(A Few) Hot Topics in Lattice QCD

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Outline

- Motivation for LQCD.
- Lattice QCD for heavy quarks.
- Leptonic b decays.
- Semileptonic c decays.
- Semileptonic b decays.
- Pitfalls: topology freezing.
- Outlook

Motivation

- Simple QCD matrix elements enter into weak decay rates (CKM, unitarity).

$$
\begin{aligned}
\mathcal{B}(B \to l\nu) &= \frac{G_F^2 |V_{ub}|^2 \tau_B}{8\pi} {f_B}^2 m_B m_l^2 \left(1 - \frac{m_l^2}{m_B^2}\right)^2 \\
\langle 0 | A^\mu | B(p) \rangle &= f_B p_\mu
\end{aligned}
$$

Motivation

- Simple QCD matrix elements enter into weak decay rates (CKM, unitarity).

$$\mathcal{B}(B \to l\nu) = \frac{G_F^2 |V_{ub}|^2 \tau_B}{8\pi} {f_B}^2 m_B m_l^2 \left(1 - \frac{m_l^2}{m_B^2}\right)^2$$

$$\langle 0|A^\mu|B(p)\rangle = f_B p_\mu$$

- For neutral mesons

$$\mathcal{B}(B_s \to \mu^+\mu^-) = \frac{G_F^2 \alpha^2 |V_{tb} V_{ts}^*|^2 \tau_{B_s}}{64\pi^3} f_{B_s}^2 m_{B_s}^3 \sqrt{1 - \frac{4{m_\mu}^2}{{m_{B_s}}^2}} \times \{\cdots\}$$

$$\mathcal{B}(Bs \to \mu^+\mu^-)_{SM} = (3.32 \pm 0.17)x10^-9$$

$\mathcal{B}(Bs \to \mu^+\mu^-)_{LHCb} = (3.2^{+1.5}_{-1.2})x10^-9$ (PRL 110, 021801 (2013))

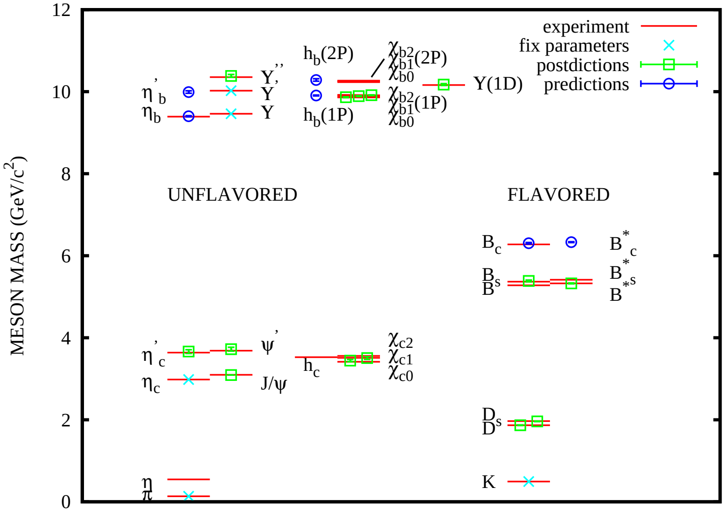
Motivation

- Lattice QCD is a first principles calculation.
- In a full simulation, in principle no uncontrollable errors should remain. Precision tool.
- Fixing the parameters
  The free parameters in the lattice formulation are fixed by setting a set of calculated quantities to their measured physical values.
  Quantities that can be accurately calculated from the lattice and are measured with good precision experimentally.
  - Scale: lattice spacing a:
  - Quark masses: $m_{u,d}, m_s, m_c, m_b$.
    Could be fixed, for example, by $m_\pi, m_K, m_{\eta_c}, m_{\eta_b}$.
- Large freedom in choosing the discretization: different systematics.

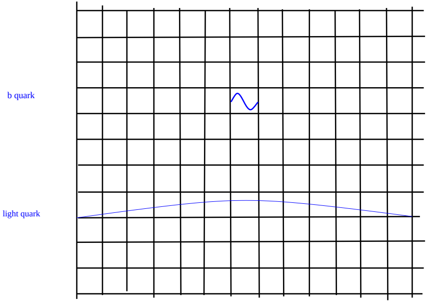
Motivation

- Only a limited amount of quantities can be calculated (precisely): spectroscopy of fundamental and first few excited states, leptonic and semileptonic decay constants, quark masses, etc.
- In the heavy quark sector (c and b) there are many gold-plated states in the spectrum. We can test our calculations.
- Precision is crucial for searches of BSM physics. We need good control over all systematic errors. Best if we have independent calculations for crosscheck.

Meson Spectrum



Heavy quarks on the lattice



$\lambda_b \approx M_b^{-1} \gtrsim a$ (0.05fm)

Discretization errors: $(aM)^k$ (k tipically 2)

For light quarks, we need $La \gg m_\pi$ (finite volume error)

For heavy quarks, we would like $aM \ll 1$

Computational cost for ensemble generation grows with the lattice spacing with $\sim a^{-k}$, with a large k (6, 7).

Heavy quarks on the lattice

Nonrelativistic effective theory

- M large: non-relativistic system ($v^2 \approx 0.1$).
- Remove M from the dynamics $\rightarrow$ effective theory (NRQCD, HQET).
- $m_b \approx 4$ GeV, binding energies much smaller.

Relativistic approach

- Use highly improved discretization + very fine lattices. We can do this already for c quarks. For b quarks, needs extrapolation in M_h.
- HISQ (highly improved staggered quarks): $\mathcal{O}(\alpha_s a^2, a^4)$
- Twisted mass action: $\mathcal{O}(a^2)$.
- Clover action: $\mathcal{O}(a^2)$.
- Domain-wall/overlap action: $\mathcal{O}(a^2)$ (charm).

Heavy quarks on the lattice

Nonrelativistic effective theory

- Computationally cheap.

- Rest mass M_0 and "kinetic mass" M_K.
- Needs matching to continuum QCD. Difficult to carry out to high orders.

Relativistic approach

- Computationally expensive

- Only one mass, M_0.
- In formalisms with enough chiral symmetry: PCAC $\rightarrow$ non-renormalization of pseudoscalar decay constants.
- Using the same action for all quarks is conceptually simpler.
- Error cancelation in ratios. Can be used as a lever.
- More predictive, same action from light to heavy sectors.

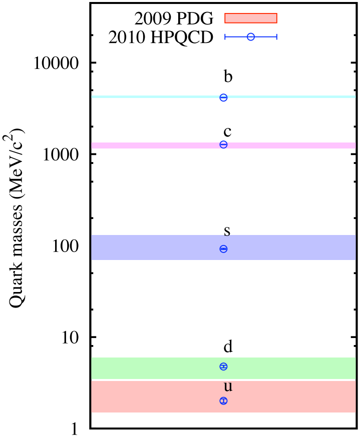
Heavy quarks on the lattice

- Relativistic calculation of ratios of quark masses: m_b/m_c, m_c/m_s.
- Renormalization constants cancel: lever.

$$\left(\frac{m_{q_1,latt}}{m_{q_2,latt}}\right)_{a\to 0} = \frac{m_{q_1,\bar{M}S}(\mu)}{m_{q_2,\bar{M}S}(\mu)}$$

- $$\frac{m_c}{m_s} = 11.85(16)$$

$$\left(\frac{m_b}{m_c}\right)_{NP} = 4.51(4)$$



b leptonic decay constants

$$\Gamma(B \to l\nu) = \frac{G_F^2 |V_{ub}|^2}{8\pi} {f_B}^2 m_B m_l^2 \left(1 - \frac{m_l^2}{m_B^2}\right)^2$$

$$\langle 0 | A^\mu | B(p) \rangle = f_B p_\mu$$

$$\langle 0 | A^\mu | B_s(p) \rangle = f_{B_s} p_\mu$$

b leptonic decay constants

HPQCD: Calculation on MILC $N_f = 2 + 1 + 1$ HISQ sea, including physical light quarks [arXiv:1309.4610].
Three lattice spacings, $a \approx 0.09$, 0.12, 0.15.
Improved NRQCD b quark, HISQ light valence quarks.
$M_{B_s} - M_B = 85(2)$ MeV

Fermilab/MILC calculation on MILC $N_f = 2 + 1$ asqtad configurations [1112.3978]. Three lattice spacings, a $\sim 0.09, 0.12, 0.15$ fm.
b quarks using Fermilab method, asqtad light valence quarks.

HPQCD
$f_B = 186(4)$ MeV
$f_{B_s} = 224(5)$ MeV
$f_{B_s}/f_B = 1.205(7)$

FERMILAB/MILC
$f_B = 196.9(8.9)$ MeV.
$f_{B_s} = 242.0(9.5)$ MeV.
$f_{B_s}/f_B = 1.229(0.026)$.

b leptonic decay constants

Alpha collaboration: calculation on CLS $Nf = 2$ configurations [1210.7932].
Three lattice spacings, $a \sim 0.05, 0.065, 0.075$ fm.

HQET for b, NP improved Wilson for the light valence quarks.

$f_B = 193(9)_{stat}(4)_{\chi}$ MeV
$f_{B_s} = 219(12)_{stat}$ MeV

b leptonic decay constants

ETM: calculation on $N_f = 2+1+1$ twisted Wilson configurations [1311.2837].

Twisted Wilson for valence light quarks, extrapolation on the heavy quark mass to m_b.

Three lattice spacings, $a \sim 0.062, 0.081, 0.089$ fm.

$f_B = 196(9)$ MeV.

$f_{B_s} = 235(9)$ MeV.

$\frac{f_{B_s}}{f_B} = 1.201(25)$.

b leptonic decay constants

RBC-UKQCD: calculation on $N_f = 2 + 1$ domain wall configurations [1311.0276].

Domain wall light valence quarks and NP-tuned clover relativistic b quarks.

Two lattice spacings, $a \sim 0.09$, 0.11 fm.

Errors are statistical for now.

$f_B = 191(6)$ MeV.
$f_{B_s} = 233(5)$ MeV.
$\frac{f_{B_s}}{f_B} = 1.20(2)$.

b leptonic decay constants

HPQCD: calculation on MILC $N_f = 2 + 1$ asqtad configurations [1110.4510].

HISQ valence quarks, extrapolation on the heavy quark to m_b.

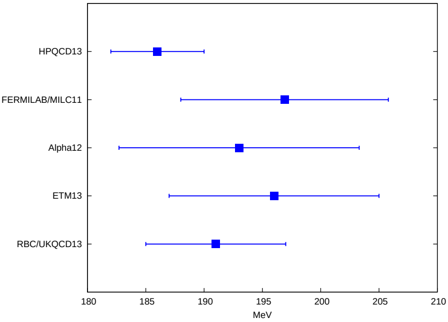
5 values of the lattice spacing, from a ~ 0.15 fm to ~ 0.045 fm.

$f_{B_s} = 225(4)$ MeV.

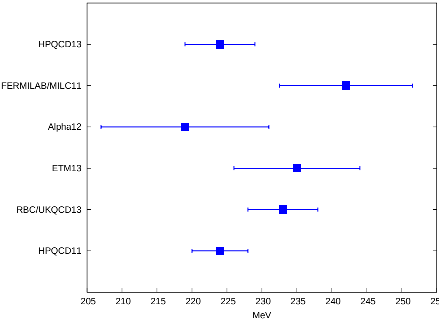
f_B could be calculated directly, but much more expensive.

b leptonic decay constants

f_B

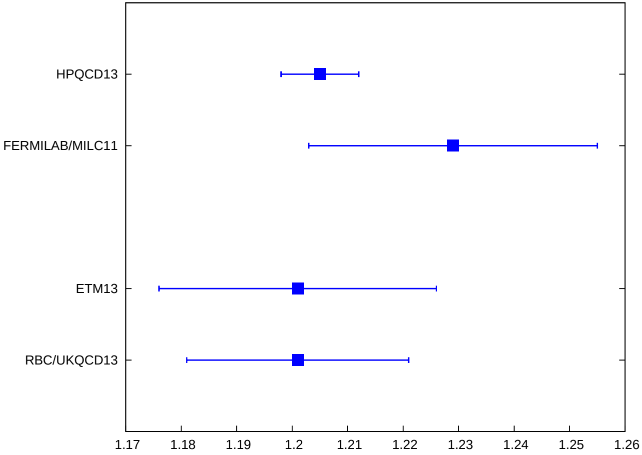


f_{B_s}



b leptonic decay constants

$$f_{B_s}/f_B$$



D semileptonic decays

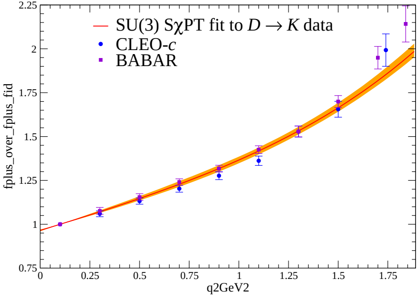
$$\langle K|V^\mu|D\rangle = f_+(q^2)\left[p_D^\mu + p_K^\mu - \frac{M_D^2 - M_K^2}{q^2} q^\mu\right] + f_0(q^2)\frac{M_D^2 - M_K^2}{q^2} q^\mu$$

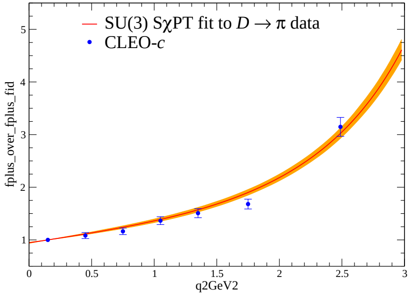
$$\frac{d\Gamma(D \to K\ell\nu)}{dq^2} = \frac{G_F^2|V_{cs}|^2}{24\pi^3} p^3 |f_+(q^2)|^2$$

- Theory/experiment comparison of functions of q^2.
- For $D \to K(\pi)$, the experiment and lattice kinematic regions mostly overlap: stringent test of LQCD.

$D \to K(\pi)l\nu$

FNAL/MILC (arXiv:1211.4964): $2+1$ asqtad sea, asqtad light valence, heavy clover c valence.





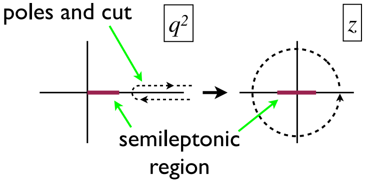
$D \to K(\pi) l\nu$

We would like a model-independent procedure for comparison of experimental and theoretical results: z-expansion:

$$z = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}, \qquad t_\pm = (m_D \pm m_K)^2$$

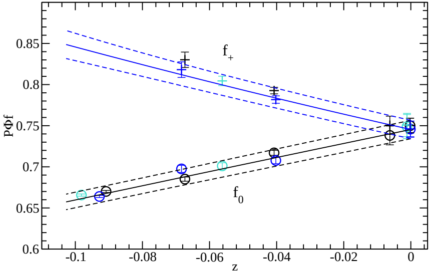
Maps the semi-leptonic region, $0 < q^2 < t_-$, to the interior of the unit circle.

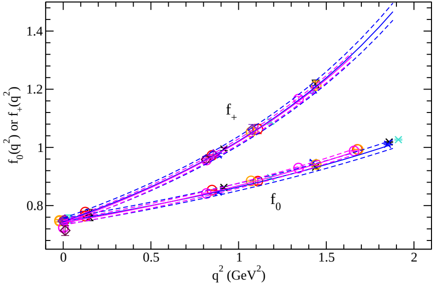
$$f(q^2) = \frac{1}{P(q^2)\Phi(q^2)} \sum_{n=0}^{N} b_n z^n$$



$D \to K l \nu$

HPQCD (arXiv:1305.1462): $2+1$ asqtad sea, HISQ valence, $D \to K$.

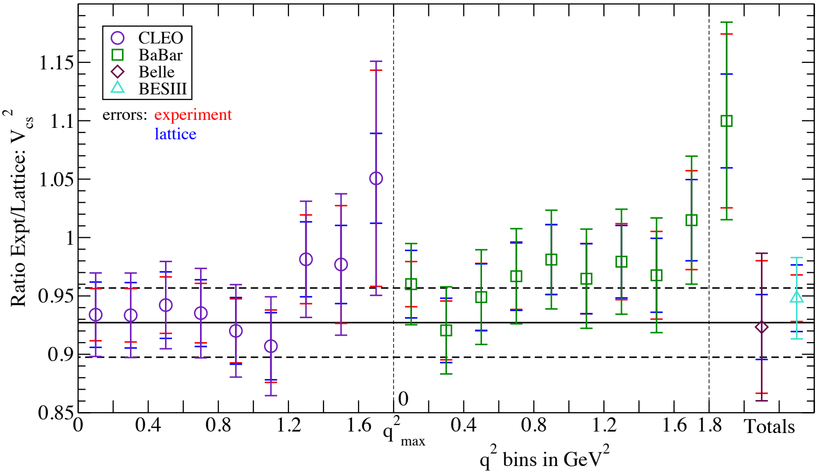




Insensitivity to the spectator quark.

$D \to K l \nu$

Bin-by-bin comparison



b semileptonic decays

- $B_s \to K l \nu$: $|V_{ub}|$
- $B \to K l l$: sensitive to new physics.

$$\langle K|V^\mu|B\rangle = f_+(q^2)\left(p_B^\mu + p_K^\mu - \frac{M_B^2 - M_K^2}{q^2}\, q^\mu\right) + f_0(q^2)\,\frac{M_B^2 - M_K^2}{q^2}\, q^\mu$$

$$\langle K|T^{\mu\nu}|B\rangle = \frac{p_B^\mu p_K^\nu - p_B^\nu p_K^\mu}{M_B + M_K}\, 2 f_T(q^2)$$

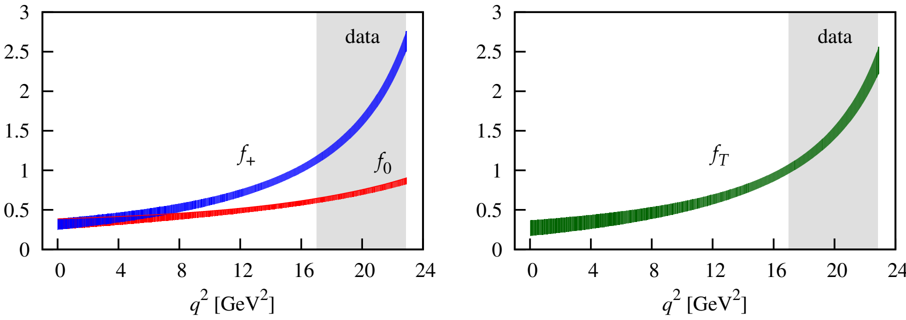
$B \to Kll$

HPQCD (Phys.Rev. D88 (2013) 054509;Phys.Rev.Lett. 111 (2013) 162002)

2+1 asqtad sea, HISQ light valence, NRQCD b.

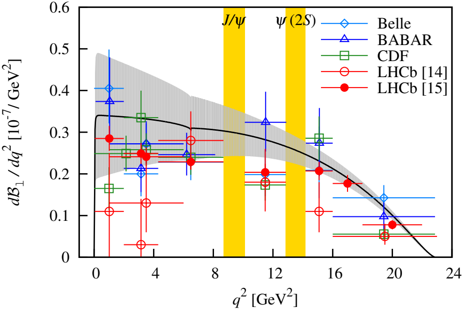
m_l/m_s down to 1/10.

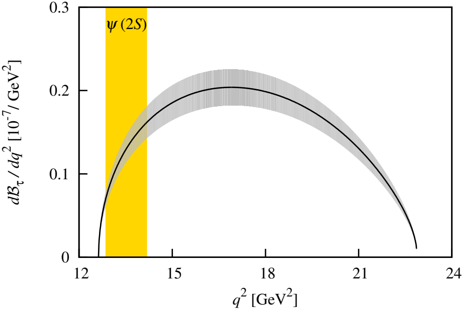
z expansion to extrapolate in q^2. $|z| < 0.16$.



$B \to Kll$

Decay rates





$B \to Kll$

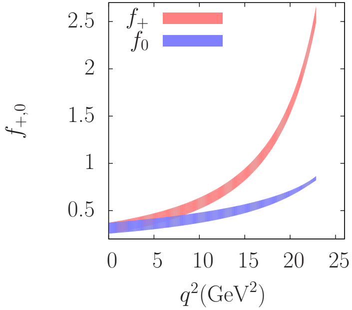
Branching fraction ratios: potentially sensitive to new physics

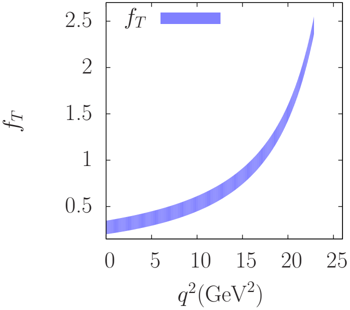
$$R_e^{\mu}(q^2_{\mathrm{low}}, q^2_{\mathrm{high}}) \equiv \frac{\int_{q^2_{\mathrm{low}}}^{q^2_{\mathrm{high}}} dq^2 \; d\mathcal{B}_{\mu}/dq^2}{\int_{q^2_{\mathrm{low}}}^{q^2_{\mathrm{high}}} dq^2 \; d\mathcal{B}_{e}/dq^2},$$

$$\begin{aligned} R_e^{\mu}(4m_{\mu}^2, q^2_{\mathrm{max}}) &= 1.00029(69), \\ R_{\mu}^{\tau}(14.18\ \mathrm{GeV}^2, q^2_{\mathrm{max}}) &= 1.174(40), \\ R_e^{\tau}(14.18\ \mathrm{GeV}^2, q^2_{\mathrm{max}}) &= 1.178(41), \\ R_{\ell}^{\tau}(14.18\ \mathrm{GeV}^2, q^2_{\mathrm{max}}) &= 1.176(40). \end{aligned}$$

$B \to Kll$

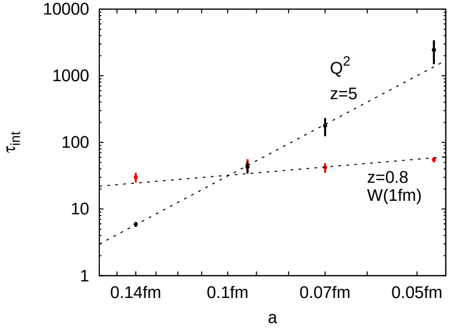
FNAL/MILC (arxiv:1312.3197)
2+1 asqtad sea, asqtad light valence, clover(FNAL) b.
0.12 to 0.06 fm, m_l/m_s down to 1/10.
z expansion to extrapolate in q^2. $|z| < 0.16$.
Errors $\equiv 3 - 8\%$ for $q^2 > 17\text{GeV}^2$.





Towards relativistic b pitfalls: Topology freezing

- In the continuum $a \to 0$, we expect the sectors of different topological charle Q to become separated by infinite barriers.
- Montecarlo integrated autocorrelation time: $\tau_{int} = a^{-z}$.
- For Q^2, z compatible with 5 (arXiv:1211.5069).



Towards relativistic b pitfalls: Topology freezing

What to do?

- New algorithms.
- Open boundary conditions (arXiv:1105.4749).
- Maybe topology change does not matter for most observables?
- Simulate in a fixed sector: larger finite volume effects.
- But how can we be sure that it is safe?

Outlook

- LQCD is by now a mature tool for QCD calculations of (some) quantities of phenomenological relevance, both as a non-perturbative test of QCD itself and as a fundamental input for BSM physics.
- Many accurate calculations across the entire QCD range, from light to heavy states, with no free parameters. With different discretizations, different systematics. Numerous crosschecks.
- Already many lattice calculations of relevant matrix elements, in particular in flavour physics.
- Effective theory methods and relativistic ones will be complementary, at least for now. Use ratios + relativistic methods. Different systematics.

Outlook

- To increase precision in relativistic calculations we will need to go to smaller lattice spacings.
  In principle straightforward (computing time), but there may be problems: topology freezing.
- We start to have ensembles at the physical light quark masses. Less dependence on chiral extrapolations, (playtool for theorists).
- In spectroscopy and some decay constants we have reached a level of precision (sub-percent) where isospin and electromagnetic effects have to be taking seriously and calculated, no only estimated. Already in progress.
- There is still much scope for improvement.