Robustness of symmetry-protected topological orders in spin-2 quantum chains

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Motivation

Topological order is a new kind of order in quantum matter associated to a non-local pattern of long-range entanglement.

The existence of such a topologically ordered Intermediate Haldane phase for a spin-2 Heisenberg chain was suggested by Oshikawa in 1992[1]. The proof for the existence of such a phase in a spin-2 quantum chain was only published recently[2].

We study the phases of this spin-2 quantum chain with uniaxial anisotropy and bilinear, biquadratic, bicubic and biquartic interactions along four projections in parameter space. We also study the robustness of these phases under external perturbations of the Hamiltonian parameters as well as its time evolution under global quenches.

The Hamiltonian of the system is

$$H=\sum_j\sum_{\gamma=1}^{4} J_\gamma(\vec{S}_j\cdot\vec{S}_{j+1})^\gamma + D\sum_j (S_j^z)^2$$

for periodic boundary conditions and in the thermodynamic limit.

Numerical technique used

Our numerical results are based on using Matrix Product States (MPS) to represent the wave function, in combination with the infinite Time-Evolving Block Decimation (iTEBD) method [3].

We use a bond dimension of 40 which is sufficient for our purpose.

Phase diagram and other properties

1. The phase diagram and time evolution of the model was studied for arbitrary values of the interaction strength J_γ and anisotropy $\boldsymbol{D}$.

2. In particular, we focus on four two-dimensional projections of the phase diagram obtained by fixing all the except one to the values in [2]. Thus we study the four two-dimensional planes $\langle J_1,D\rangle$, $\langle J_2,D\rangle$, $\langle J_3,D\rangle$ and $\langle J_4,D\rangle$.

3. For each plane, we compute the expectation value of string order parameters defined as follows $O^{12}=\lim_{|k-j|\to\infty}\left\langle L_j^{12}\prod_{l=j+1}^{k-1}\exp(i\pi L_l^{12})L_k^{12}\right\rangle$ and O^{34} (where L^{12} is replaced by L^{34}), with $L^{12}=|2\rangle\langle 2|-|-2\rangle\langle -2|$ and $L^{34}=|1\rangle\langle 1|-|-1\rangle\langle -1|$ in the basis of spin-2.

4. We also compute the degeneracies in the entanglement spectrum, the entanglement entropy of half an infinite chain and the ground state fidelity to help us certify a consistent picture.

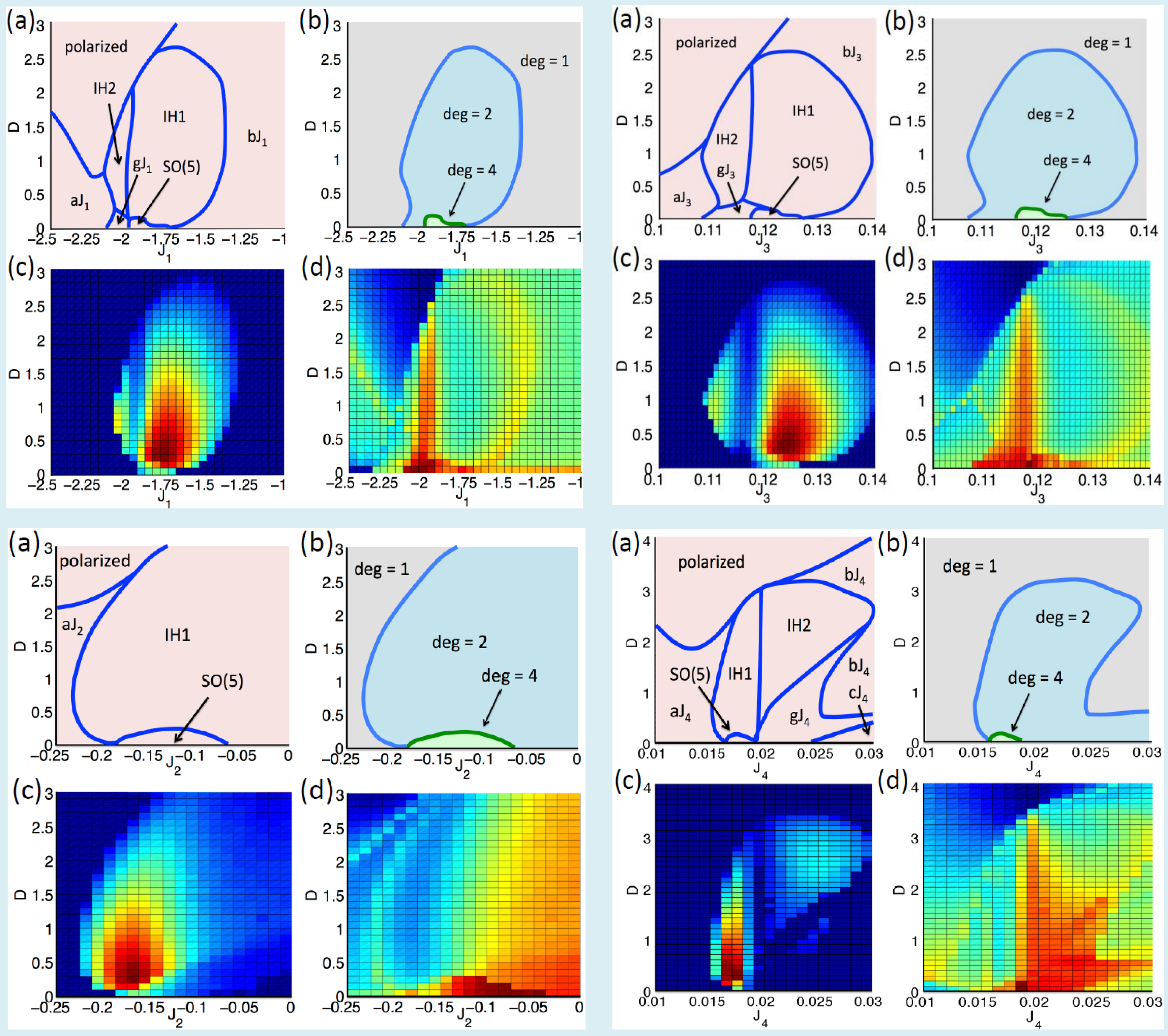

FIG. 1: (a) Phase diagram for the $\langle J_1,D\rangle$, $\langle J_2,D\rangle$, $\langle J_3,D\rangle$ and $\langle J_4,D\rangle$ plane; (b) degeneracies in the entanglement spectrum; (c)string order parameter O^{34} (aerial view); (d) entanglement entropy of half an infinite chain (aerial view).

Response to global quenches

1. We studied the time evolution of SO(5) and IH phases in the model. In particular, we focused on the following points: A and A′ in the SO(5) phase, B and B′ in the IH1 phase and C in the polarized phase.

2. We take the ground state for A and time-evolve it with the Hamiltonians for A′, B and C. We do the same with other points in the plane with respect to the Hamiltonian of other points.

3. The time evolution of the string order parameters O^{12} and O^{34} and the degeneracy of the entanglement spectrum were computed for short time scales. The results for the string order parameters are shown below:

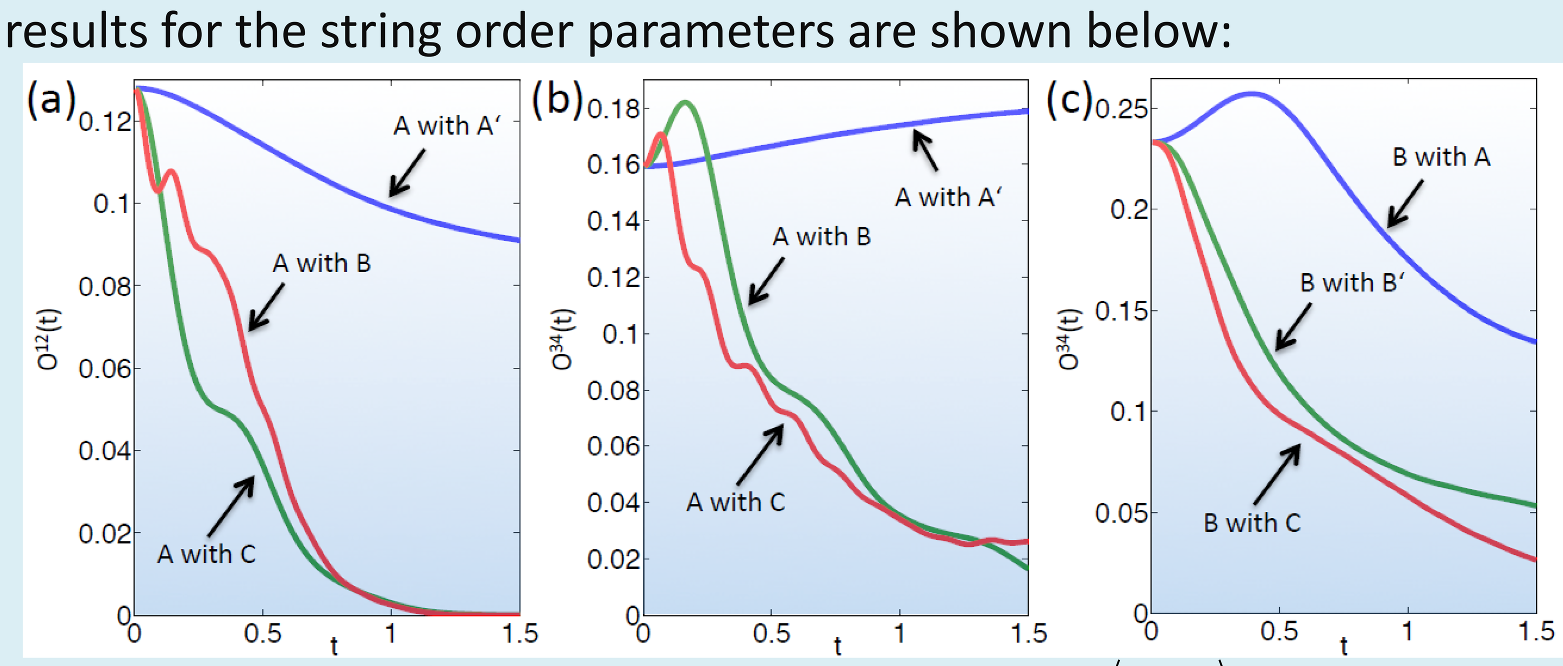

FIG. 2: Time evolution of string order parameters in the $\langle J_1,D\rangle$ plane under global quenches: (a) O^{12} for an initial state in the SO(5) phase; (b) O^{34} for an initial state in the SO(5) phase; (c) O^{34} for an initial state in the IH1 phase

Results

1. While studying the phase diagram of the four planes, we find many phases yet to be fully characterized. We also find small SO(5) phase with degeneracy 4 in the entanglement spectrum in all the cases. This SO(5)region also has non zero value of O^{12} (not shown) and non zero value of O^{34}.

2. There is also a large IH phase with degeneracy 2 in all the four cases. This phase is characterized by a non zero values of O^{34}.

3. From FIG. 2, we see that the SO(5) Haldane phase remains stable for short time scales as long as the evolving Hamiltonian is in the same phase. This is compatible with recent conclusions for a spin-1 quantum chain [4].

4. However, the same does not apply to the IH1 phase. For this phase, the symmetry protected topological order is already lost for short time scales even if the Hamiltonian has the same symmetries as the initial state.

Conclusions and further work

1. Beside showing the robustness of the SO(5) and Intermediate Haldane phase of the spin-2 quantum chain, we also find that the model has a very rich phase diagram including some new phases which according to the best of our knowledge has not yet been fully discussed in the literature. It will be interesting to understand these new phases and their properties.

2. From the time evolution, it also remains to be understood why the IH are unstable under global quenches unlike the SO(5) Haldane whenever the Hamiltonian have the same symmetry as the initial state.

References

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