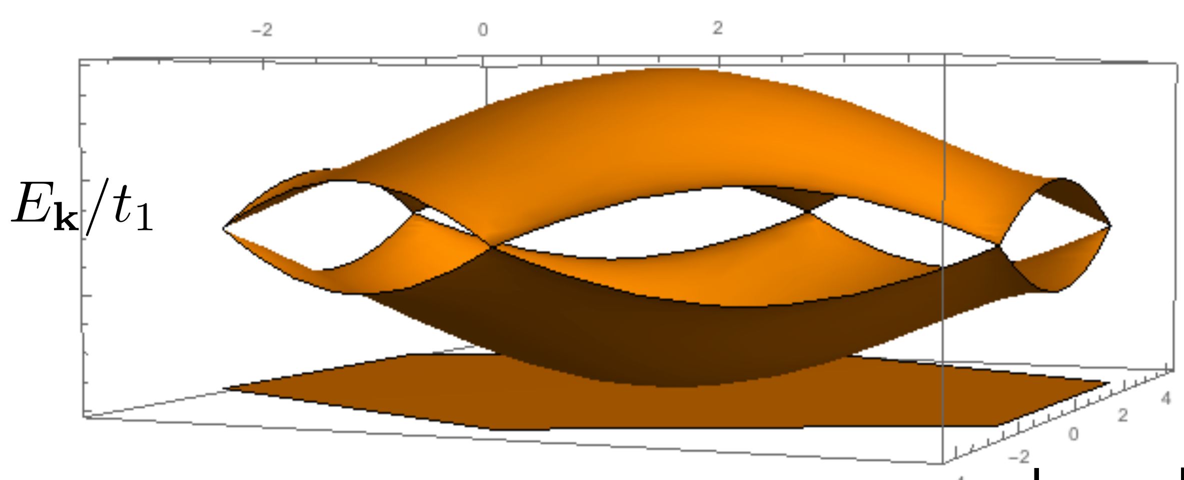
Can we analytically understand the C=N states?

- Quick recap: Flat bands and localized modes on frustrated lattices

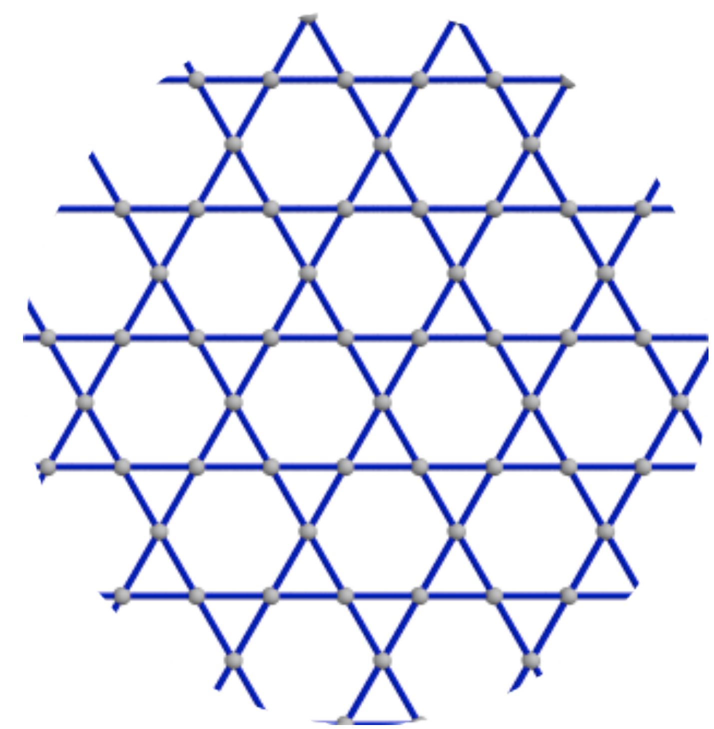
  - Example: nearest neighbor hopping on a kagome lattice

$$H = t_1 \sum_{\langle i,j \rangle} c_i^\dagger c_j$$

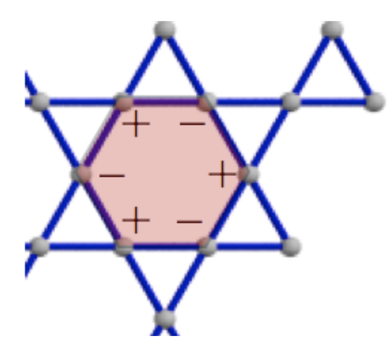
Bloch Hamiltonian: $$\mathcal{H}_{\mathbf{k}} = t_1 \begin{pmatrix} 0 & 1+e^{ik_1} & 1+e^{ik_2} \\ 1+e^{-ik_1} & 0 & 1+e^{-ik_3} \\ 1+e^{-ik_2} & 1+e^{ik_3} & 0 \end{pmatrix}$$



"Graphene + a flat band"



Localized modes explain the flat band



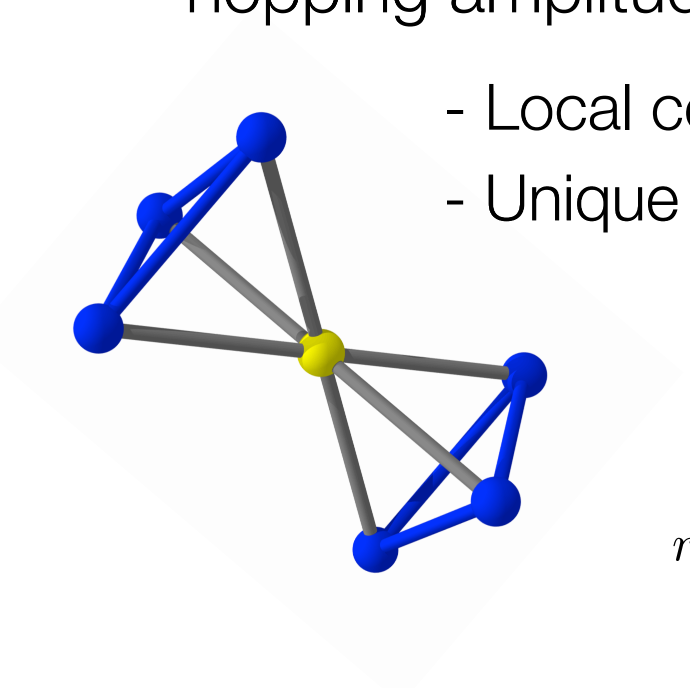
- But these states are neither topological nor and not Wannier functions!
  - Need a slightly refined concept...

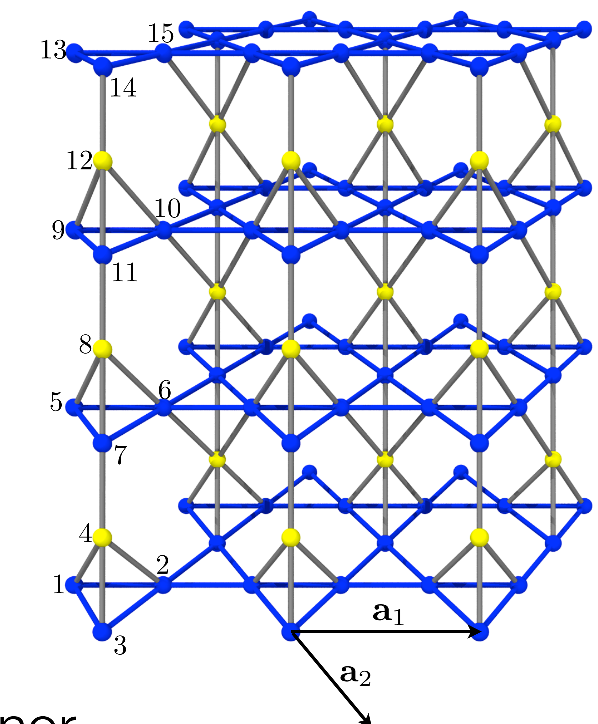
The C>1 bands are surface bands!

M. Trescher and E.J. Bergholtz, Phys. Rev. B 86, 241111(R) (2012)

E.J. Bergholtz, Z. Liu, M. Trescher, R. Moessner, and M. Udagawa, arXiv:1408.3669

- Despite the non-locality of the Wannier functions, geometrical frustration provides an avenue to novel surface states
- Crucial insight: surface bands localized to the kagome layers only if the total hopping amplitude to the triangular layer vanish.





- Local constraint, destructive interference
- Unique solution, independent of details!

$$|\psi^i(\mathbf{k})\rangle = \mathcal{N}(\mathbf{k}) \sum_{m=1}^{N} \Big(r(\mathbf{k})\Big)^m |\phi^i(\mathbf{k})\rangle_m$$

$$r(\mathbf{k}) = -\frac{\phi_1^i(\mathbf{k}) + \phi_2^i(\mathbf{k}) + \phi_3^i(\mathbf{k})}{e^{-ik_2}\phi_1^i(\mathbf{k}) + e^{i(k_1-k_2)}\phi_2^i(\mathbf{k}) + \phi_3^i(\mathbf{k})}$$

components of the single layer Bloch spinor

- Inherits the dispersion of the single layer model
- Localized to top or bottom layer, depending on $|r(\mathbf{k})|$

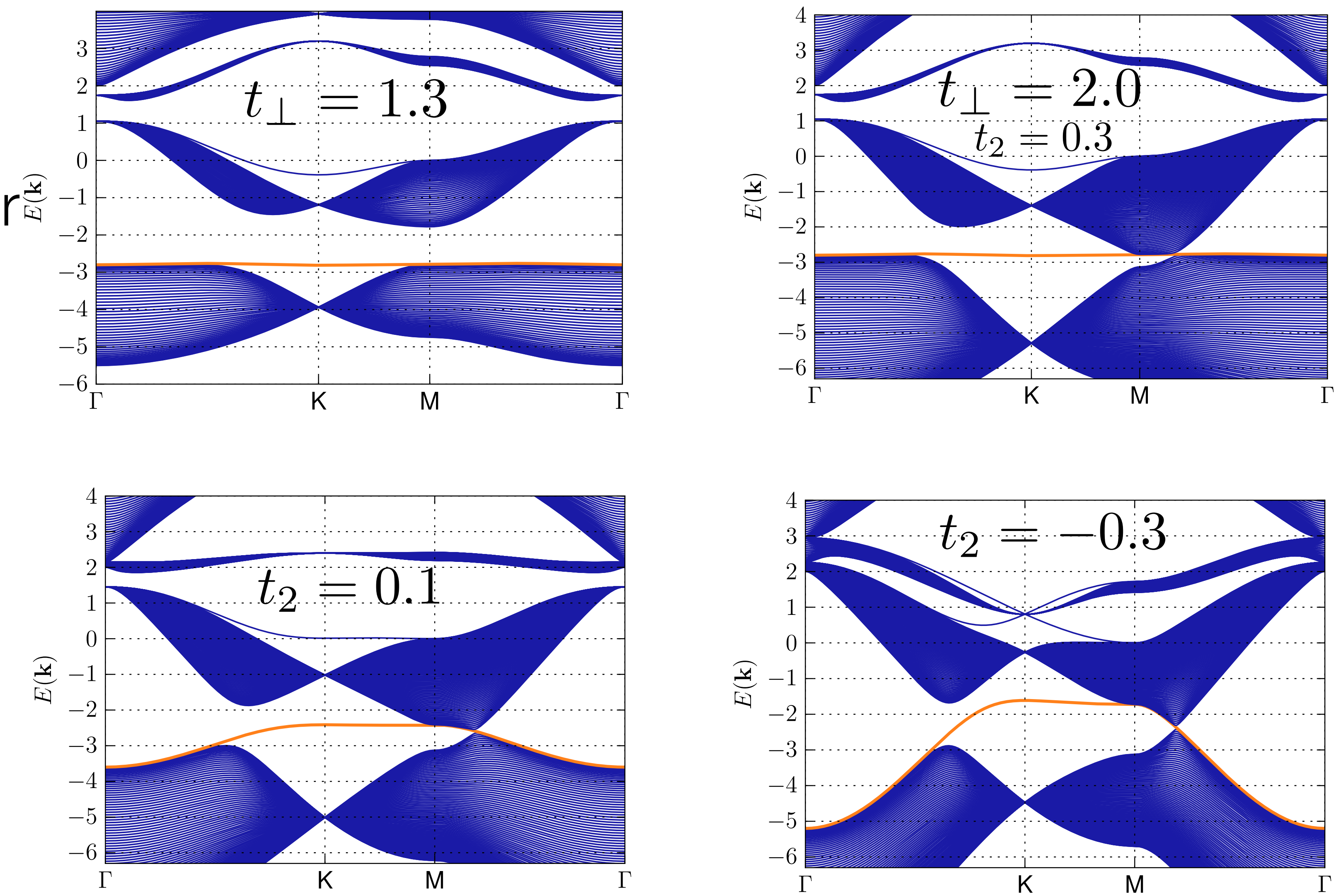
- When each kagome layer is a Chern insulator, these are precisely the states with Chern number C=N !

  - Simple way of generating (flat) bands with any Chern number

What's the nature of the surface states?

E.J. Bergholtz, Z. Liu, M. Trescher, R. Moessner, and M. Udagawa, arXiv:1408.3669

- Another look at the bulk spectrum...
- Increase the interlayer tunneling -- this leaves the flat band unchanged



- Change the nearest neighbor hopping

- Band touching described by the Weyl Hamiltonian

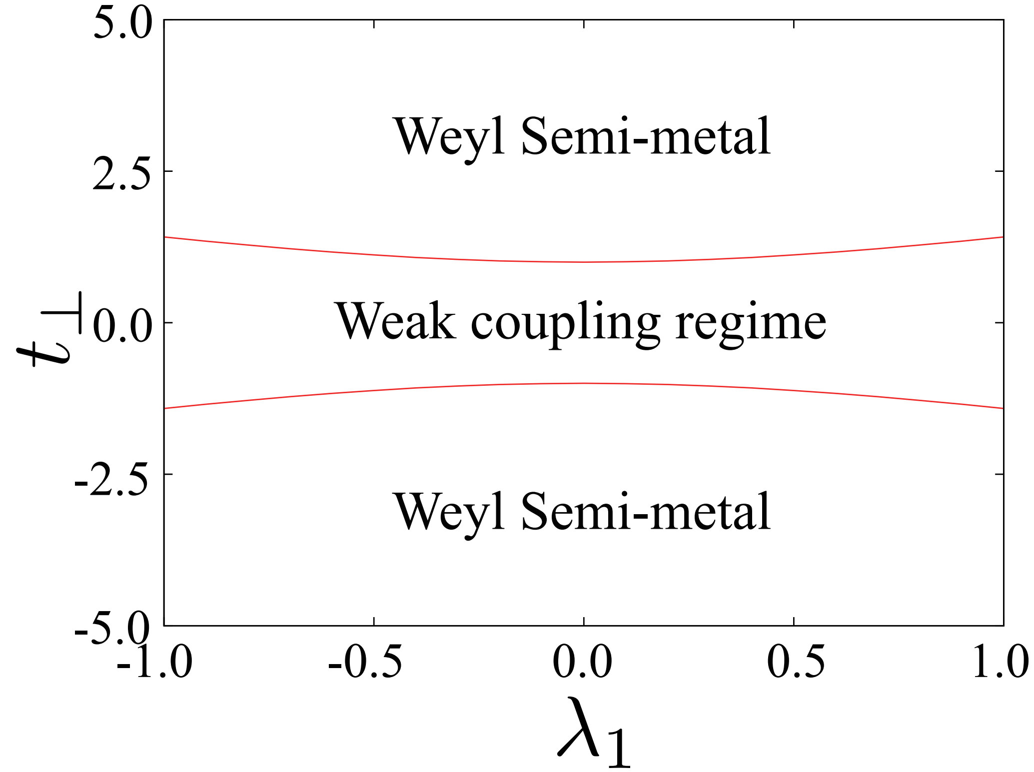
$$H_{\mathrm{Weyl}} = \sum_i v_i \sigma_i k_i + E_0(\mathbf{k})\mathbb{I}$$

  - Nb. this holds in each case, also when the touching cone is nearly flat!

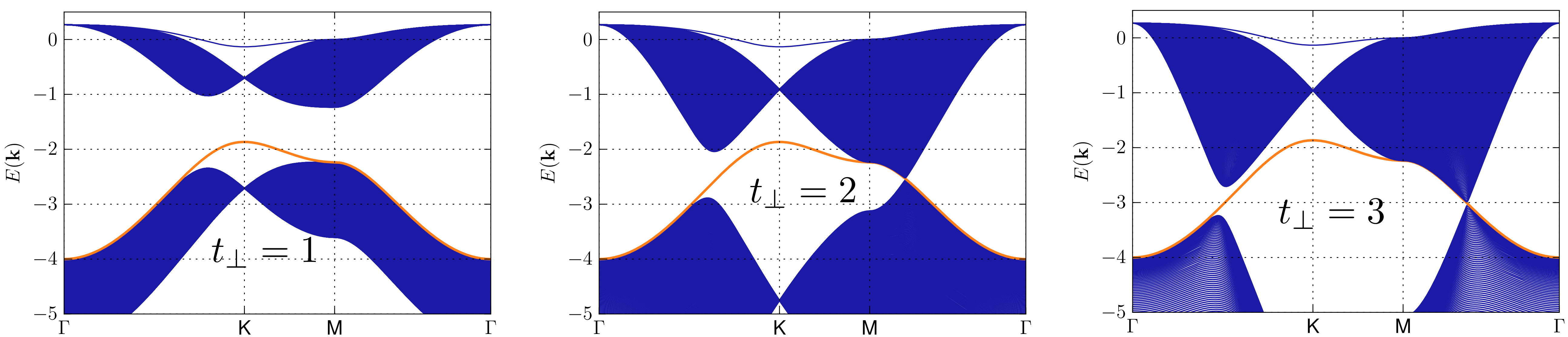
Works generically

E.J. Bergholtz, Z. Liu, M. Trescher, R. Moessner, and M. Udagawa, arXiv:1408.3669

- Phase diagram (more complicated for very strong spin-orbit coupling)



- Generic example ($t_1 = -1, \lambda_1 = 0.5$)



- Nb. the surface states are entirely independent on $t_\perp$

Weyl semi-metals

- Topological gapless phase in three dimensions

  - stable touching points, protected by a Chern number

  - half a gapless Dirac theory

$$H_{\mathrm{Weyl}} = \sum_i v_i \sigma_i k_i + E_0(\mathbf{k})\mathbb{I}$$

  - many fascinating transport phenomena, e.g., the chiral anomaly

Reviews:

A.M. Turner and A. Vishwanath, Beyond Band Insulators: Topology of Semi-metals and Interacting Phases, arXiv:1301.0330

P. Hosur and X. Qi, Recent developments in transport phenomena in Weyl semimetals, arXiv:1309:4464

- Subject to intense experimental (and theoretical) activity

  - no ideal realization yet, but similar "Dirac metals" with symmetry protected Weyl features found recently

  - many new ideas and and engineered structures materials are being tested...

- Predicted in pyrochlore iridates

- Alternative layer prescription for WSMs exist

  - details and ingredients are however very different

X. Wan, A. M. Turner, A. Vishwanath, and S. Y. Savrasov, Phys. Rev. B 83, 205101 (2011).

A. A. Burkov and L. Balents,, Phys. Rev. Lett. 107, 127205 (2011).

- The topology is manifested through exotic surface states, "Fermi arcs"

Fermi arcs in the pyrochlore slab

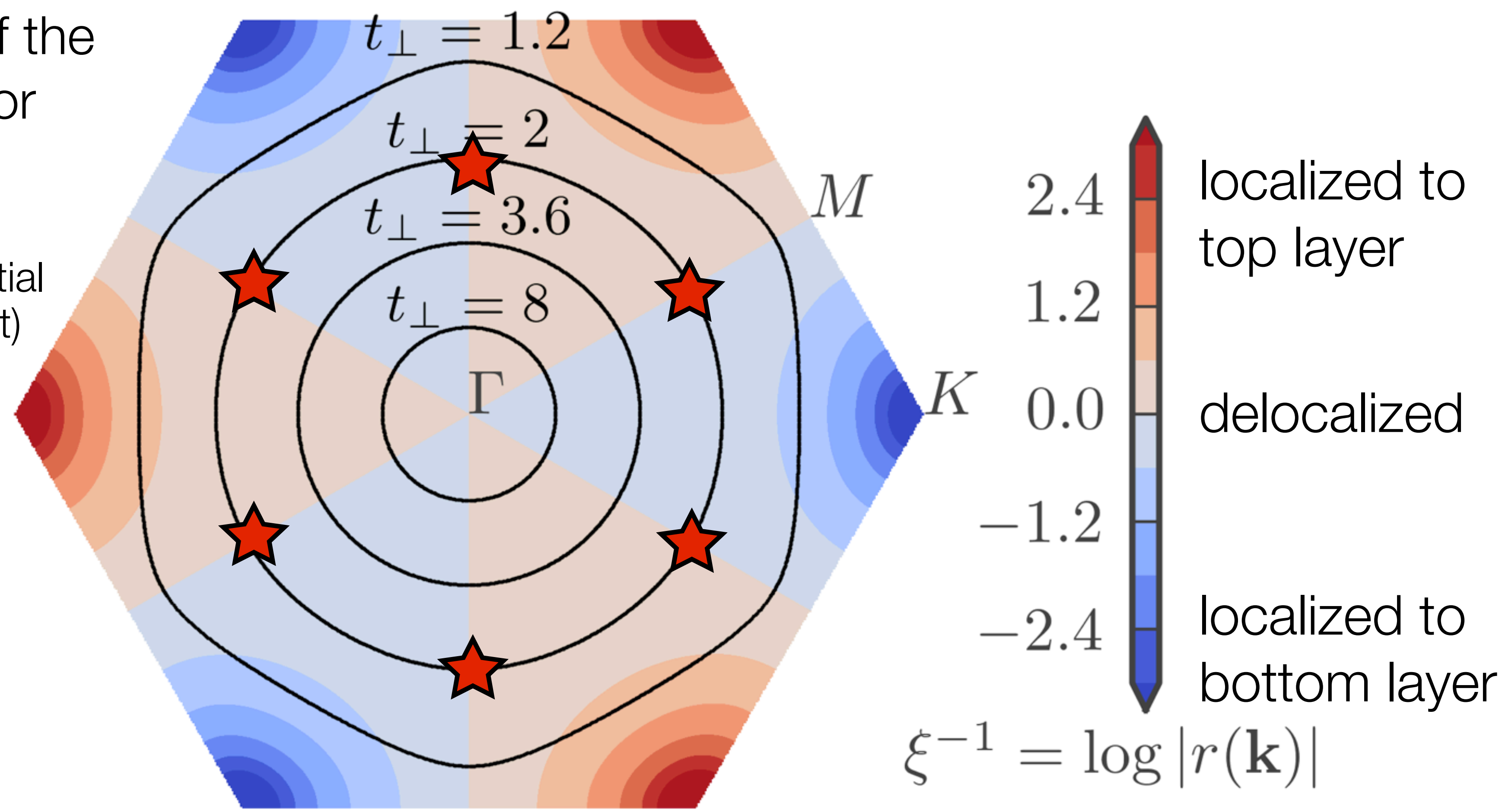
E.J. Bergholtz, Z. Liu, M. Trescher, R. Moessner, and M. Udagawa, arXiv:1408.3669

- Constant energy lines, “Fermi circles”, are split into Fermi arcs!

Projections of the Weyl points for

$$t_\perp = 2$$

(chemical potential at the Weyl point)



- Here we have an exact solutions for the Fermi arcs, and seen as a family, they carry a huge Chern number.
- The Fermi arcs also exist in absence of Weyl nodes in the bulk!

Experiments?

Oxide interfaces & geometrically frustrated systems with strong spin-orbit coupling

- Perovskite materials, ABO_3, routinely grown in sandwich structures in the [100] direction

  - Instead (111) slabs would be good for topological physics (relatively flat C=1 bands).

Epitaxial growth of (111)-oriented $LaAlO_3/LaNiO_3$ ultra-thin superlattices

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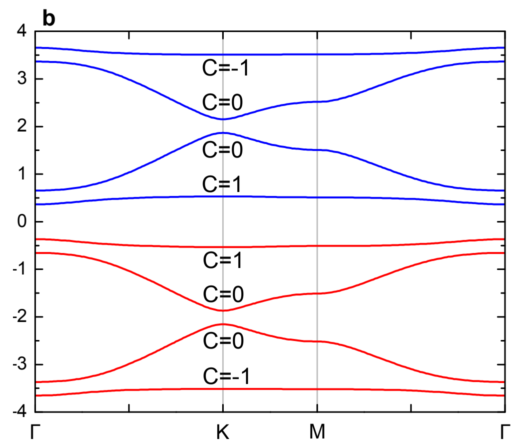
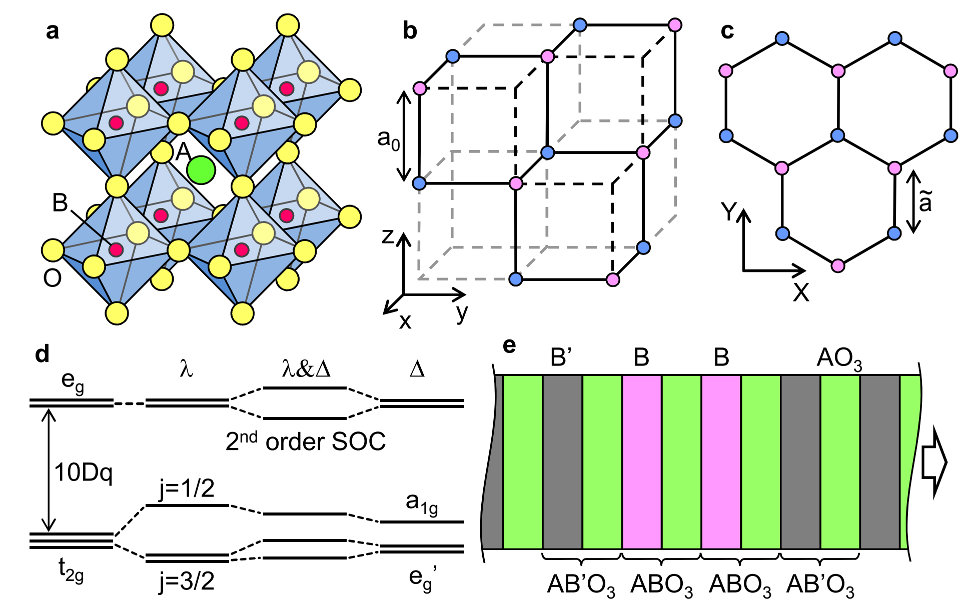
[2)] Advanced Photon Source, Argonne National Laboratory, Argonne, Illinois 60439, USA

arXiv:1212.0590v1



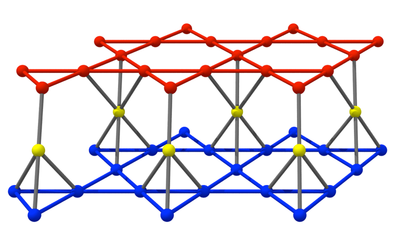


D. Xiao, W. Zhu, Y. Ran, N. Nagaosa, and S. Okamoto, Nature Commun. **2**, 596 (2011).

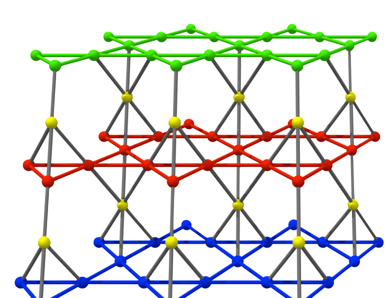
  - But [111] is not a natural cleavage/growth direction...

- **Suggestion:** *Grow (111) slabs of pyrochlore transition metal oxides, in particular* $A_2Ir_2O_7$ *iridate thin films*

  - Natural cleavage/growth direction!
  - Currently pursued by several experimental groups



N=2



N=3

- Might, *mutatis mutandis*, realize the pyrochlore slab model and the C=N bands
- Stable surface states likely to be observed
- Stoichiometry enforce chemical potential at the Weyl point, i.e., in the surface band!
- Novel FCIs in thin slabs!?!

...much further work need

Topological flat bands in realistic cold atom/ molecule setups

Selected for a Viewpoint in *Physics*
PRL **110,** 185302 (2013) PHYSICAL REVIEW LETTERS week ending 3 MAY 2013

Realizing Fractional Chern Insulators in Dipolar Spin Systems

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(Received 11 January 2013; published 29 April 2013)

Selected for a Viewpoint in *Physics*
PRL **110,** 185301 (2013) PHYSICAL REVIEW LETTERS week ending 3 MAY 2013

Reaching Fractional Quantum Hall States with Optical Flux Lattices

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(Received 14 December 2012; published 29 April 2013)

- The rotational degrees of freedom in spin-models (in optical lattices) freedom can form FCIs
- Artificial gauge fields by applying lasers with spatially modulated frequency

- Both setups give rather flat bands “for free”

(...well not quite, but still promising)

- Several other promising ideas around...

Outlook

- Experimental realizations?!

  Many possible directions here...

- Time-reversal invariant states?

  Preliminary results are slightly disappointing -- exponentially localized Wannier functions as an explanation? (See Levin & Stern, PRL '09 for alternative picture)

- Detailed theory of anomalous states in higher-C bands
- Lattice defects, dislocations and disorder
- MPS and tensor network approaches

  - Local finite-dimensional Hamiltonian without projection!
  - Entanglement (area law) estimates indicate that this is feasible but there are at the same time considerable issues to solve for chiral phases described by 2D networks

- Three-dimensional generalizations and strongly correlated phases in other surface bands
- ....

thank you!