Rényi Entanglement Entropy of Interacting Fermions

A Continuous-time Quantum Monte Carlo Approach

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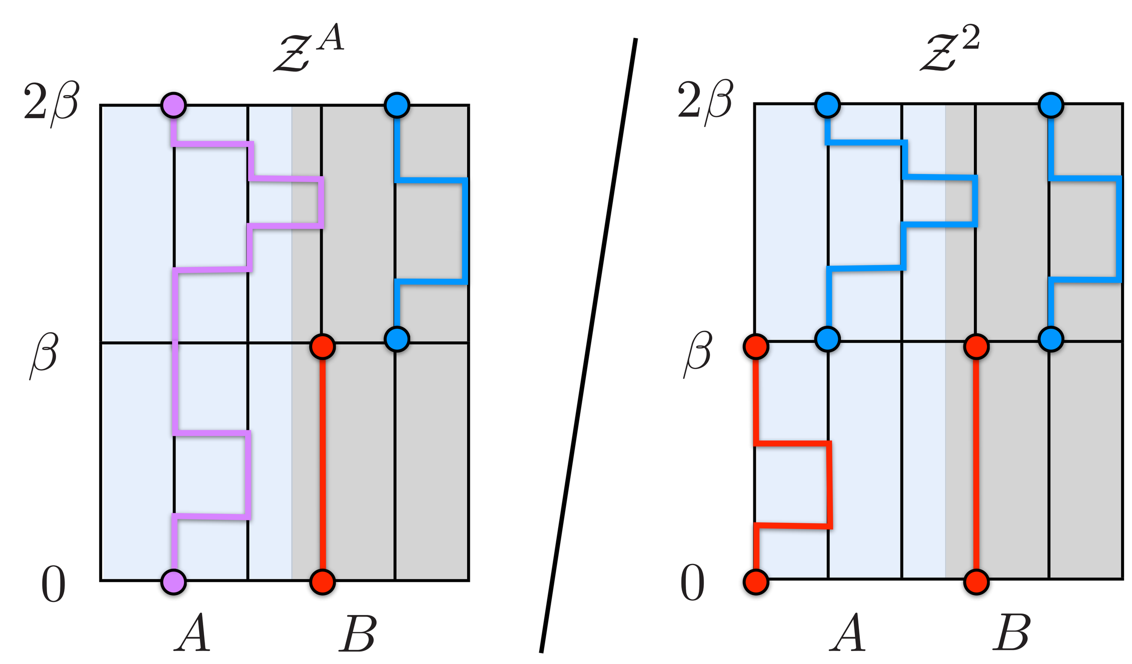
Rényi Entanglement Entropy

Rényi entanglement entropy is calculated using replica method in quantum Monte Carlo simulation

$$S_2 = -\ln\left[\mathrm{Tr}_A(\hat{\rho}_A^2)\right] = -\ln\left(\frac{\mathcal{Z}^A}{\mathcal{Z}^2}\right)$$

$$\hat{\rho}_A = \frac{1}{\mathcal{Z}}\mathrm{Tr}_B(e^{-\beta\hat{H}}) \qquad \mathcal{Z} = \mathrm{Tr}(e^{-\beta\hat{H}})$$

$$e^{-S_2} =$$



Implemented for interacting fermions [1], performs better than free fermion decomposition approach [2].

Area law scaling of entanglement entropy implies *exponentially* vanishing Monte Carlo signal.

CTQMC

$$\frac{\mathcal{Z}}{\mathcal{Z}_0} = \frac{1}{\mathcal{Z}_0}\mathrm{Tr}\left[e^{-\beta\hat{H}_0}\mathcal{T}e^{-\int_0^\beta d\tau\,\hat{H}_1(\tau)}\right] \qquad \hat{H} = \hat{H}_0 + \hat{H}_1$$

$$= \sum_{k=0}^{\infty}\int_0^\beta d\tau_1 \ldots \int_{\tau_{k-1}}^\beta d\tau_k \frac{(-1)^k}{\mathcal{Z}_0}\mathrm{Tr}\left[e^{-\beta\hat{H}_0}\hat{H}_1(\tau_k)\ldots\hat{H}_1(\tau_1)\right]$$

$$= \sum_{\mathcal{C}} w(\mathcal{C})$$

CTQMC offers an unbiased way to sample the ratio between *interacting* and *noninteracting* partition functions

Algorithm

$$S_2 = S_2^0 + \Delta S_2$$

Noninteracting entanglement entropy

$$S_2^0 = -\ln\left(\frac{\mathcal{Z}_0^A}{\mathcal{Z}_0^2}\right)$$

Easily calculated by correlation matrix method [3]

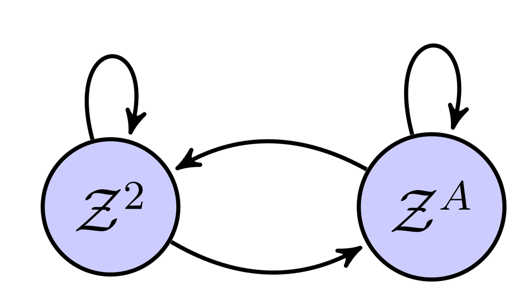
Interaction Corrections

$$\Delta S_2 = -\ln\left[\eta\frac{\mathcal{Z}^A/\mathcal{Z}_0^A}{(\mathcal{Z}/\mathcal{Z}_0)^2}\right] + \ln(\eta)$$

Sampled use CTQMC

Extended ensemble simulation

$$(\mathcal{Z}/\mathcal{Z}_0)^2 + \eta(\mathcal{Z}^A/\mathcal{Z}_0^A) = \sum_{\mathcal{C}}\left[w_{\mathcal{Z}^2}(\mathcal{C}) + \eta w_{\mathcal{Z}^A}(\mathcal{C})\right]$$



$$\Delta S_2 = -\ln\left[\frac{\langle\delta_{\mathcal{Z}^A}\rangle_{\mathrm{MC}}}{\langle\delta_{\mathcal{Z}^2}\rangle_{\mathrm{MC}}}\right] + \ln(\eta)$$

References

[1] P. Broecker and S. Trebst, J. Stat. Mech., P08015 (2014)

[2] T. Grover, Phys. Rev. Lett. **111**, 130402 (2013)

[3] I. Peschel, J. Phys. A: Math. Gen. **36**, L205 (2003)

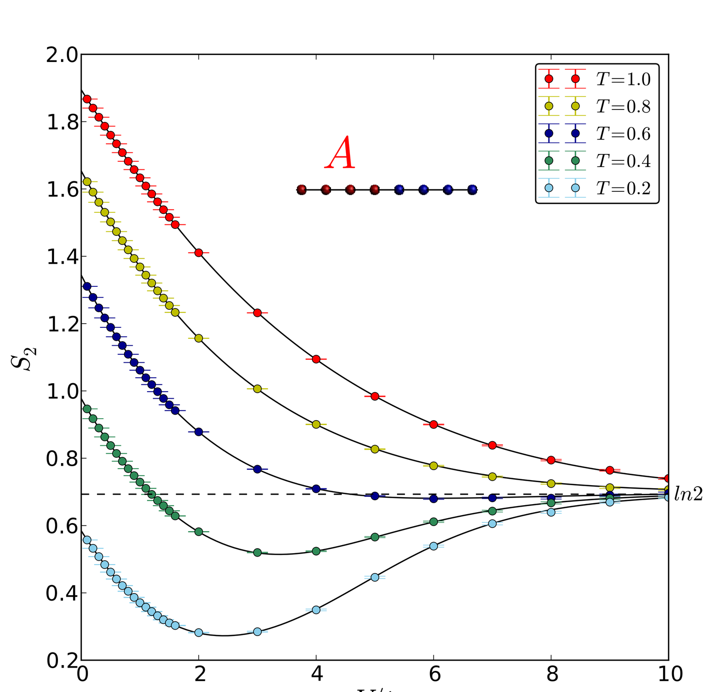
[4] J. E. Gubernatis, D. J. Scalapino, R. L. Sugar, and W. D. Toussaint, Phys. Rev. B **32**, 103 (1985)

arXiv:1407.0707
PRL in press

Benchmarks

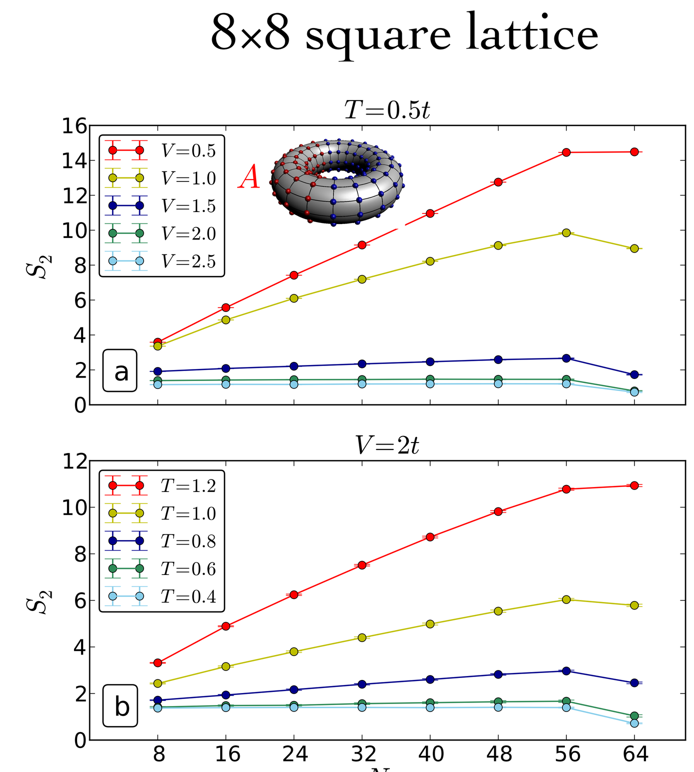
$$\hat{H} = -t\sum_{\langle\mathbf{i},\mathbf{j}\rangle}\left(\hat{c}_\mathbf{i}^\dagger\hat{c}_\mathbf{j} + \hat{c}_\mathbf{j}^\dagger\hat{c}_\mathbf{i}\right) + V\sum_{\langle\mathbf{i},\mathbf{j}\rangle}\left(\hat{n}_\mathbf{i} - \frac{1}{2}\right)\left(\hat{n}_\mathbf{j} - \frac{1}{2}\right)$$

8-site open chain

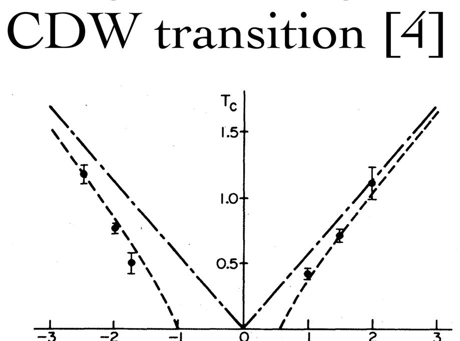


Agree perfectly with exact diagonalization (solid lines)

8×8 square lattice



Entanglement signature of a CDW transition [4]

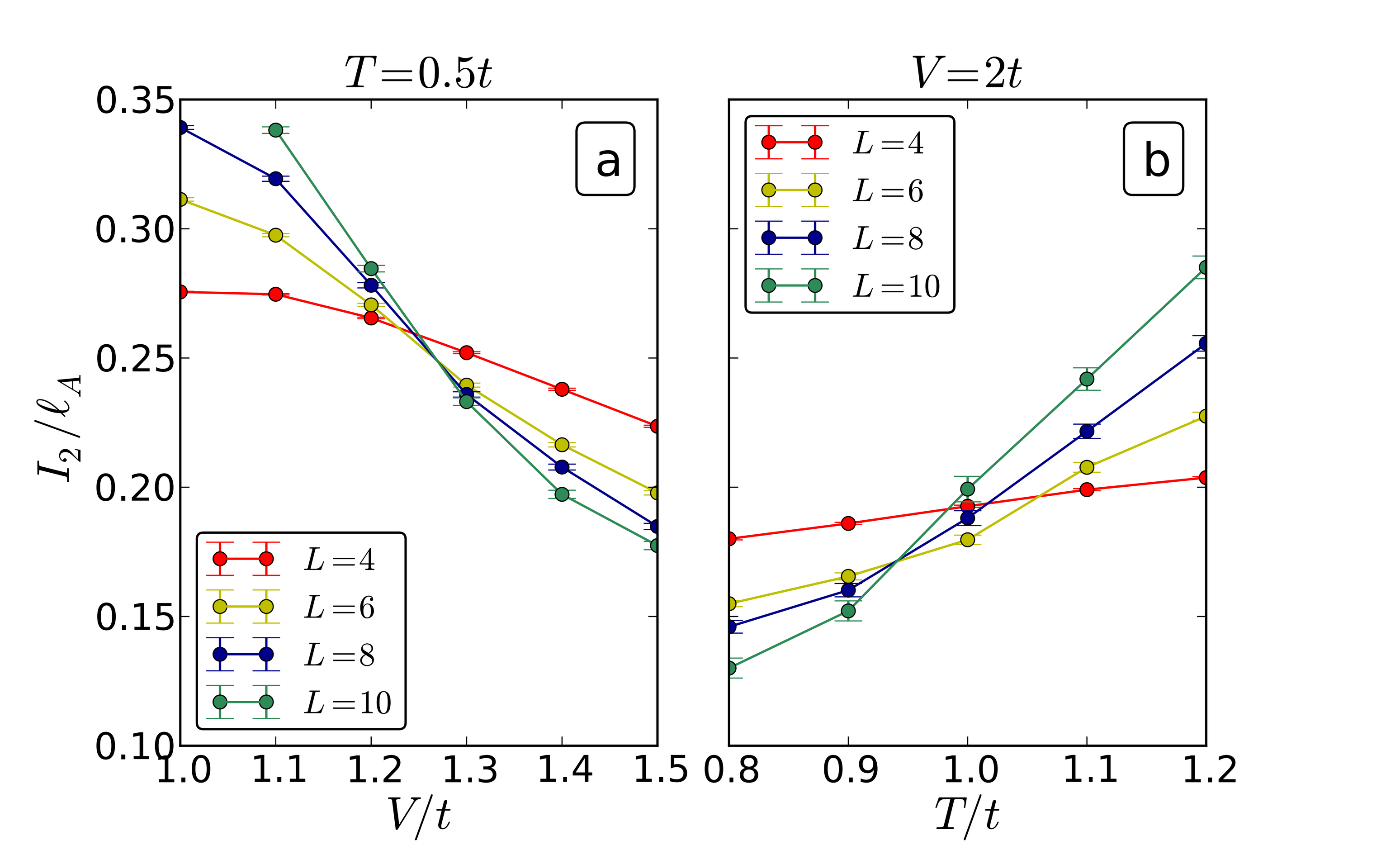


Critical Point

Square lattice t-V model undergoes an Ising phase transition to a CDW phase at large interaction or low temperature.

Mutual information crossing locates the critical point

$$I_2(A:B) = S_2(\hat{\rho}_A) + S_2(\hat{\rho}_B) - S_2(\hat{\rho}_{A\cup B})$$



FNSNF Swiss National Science Foundation
MaNEP
OLE Optical Lattice Emulator
QSIT Quantum Science and Technology, National Centre of Competence in Research