MPS: (1+1)-D QED

Buyens, B., Haegeman, J., Van Acoleyen, K., Verschelde, H., & Verstraete, F., Phys. Rev. Lett. 113, 091601 (2014)

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Introduction

$SU(N)$ Yang-Mills-theories: Yang, C. N., & Mills, R. L. 1954, Physical Review, 96, 191

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}, D_\mu = \partial_\mu - igA_\mu, F_{\mu\nu} = \frac{i}{g}[D_\mu, D_\nu]$$

Aim: Tensor networks (TN) to tackle strong coupling regime numerically

- TN allow efficient representation of low-energy physics
- Hamiltonian formulation $\rightarrow$ suited for real-time evolution
- Construction in manifast gauge invariant way

The Schwinger model

= (1+1)-D QED with 1 flavour J. S. Schwinger, Phys. Rev. **125** (1962) 397.

$$\frac{H}{g} = \frac{1}{2\sqrt{x}}\sum_n \left(L_n^2 + \frac{\mu}{2}(-1)^n(\sigma_n^z + (-1)^n) + x(\sigma_n^+ e^{i\theta_n}\sigma_{n+1}^- + h.c.) \right),$$

Banks, T. and Susskind, Leonard and Kogut, John, 1976, Phys. Rev. D, 13, 1043-1053

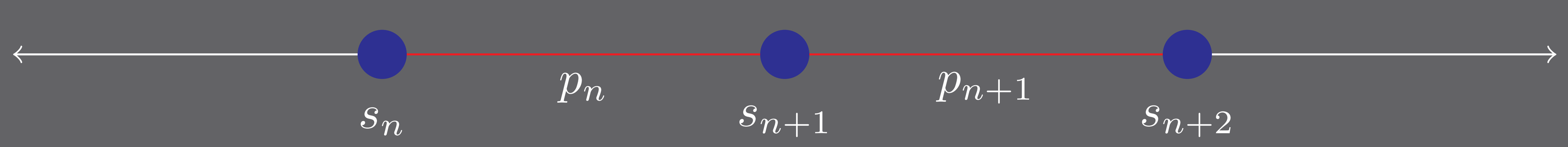
- Matter: spin operators σ_z on sites, $\sigma^\pm$ ladder operators
- Gauge fields: L angular operator on links, $e^{\pm i\theta}$ ladder operators
- $x = 1/(g^2a^2), \mu = 2m/(g^2a), a$ lattice spacing
- Continuum limit: $x = +\infty$

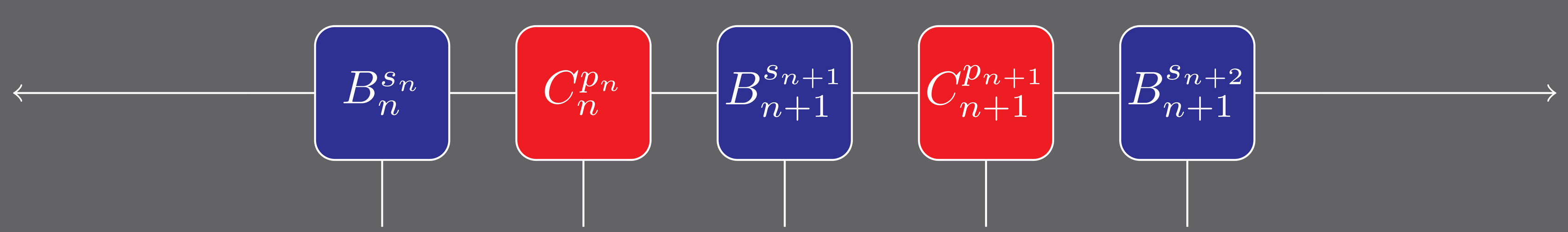
Symmetries:

- Local $U(1)$: $G_n = (L_n - L_{n-1} - [\sigma_n^z + (-1)^n]/2) = 0.$
- Translation symmetry over two sites
- CT-symmetry
  $\rightarrow C$ charge conjugation, T translation

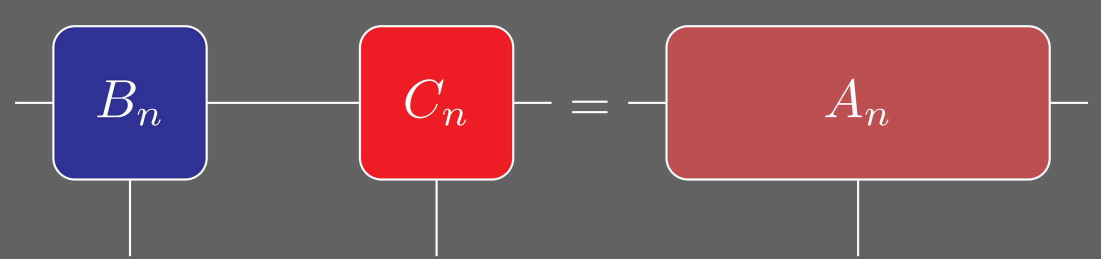
Ground state

MPS-ansatz: M. Fannes, B. Nachtergaele, and R.F., Werner, Comm. Math. Phys. **144**, 443 (1992)

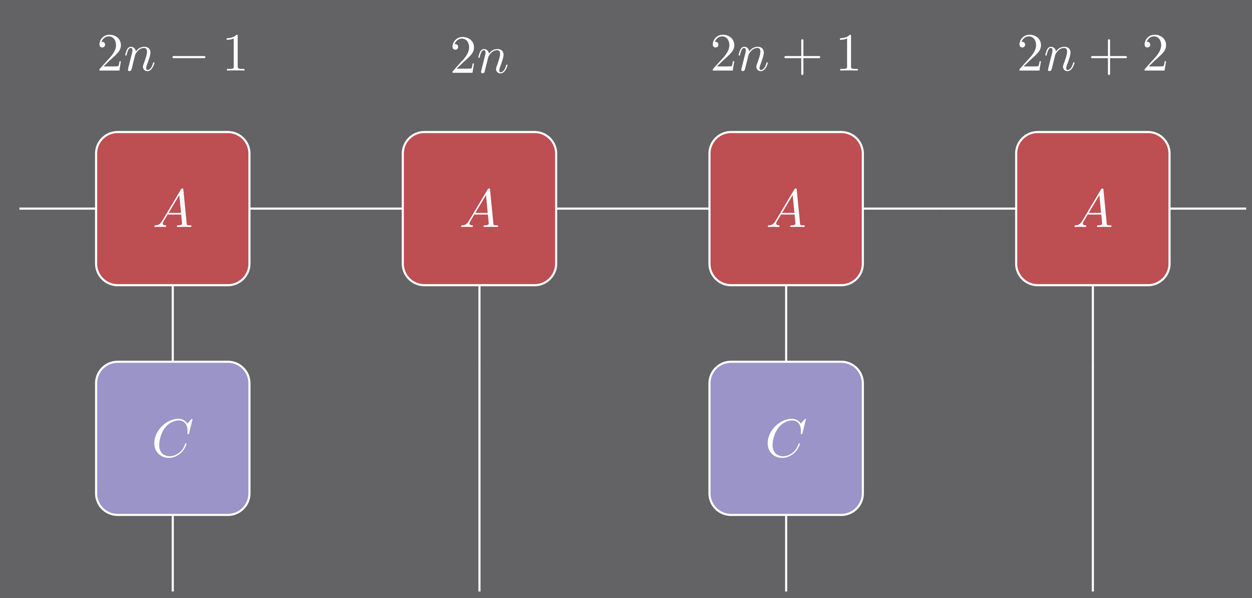




Block site n and link n:



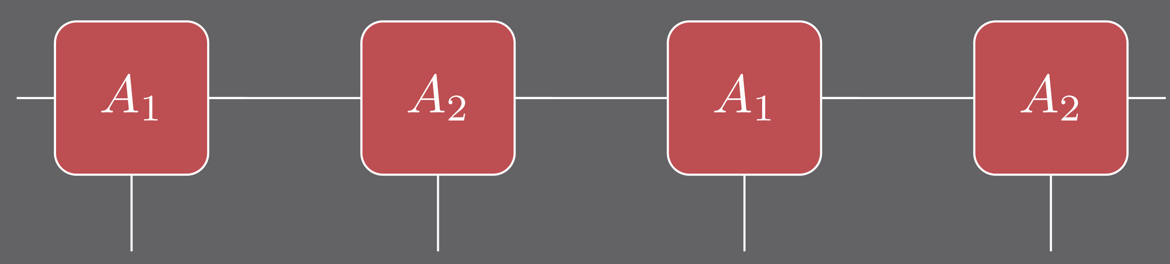
CT-symmetry:



- Gauge invariance: $[A^{s,p}]_{(q,\alpha);(r,\beta)} = \delta_{p,q+(s+1)/2}\delta_{r,-p}[a^{\{s,p\}}]_{\alpha,\beta}$
- Optimization via TDVP Haegeman, J., Osborne, T. J., & Verstraete, F. 2013, Phys. Rev. B, 88, 075133

Results

Real-time evolution. Quench α: $L_n \rightarrow L_n + \alpha$ in H : CT broken!
MPS:



- Invariant under translations over two sites
- Gauge invariance: $[A_n^{s,p}]_{(q,\alpha);(r,\beta)} = [a_n^{s,p}]_{\alpha,\beta}\delta_{p,q+(s+(-1)^n)/2}\delta_{r,p}$
- Real-time evolution with iTEBD Vidal, G. 2007, Physical Review Letters, 98, 070201

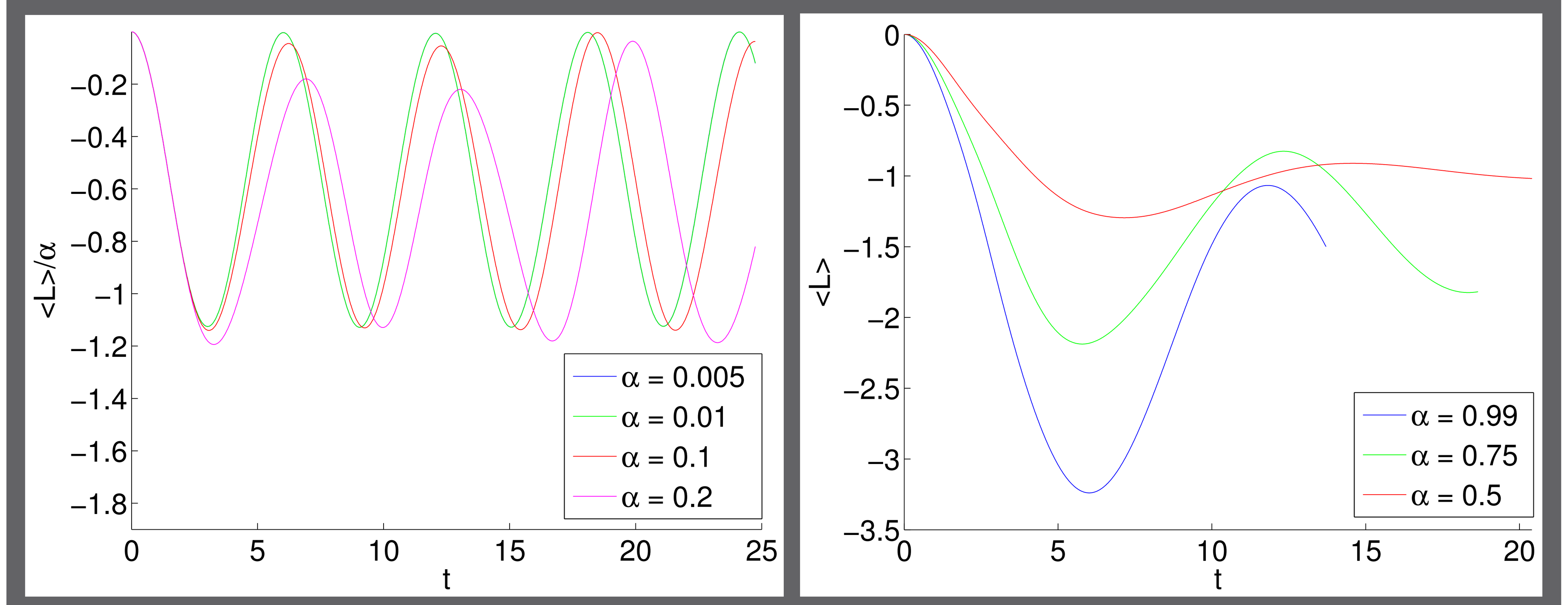
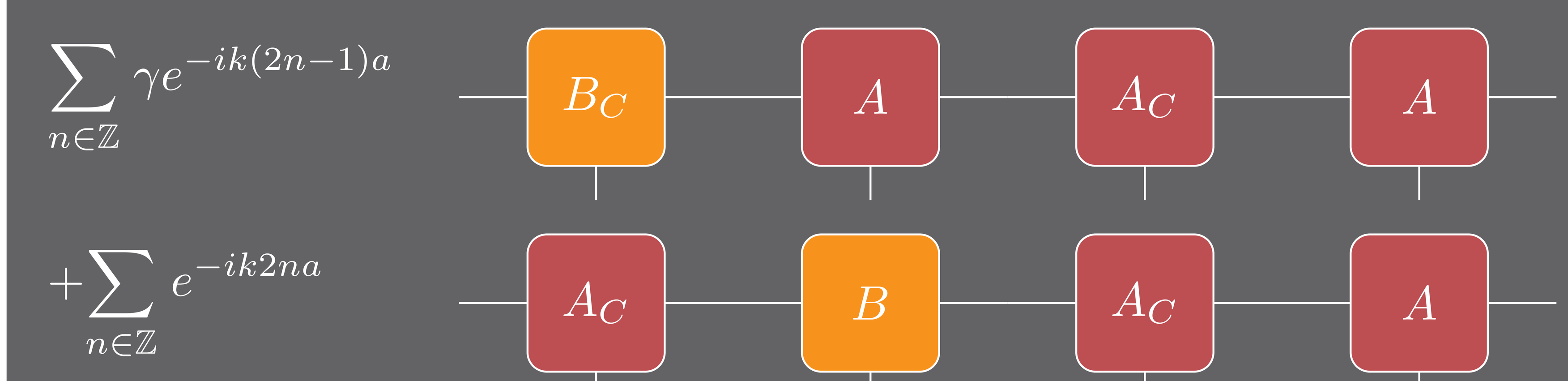


Figure: Real-time evolution ($g = 1, x = 100, m = 0.25$): Evolution of electric field within linear response regime (left) and beyond (right).

Excitations. Ansatz with momentum $k \in [-\pi/2a, \pi/2a[$ and C-number $\gamma \in \{\pm 1\}$. Haegeman, J., Pirvu, B., Weir, D. J., et al. 2012, Phys. Rev. B, 85, 100408



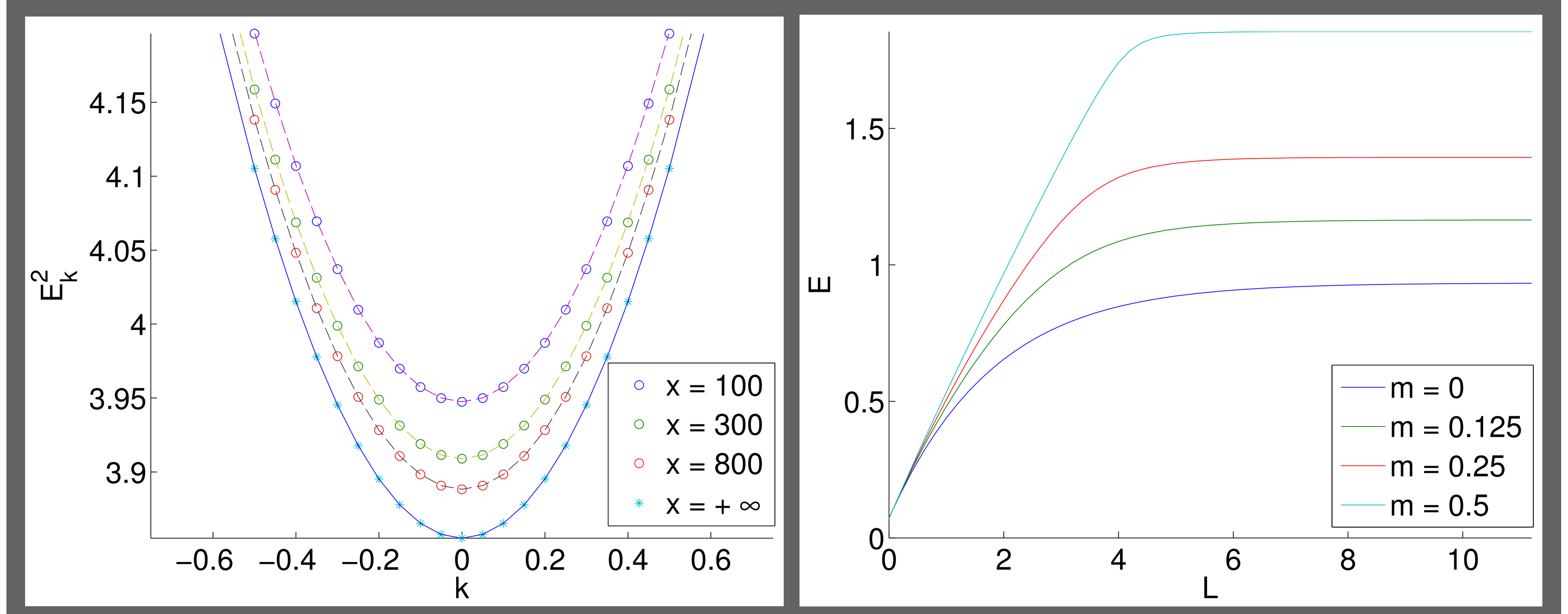
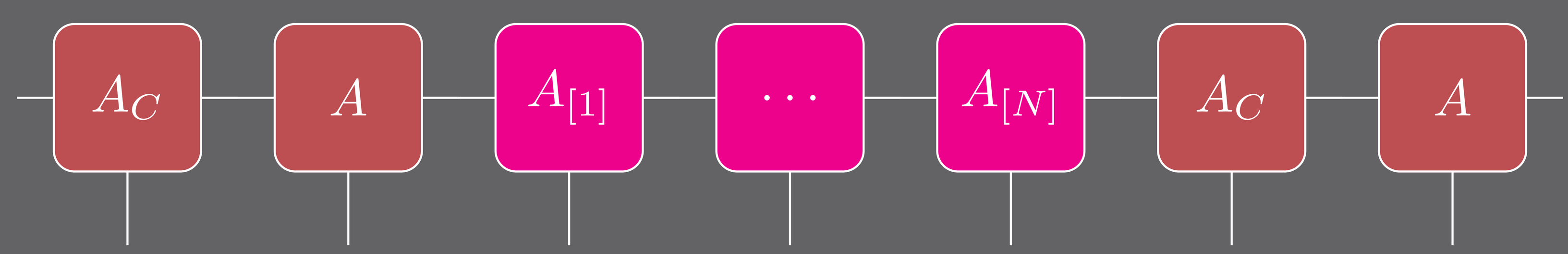


Figure: Left (a): Lorentz-dispersion relation for excitations. Energy squared vs. momentum ($g = 1, m = 0.75, \gamma = -1$). Right (b): String breaking. Energy vs. seperation length L between fermion and antifermion ($x = 100$, $g = 1$).

String breaking. Massive fermion-antifermion pair in the vacuum.

- Anti-fermion on site j_-, fermion on site j_+, charge q^{ext}
- Not dynamical, influence Gauss' law: $G_n + q^{ext}(\delta_{n,j_+} - \delta_{n,j_-}) = 0$

MPS-ansatz:



- Asymptotically: ground state
- Around fermion-antifermion pair: new variatonal freedom $A_{[n]}$
  $\rightarrow$ DMRG White, S. R. 1992, Physical Review Letters, 69, 2863
- $[A_{[n]}^{s,p}]_{(q,\alpha)(r,\beta)} = [a_{[n]}^{s,p}]_{\alpha,\beta}\ \delta_{p,q-\delta_{n,j_-}+\delta_{n,j_+}+(s+(-1)^n)/2}$