Entanglement detection in coupled particle plasmons

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Introduction

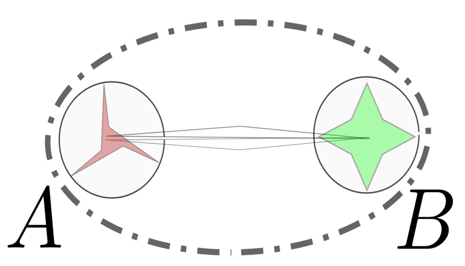
Entanglement: main concepts

Definition: a quantum state ρ in a composite Hilbert space $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$ is separable if $\rho = \sum_i p_i \rho_i^A \otimes \rho_i^B$ $(\sum_i p_i = 1)$

Otherwise the state is **entangled**; generalization to N subsystems gives definition of **multipartite** entangled state

Peres-Horodecki criterion: If a state is separable: **partial transpose** over one subsystem ***has positive eigenvalues.***

Entanglement witness: observable that signals the presence of many-body entanglement when its value lies above some threshold



Formalism: Gaussian states

Provides tools for studying ground state properties of Hamiltonians which are **second order polynomials of bosonic operators**: all properties of the system can be deduced from expectation values of a **finite** set of operators O or **moments**:

$$O \in \{x_n, p_n, x_n x_m, p_n p_m, x_n p_m\}$$

In particular, for correlation properties, such as entanglement, we only need one ingredient, **the covariance matrix** σ

$$\sigma_{ij} = \tfrac{1}{2}\langle R_i R_j + R_j R_i\rangle - \langle R_i\rangle\langle R_j\rangle$$

where $\boldsymbol{R^T} := (x_1, x_2, ..., x_N, p_1, p_2, ..., p_N)$

and $x_i = \frac{1}{\sqrt{2}}(a_i + a_i^\dagger)$, $p_i = \frac{1}{\sqrt{2}} i(a_i^\dagger - a_i)$

$a_n :=$ bosonic operators

C. Weedbrook *et al.* Reviews of Modern Physics 84, 621 (2012)

- **Alternative:**

Use an **entanglement witness** generalizing the idea of ***Duan et al.*** (2 modes),

$$W = \langle(\Delta(x_1 - x_2))^2\rangle_\rho + \langle(\Delta(p_1 + p_2))^2\rangle_\rho$$

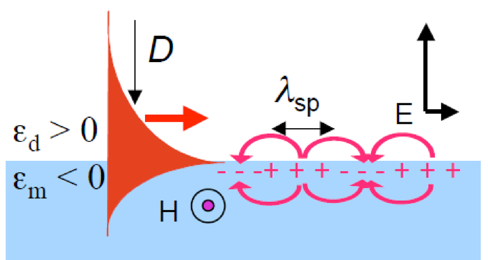
(W<1 = entanglement)

L. Duan *et al.* (PRL 84, 2722 (2000))

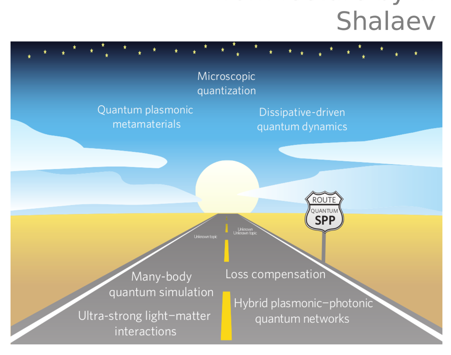
Why use plasmons

- **Achievements of Quantum Plasmonics:**

Many quantum properties of photons are preserved when these photons are coupled in and out of ***plasmonic modes*** (plasmons used as carriers)



from lecture by V. Shalaev



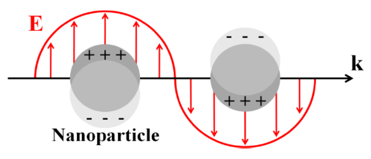
from M. S. Tame *et al.* Nature Physics 9, 329 (2013)

- **Our proposal**: use of quantum properties of plasmons ***themselves***; localized plasmonic resonances (**SPPs**) as entangled parties.

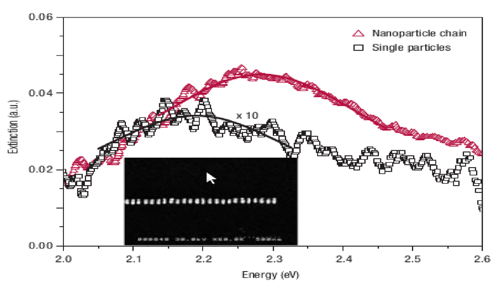
D. E. Chang, *et al*, Phys. Rev. Lett. 97, 053002 (2006).
D. Dzsotjan, *et al*, Phys. Rev. B 82, 075427 (2010).
A. Gonzalez-Tudela *et al.* (PRL 106, 020501 (2011))
E. Altewischer *et al.* (Nature 418,304 (2002))

Nanoparticle arrays

- **Nanoparticles: localized SP resonance** strong absorption/scattering strong local field enhancement high losses



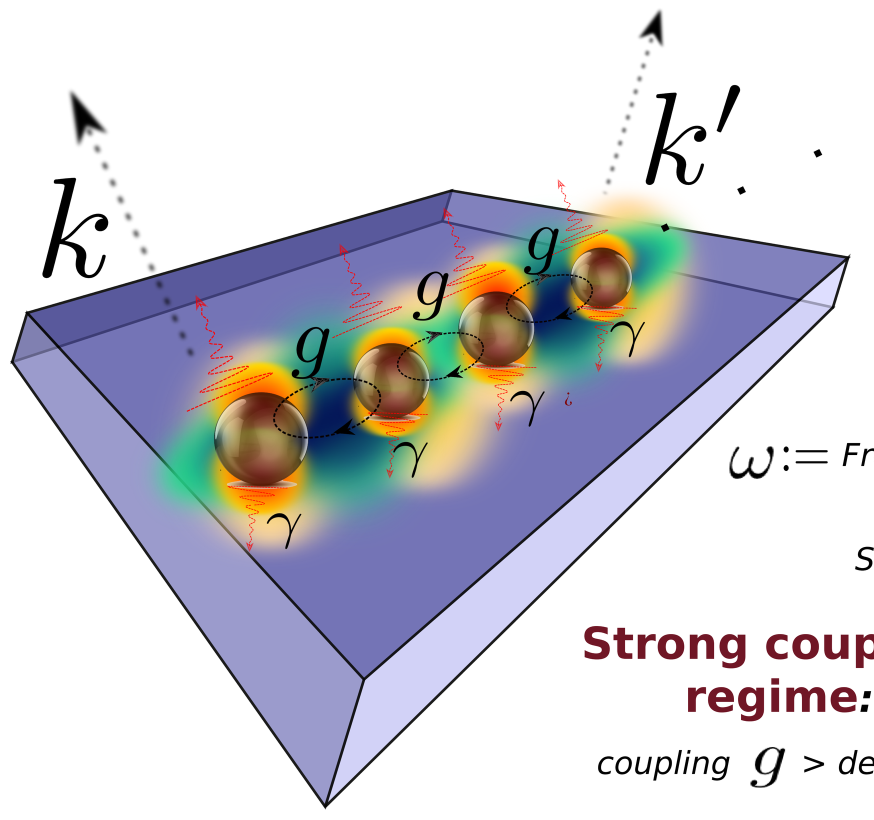
- **Nanoparticles arrays**: Earlier experiments revealed **short propagation lengths**, discouraging the use of such arrays for transport of quantum information



N. Harris *et al.* J. Phys. Chem. C 113, 2784 (2009);
S. A. Maier *et al*, PRB 65, 193408 (2002), Phys. Lett. 81,1714 (2002).
M. L. Brongersma *et al.*, PRB 62, R16356 (2000).

Effective model

1D Array of **metallic nanoparticles (Ag, Au)**: **coupling** of localized **plasmons**



$\omega :=$ *Frequency of the localized SPP resonance*

Strong coupling regime:

coupling g > *decay rate* γ

- **Coupled particle plasmons system**: oscillating dipoles forming a 1D array (open boundary conditions), which interact through a nearest-neighbor dipole coupling $(\hbar = 1)$

$$H = \sum_{n=1}^{N} \tfrac{\omega}{2}(x_n^2 + p_n^2) + \sum_{\langle n,m\rangle} g x_n x_m + \sum_n f_n(t) x_n$$

$x_n :=$ *dipole operator* $\quad p_n :=$ *conjugate momentum*

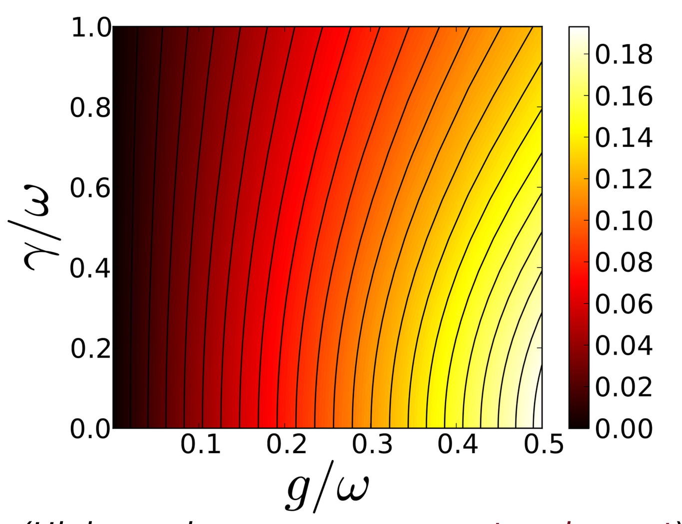
$f_n(t) :=$ *external driving*

- **Local dissipative dynamics** (spontaneous emission, absorption, etc) modeled through a **master equation** in Lindblad form

$$\partial_t \rho = -i[\rho, H] + \sum_{n=1}^{N} \tfrac{\gamma}{2}\left(2a_n\rho a_n^\dagger - a_n^\dagger a_n \rho - \rho a_n^\dagger a_n\right)$$

where $a_n = \frac{1}{\sqrt{2}}(x_n + ip_n)$

From **steady state (SS)** σ **under no driving**, ***(no effect on quantum correlations for Gaussian states)*** we calculate amount of entanglement: **logarithmic negativity (LN)**, between 2 partitions of 10 consecutive particles.



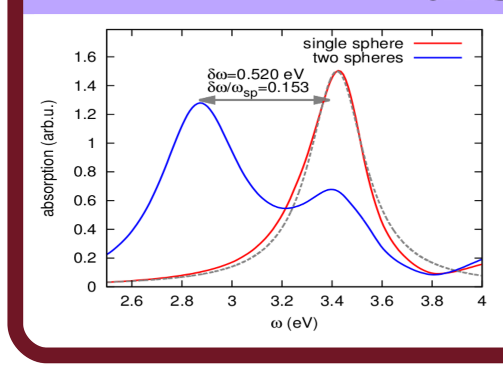
(Higher values mean more entanglement)

- **Conclussion**: both parties are **entangled**
- We check that any other bipartition is *entangled as well*: **Many-body multipartite entanglement**
- **Problem** if ***N*** is **large**

Needs sufficiently accurate reconstruction of σ

(**Experimentally difficult**)

System parameters: Coupling, dissipation



Lorentzian fit of **absorption spectrum** for Ag nanopart.

(radius $R = 25$ nm, *separation* $\Lambda = 2$ nm)

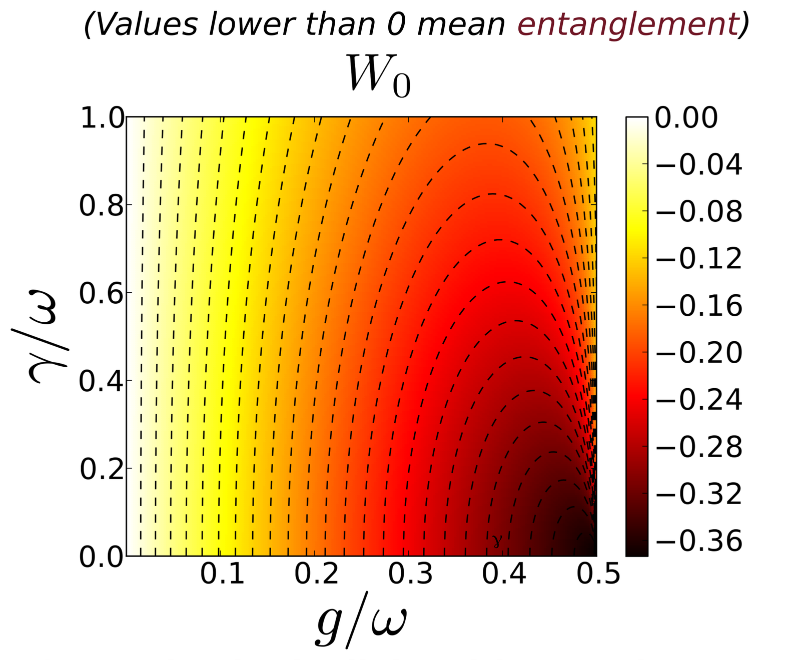
- Decay rate $\frac{\gamma}{\omega} \simeq 0.078$
- Coupling $\frac{g}{\omega} \simeq 0.153$

Detecting entanglement (SS)

- To measure **LN** is experimentally **difficult**
- We want to detect the **presence** of Entanglement with the **least number of measurements,** while being **robust to noise and imperfections: We propose** a witness, in the normal-mode (momentum) basis

$$W_k := \min\left\{0, \langle\Delta X_k^2\rangle + \langle\Delta P^2_{(\pi/\Lambda)-k}\rangle - 1\right\}$$

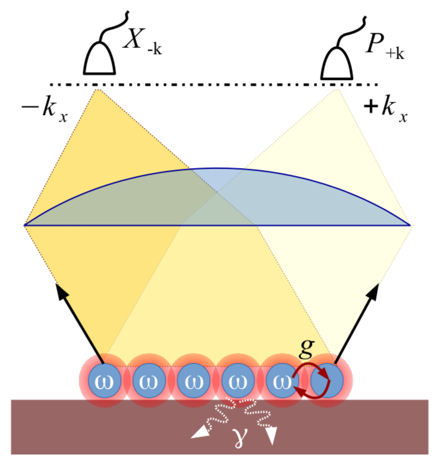
(Values lower than 0 mean entanglement)



- **Also valid for non-gaussian states :** irrespective of our model
- We also proof: **weakly dependence** on Temperature and also on k

- **Squeezing in the light with opposite momenta** is a signature of entanglement. Using fitted g and γ, we obtain $\simeq 12\%$ **of squeezing** which is **measurable**

- **Experimental setup:** measurements of fluctuations (homodyne) from **emitted light** collected by a lens at different points in the focal plane



Transport: propagation length

- We describe the **transport** of plasmonic excitations that are introduced by driving a single plasmon close to resonance

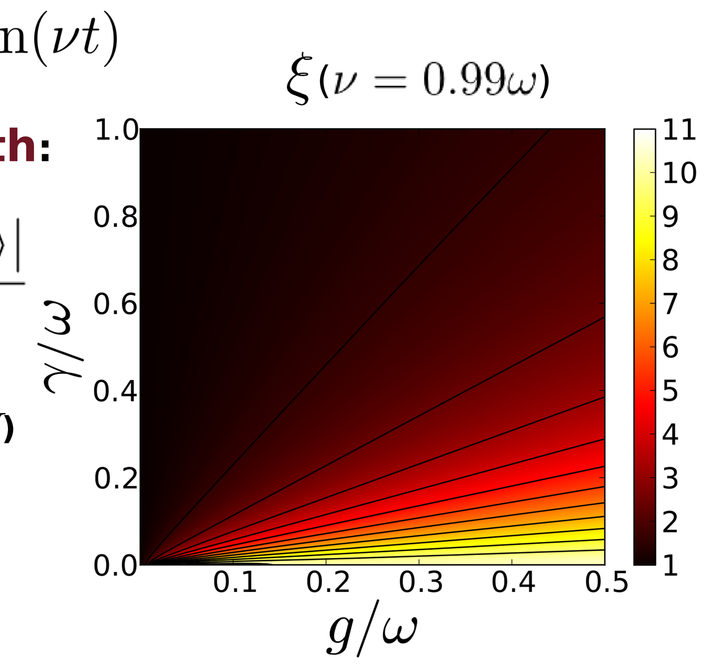
$$f_1(t) \sim \sin(\nu t)$$

Propagation length:

$$\xi = \frac{\sum_{n=1}^{N} n\Lambda|\langle x_n\rangle|}{\sum_{n=1}^{N} |\langle x_n\rangle|}$$

(Provided that, for big N)

$$\langle x_n\rangle \sim e^{-n\Lambda/\xi}$$



- Even when energy transport is not very efficient, (**as observed in experiment**) the array of plasmons **becomes entangled**!

Summary & Outlook

- We model a setup in which **multipartite entanglement** arises, estimating the strength of the measurement outcomes for realistic setups.
- We develop a quite general measure which detects it, **independent** of our modelling.
- Our predictions could be tested using **present-day state of the art technology,** and some of the ideas can be exported to nanophotonics, matter waves and coupled resonators in superconducting circuits.

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