Flavour physics (2)

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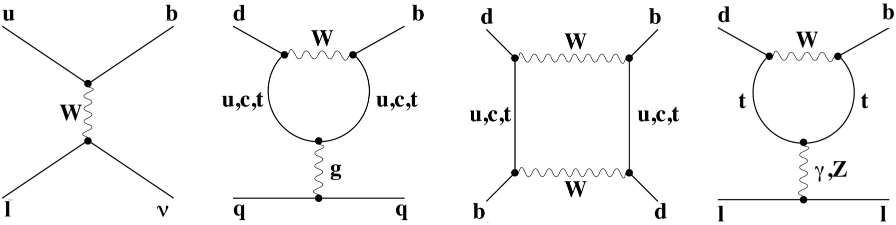
Outline

- Why and how flavour is useful
- Flavour in the Standard Model
  - $\Delta F = 1$ Flavour Changing Charged Currents: leptonic and semileptonic decays
  - $\Delta F = 2$ Flavour Changing Neutral Currents: neutral-meson mixing
  - $\Delta F = 1$ Flavour Changing Neutral Currents: strong penguins
- Hints of NP in flavour data

Processes of interest

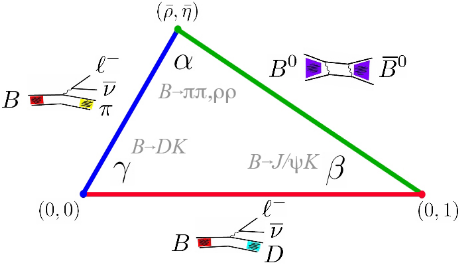


Semi/leptonic Penguins Mixing Radiative

	Semi/leptonic	Penguins	Mixing	Radiative
Process	$\Delta F = 1$ FCCC	$\Delta F = 1$ FCCC	$\Delta F = 2$ FCNC	$\Delta F = 2$ FCNC
NP sensitiv.	Small	Large ?	Large	Large
B	$B \to D\ell\nu$, $B \to \tau\nu$	$B \to \pi\pi$	$\Delta m_d, \Delta m_s$	$B \to K^* \mu\mu, B_s \to \mu\mu$
D	$D \to K\ell\nu$, $D_s \to \mu\nu$	$D \to K\pi$	x, y, ϕ	$D \to X_u \ell\ell$
K	$K \to \pi\ell\nu$, $\tau \to K\nu$	$K \to \pi\pi$	ϵ_K	$K \to \pi\nu\nu, K \to \mu\mu$

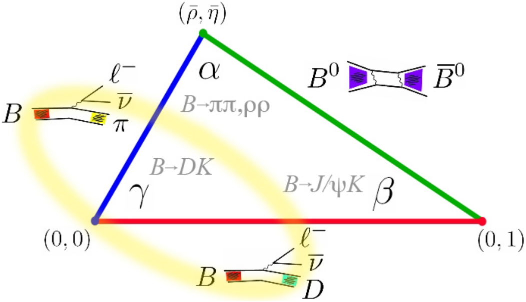
The SM point of view: the unitarity triangle

$$V = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \simeq \begin{bmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\bar\rho - i\bar\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\bar\rho-i\bar\eta) & -A\lambda^2 & 1 \end{bmatrix} + O(\lambda^4)$$

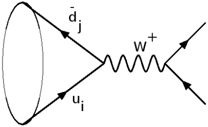


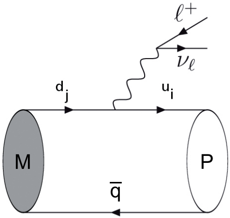
Exp. uncertainties		(Controlled) th. uncertainties	
$B \to \pi\pi, \rho\rho$	α	$B(b) \to D(c)\ell\nu$	$\lvert V_{cb}\rvert$ vs form factor (OPE)
$B \to DK$	γ	$B(b) \to \pi(u)\ell\nu$	$\lvert V_{ub}\rvert$ vs form factor (OPE)
		$M \to \ell\nu(\gamma)$	$\lvert V_{UD}\rvert$ vs f_M (decay cst)
$B \to J/\Psi K_s$	β	ϵ_K	$(\bar\rho, \bar\eta)$ vs B_K (bag parameter)
$B_s \to J/\Psi\phi$	β_s	$B_d\bar{B}_d$, $B_s\bar{B}_s$ mix	$\lvert V_{tb}V_{tq}\rvert$ vs $f_B^2 B_B$ (bag param)

$\Delta F = 1$ FCCC



Leptonic and semileptonic decays





- Leptonic, with f_M decay constant

$$B[M \to \ell\nu_\ell]_{\rm SM} = \frac{G_F^2 m_M m_\ell^2}{8\pi}\left(1-\frac{m_\ell^2}{m_M^2}\right)^2 |V_{q_u q_d}|^2 f_M^2 \tau_M (1+\delta_{em}^{M\ell 2})$$

- Semileptonic, with 2 form factors f_+ and f_0 (helicity suppression)

$$\frac{d\Gamma(M\to P\ell\nu)}{dq^2} = \frac{G_F^2|V_{q_u q_d}|^2}{24\pi^3}\frac{(q^2-m_\ell^2)^2\sqrt{E_P^2-m_P^2}}{q^4 m_H^2}$$
$$\times\left[\left(1+\frac{m_\ell^2}{2q^2}\right)m_H^2(E_P^2-m_P^2)|f_+(q^2)|^2+\frac{3m_\ell^2}{8q^2}(m_H^2-m_P^2)^2|f_0(q^2)|^2\right]$$

Processes of interest

	Leptonic	Semileptonic	Others
$\|V_{ud}\|$	$\pi \to \ell\nu_\ell, \tau \to \pi\nu_\tau$	$\pi^+ \to \pi^0 e^+ \nu_e$	nuclear β decays, neutron lifetime
$\|V_{us}\|$	$K \to \ell\nu_\ell, \tau \to K\nu_\tau$	$K \to \pi\ell\nu$	inclusive τ decays
$\|V_{cd}\|$	$D^+ \to \ell\nu_\ell$	$D \to \pi\ell\nu_\ell$	μ production by ν beams
$\|V_{cs}\|$	$D_s \to \ell\nu_\ell$	$D \to K\ell\nu_\ell$	$W \to cs$
$\|V_{ub}\|$	$B \to \tau\nu$	$B \to \pi\ell\nu_\ell$	$B \to X_u\ell\nu_\ell$
$\|V_{cb}\|$	$(B_c \to \tau\nu_\tau)$	$B \to D(^*)\ell\nu$	$B \to X_c\ell\nu_\ell$
$\|V_{tb}\|$	–	–	$t \to Wb$

- No direct handle on V_{td}, V_{ts}
- $B_c \to \tau\nu_\tau$ hard to reach in a foreseeable future
- Some processes not competitive in terms of theoretical/experimental accuracy

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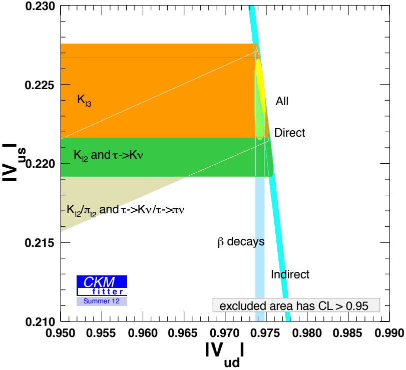
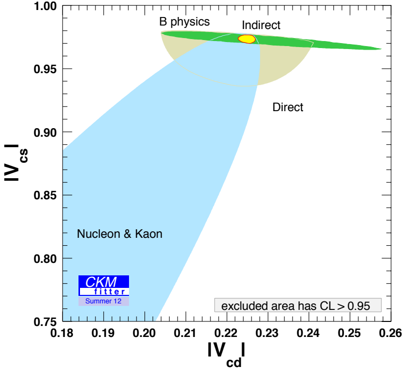
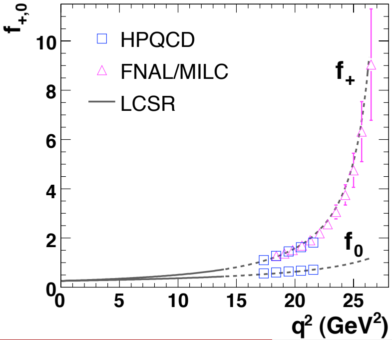




- "Direct" (semi- and leptonic) vs "indirect" (other sectors)
- $(|V_{ud}|, |V_{us}|)$: mostly from nuclear β decays + $K \to \pi\ell\nu$
- $(|V_{cd}|, |V_{cs}|)$: mostly from leptonic

More on hadronic quantities

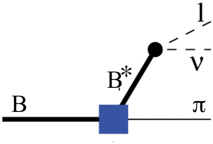
- Leptonic: decay constants easy to compute with lattice QCD
- Semileptonic: form factors more challenging, due to q^2-dep
  - experiment: $f_+(q^2)|V_{q_1 q_2}|$, theory: f_+ normalisation
  - $|V_{us}|, |V_{cd}|, |V_{cs}|$: $f_+(0)$ from lattice QCD, q^2-dependence fairly easy (from experiment, simple ansätze with smooth functions)
  - $|V_{ub}|, |V_{cb}|$: q^2 dependence trickier (B^* pole, large q^2 range)



- lattice simulations prefer small recoil situation, i.e. high q^2
- whereas experiments get data over the whole range, but more precise at low q^2
- worst case: $B \to \pi \ell \nu$: range of momenta (up to $q^2_{max} = 26$ GeV^2) too large for lattice

$|V_{ub}|$: parametrisations of form factors

- Pole, or modified pole parametrization



$$f_+(q^2) = \frac{f(0)}{(1-q^2/m^2_{B^*})(1-\alpha q^2/m^2_{B^*})}$$
$$f_0(q^2) = \frac{f(0)}{(1-1/\beta\cdot q^2/m^2_{B^*})}$$

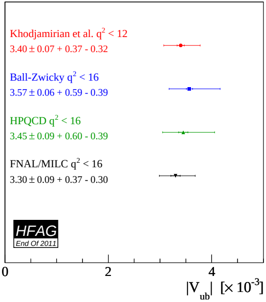
  - Simple with a limited number of elements
  - Good properties for $m_B \to \infty$

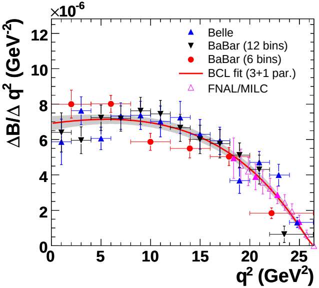
- z-expansion $z(q^2,t_0) = \frac{\sqrt{t_+-q^2}-\sqrt{t_+-t_0}}{\sqrt{t_+-q^2}+\sqrt{t_+-t_0}}$ $\quad t_+ = (m_B+m_\pi)^2 > t_0$

$$f_+(q^2) = \frac{1}{1-q^2/m^2_{B^*}}\sum_{n=0}^{N} a_n(t_0) z(q^2,t_0)^n$$

  - z maps analyticity region of f_+ into unit disc: series converges quickly (and choice of t_0 to improve the convergence)
  - More parameters, in principle improvable order by order
  - Bounds on coefficients a from asymptotic behaviour of $f_+(q^2)$

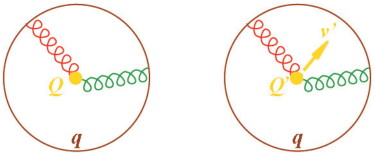
$|V_{ub}|$: exclusive determination





- Normalisation using lattice QCD (large q^2) or light-cone sum rules (small q^2)
- Fit to expermental data + lattice data + parametrisation of data

$|V_{cb}|$: parametrisation of form factors (1)



- Velocities $v_\mu = p_{B\mu}/M_B$, $v'_\mu = p_{D\mu}/M_D$
- Recoil energy of D in B rest frame $E = m_D(v \cdot v' - 1)$, with $v \cdot v'$ between 1 and 1.6

In the heavy quark limit $m_b, m_c \to \infty$, for $B \to D^{(*)}\ell\nu$

- Relations between D and D^* by heavy-quark symmetry on c spin
- In no-recoil limit $v = v'$, $b \to c$ unnoticed by light quark
- For $v \neq v'$, exchange of (soft) gluons to reorganise light cloud
- ...decreasing the overlap between initial B and final D

Form factors in terms of the Isgur-Wise function $\xi(v \cdot v')$, with $\xi(1) = 1$

$$B \to D: \quad \frac{M_B + M_D}{2\sqrt{M_B M_D}}\xi(v \cdot v') \quad = \quad f_+ = \left(1 - \frac{q^2}{M_B + M_D)^2}\right)^{-1} f_0$$

$$B \to D^*: \quad \frac{M_{B^*} + M_D}{2\sqrt{M_{B^*} M_D}}\xi(v \cdot v') \quad = \quad V = A_0 = A_2 = \left(1 - \frac{q^2}{M_{B^*} + M_D)^2}\right)^{-1} A_1$$

$|V_{cb}|$: parametrisation of form factors (2)

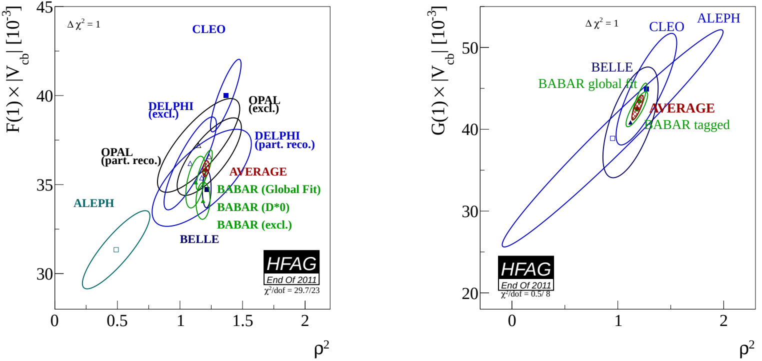
$$\frac{d\Gamma(B \to D^*\ell\nu)}{d\omega} = \frac{G_F^2}{48\pi^3}|V_{cb}|^2 m_{D^*}^3 \sqrt{\omega^2-1}P(\omega)|\mathcal{F}(\omega)|^2$$
$$\frac{d\Gamma(B \to D\ell\nu)}{d\omega} = \frac{G_F^2}{48\pi^3}|V_{cb}|^2 (m_B+m_D)^2 m_D^3 (\omega^2-1)^{3/2}|\mathcal{G}(\omega)|^2$$

- $\omega = v \cdot v'$, $P(\omega)$ phase space
- models for ξ: $\xi(\omega) = 1 - \rho^2(\omega-1) + O[(\omega-1)^2]$
- $\mathcal{F}$ and $\mathcal{G}$ form factors, related to ξ, include $1/M$-corrections

$$\mathcal{F}(\omega) = \eta_{QED}\eta_A\left[1 + O\left(\left(\frac{1}{m_b}-\frac{1}{m_c}\right)^2\right)\right] + (\omega-1)\rho^2 + O[(\omega-1)^2]$$
$$\mathcal{G}(1) = \eta_{QED}\eta_V\left[1 + O\left(\frac{M_B-M_D}{M_B+M_D}\right)\right]$$

- η_{QED} and $\eta_{A,V}$ perturbative corrections
- $\mathcal{F}(1)$ has no $1/m_Q$ corrections (Luke's theorem)

$|V_{cb}|$: exclusive determination



- Based on z-expansion version of $\mathcal{F}$ and $\mathcal{G}$
- Need $\mathcal{F}(1)$ and $\mathcal{G}(1)$ from lattice or sum rules
- Determination of $|V_{cb}|$ at 2% ($B \to D^*$) and 5% ($B \to D$)

$|V_{cb}|$ and $|V_{ub}|$ also extracted
from inclusive $B \to X_c \ell \nu$ and $B \to X_u \ell \nu$

Inclusive transitions

$$d\Gamma(B \to X_c \ell \nu) \propto \sum_X |\langle X|j_\mu|\bar{B}\rangle \bar{v}_\nu \gamma_\mu \gamma_5 u_e|^2 d\Phi_X$$

with $d\Phi_X$ phase space, $j_\mu = \bar{c}\gamma_\mu b$. It can be reexpressed as

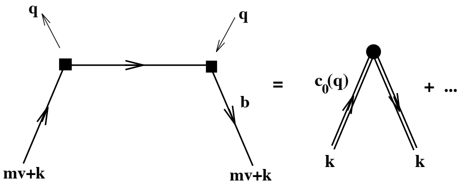
$$h_{\mu\nu} = \sum_X \delta^{(4)}(P - q_X - q)\langle \bar{B}(P)|j_\nu(0)|X\rangle\langle X|j_\mu(0)|\bar{B}\rangle$$

$$T_{\mu\nu} = i\int d^4x e^{-iq\cdot x}\langle \bar{B}(P)|T(j_\nu^\dagger(x) j_\mu(0)|\bar{B}\rangle$$

Optical theorem (cons. of probabilities)

$$h = \frac{1}{\pi}\mathrm{Im}\ T$$

expanded in $1/m_b$ and α_s



Operator product expansion

Performing a $1/m_b$-expansion of $\mathrm{Im}\ T$, integrating over phase space

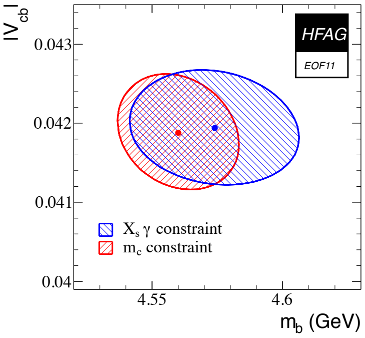
$$\Gamma(B \to X_c \ell\nu) = \frac{G_F |V_{cb}|^2}{192\pi^2} \left[z_0 \left(1 - \frac{\mu_\pi^2 - \mu_G^2}{2m_b^2}\right) - 2\left(1 - \frac{m_c^2}{m_b^2}\right)^4 \frac{\mu_G^2}{m_b^2} + \ldots \right]$$

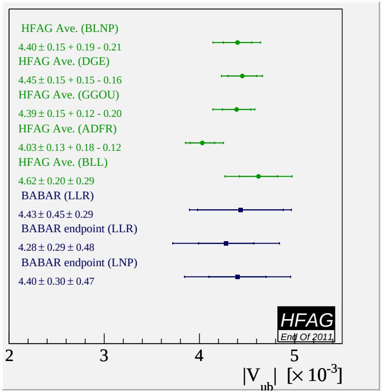
- z_0 function of $\frac{m_c^2}{m_b^2}$, ellipsis: higher orders in α_s and $1/m_b$
- μ_π^2 linked to movement of heavy quark inside meson
- μ_G^2 linked to b-spin (B, B^* splitting)

One can also compute moments of the differential decay rate

- Quark-hadron duality (sum of exclusive states can be computed at parton level) assumed valid for sufficiently inclusive quantities
- $B \to X_u \ell\nu$ more difficult: kinematical cuts to eliminate $B \to X_c \ell\nu$ need distribution of partons in B-meson (shape functions)

Determinations of $|V_{cb}|$ and $|V_{ub}|$





$|V_{cb}| \cdot 10^{-3}$

- inclusive 42.4 ± 0.9
- exclusive 39.5 ± 0.8

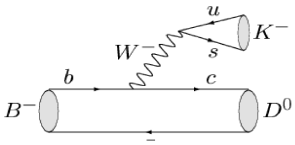
$|V_{ub}| \cdot 10^{-3}$

- inclusive $4.41 \pm 0.15^{+0.15}_{-0.17}$
- exclusive 3.23 ± 0.31

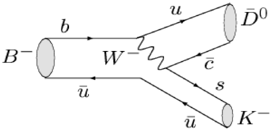
Only a marginal agreement in both cases (averaging procedure)

$\arg(V_{ub}^*) = \gamma$

γ angle from interference between $B^- \to D^0 K^-$ and $B^- \to \bar{D}^0 K^-$



colour allowed
$V_{cb} V_{us}^* \sim A\lambda^3$



colour suppressed
$V_{ub} V_{cs}^* \sim A\lambda^3(\rho - i\eta)$

Sensitivity depending on size of hadronic ratio

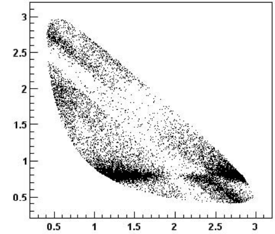
$$r_B e^{i\delta_B} = \frac{A_{supp}}{A_{fav}} \qquad r_B \sim \left| \frac{V_{ub} V_{cs}^*}{V_{cb} V_{us}^*} \right| \times 1/N_c \sim 0.1 - 0.2$$

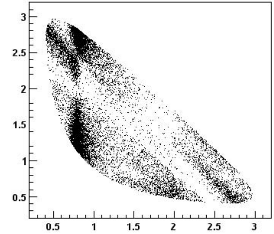
- GLW : D into CP eigenstates (KK, $\pi\pi$, $K_S\pi^0$, $K_S\omega$, $K_S\phi$)
- ADS : $D^{(*)}$ into doubly Cabibbo suppressed states
- GGSZ : $D^{(*)}$ into 3-body state and Dalitz analysis

GGSZ: $D^0, \bar{D}^0 \to K_S \pi^+ \pi^-$

Dalitz amplitude: function of $m_+^2 = s_{K_s\pi^+}$ and $m_-^2 = s_{K_S\pi^-}$

$$\bar{D}_0 \to K_S\pi^+\pi^- \sim f(m_+^2, m_-^2) \qquad D_0 \to K_S\pi^+\pi^- \sim f(m_-^2, m_+^2)$$



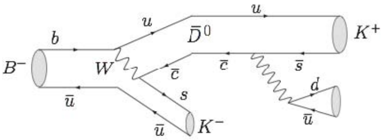


$$B^+ \to (K_S\pi^+\pi^-)_D K^+ \quad : \quad f(m_+^2, m_-^2) + r_B e^{i(\delta_B+\gamma)} f(m_-^2, m_+^2)$$
$$B^- \to (K_S\pi^+\pi^-)_D K^- \quad : \quad f(m_-^2, m_+^2) + r_B e^{i(\delta_B-\gamma)} f(m_+^2, m_-^2)$$

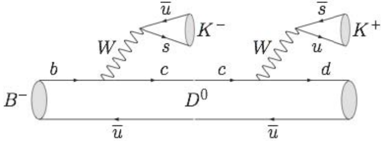
$\Longrightarrow$simultaneous fit of γ, r_B, δ_B + function f (model dependence)

ADS and GLW

- ADS: interf in rare $B^- \to [K^+\pi^-]_D K^-$ normalised to common rate



Colour suppressed +
Cabibbo favoured $V_{cs}V_{ud}^*$



Colour allowed +
Cabibbo suppressed $V_{cd}V_{us}^*$

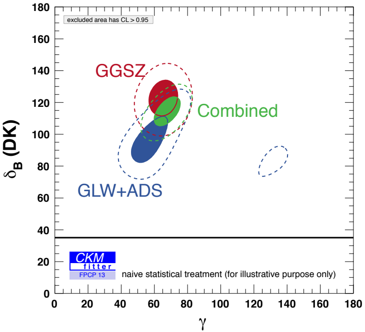
$$R_{DK} = \frac{\Gamma([K^+\pi^-]K^-) + \Gamma([K^-\pi^+]K^+)}{\Gamma([K^-\pi^+]K^-) + \Gamma([K^+\pi^-]K^+)} = r_B^2 + r_D^2 + 2 r_B r_D \cos(\delta_B + \delta_D)\cos\gamma$$

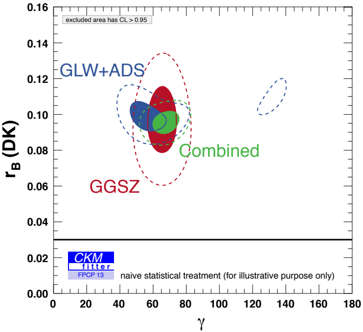
$$A_{DK} = \frac{\Gamma([K^+\pi^-]K^-) - \Gamma([K^-\pi^+]K^+)}{\Gamma([K^-\pi^+]K^-) + \Gamma([K^+\pi^-]K^+)} = 2 r_B r_D \sin(\delta_B + \delta_D)\sin\gamma / R_{DK}$$

with $r_D e^{i\delta_D} = A(D^0 \to K^+\pi^-)/A(\bar{D}^0 \to K^+\pi^-)$ and $r_D \simeq 0.06$

- GLW: decay into CP-eigenstate, with $R_{CP,\pm}, A_{CP,\pm}$
  - same structure as ADS, but CP eigenstate $\delta_D = 0, \pi$, $r_D = 1$
  - possible also with $D^* \to D^0\pi^0$ or $D^0\gamma$

γ and hadronic parameters





- GLW : D into CP eigenstates (KK, $\pi\pi$, $K_S\pi^0$, $K_S\omega$, $K_S\phi$)

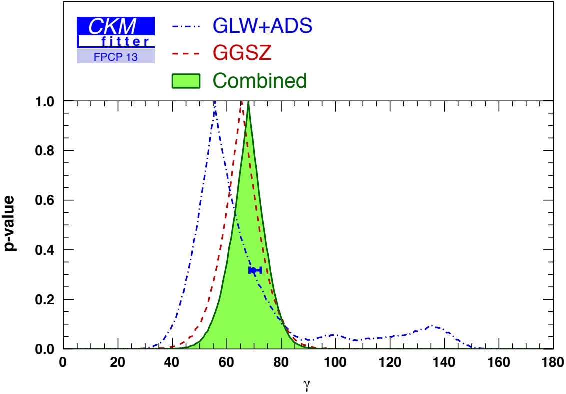
  r_B, δ_B, γ

- ADS : $D^{(*)}$ into doubly Cabibbo suppressed states

  $r_B, \delta_B, \gamma + r_D, \delta_D$ for $A(D \to f)$

- GGSZ : $D^{(*)}$ into 3-body state and Dalitz analysis

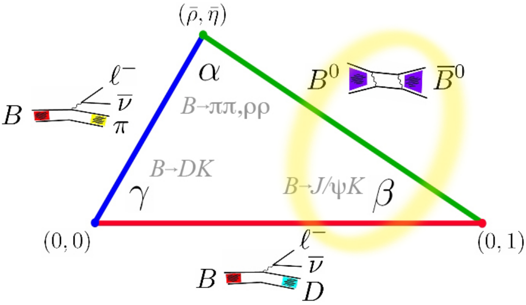
  $r_B, \delta_B, \gamma + A(D \to K_S h^+ h^-) = f(s_{Kh^+}, s_{Kh^-})$



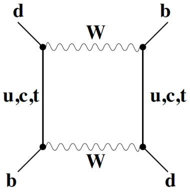
Including all modes for $B \to D(^*)K(^*)$: $\gamma = (68.0^{+8.0}_{-8.5})^\circ$

$\Longrightarrow$Same exercise possible for $B \to D(^*)\pi$ (but r_B 10 times smaller)

$\Delta F = 2$ FCNC



SM neutral-meson mixing

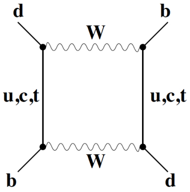


Neutral $B_q\bar{B}_q$ mixing in Standard Model, using effective Hamiltonian approach

$$A_{\Delta B=2} = \langle B_q|\mathcal{H}_{eff}|\bar{B}_q\rangle = \sum_{q,q'=u,c,t} (V^*_{qb} V_{qd})(V^*_{q'b} V_{q'd}) A_{qq'}$$

$$A_{\Delta B=2} = \sum_{q=u,c,t} V^*_{qb} V_{qd}[V^*_{tb} V_{td}[A_{tq} - A_{uq}] + V^*_{cb} V_{cd}[A_{cq} - A_{uq}]]$$

SM neutral-meson mixing



Neutral $B_q\bar{B}_q$ mixing in Standard Model, using effective Hamiltonian approach

$$A_{\Delta B=2} = \langle B_q|\mathcal{H}_{eff}|\bar{B}_q\rangle = \sum_{q,q'=u,c,t} (V_{qb}^* V_{qd})(V_{q'b}^* V_{q'd}) A_{qq'}$$

$$A_{\Delta B=2} = \sum_{q=u,c,t} V_{qb}^* V_{qd}[V_{tb}^* V_{td}[A_{tq} - A_{uq}] + V_{cb}^* V_{cd}[A_{cq} - A_{uq}]]$$

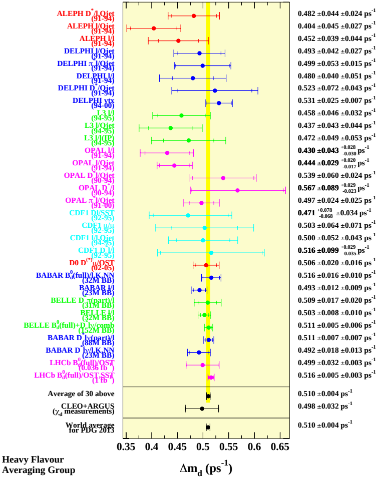
$$A_{qq'} \propto \frac{g^4}{16\pi^2 m_W^2}\left[c^{st} + \frac{m_q m_q'}{m_W^2} + \dots\right]\langle\bar{B}|(\bar{b}_L\gamma_\mu d_L)^2|B\rangle$$

Cst killed by GIM, and hierarchy of masses and CKM matrix elements:

$$A_{\Delta B=2} \propto (V_{tb}^* V_{tq})^2 \frac{g^4 m_t^2}{16\pi^2 m_W^4}\langle\bar{B}_q|(\bar{b}_L\gamma_\mu d_L)^2|B_q\rangle + \dots$$

- Δm_q related to $\langle\bar{B}_q|(\bar{b}_L\gamma_\mu d_L)^2|B_q\rangle$ (bag parameter B_{B_d})
- single weak phase: $q/p = \exp[i\arg[(V_{tb}^* V_{tq})^2]$

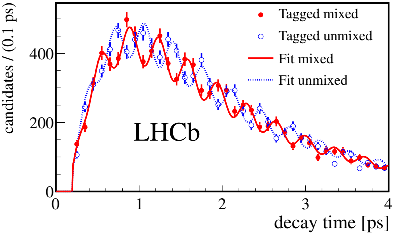
$\Delta m_d, \Delta m_s$



Flavour-specific decay $B_s \to D_s^- \pi^+$

$$\Gamma(B_s(t) \to f) \sim e^{-\Gamma_s t}[\cosh \frac{\Delta\Gamma_s t}{2} + \cos(\Delta m_s t)]$$

$$\Gamma(B_s(t) \to \bar{f}) \sim e^{-\Gamma_s t}[\cosh \frac{\Delta\Gamma_s t}{2} - \cos(\Delta m_s t)]$$



$$\Delta m_d = 0.510 \pm 0.004 \mathrm{ps}^{-1} \qquad \Delta m_s = 17.69 \pm 0.08 \mathrm{ps}^{-1}$$

B_d mixing: $B_d \to J/\psi K_S$

Interference between $B_d\bar{B}_d$ mixing and $\bar{b} \to \bar{c}s\bar{s}$ decay

$$\frac{\Gamma(\bar{B}^0(t) \to f_{CP}) - \Gamma(B^0(t) \to f_{CP})}{\Gamma(\bar{B}^0(t) \to f_{CP}) + \Gamma(B^0(t) \to f_{CP})} = S \sin(\Delta m t) - C \cos(\Delta m t)$$

$S \leftrightarrow$ weak phases due to mixing + decay (if dominated by 1 phase)

B_d mixing: $B_d \to J/\psi K_S$

Interference between $B_d\bar{B}_d$ mixing and $\bar{b} \to \bar{c}s\bar{s}$ decay

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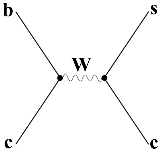
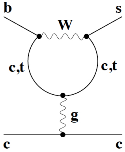
$S \leftrightarrow$ weak phases due to mixing + decay (if dominated by 1 phase)



tree

$V_{cb}V_{cs}^*$

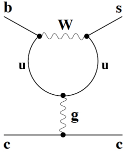
$O(\lambda^2)$ real



c,t penguins

$V_{cb}V_{cs}^*$ and $V_{tb}V_{ts}^*$

$O(\alpha_s\lambda^2)$ real



u penguins

$V_{ub}V_{us}^*$

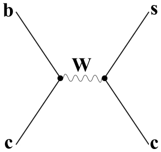
$O(\alpha_s\lambda^4)$

B_d mixing: $B_d \to J/\psi K_S$

Interference between $B_d \bar{B}_d$ mixing and $\bar{b} \to \bar{c} s \bar{s}$ decay

$$\frac{\Gamma(\bar{B}^0(t) \to f_{CP}) - \Gamma(B^0(t) \to f_{CP})}{\Gamma(\bar{B}^0(t) \to f_{CP}) + \Gamma(B^0(t) \to f_{CP})} = S \sin(\Delta m t) - C \cos(\Delta m t)$$

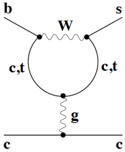
$S \leftrightarrow$ weak phases due to mixing + decay (if dominated by 1 phase)



tree

$V_{cb} V_{cs}^*$

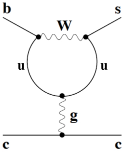
$O(\lambda^2)$ real



c,t penguins

$V_{cb} V_{cs}^*$ and $V_{tb} V_{ts}^*$

$O(\alpha_s \lambda^2)$ real



u penguins

$V_{ub} V_{us}^*$

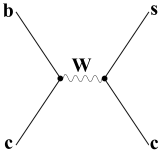
$O(\alpha_s \lambda^4)$

$S = \sin(\phi_{B_d}) = \sin(2\beta)$ in SM: golden channel for B factories

$\Longrightarrow \sin(2\beta) = 0.679 \pm 0.020$

B_s mixing: $B_s \to J/\psi\phi$

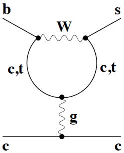
Interference between $B_s\bar{B}_s$ mixing and $\bar{b} \to \bar{c}s\bar{s}$ decay



tree

$V_{cb}V^*_{cs}$

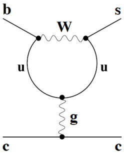
$O(\lambda^2)$ real



c,t penguins

$V_{cb}V^*_{cs}$ and $V_{tb}V^*_{ts}$

$O(\alpha_s\lambda^2)$ real



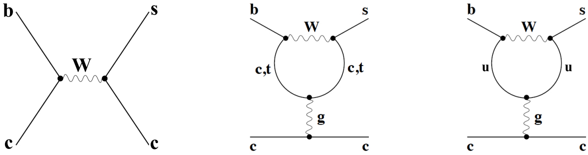
u penguins

$V_{ub}V^*_{us}$

$O(\alpha_s\lambda^4)$

B_s mixing: $B_s \to J/\psi\phi$

Interference between $B_s\bar{B}_s$ mixing and $\bar{b} \to \bar{c}c\bar{s}$ decay



tree	c,t penguins	u penguins
$V_{cb}V_{cs}^*$	$V_{cb}V_{cs}^*$ and $V_{tb}V_{ts}^*$	$V_{ub}V_{us}^*$
$O(\lambda^2)$ real	$O(\alpha_s\lambda^2)$ real	$O(\alpha_s\lambda^4)$

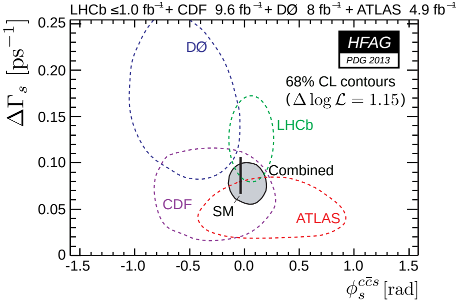
$S = \sin(\phi_{B_s})$: golden channel for Tevatron and LHC

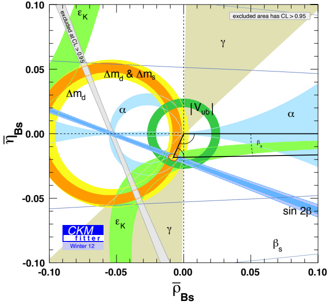
- $\phi_{B_s} = 2\beta_s = O(\lambda^2)$, and thus very small in SM
- Very strong constraint on NP
- $J/\psi\phi$ not CP-eigenstate
  - 3 possible helicity states (same for J/ψ and ϕ)
  - three amplitudes $A_0, A_\perp, A_{||}$ with definite CP-parity
  - ang. analysis to separate $A_{0,\perp,||}$, determine relative strong phases

B_s mixing

- $\Delta\Gamma_s$ difference of widths
- ϕ_{Bs} mixing phase describing B_{sH}, B_{sL} in terms of $B_s, \bar{B}_s$
  - $\phi_{Bs} = (2.3^{+5.7}_{-7.4})^\circ$
  - In SM, $\phi_{Bs} = 2\arg(V_{cs}V_{cb}^*/V_{ts}V_{tb}^*) = -2.1^\circ \pm 0.1^\circ$
- stringent constraint on NP models, but not competitive in SM

(cf B_s unitarity triangle)





K mixing

If K mass eigenstates were CP-eigenstates

- $|K_S\rangle \to (|K_0\rangle + |\bar{K}_0\rangle)/\sqrt{2} \to 2\pi$
- $|K_L\rangle \to (|K_0\rangle - |\bar{K}_0\rangle)/\sqrt{2} \to 3\pi$

$$\eta_{00} = \frac{\langle \pi^0\pi^0|\mathcal{H}|K_L\rangle}{\langle \pi^0\pi^0|\mathcal{H}|K_S\rangle} \qquad \eta_{+-} = \frac{\langle \pi^+\pi^-|\mathcal{H}|K_L\rangle}{\langle \pi^+\pi^-|\mathcal{H}|K_S\rangle}$$

measure CP-violation and can be analysed in isospin final states

$$\langle(\pi\pi)_I|\mathcal{H}|K_0\rangle = a_I e^{i\delta_I} \qquad \langle(\pi\pi)_I|\mathcal{H}|\bar{K}_0\rangle = a_I^* e^{i\delta_I} \qquad a_I = |a_I| e^{i\theta_I}$$

with δ, θ strong, weak phases

One can decompose $\pi^0\pi^0$ and $\pi^+\pi^-$ over $\pi\pi$ isospin states

$$\epsilon_K = (\eta_{00} + 2\eta_{+-})/3 \qquad \epsilon'_K = (-\eta_{00} + \eta_{+-})/3$$

to "separate" the contributions from a_0 and a_2 to the decay amplitudes

$$\epsilon_K = \frac{1-\lambda_0}{1+\lambda_0} = \frac{\langle(\pi\pi)_{I=0}|K_L\rangle}{\langle(\pi\pi)_{I=0}|K_S\rangle} \qquad \lambda_0 = \frac{q}{p}\frac{\bar{A}_{(\pi\pi)_{I=0}}}{A_{(\pi\pi)_{I=0}}}$$

$$\epsilon_K \simeq \frac{1}{2}[1-\lambda_0] \simeq \frac{1}{2}\left(1-\left|\frac{q}{p}\right| - i\mathrm{Im}\lambda_0\right) + O(\epsilon_K^2)$$

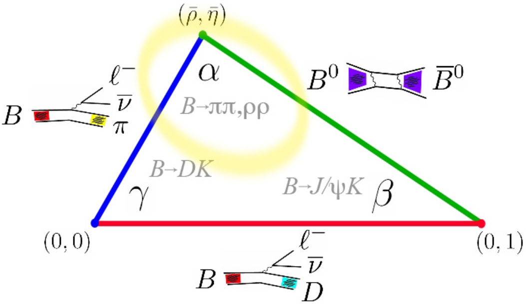
ϵ_K can be reexpressed as $\quad \epsilon_K = \sin\phi_\epsilon e^{i\phi_\epsilon}\left[\frac{\mathrm{Im}M_{12}}{\Delta M_K} + \xi\right]$

- ΔM_K and $\phi_\epsilon = \arctan(-2\Delta M/\Delta\Gamma) \simeq \pi/4$ from experiment
- $\xi = \mathrm{Im}A_0/\mathrm{ReA}_0$ estimated from ϵ'/ϵ
- $\mathrm{Im}M_{12}$ from effective Hamiltonian
  - keep only lowest-dimension contribution $\mathrm{Im}M_{12}^{(6)}$
  - tt, ct, cc boxes $\otimes$ $\hat{B}_K \propto \langle\bar{K}^0|(\bar{s}d)_{V-A}(\bar{s}d)_{V-A}|K^0\rangle$ bag parameter

$$\epsilon_K \simeq C_\epsilon \hat{B}_K \lambda^2 \bar{\eta}^2 |V_{cb}|^2 [|V_{cb}|^2(1-\bar{\rho})\eta_{tt}S_0(x_t) + \eta_{ct}S_0(x_t,x_c) - \eta_{cc}S_0(x_c)]$$

- hyperbola shape in $(\bar{\rho},\bar{\eta})$
- short-distance (Inami-Lim) functions $S_0(x_q = m_q^2/m_W^2)$

$\Delta F = 1$ FCNC



Strong penguins: $\bar{b} \to \bar{s}s\bar{s}$

Interference between $B_d\bar{B}_d$ mixing and $\bar{b} \to \bar{s}s\bar{s}$ decay

$$\frac{\Gamma(\bar{B}^0(t) \to f_{CP}) - \Gamma(B^0(t) \to f_{CP})}{\Gamma(\bar{B}^0(t) \to f_{CP}) + \Gamma(B^0(t) \to f_{CP})} = S \sin(\Delta m t) - C \cos(\Delta m t)$$

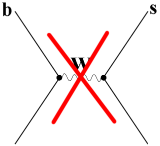
$S \leftrightarrow$ weak phases due to mixing + decay (if dominated by 1 phase)

Strong penguins: $\bar{b} \to \bar{s}s\bar{s}$

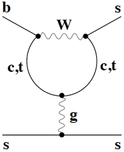
Interference between $B_d \bar{B}_d$ mixing and $\bar{b} \to \bar{s}s\bar{s}$ decay

$$\frac{\Gamma(\bar{B}^0(t) \to f_{CP}) - \Gamma(B^0(t) \to f_{CP})}{\Gamma(\bar{B}^0(t) \to f_{CP}) + \Gamma(B^0(t) \to f_{CP})} = S \sin(\Delta m t) - C \cos(\Delta m t)$$

$S \leftrightarrow$ weak phases due to mixing + decay (if dominated by 1 phase)



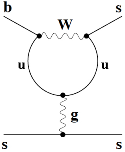
no tree



c,t penguins

$V_{cb} V_{cs}^*$ and $V_{tb} V_{ts}^*$

$O(\alpha_s \lambda^2)$ real



u penguins

$V_{ub} V_{us}^*$

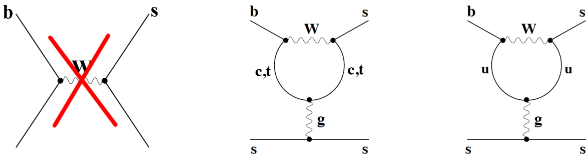
$O(\alpha_s \lambda^4)$

Strong penguins: $\bar{b} \to \bar{s}s\bar{s}$

Interference between $B_d\bar{B}_d$ mixing and $\bar{b} \to \bar{s}s\bar{s}$ decay

$$\frac{\Gamma(\bar{B}^0(t) \to f_{CP}) - \Gamma(B^0(t) \to f_{CP})}{\Gamma(\bar{B}^0(t) \to f_{CP}) + \Gamma(B^0(t) \to f_{CP})} = S \sin(\Delta m t) - C \cos(\Delta m t)$$

$S \leftrightarrow$ weak phases due to mixing + decay (if dominated by 1 phase)



no tree

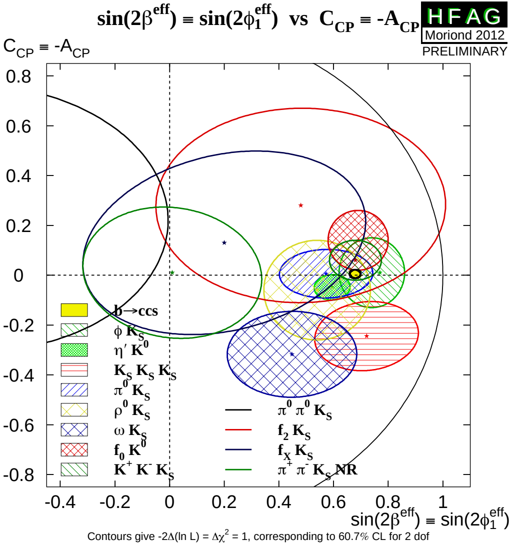
c,t penguins
$V_{cb}V^*_{cs}$ and $V_{tb}V^*_{ts}$
$O(\alpha_s\lambda^2)$ real

u penguins
$V_{ub}V^*_{us}$
$O(\alpha_s\lambda^4)$

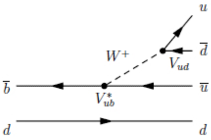
Pollution from u penguins (10% ?): $|\sin 2\beta_{eff} - \sin 2\beta| \leq O(0.1)$ in SM

sin(2β) and sin(2β_{eff})

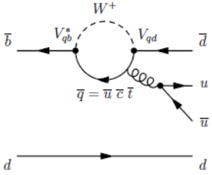
- Used to be followed closely because of discrepancies with sin(2β)
- Best measurements (high stat, control of systematics) do not indicate significant deviation
- Possiblity to combine B_d and B_s penguin-mediated modes to improve theoretical control



α: penguin pollution in $B \to \pi^+\pi^-$



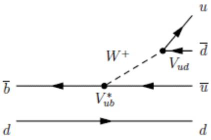
Tree



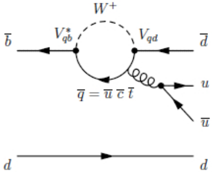
Penguin

$$A(B^0 \to \pi^+\pi^-) = V_{ud} V^*_{ub} t + \sum_{q=u,c,t} V_{qd} V^*_{qb} p_q$$

α: penguin pollution in $B \to \pi^+\pi^-$



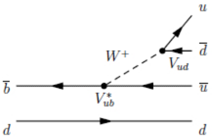
Tree



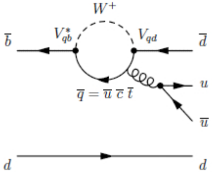
Penguin

$$A(B^0 \to \pi^+\pi^-) = V_{ud}V_{ub}^*(t + p_u) + (-V_{ud}V_{ub}^* - V_{td}V_{tb}^*)p_c + V_{td}V_{tb}^* p_t$$

α: penguin pollution in $B \to \pi^+\pi^-$



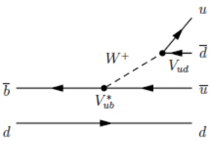
Tree



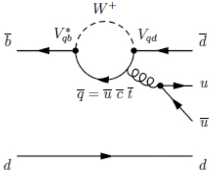
Penguin

$$A(B^0 \to \pi^+\pi^-) = V_{ud} V^*_{ub} t^{+-} + V_{td} V^*_{tb} p^{+-}$$

α: penguin pollution in $B \to \pi^+\pi^-$



Tree



Penguin

$$A(B^0 \to \pi^+\pi^-) = V_{ud} V_{ub}^* t^{+-} + V_{td} V_{tb}^* p^{+-}$$

Time-dependent asymmetry

$$\begin{aligned} A(t) &= S_{\pi^+\pi^-} \sin(\Delta m t) - C_{\pi+\pi-} \cos(\Delta m t) \\ &= \sqrt{1 - C^2_{\pi^+\pi^-}} \sin 2\alpha_{eff} \sin(\Delta m t) - C_{\pi+\pi-} \cos(\Delta m t) \end{aligned}$$

Combining CKM for t^{+-} and B-$\bar{B}$ mixing: $S_{\pi+\pi-} = \sin(2\alpha) + O(\frac{p^{+-}}{t^{+-}})$
$\Longrightarrow$Penguin pollution: handle on p^{+-} and t^{+-} to extract $\sin(2\alpha)$?

α: Isospin analysis for $B \to \pi\pi$

In terms of isospin quantities $Q^{(I)}_{I_z}$

- Two operators for $\bar{b} \to \bar{u}u\bar{d}$: $O^{(3/2)}_{1/2}$ and $O^{(1/2)}_{1/2}$ (adding 3 $I = 1/2$)
- Two inital states: $|B^+\rangle = |B^{(1/2)}_{1/2}\rangle$ and $|B^0\rangle = |B^{(1/2)}_{-1/2}\rangle$
- Three final states: [$I = 1$ forbidden by Bose symmetry]

$$\langle \pi^+\pi^0| = \langle \pi\pi^{(2)}_1| \qquad \begin{aligned} \langle \pi^+\pi^-| &= \sqrt{\frac{1}{3}}\langle \pi\pi^{(2)}_0| + \sqrt{\frac{2}{3}}\langle \pi\pi^{(0)}_0| \\ \langle \pi^0\pi^0| &= \sqrt{\frac{2}{3}}\langle \pi\pi^{(2)}_0| - \sqrt{\frac{1}{3}}\langle \pi\pi^{(0)}_0| \end{aligned}$$

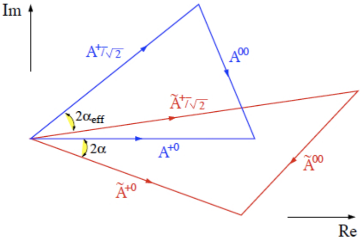
From $B^{(1/2)}$, $O^{(3/2)}$ can only yield $I = 2$ final states,
and $O^{(1/2)}$ only $I = 0$, so two reduced amplitudes

$$A_2 = \frac{1}{2\sqrt{3}}\langle \pi\pi^{(2)}||O^{(3/2)}||B^{(1/2)}\rangle \qquad A_0 = -\frac{1}{\sqrt{6}}\langle \pi\pi^{(0)}||O^{(1/2)}||B^{(1/2)}\rangle$$

Trapping the penguin in $B^0 \to \pi^+\pi^-$

$$B^+, B^0 \;:\; A^{+0} = 3A_2 \qquad A^{+-} = \sqrt{2}(A_2 - A_0) \qquad A^{00} = 2A_2 + A_0$$
$$B^-, B^0 \;:\; \bar{A}^{+0} = 3\bar{A}_2 \qquad \bar{A}^{+-} = \sqrt{2}(\bar{A}_2 - \bar{A}_0) \qquad \bar{A}^{00} = 2\bar{A}_2 + \bar{A}_0$$

A^{+0} is $I = 2$ $\pi\pi$, only from tree and (negligible) $I = 3/2$ penguins



Two triangular relations

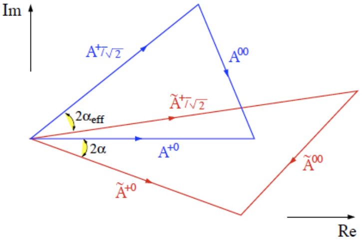
$$A^{+-} + \sqrt{2}A^{00} = \sqrt{2}A^{+0}$$
$$\bar{A}^{+-} + \sqrt{2}\bar{A}^{00} = \sqrt{2}\bar{A}^{+0}$$

from Br and CP-asymmetries

Trapping the penguin in $B^0 \to \pi^+\pi^-$

$$
\begin{array}{llll}
B^+, B^0 \; : & A^{+0} = 3A_2 & A^{+-} = \sqrt{2}(A_2 - A_0) & A^{00} = 2A_2 + A_0 \\
B^-, \bar{B}^0 \; : & \bar{A}^{+0} = 3\bar{A}_2 & \bar{A}^{+-} = \sqrt{2}(\bar{A}_2 - \bar{A}_0) & \bar{A}^{00} = 2\bar{A}_2 + \bar{A}_0
\end{array}
$$

A^{+0} is $I = 2$ $\pi\pi$, only from tree and (negligible) $I = 3/2$ penguins



Two triangular relations

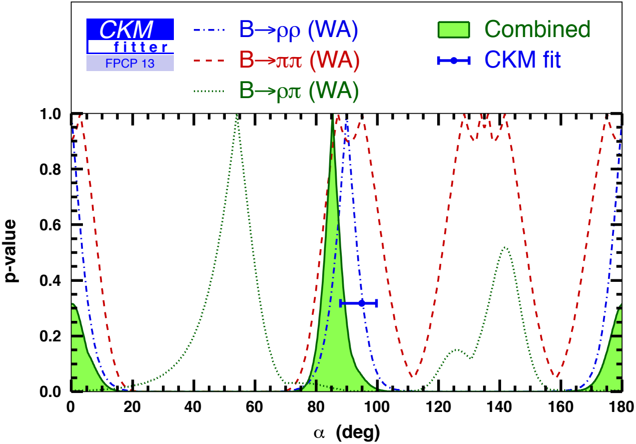
$$
\begin{array}{rcl}
A^{+-} + \sqrt{2}A^{00} & = & \sqrt{2}A^{+0} \\
\bar{A}^{+-} + \sqrt{2}\bar{A}^{00} & = & \sqrt{2}\bar{A}^{+0}
\end{array}
$$

from Br and CP-asymmetries

Introducing $\tilde{A}^{ij} = exp(-2i\beta)\bar{A}^{ij}$,

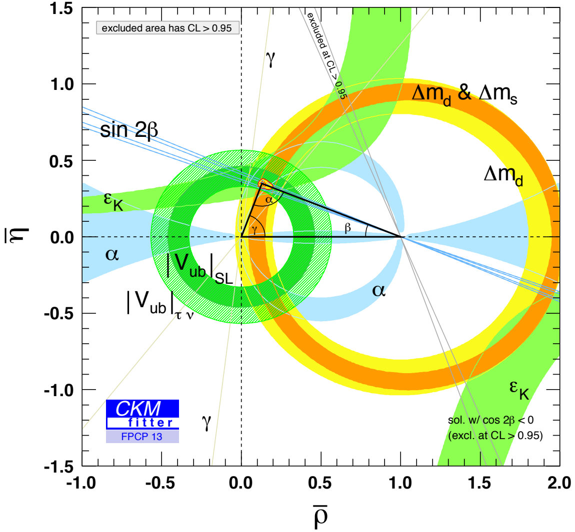
2α between A^{+0} and $\tilde{A}^{+0}$, $\quad 2\alpha_{eff}$ between A^{+-} and $\tilde{A}^{+-}$

- Measure mixed CP-asymmetry in $\pi^+\pi^-$ as $\sin(2\alpha_{eff})$
- Up to discrete ambiguity, determine $\sin(2\alpha)$
- Can be extended to $\rho\pi$ and $\rho\rho$



$$\alpha = (85.4^{+4.0}_{-3.8})^\circ \qquad (\pi\pi, \rho\pi, \rho\rho)$$

The current status of CKM



$|V_{ud}|, |V_{us}|, |V_{cb}|, |V_{ub}|_{SL}$

$B \to \tau\nu$

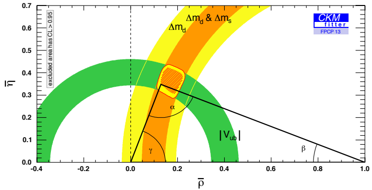
$\Delta m_d, \Delta m_s, \epsilon_K$

$\alpha, \sin 2\beta, \gamma$

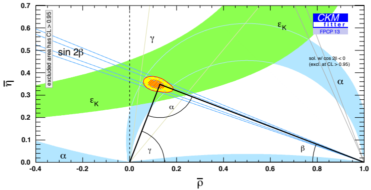
$$A = 0.823^{+0.012}_{-0.033}$$
$$\lambda = 0.2246^{+0.0019}_{-0.0001}$$
$$\bar\rho = 0.129^{+0.018}_{-0.009}$$
$$\bar\eta = 0.348^{+0.012}_{-0.012}$$

(68% CL)

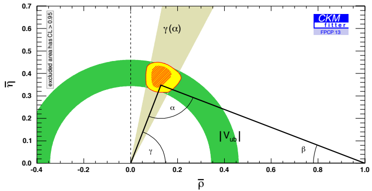
A very consistent picture



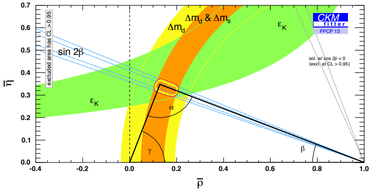
CP-conserving



CP-violating



Tree observables



Loop observables

…and thus a very strong constraint on NP extensions