Numerical and Analytical Methods in Correlated Systems, Benasque, Spain

Quantum Impurity in a Luttinger Liquid

Universal Conductance with Entanglement Renormalization

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Acknowledgements

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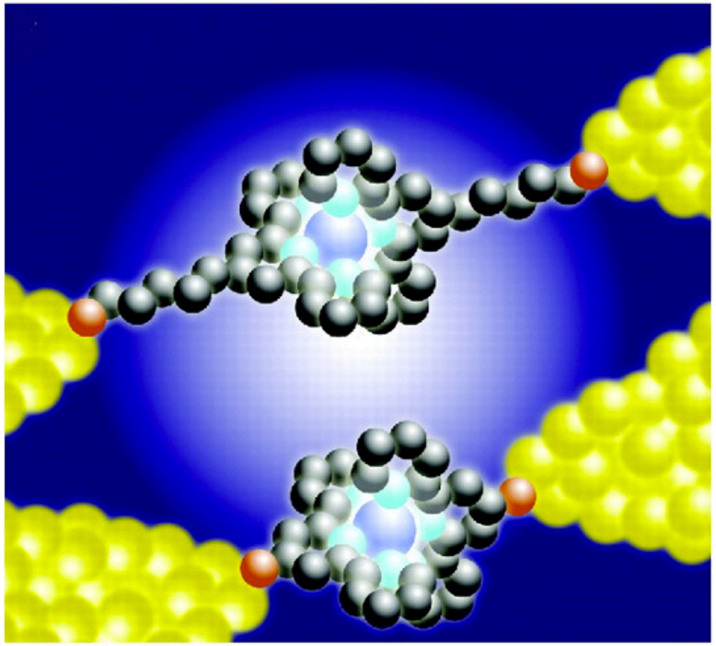
Motivation

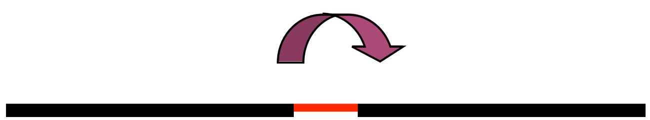


- At nano-scale, quantum mechanics and interaction play important roles in the transport properties.
- What are the universal properties and how to study them?

A. Nitzan and M.A. Ratner, Science 300, 1384 (2003)

Universal Properties

1D Luttinger liquid wires with a weak link



Kane and Fisher, PRL; PRB (1992)

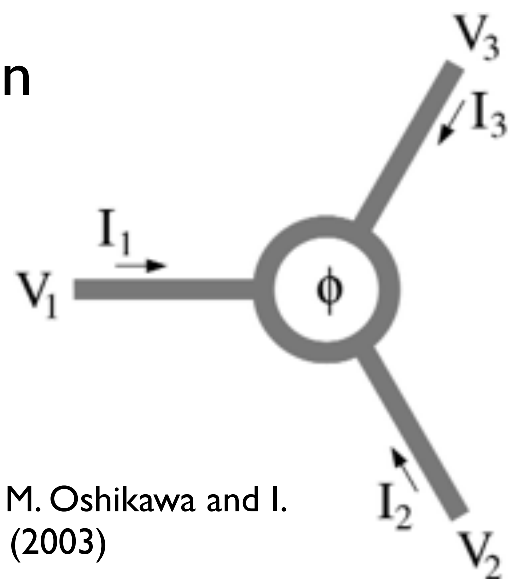
Repulsive interaction: $g < 1$

$$G = 0$$

Attractive interaction: $g > 1$

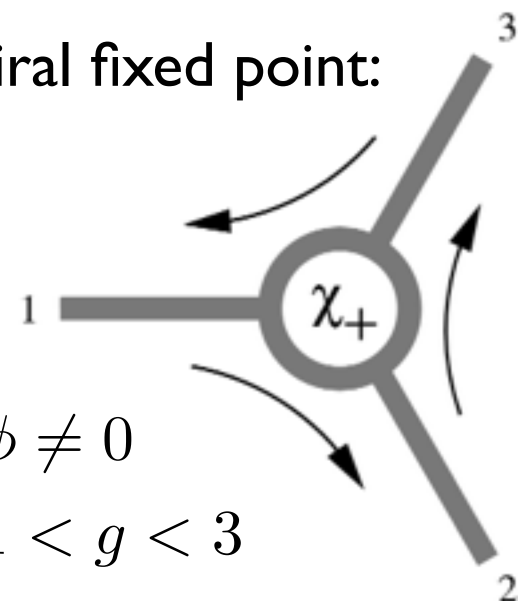
$$G = ge^2/h$$

Y-junction



C. Chamon, M. Oshikawa and I. Affleck, PRL (2003)

Chiral fixed point:

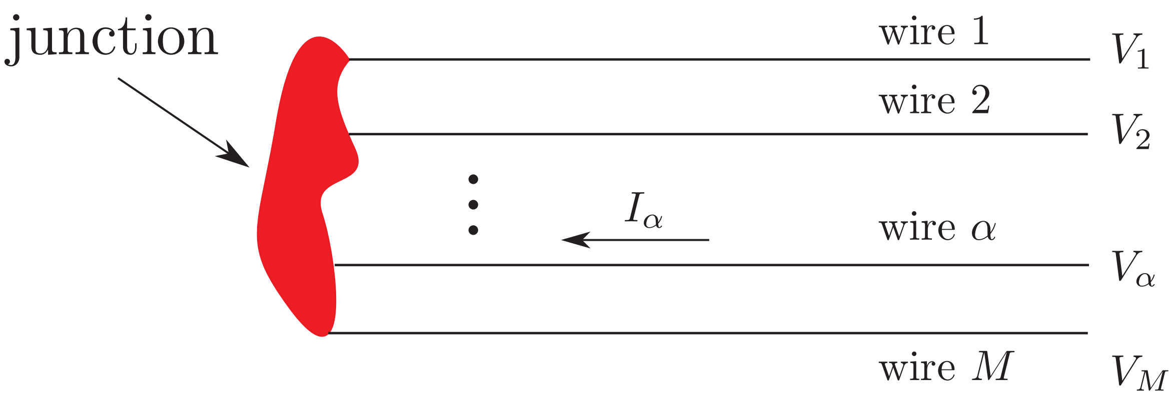


$$\phi \neq 0$$

$$1 < g < 3$$

Universal Conductance

Given an arbitrary junction with some structure and interactions, how to determine the universal conductance of the junction?

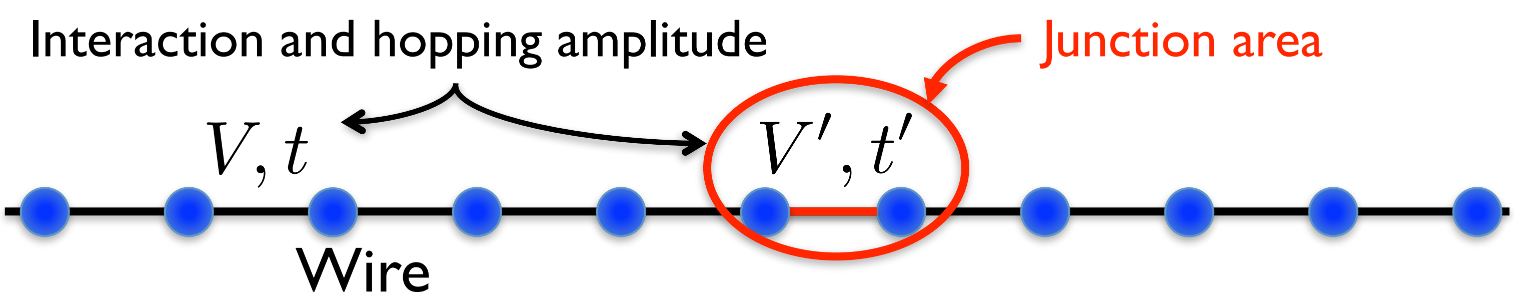


Analytical Methods: BCFT

Numerical Methods: DMRG, MERA

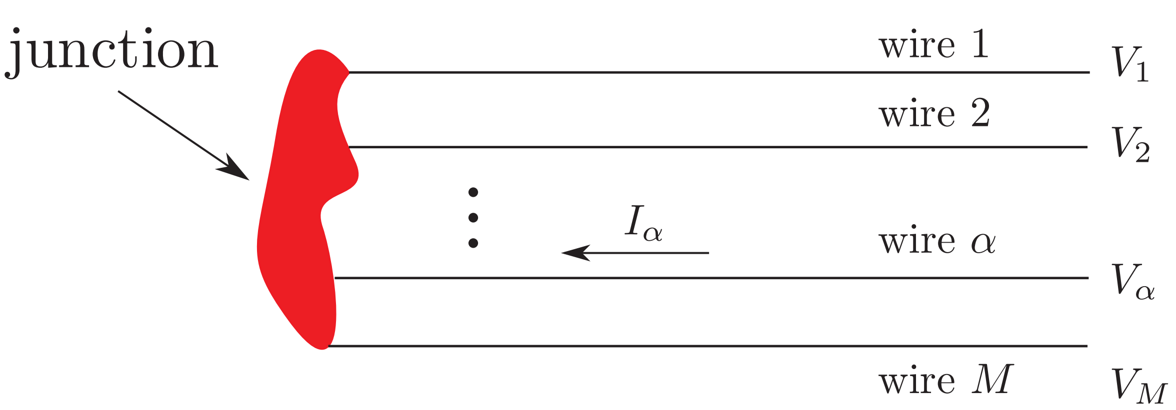
Quantum Wire Junction

Tight-binding model



Kane and Fisher, PRL; PRB (1992)

Transport properties through a general junction



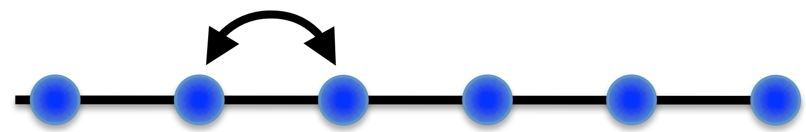
Linear response regime

$$I_i = \sum_j \boxed{G_{ij}} V_j$$

conductance tensor

Interacting Quantum Wire

Luttinger liquid wire



$t,\ V$

$$H = \sum_i \left[-t c_i^\dagger c_{i+1} + h.c. + V(n_i - \frac{1}{2})(n_{i+1} - \frac{1}{2}) \right] \qquad n_j = c_j^\dagger c_j$$

$$-2 < V < 2$$

spinless fermion

Bosonization

$$H = \frac{v}{4\pi} \int dx \left[g(\partial_x \varphi)^2 + \frac{1}{g}(\partial_x \theta)^2 \right]$$

$$\rho(x) = \partial_x \theta(x)/(\sqrt{2}\pi)$$

$$[\varphi(x), \theta(x')] = i\pi \mathrm{sgn}(x' - x)$$

$$S = \frac{1}{4\pi g} \int d\tau dx\, \partial_\mu \theta \partial^\mu \theta$$

$$= \frac{g}{4\pi} \int d\tau dx\, \partial_\mu \varphi \partial^\mu \varphi$$

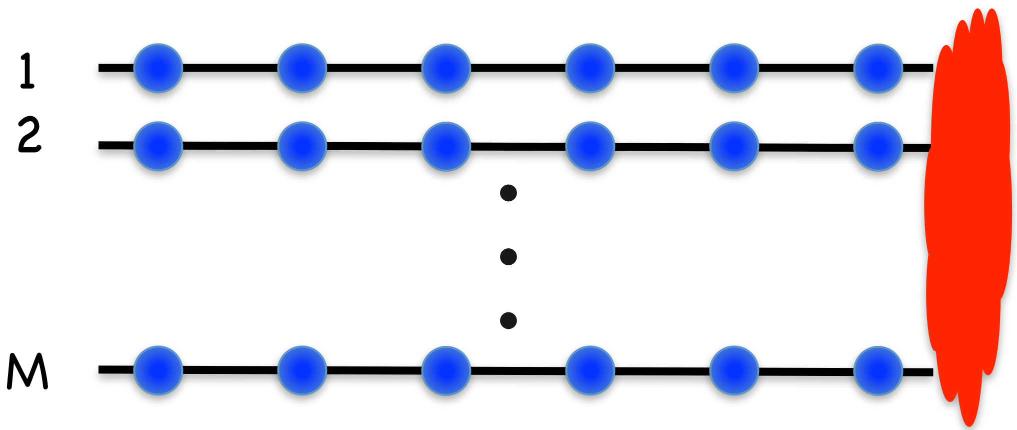
Half-filling, Bethe-Ansatz

$$v = \pi \frac{\sqrt{1 - (V/2)^2}}{\arccos{(V/2)}}$$

$$g = \frac{\pi}{2\arccos{(-V/2)}}$$

g < 1: repulsive
g = 1: non-interacting
g > 1: attractive

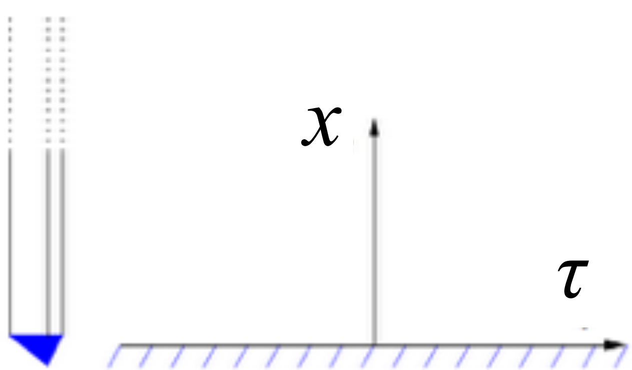
Junction: Boundary CFT



Without junction, CFT with M species of boson fields lives on the infinite complex plane.

Hypothesis: At RG fixed points, the universal behaviors of junction can be described by a conformally invariant boundary conditions.

$$z = \tau + ix$$



$$\theta_j, \; j = 1, \cdots, M$$

BCFT with M species of boson fields lives on the upper-half complex plane.

Universal Conductance

How does the boundary condition of junction affect transport?

Kubo formula

$$G_{ij} = \lim_{\omega \to 0_+} -\frac{e^2}{\hbar}\frac{1}{\omega L}\int_{-\infty}^{\infty} d\tau\, e^{i\omega\tau}\int_0^L dx\, \langle \mathcal{T}_\tau J^i(y,\tau) J^j(x,0)\rangle$$

current-current correlation function has the information about BC

Current operator for a given wire j

$$J^j = J^j_R - J^j_L$$

Primary fields

$$J^j_L(z) = \frac{i}{\sqrt{2}\pi}\,\partial\,\theta^j(z,\bar{z})$$

$$J^j_R(\bar{z}) = -\frac{i}{\sqrt{2}\pi}\,\bar{\partial}\,\theta^j(z,\bar{z}),$$

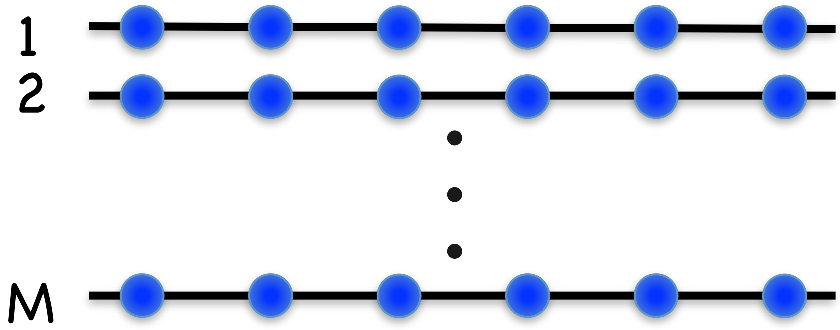
Universal Conductance in BCFT

LL and RR correlators

$$\langle \mathcal{T}_\tau J^i_L(z_1) J^j_L(z_2)\rangle = \frac{g}{4\pi^2}\frac{\delta_{ij}}{(z_1-z_2)^2},$$

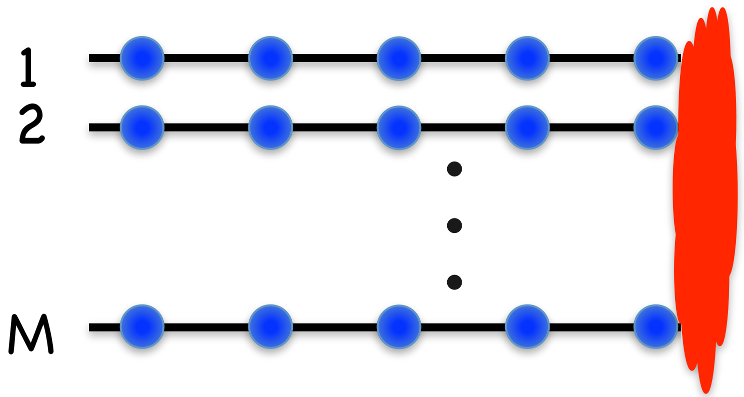
$$\langle \mathcal{T}_\tau J^i_R(\bar{z}_1) J^j_R(\bar{z}_2)\rangle = \frac{g}{4\pi^2}\frac{\delta_{ij}}{(\bar{z}_1-\bar{z}_2)^2}.$$

same as no junction



LR-RL correlators:

$$\langle \mathcal{T}_\tau J^i_R(\bar{z_1}) J^j_L(z_2)\rangle = -\frac{g}{4\pi^2} A^{ij}_{\mathcal{B}} \frac{1}{(\bar{z_1}-z_2)^2}$$



- The boundary does not change the scaling dimensions of the operators.
- Information of BC is encoded in A

J. L. Cardy and D. C. Lewellen, Phys. Lett. B 259, 274 (1991).

Universal Conductance

Conductance (from Kubo formula): $$G_{ij} = g\frac{e^2}{h}(\delta_{ij} + A_{\mathcal{B}}^{ij})$$

How to compute $A_{\mathcal{B}}^{ij}$?

Cardy's BCFT:

Conformally invariant BC

Boundary states $|B\rangle$

$$\Rightarrow \quad A_B^{ij} = \frac{\langle J_L^i J_R^j, 0|B\rangle}{\langle 1, 0|B\rangle}$$

J. L. Cardy and D. C. Lewellen, Phys. Lett. B 259, 274 (1991).

Numerical: $$\langle \mathcal{T}_\tau J_R^i(\bar{z}_1) J_L^j(z_2)\rangle = -\frac{g}{4\pi^2} A_{\mathcal{B}}^{ij} \frac{1}{(\bar{z}_1 - z_2)^2}$$

Challenge:

- Dynamical correlators
- Open quantum system

$\Rightarrow$

Time-dependent

Infinitely large system

Challenges

Time-dependent calculations?

Conformal symmetry ties
space and time together.

$$\langle J_R^i(x) J_L^j(x) \rangle = \frac{g}{4\pi^2} A_{\mathcal{B}}^{ij} \frac{1}{(2x)^2}$$

⇨ A static ground state expectation value

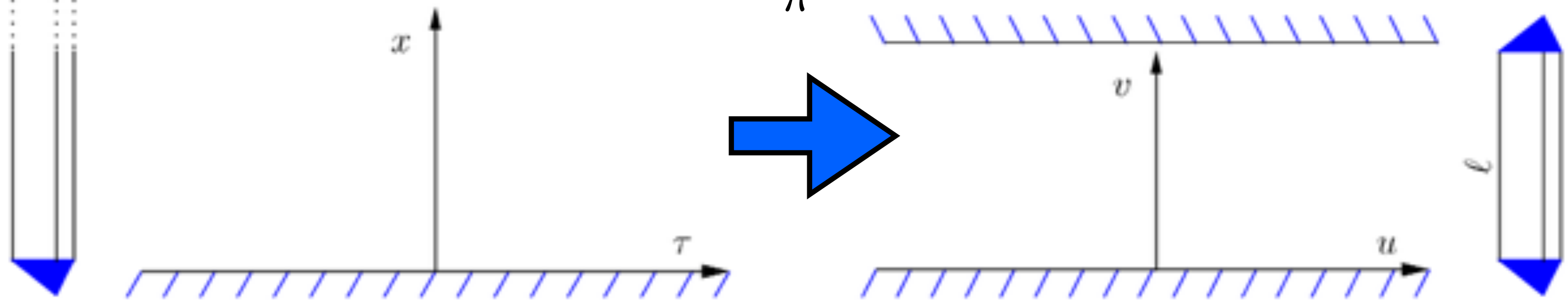
Infinitely large system?

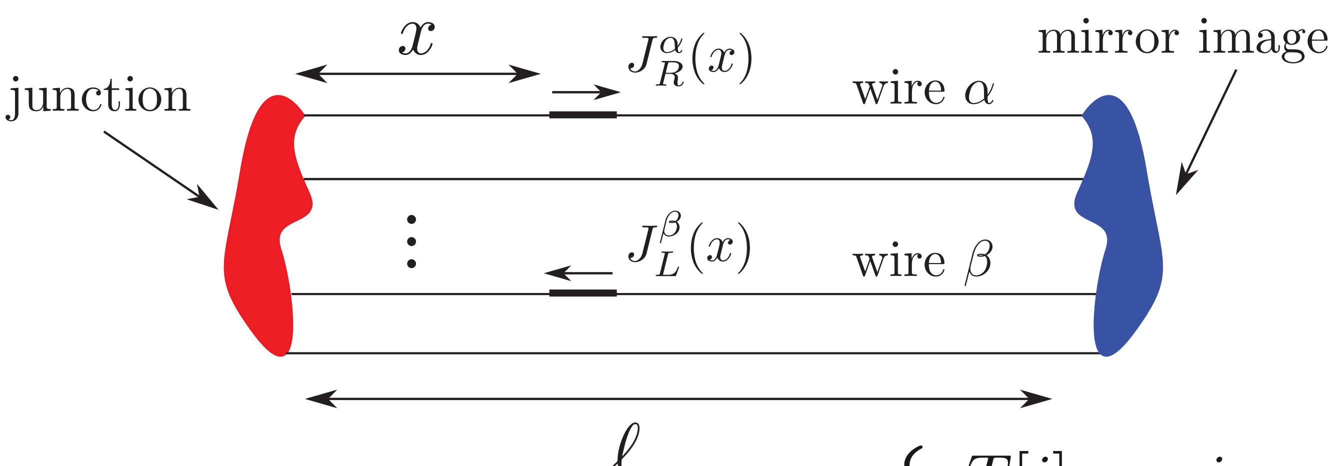
- Map to a finite system (DMRG)
- Deal with it directly (MERA)

DMRG approach

Conformal mapping of upper complex plane to a finite strip

$$\omega = \frac{\ell}{\pi} \ln z$$





$$\color{blue}{H_R} = T[C(\color{red}{H_L})] \text{ where } \begin{cases} T[i] = -i \\ C(c) = c^\dagger \end{cases}$$

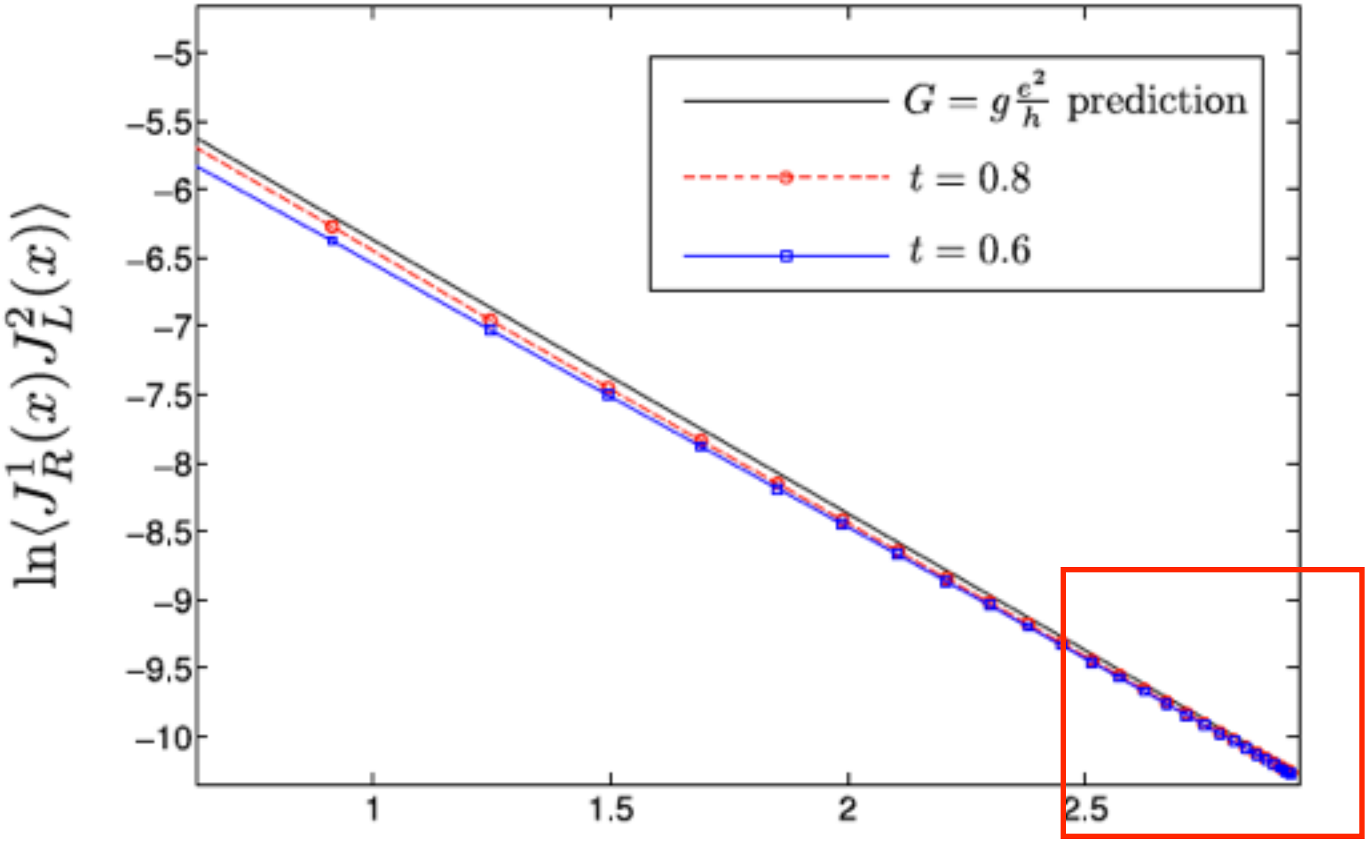
A. Rahmani et al. PRL (2010); PRB (2012)

DMRG results

Two wires (Kane and Fisher's problem)

Attractive interaction: g>1

$g = 2.0,\ \ell = 60$



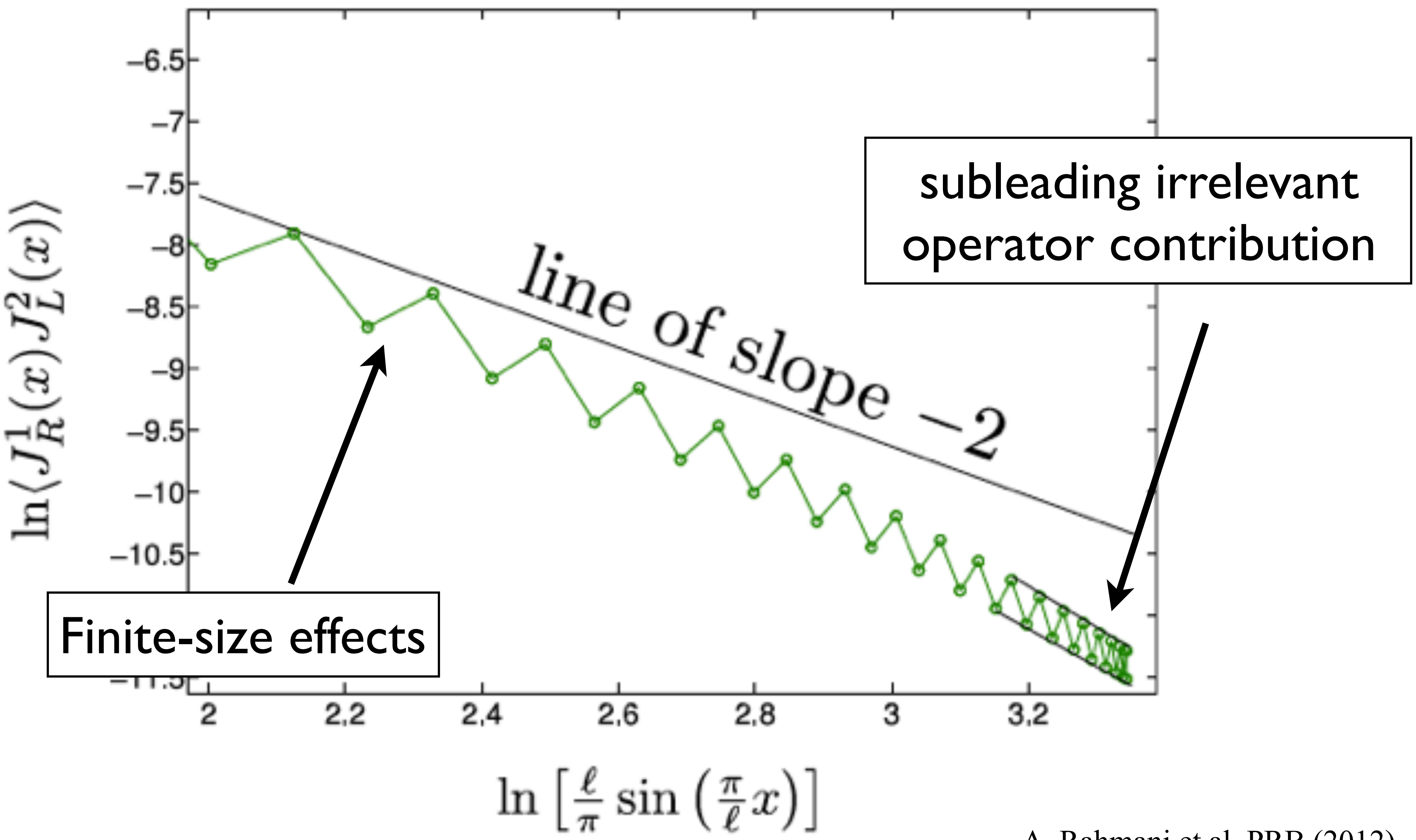
$\ln\left[\frac{\ell}{\pi}\sin\left(\frac{\pi}{\ell}x\right)\right]$

A. Rahmani et al. PRL (2010); PRB (2012)

DMRG results

Repulsive interaction: g<1

$$g = 0.65,\ \ell = 90,\ t = 0.3$$

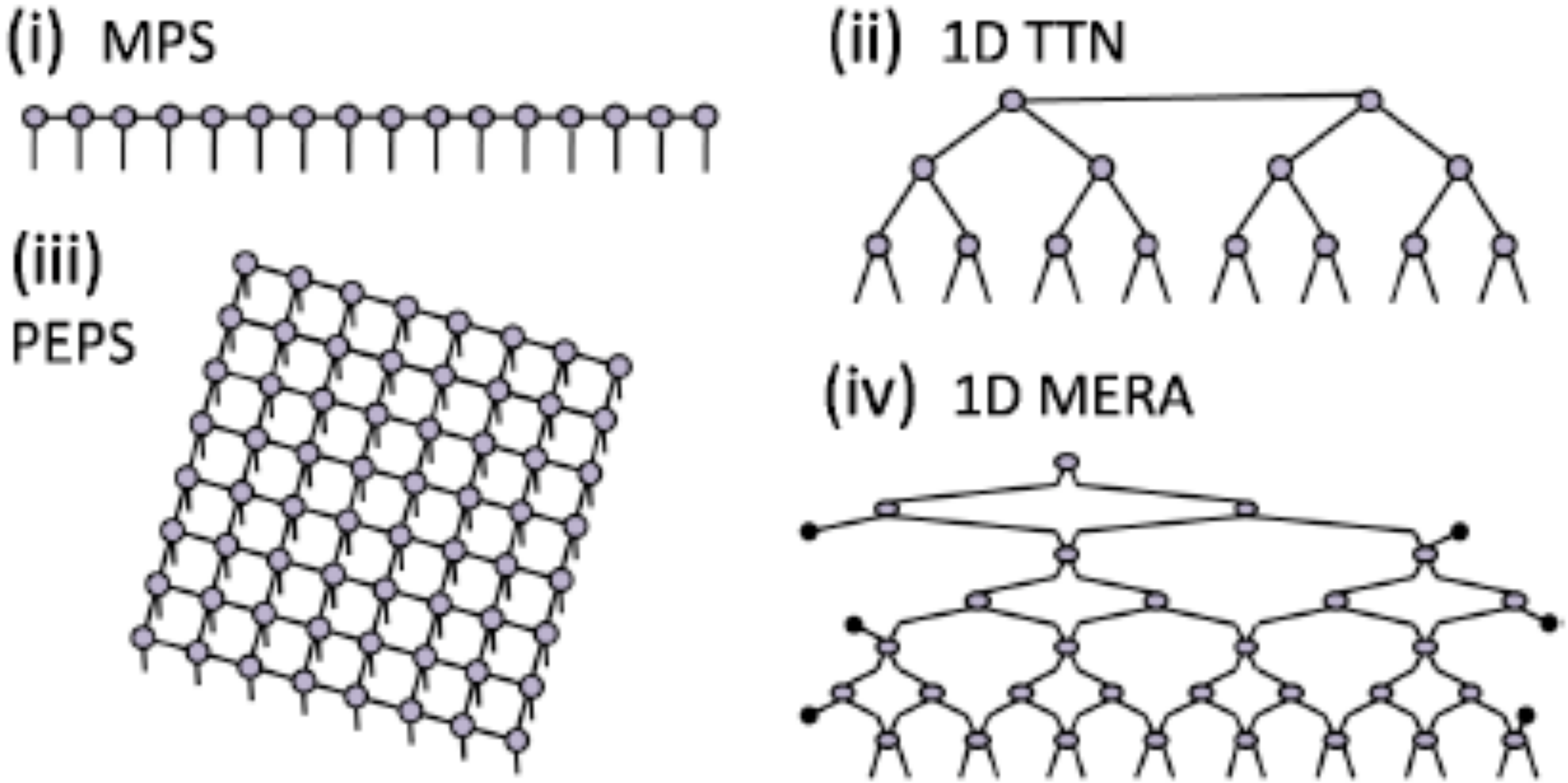


A. Rahmani et al. PRB (2012)

Limitation

- Limitation of DMRG approach:
  - ⋆ "Small" system sizes; finite size effects. Universal regime reached?
  - ⋆ No information about the scaling dimensions of the operators at the junction.
- Alternatives?
  - ⋆ Entanglement Renormalization (MERA)

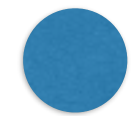
Tensor Network States



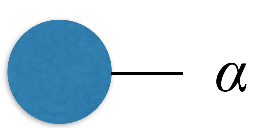
G. Vidal

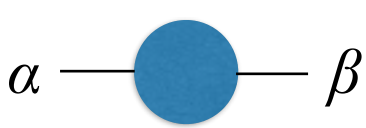
Graphical Representation

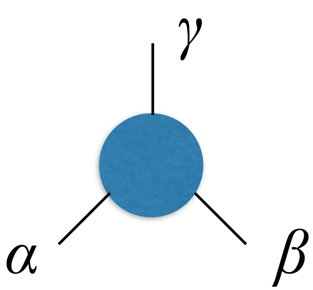
$$A = \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ \vdots \\ A_N \end{bmatrix} \quad B = \begin{bmatrix} B_{11} & \cdots & B_{1n} \\ \vdots & \ddots & \vdots \\ B_{m1} & \cdots & B_{mn} \end{bmatrix}$$

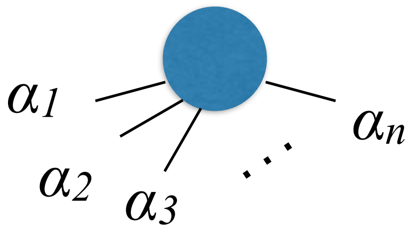
scalar  S

vector	matrix	rank-3 tensor	rank-n tensor
A_α	$B_{\alpha\beta}$	$C_{\alpha\beta\gamma}$	$T_{\alpha_1\alpha_2\alpha_3\ldots\alpha_n}$



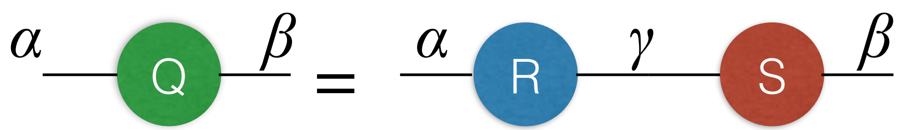




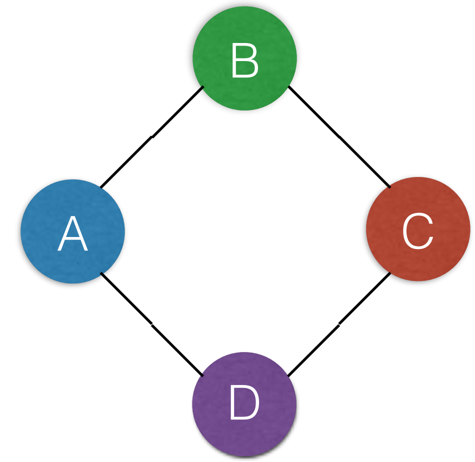


Graphical Representation

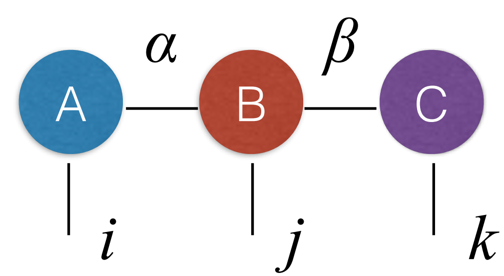
product of tensors (matrices)



$$Q_{\alpha\beta} = \sum_{\gamma} R_{\alpha\gamma} S_{\gamma\beta}$$



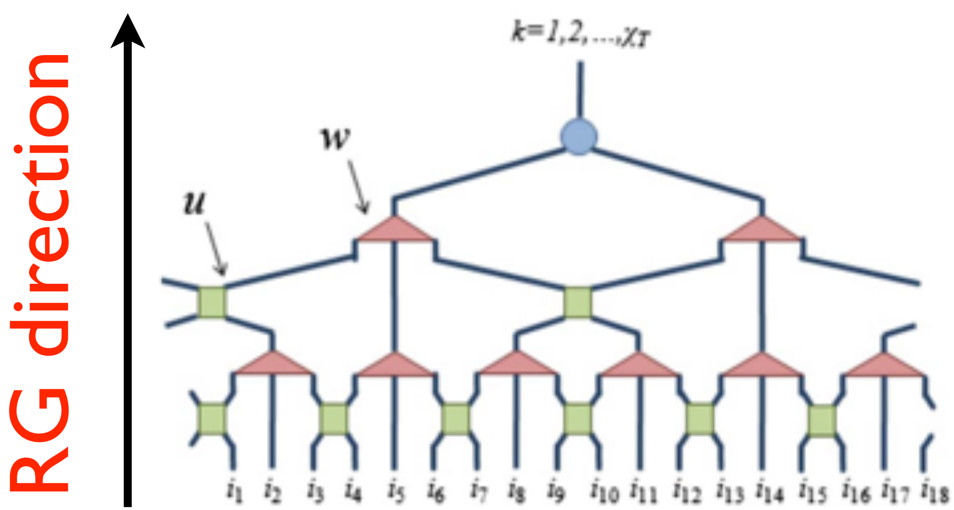
$$\mathrm{Tr}(ABCD)$$

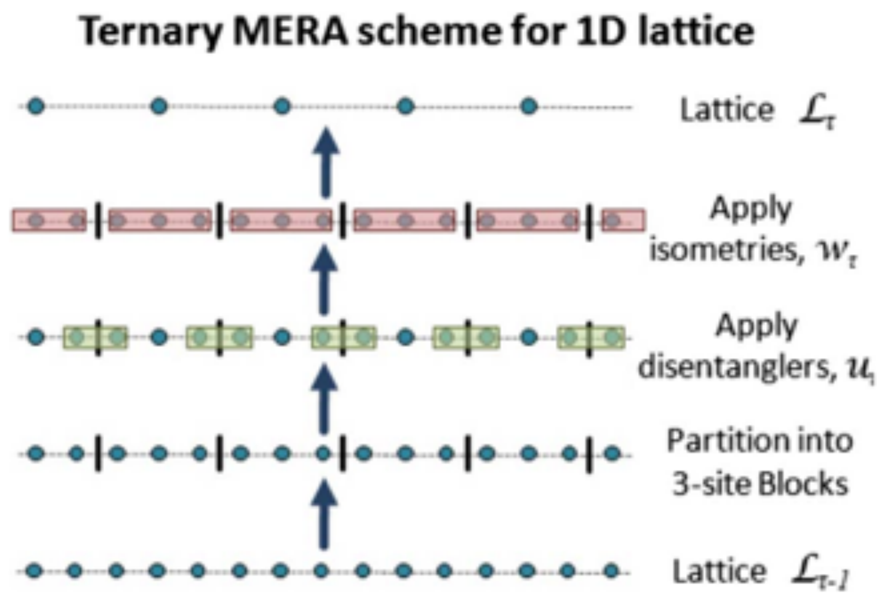


- Internal lines are summed over
- External lines are external indices

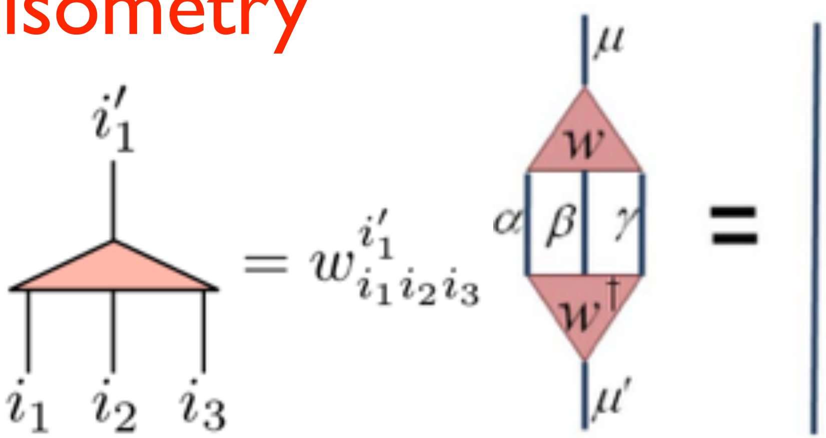
$$T_{ijk} = \sum_{\alpha\beta} A_{\alpha i} B_{\alpha\beta j} C_{\beta k}$$

Multiscale Entanglement Renormalization Ansatz

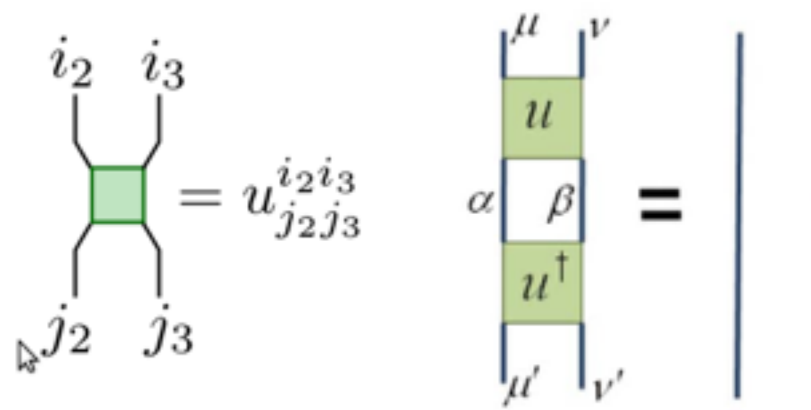




isometry

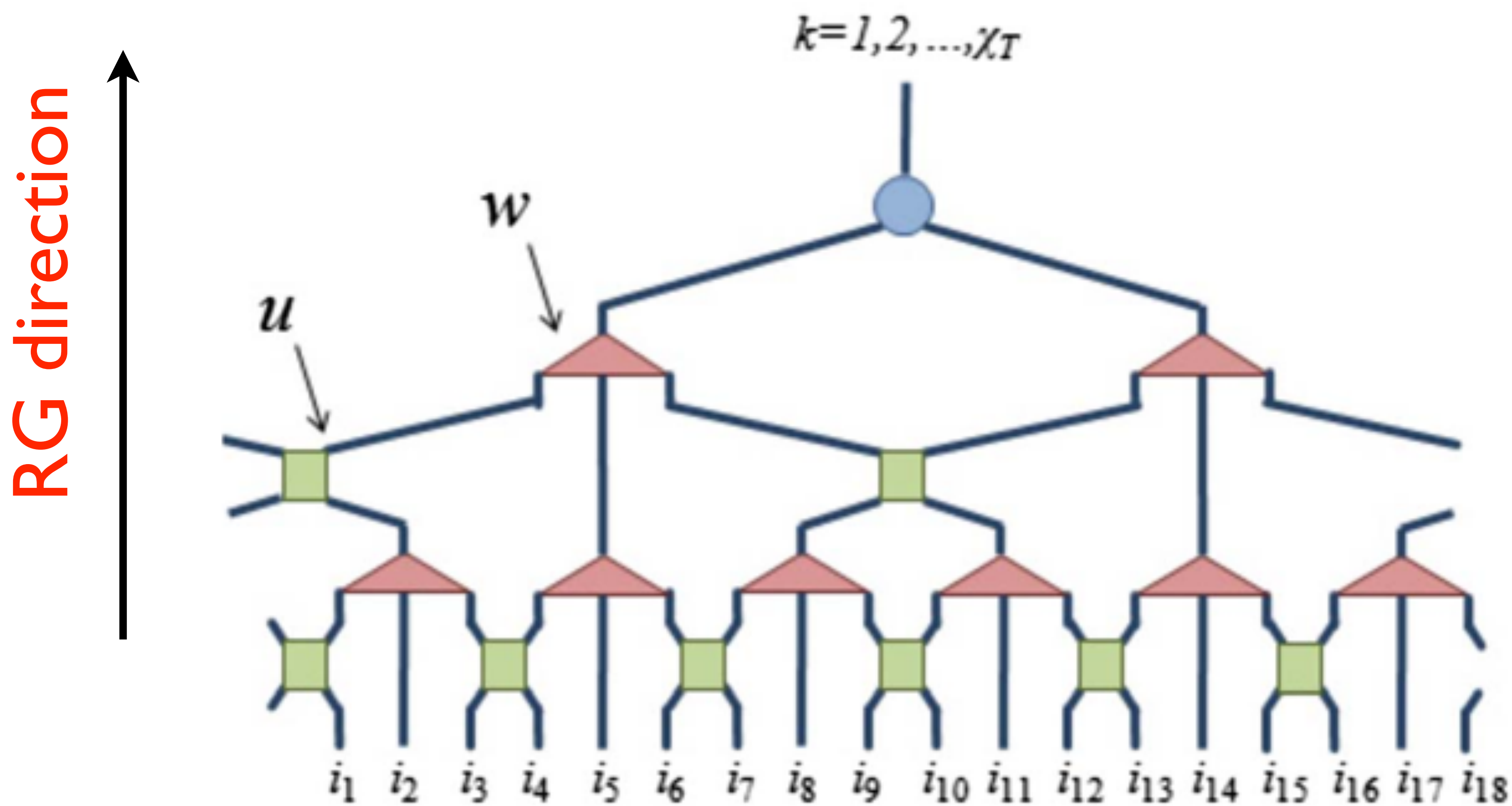


disentangler



G. Evenbly and G. Vidal, Phys. Rev. B, 79,144108(2009)

Scale-invariant MERA



All the w_τ and u_τ are the same at all layers

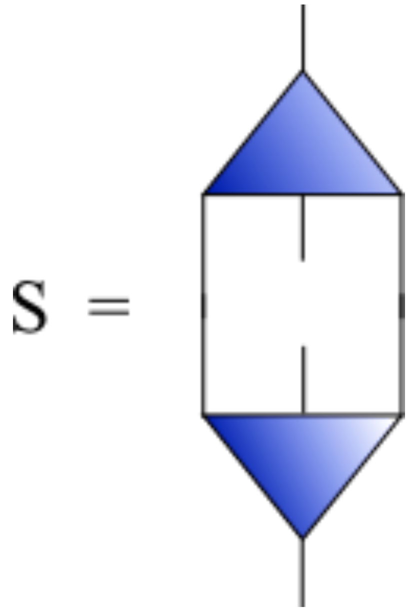
Describe one-dimensional critical systems

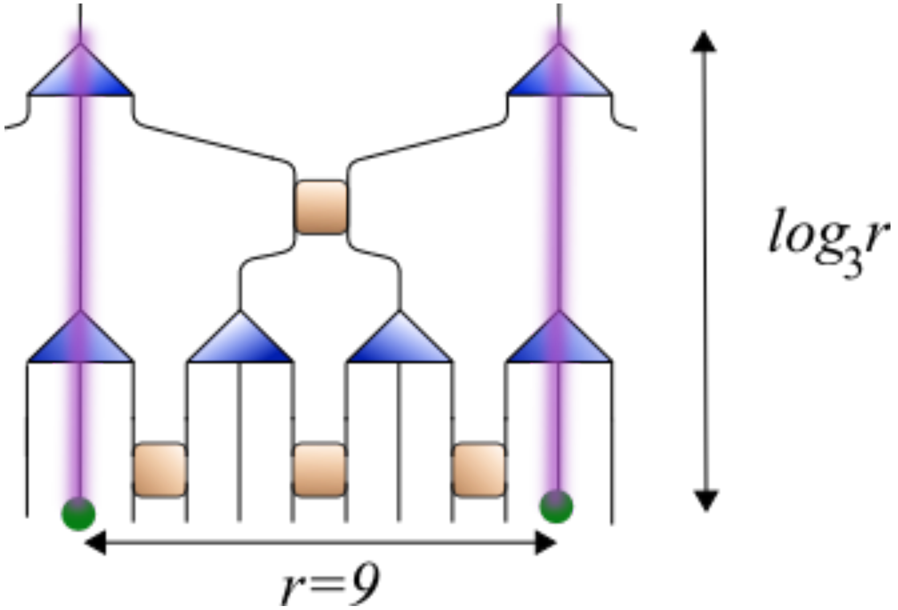
G. Evenbly and G. Vidal, Phys. Rev. B, 79,144108(2009)

Scaling Dimensions

Scaling operator $$S(O_s) = \lambda O_s$$

Eigenvalues of S gives scaling dimensions

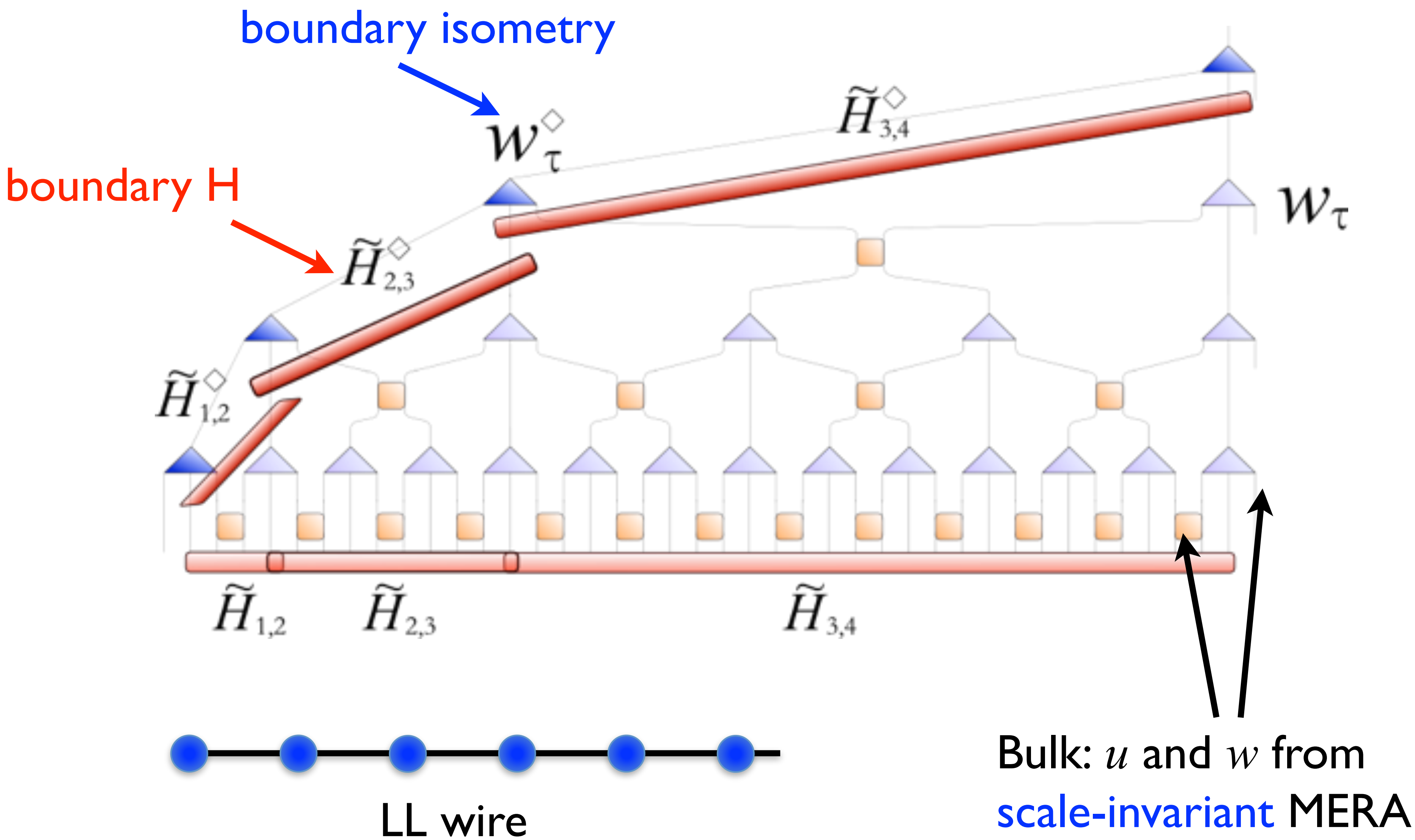




$$S(\phi) = 3^{-\Delta}\phi \qquad \langle \phi_i \phi_j \rangle = \frac{1}{9^{2\Delta}} \mathrm{tr}(\rho \phi_i \phi_j) = \frac{C}{r^{2\Delta}}$$

G. Evenbly and G. Vidal, Phys. Rev. B, 79,144108(2009)

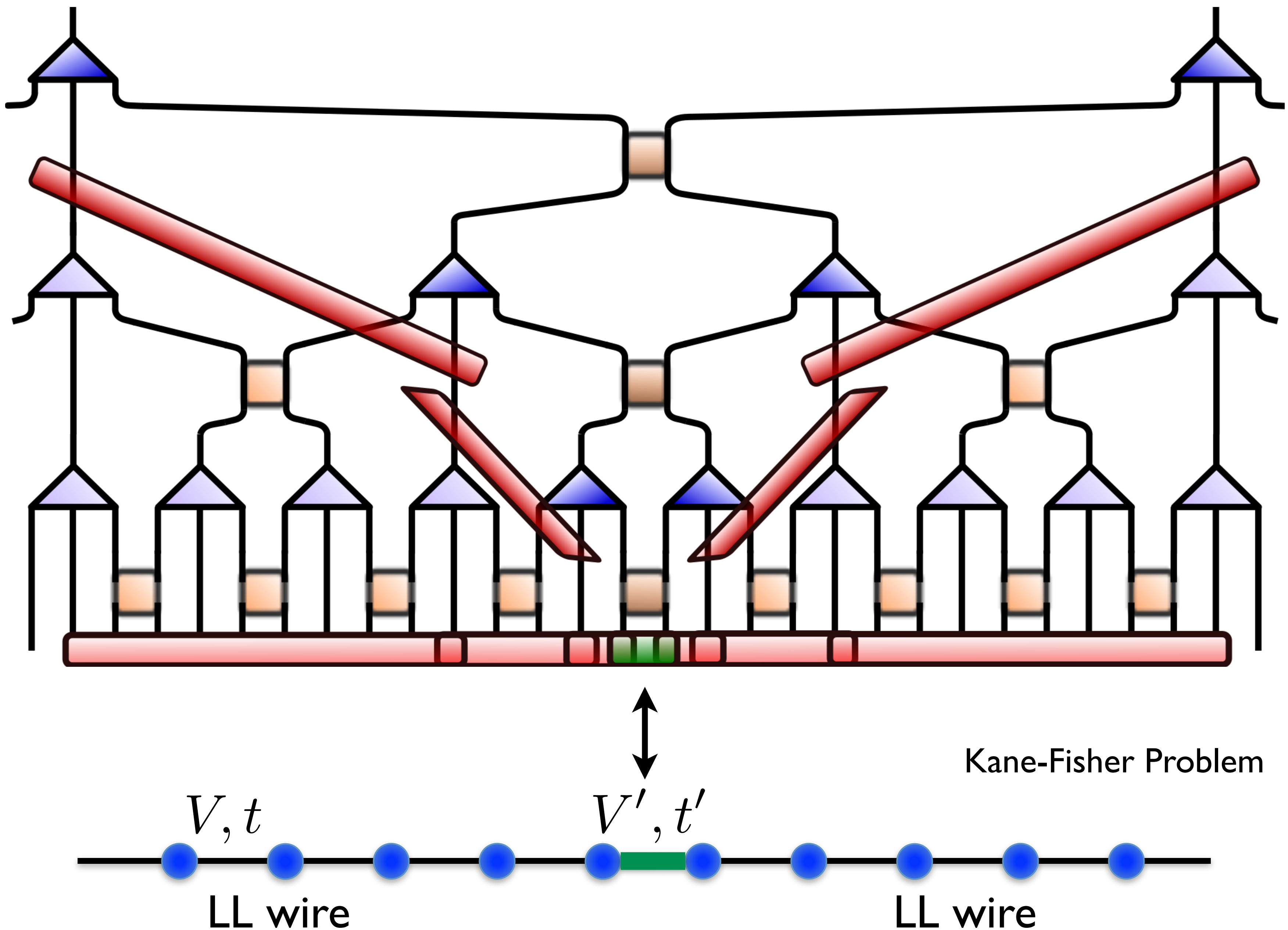
Semi-infinite Wire



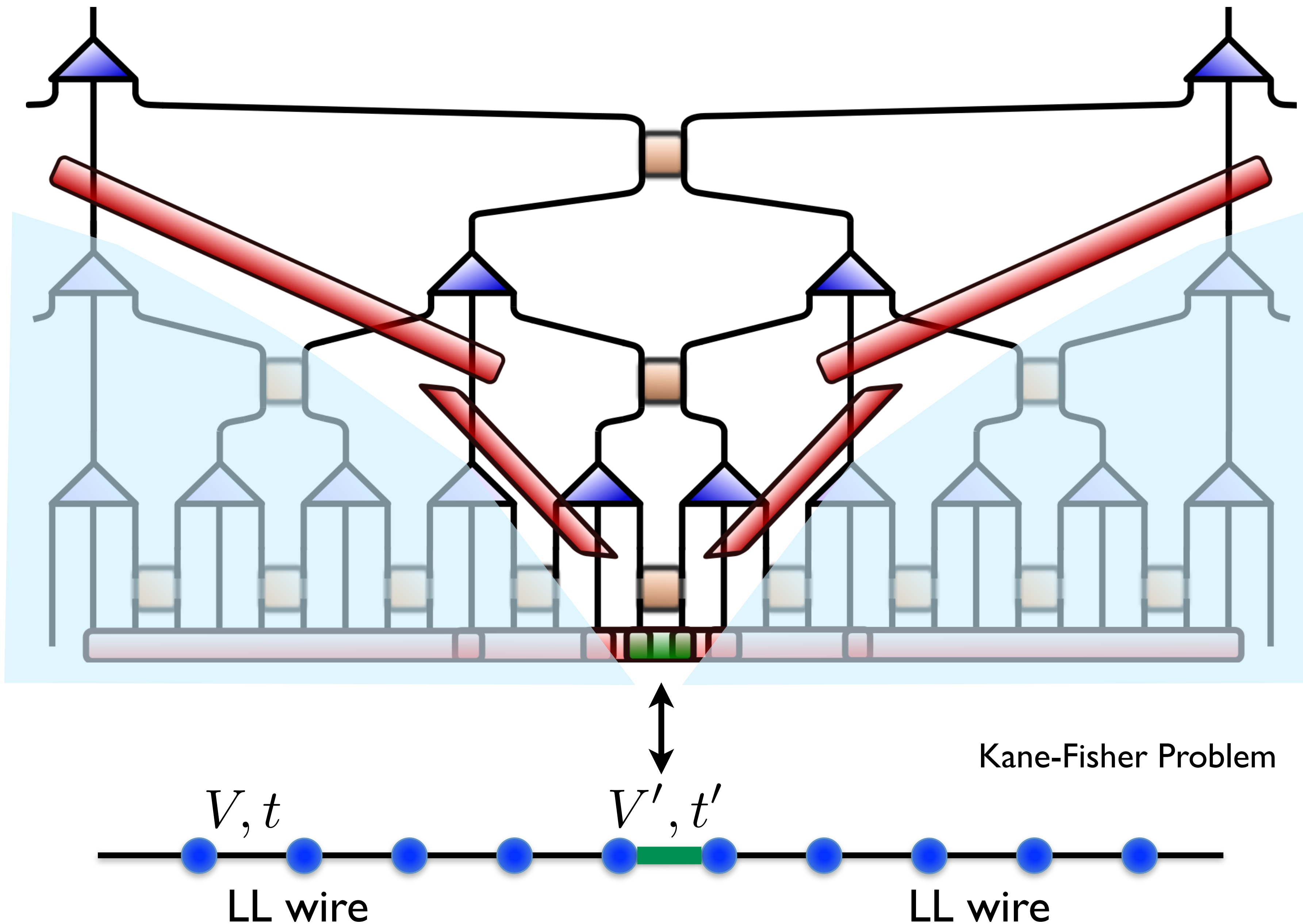
Boundary tensors form an MPS

Evenbly et al. PRB (2010)

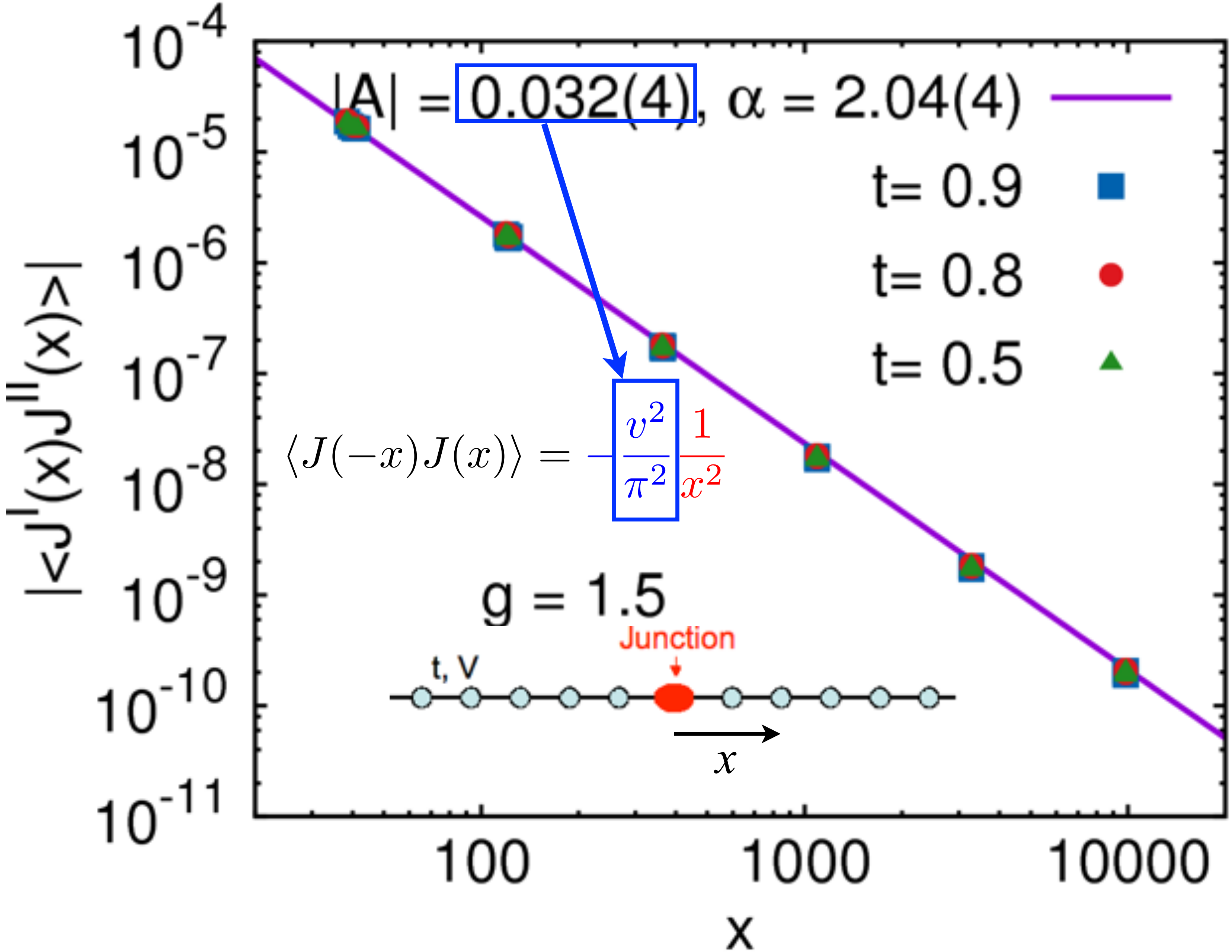
Impurity in LL Wire



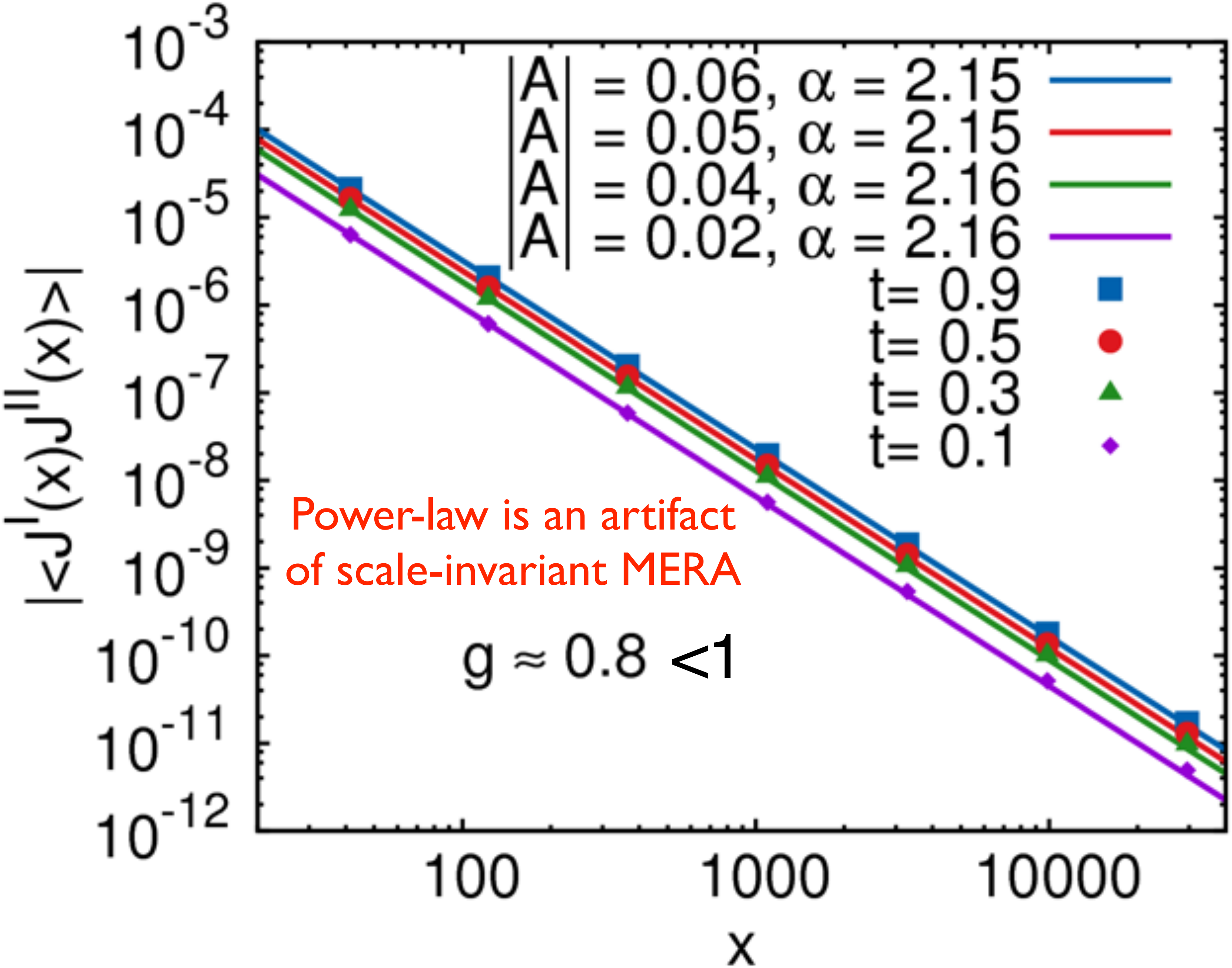
Impurity in LL Wire



Universal Conductance

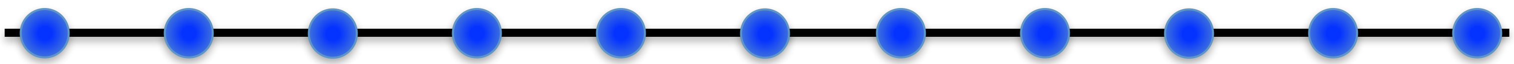


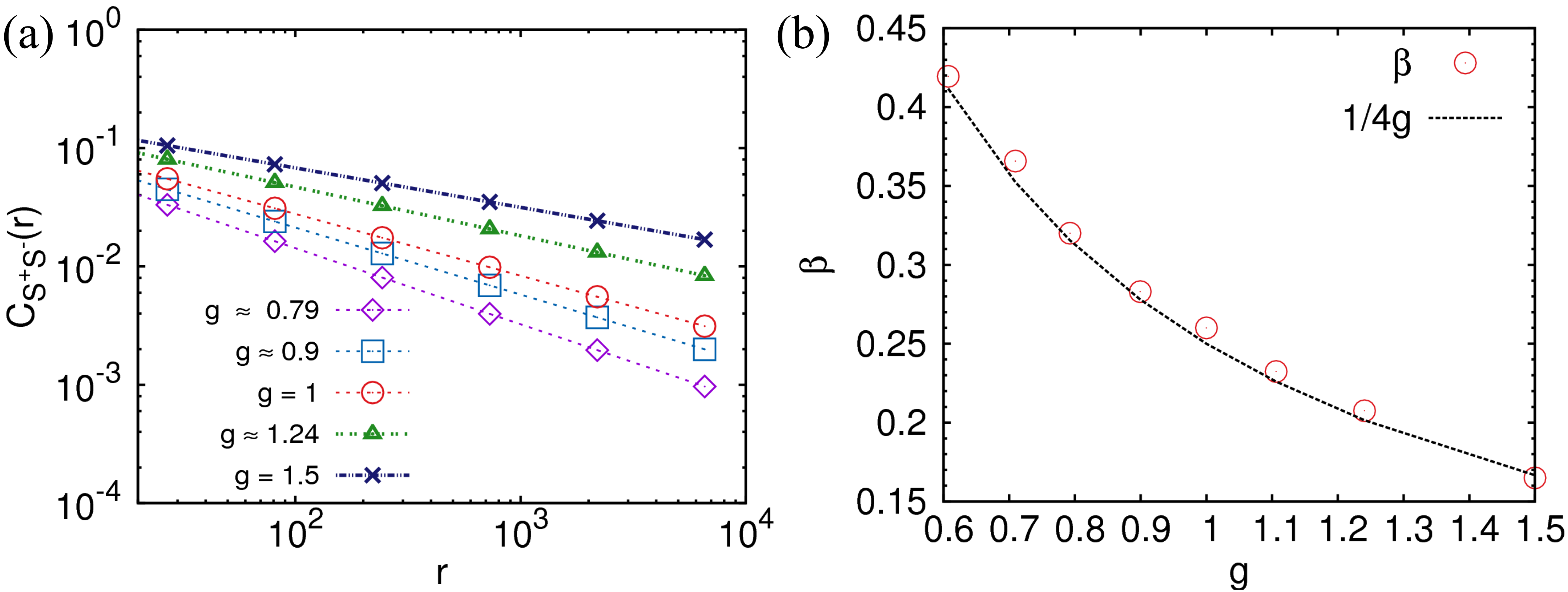
Non-universal Behavior



spin-spin correlation

Bulk Wire

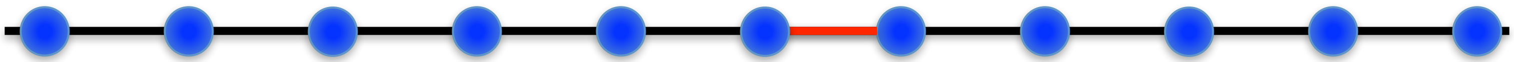


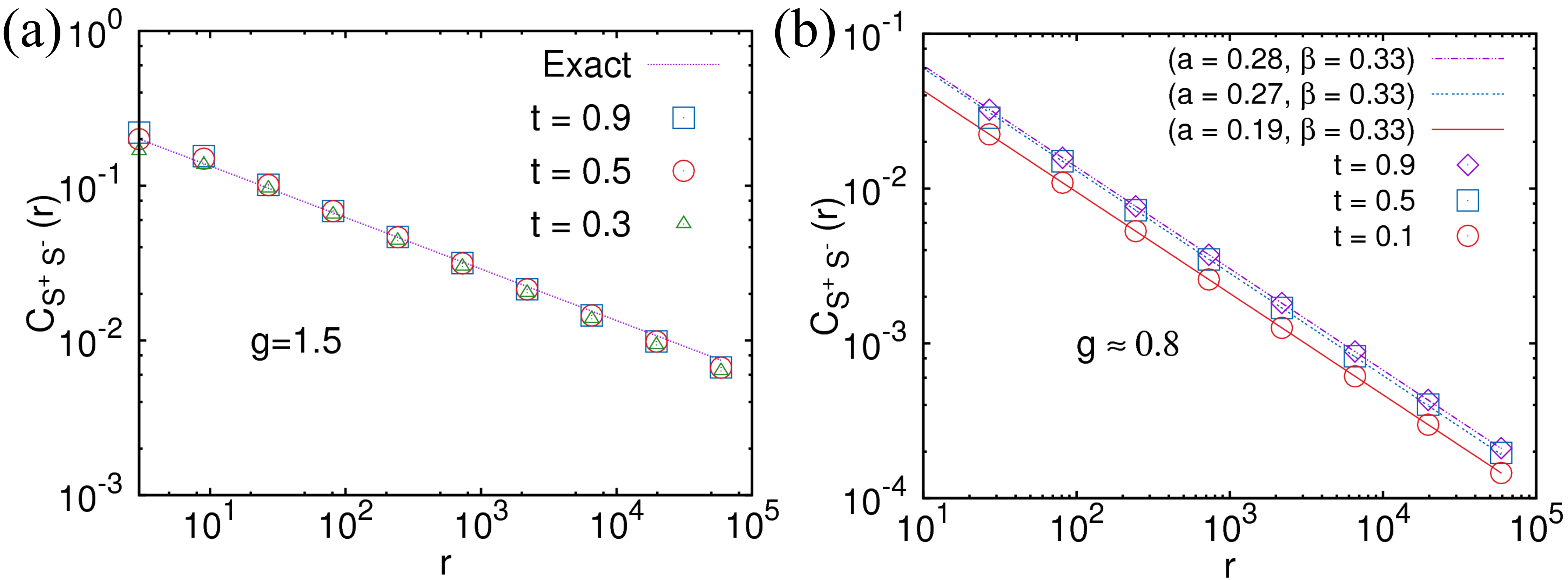


$$C_{S^+S^-}(r) = \left|\langle S^+(r_1)S^-(r_2)\rangle - \langle S^+\rangle\langle S^-\rangle\right| = \alpha r^{-2\beta}$$

spin-spin correlation

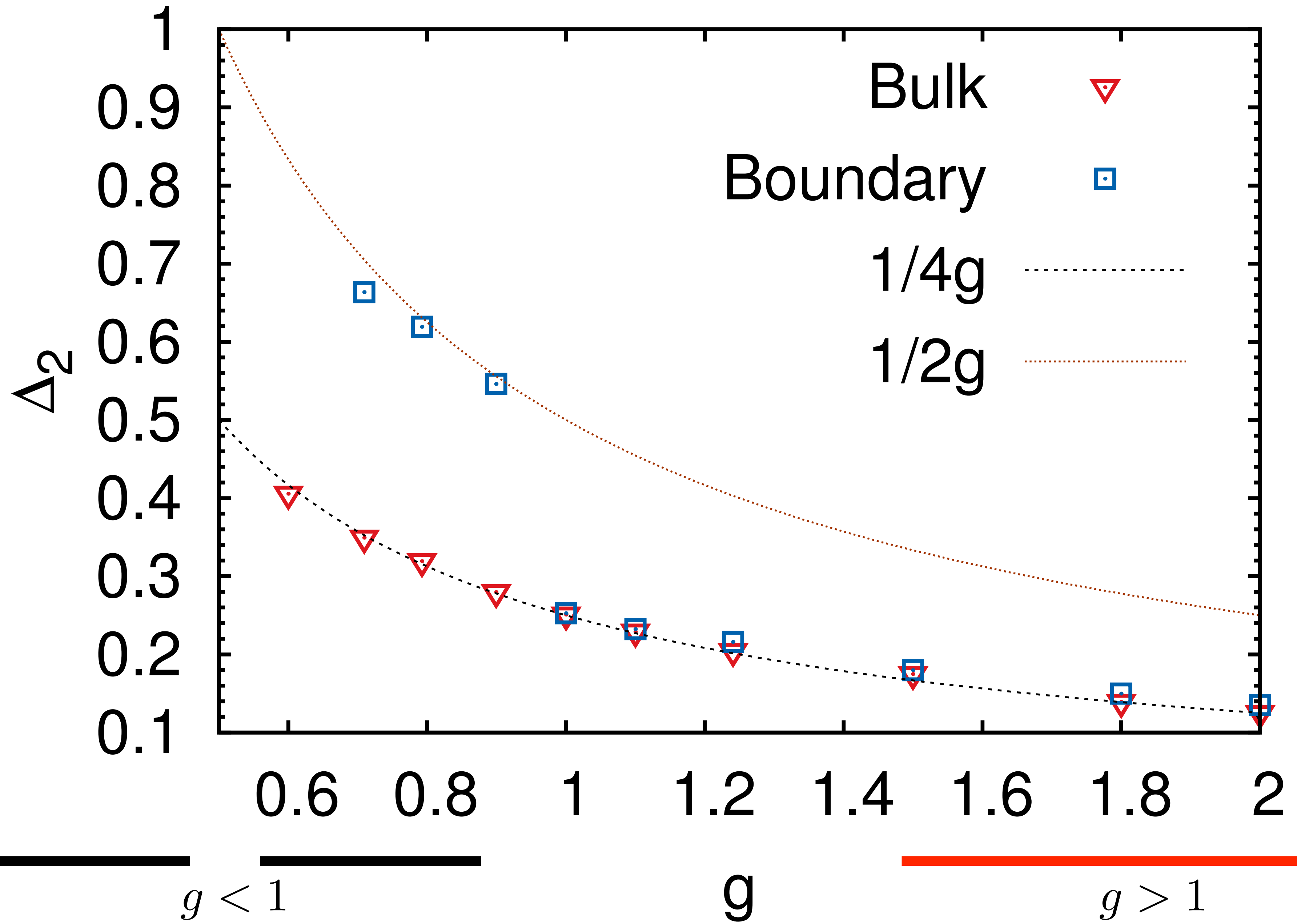
With impurity





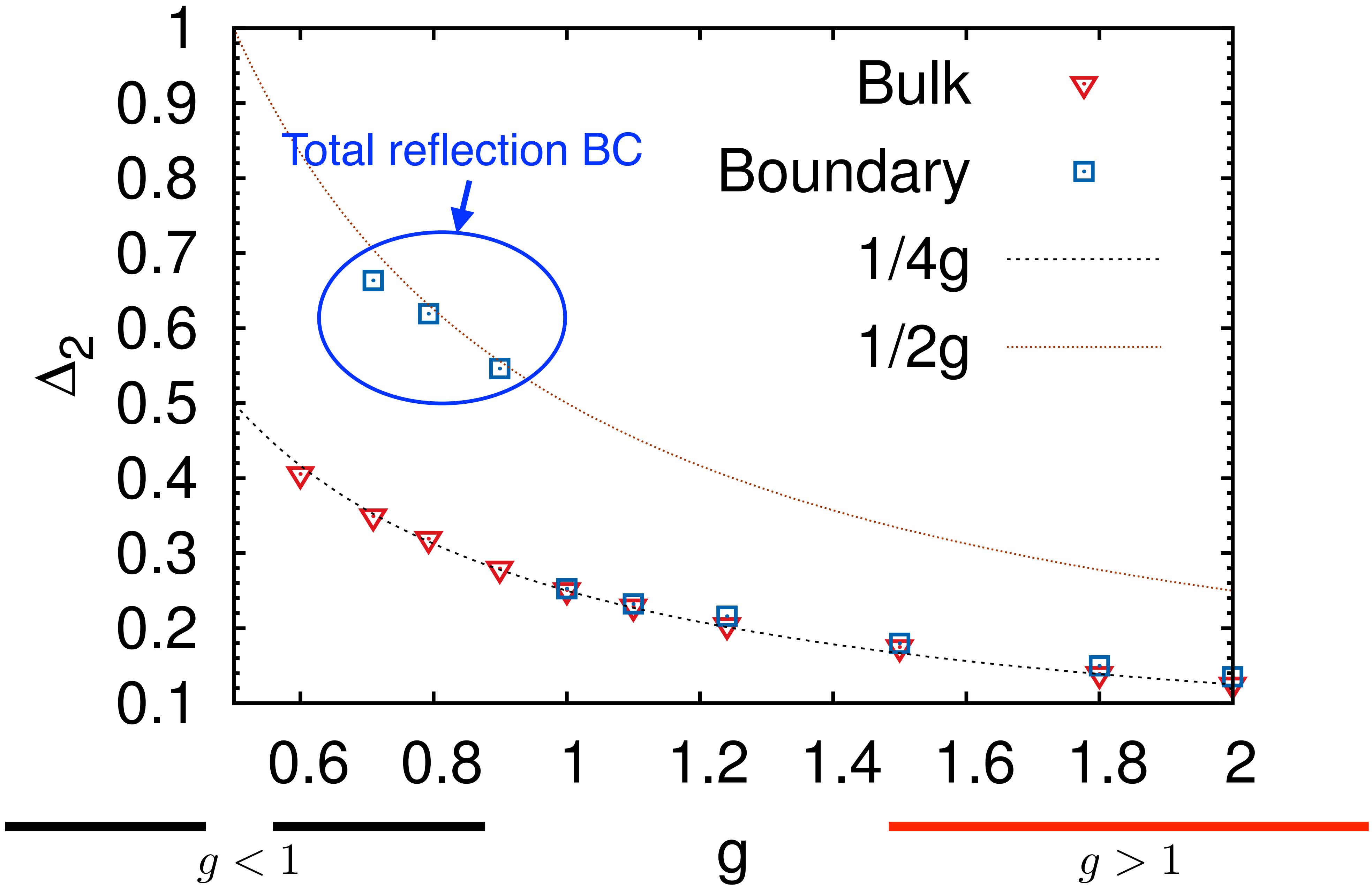
Universal $\beta = 1/4g$ **non-Universal**

Scaling Dimensions



$g < 1$ $g > 1$

Scaling Dimensions



Conclusions

- Applying boundary MERA to a Luttinger liquid wire with a single impurity, we are able to access the universal regime for $g > 1$ (attractive) with various t over a long distance and confirm the prediction from BCFT.
- Scaling dimensions of boundary operators are obtained directly.
- Non-universal behaviors for $g < 1$ (repulsive) with various t.
- Extension to Y-junction, other types of wires

Uni10

- Fully implemented in objected-oriented C++
- Aimed toward applications in **tensor network algorithms**
- Provides basic tensor operations with easy-to-use interface
  - A symmetric tensor class **UniTensor** (Abelian symmetry) with auxiliary classes for quantum numbers, **Qnum**, blocks **Block** and bond labels, **Bond** and functions performing tensor operations.
  - A network class **Network**, where details of the graphical representations of the networks are processed and stored.
  - An **engine** to construct and analyze the **contraction tree** for a given network.
  - A **heuristic algorithm** to search for an optimal **binary contraction order** based on the computational and memory constraints.
- Provides wrappers for **Matlab** and **Python** (soon).
- Supports acceleration with Nvidia Cuda based **GPU**.
- Open source LGPL with cite-me license.

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